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DOUBLY DYNAMIC TRAFFIC ASSIGNMENT: SIMULATION MODELLING FRAMEWORK AND EXPERIMENTAL RESULTS

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ABSTRACT

This paper presents, and investigates properties of, a doubly dynamic simulation assignment model which involves specifying a day-to-day route choice model as a discrete time stochastic process, combining a between-day driver learning and adjusting model with a continuous time, within-day dynamic network loading. Such a simulation model may be regarded as the realisation of a stochastic process, which under certain mild conditions, admits a unique stationary probability distribution (i.e. an invariant probability distribution over time of network flows and travel times). Such a stationary state of the stochastic process is of interest to transport modellers, as one can then describe the stochastic process by its moments such as the means, variances and covariances of the flow and travel time profiles. The results of a simulation experiment are reported in which the process of individual drivers' day-to-day route choices are based on the aggregate learning of the experienced within-day route costs by all drivers departing in the same period. Experimental results of the stationarity of the stochastic process are discussed, along with an analysis of the sensitivity of autocorrelations of the route flows to the route choice model parameters. The results also illustrate the consistency of the link flow model with properties such as First-In, First-Out (FIFO), and a simple network is used to illustrate the properties.

INTRODUCTION

Traditionally, dynamic traffic assignment in the literature refers to the modelling of traffic flows on street networks due to the variations in the demand within a day, and capturing the spatio-temporal congestion effects through suitable dynamic link travel time functions. Usually such models are aimed at solving for either dynamic system optimal or dynamic user equilibrium problems. As they consider deterministic flow variables, the solutions naturally tend to be deterministic representing an average situation at each moment. As a result, the within-day deterministic models cannot explain the random variations in traffic flow, besides being unable to represent the transient states in the evolution towards equilibrium (*1*). In fact, the purview of dynamic traffic assignment is much wider and includes day-to-day variations in the demand in addition to the usual within-day variations. Day-to-day evolution of traffic flows was considered by several authors in the past (*1-4*), all of whom focused on the evolution of the traffic flows across the days either as a stochastic or a deterministic process, but primarily based on *static* within-day cost-flow functions. On the contrary, nowadays, more generalised traffic assignment models are being developed which are aimed at addressing both the day-to-day and within-day variations in route flows and such models are called doubly dynamic traffic assignment models, which are the main subject of the present paper.

Cascetta and Cantarella (*5*) developed such a doubly dynamic simulation model in which they defined the route flows on any day as a stochastic process and included a queuing model to capture the delays on the links. Friesz et al (*6*) considered deterministic flow variables and defined implicitly a doubly dynamic assignment model considering the day-to-day and within-day dynamics simultaneously, but the model carries with it the usual limitations associated with the deterministic approaches described in the previous paragraph. Balijepalli and Watling (*7*) developed a variance approximation method to estimate the properties of a stationary probability distribution of a stochastic process in a doubly dynamic environment. Their model was developed as an alternative to the simulation model, based on a deterministic approximation approach. On the other hand, the present paper considers the simulation of route choice process based on a Monte Carlo technique. This paper focuses further on the concept of stationarity of stochastic processes and analyses correlograms as a way of detecting the stationarity based on a necessary condition.

This work builds on the findings of Cantarella and Cascetta (*8*), and the particular aims of this paper being to specify and investigate the properties of a doubly dynamic simulation model for the route choice process incorporating the drivers' day-to-day learning and adjustment process through an aggregate memory model, combined with a continuous time dynamic network loading method to obtain the drivers' experienced travel costs within a day. The doubly dynamic traffic assignment problem is solved using a Monte Carlo simulation method. Given the time varying demand profile and the network specification, the expected output of the stochastic process model includes, during each departure period within a day, the mean traffic flow, the variance of route flows, as well as estimates of the covariance in flows between days and time periods when the process is stationary. Thus this research provides a further step in advancing our understanding of the modelling of variability of traffic flows on street networks. A simple grid network with multiple origins and destinations is used to illustrate the principles described.

SIMULATION MODELLING FRAMEWORK

Preliminaries and Modelling Assumptions

Consider a network of directed links serving O-D demand represented by $\boldsymbol{Q} = \{...., q_k ,...\}$ where q_k is the O-D demand for a particular commodity *k*, each commodity defining a combination of origin, destination and (discrete) departure period. It is assumed that the total period of analysis is divided into *L* departure periods. Each commodity k is served by a set of routes R_k with $|R_k|$ elements; the full route set across all commodities thus has dimension $\rho = \sum_{k=1}^{K} |R_k|$. Let ***f*** be the *ρ*- vector of commodity route flows and ***c(f)*** the vector of commodity route costs.

It is assumed that all the trip makers of commodity *k* are rational in their behaviour when choosing their route, in an attempt to minimise their perceived cost of travel. For each commodity k and route $r \in R_k$, the perceived travel cost $\hat{C}_r^{(n)k}$ at the start of day *k* is given by

$$\hat{C}_r^{(n)k} = C_r^{(n-1)k} + \eta_r^{(n)k} \tag{1}$$

where $C_r^{(n-1)k}$ is the population-mean perceived cost for commodity k and route r at the end of day *n-1*, and $\eta_r^{(n)k}$ is a random variable describing unobserved attributes contributing to the population-dispersion of the perceived attractiveness of route r by commodity *k*. The ρ-vector $\boldsymbol{C}^{(n-1)}$ represents the collection of population-mean perceived costs across all commodities. The probability of choosing route *r* on day *n* is then given by:

$$p_r^k(\boldsymbol{C}^{(n-1)}) = Pr\left\{ C_r^{(n-1)k} + \eta_r^{(n)k} < C_i^{(n-1)k} + \eta_i^{(n)k} \right\} \qquad \forall\ i \neq r \tag{2}$$

$\boldsymbol{p}^k(.)$ then represents the vector (of dimension $|R_k|$) of route choice probabilities for the commodity *k*, and ***p(.)*** denotes the collection of these choice probability vectors over all the commodities so is a vector of dimension *ρ*. The functional form of the path choice probabilities depends on the joint probability density function assumed for the residuals $\left\{\eta_r^{(n)k} : r \in R_k\right\}$ for each commodity *k*, resulting (for example) in a logit model, if independent Gumbel distributions are assumed, and a probit model for a multivariate normal distribution.

Day-to-day Learning Model

While the behavioural choice-side of the model is quite conventional, a simple linear learning filter is used to replicate drivers building up their experience of travel costs on a day-by-day basis following the completion of each day's trip. In this research, we assume a simple weighted average approach akin to many other simulation experiments, for example, Horowitz (*9*), Cascetta (*1*) and Nakayama et al (*10*). Thus following the completion of trips on any day *(n-1)*, the population-mean experienced costs are updated based on a weighted average of costs actually incurred in a finite number of previous days *m*, using the form:

$$\boldsymbol{C}^{(n)} = s(\lambda)^{-1}\left\{\boldsymbol{c}(\boldsymbol{F}^{n-1}) + \lambda\boldsymbol{c}(\boldsymbol{F}^{n-2}) + \dots\dots + \lambda^{m-1}\boldsymbol{c}(\boldsymbol{F}^{n-m})\right\} \tag{3}$$

$$s(\lambda) = \sum_{j=1}^{m} \lambda^{j-1} = (1-\lambda^{m})/(1-\lambda) \tag{4}$$

where, $s(\lambda)$ is simply a scaling factor to make the weights sum to unity, $\boldsymbol{c}(.)$ the commodity route cost-flow function as defined above, and $\boldsymbol{F}^{n}$ a vector random variable of dimension ρ denoting the network path flows by commodity on day *n*.

Stochastic Process Model

The number of drivers in each commodity, as defined by the combination of OD pair and departure period, choose routes independently on day *n* based on the experienced costs, implying a probability distribution in the space of commodity flows. Assuming that for any day *n* and for each commodity *k*, all q^k drivers wishing to travel make their travel choices independently conditional on their experiences in the past days, then the number of drivers taking each possible route on day *n* by each commodity *k*, conditional on the costs (3) experienced in the past, is obtained as:

$$\boldsymbol{F}^{(n)k} \Big| \boldsymbol{C}^{(n-1)} \sim Multinomial\left(q^{k}, \boldsymbol{p}^{k}(\boldsymbol{C}^{(n-1)})\right) \quad \text{independently for k = 1,2,,....K} \tag{5}$$

where $\boldsymbol{F}^{(n)k}$ is the vector of route flows on day *n* by the commodity *k*. The route choice probabilities in equation (5) are computed based on the experienced costs up to the end of the previous day. Individual differences among users in the same commodity are taken into account through random residuals around the population-mean experienced costs defined by equation (3), and hence models of this form are called aggregate memory models. However, in a more general situation, each driver's perceptions can be modelled through individual learning models, which are called disaggregate memory models, but at the cost of significant computing time (*8*). It is also noted that OD demand is assumed to be constant, but could be readily extended to the case of uncertain demand as in (*11) and (12*).

Dynamic Network Loading Model

In order to be able to capture the interactions amongst the vehicles departing in the same/successive departure periods, we need to subdivide each departure period into a number of smaller time steps. Let δ be the time increment of this discretisation, and denote the complete analysis period by ($0, N\delta$] for some positive integer *N*. The time increments are thus the intervals ($t-\delta, t$] for $t = \delta, 2\delta, \dots, N\delta$, which are referred to as minor time steps. Below, when we refer to a time step (or interval) *t*, it is to be understood that we are referring to the period ($t-\delta, t$]. We assume that δ is chosen so as to be smaller than the free flow time to traverse any link. The OD demand rates are assumed (for notational convenience) to be specified over a common discretisation of the whole analysis period ($0,N\delta$], divided it into *L* major time periods, also referred to as departure periods (w_{j-1}, w_j] (for $j = 1,2,..,L$) such that $(w_0, w_1] \cup (w_1, w_2] \cup \dots\dots \cup (w_{L-1}, w_L] = (0, N\delta]$. These match exactly the departure periods defined in the previous section, and for convenience are assumed to be of the same duration, i.e. $w_j - w_{j-1} = \kappa$ for all $j = 1,2,,...L$ and some given κ.

Link and Path Travel Times

Assuming that whole link travel time models of linear form (*13*) are defined on each of the links on any route *r*, and that for this route r the links traversed in order are numbered $\{a_1, a_2, ..., a_r\}$ the travel time function and exit time functions for any link a_i on this path may be expressed as a nested path cost operator. It is noted that the assumption of linear travel time models is not necessary for the application of our model, as the simulation models can be coupled with any type of link travel time models based on linear or non-linear, continuous or discrete time approaches. However, it is important to note that only the linear travel time function is guaranteed to satisfy the desirable properties such as FIFO (*14),* and hence has been the choice here. Then the expressions for travel time and the exit time are as given below:

$$\tau_{a_i}\left(g_{a_{i-1}}(t)\right) = \alpha_{a_i} + \beta_{a_i}\left(g_{a_{i-1}}(t)\right) \qquad \left(i = 1,2,...,r; \quad g_{a_0}(t) \equiv t \right) \tag{6}$$

$$g_{a_i}(t) = g_{a_{i-1}}(t) + \tau_{a_i}\left(g_{a_{i-1}}(t)\right) \tag{7}$$

where $\tau_{a_i}(.)$ is the travel time on the link a_i , α_{a_i} the free flow time on the link, β_{a_i} the inverse of the exit capacity of the link a_i, $x_{a_i}(.)$ the number of vehicles on the link a_i , and $g_{a_i}(.)$ the exit time from the link a_i.

As the model discretises time into a finite number of minor time steps, we have the knowledge of travel times computed only at the discrete time steps. But this will be insufficient to compute the path travel time on any path with multiple links, especially from the second link onwards where the travel time needs to be computed at some real time and not just integers. This is countered by computing the travel time in equation (6) using linear interpolation, which is given below:

$$\tau_{a_i}(t) \approx \hat{\tau}_{a_i}(<t/\delta>.\delta) + \frac{t-<t/\delta>.\delta}{\delta}\left[\hat{\tau}_{a_i}\left((<t/\delta>+1).\delta\right) - \hat{\tau}_{a_i}\left(<t/\delta>.\delta\right)\right] \tag{8}$$

for ($t \geq 0$; $i = 1,2,...,n$), where, $\hat{\tau}_{a_i}(.)$ is the travel time on link a_i at integer time, and $<t/\delta>$ the integer part of time t/δ.

Then for example, the path travel time for vehicles entering the link a_1 at time *t* on route *r* (with a_r being the last link on route *r* before discharging the vehicles to their destination) is simply given as the difference between the exit time from link *n* and the entry time at the origin, expressed as:

$$c(t) = \lfloor g_{a_r}(t) - t \rfloor . \tag{9}$$

Finally, the departure-time dependent mean travel time for route r with uniform inflow rate in any departure time period T bounded by $\left(w_{j-1}, w_j\right]$ may be expressed as,

$$c_r^T = \left\{ \frac{1}{w_j - w_{j-1}} \right\} \sum_{t=(j-1)n\delta+1}^{jn\delta} c(t) \tag{10}$$

where *n* is the number of minor time steps in major time period *T*. The departure-time dependent mean travel time obtained from equation (10) is used for updating the drivers' memorised travel cost in equation (3), which then determines the route choice probability distribution for the following day through (5).

Experimental Set Up

A Monte Carlo simulation method has been used to solve the doubly dynamic assignment problem described. This means that the drivers are allocated to the routes based on pseudo-random numbers generated from a pre-specified distribution with the expected values given by the route choice probabilities. The steps in the simulation are listed below:

[1] initialise the route choice probabilities based on free flow costs (initialisation of (3));
[2] allocate the drivers in various departure periods to routes based on random multinomial experiments (implementation of (5));
[3] compute the departure period dependent experienced route costs based on a dynamic network loading map, (6)-(10);
[4] at the end of day n-1, the population mean experienced route costs are updated using the learning model (3) and the costs fed back to the first step above; and
[5] compute the summaries viz., means, variances and covariances of route flows at the end of the realisation.

EXPERIMENTAL RESULTS

Network Supply and Demand Characteristics

In order to illustrate the principles described in the previous section, a simple grid network of 12 links serving two origins and three destinations is used (Figure 1). Note that all the links are one-way, and there are 14 routes in all and the link-path incidence is shown in Table 1. It is assumed that dynamic linear travel time functions with parameters shown in Table 2 are defined on all the links of the network. The demand for each of the six possible O-D pairs is assumed to be known a-priori in each departure period, (Table 1). In this example, we included four departure periods of 15 minutes duration each and we assumed a minor step length of one minute each for dynamic network loading purposes. The route choice probability model is assumed to follow the logit principle with the dispersion parameter θ = 0.1, unless otherwise mentioned. Drivers were assumed to remember up to m = 2 days, and the memory weight was taken to be λ = 0.5.

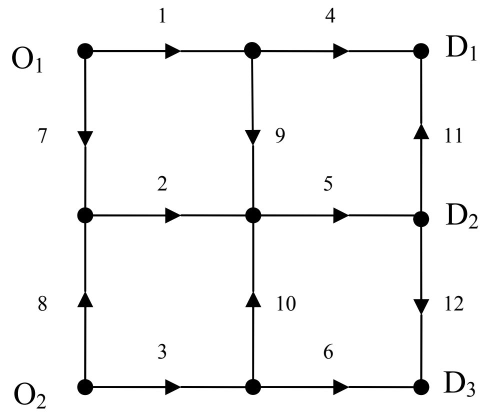

FIGURE 1 Grid Network

TABLE 1 OD Demand and Link – Path Incidence

OD Pair	OD Demand (No. of Drivers per Departure Period)				Available Paths	Links Used by Paths
	1	2	3	4		
O_1-D_1	400	700	500	300	1	1-4
					2	1-9-5-11
					3	7-2-5-11
O_1-D_2	100	125	155	195	4	1-9-5
					5	7-2-5
O_1-D_3	225	182	142	112	6	1-9-5-12
					7	7-2-5-12
O_2-D_1	121	144	199	137	8	8-2-5-11
					9	3-10-5-11
O_2-D_2	165	165	165	165	10	8-2-5
					11	3-10-5
O_2-D_3	325	267	319	255	12	3-6
					13	8-2-5-12
					14	3-10-5-12

TABLE 2 Network Link Parameters

Link	Free flow time, α_a minutes	Service Rate, β_a minutes/vehicle	Exit Capacity, Vehicles/hour
1	6	0.025	2400
2	4	0.040	1500
3	5	0.029	2069
4	4	0.021	2857
5	5	0.015	4000
6	5	0.030	2000
7	3	0.018	3333
8	2	0.024	2500
9	4	0.019	3158
10	3	0.022	2727
11	6	0.01	6000
12	5	0.01	6000

Total Travel Time

Total travel time measured by the vehicle-hours on the network indicates the intensity of travel over the network, and if monitored over the period of simulation, will indicate the day-to-day evolution of the intensity of travel. Figure 2 shows the day-to-day total travel on the network.

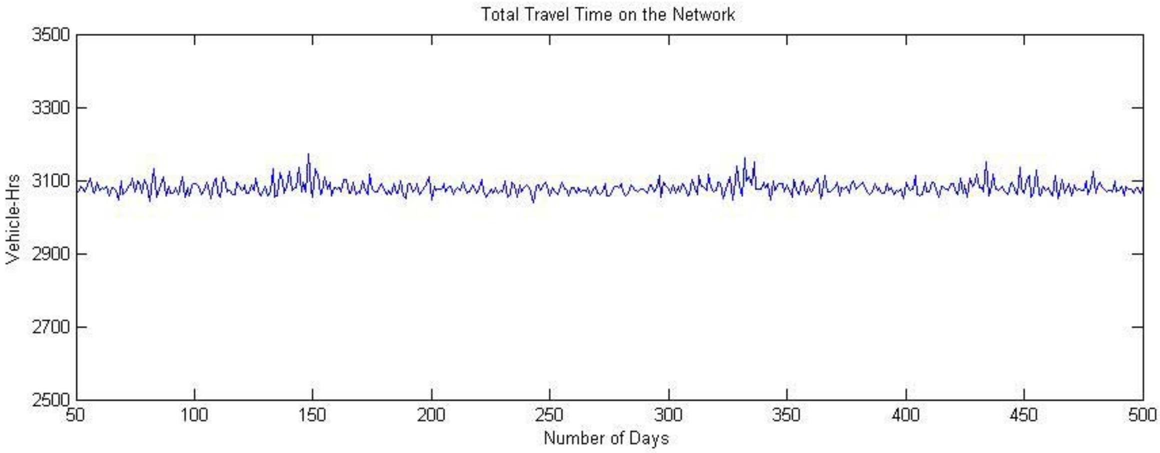

FIGURE 2 Total Travel time on the Network over 500 Days

Figure 2 indicates that in a realisation of 500 days, the total travel time on the network settles down to a mean value of around 3079 veh-hrs, with a standard deviation of 18.4 veh-hrs. If it were a stationary deterministic process then the graph of total travel time would have been a horizontal line over the number of simulated days, but a stochastic process even when stationary will always exhibit some variation due to the inherent nature of randomness in the variable being studied. A stochastic process is said to be in equilibrium if the probability distribution remains unaltered with time shifts. Therefore, it is important to draw the histograms of route flows to ensure the stationarity of the process. While monitoring the total travel on the network, in order to account for the empty network conditions, an assumed initial burn in period equivalent to 10% of the simulated days has been discounted. Figure 3 shows the day-to-day evolution of travel over 1000 days and provides a sustained visual reassurance that the process is stable. Somewhat more formally, we could apply statistical hypothesis tests: e.g., a t-test to check whether the means of two samples from a realisation are the same. However, this is not the same as testing for stationarity, a more complex task yet crucial for the practical application of such models – this is therefore explored in more detail below.

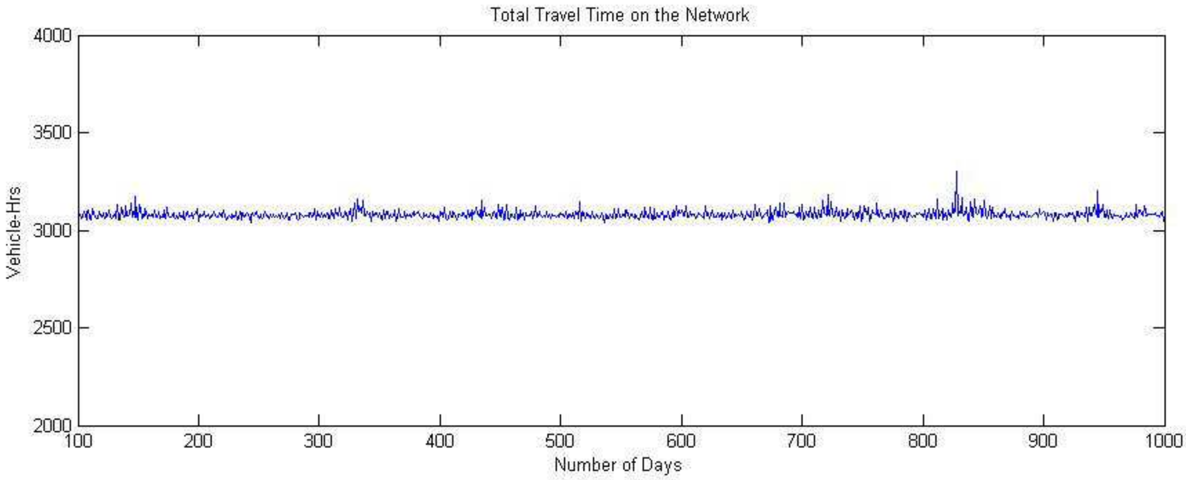

FIGURE 3 Total Travel time on the Network over 1000 Days

Stationarity of Stochastic Process

A stochastic process is said to be strictly stationary if its properties remain unaffected by a change of time origin, or in other words, the joint probability distribution of m observations made at any set of times **t** (for t = 1,2,..,m) is the same as that associated with m observations separated by an integer k made at set of times **t**+k (for t = 1,2,..,m and k is an integer) where k is called the lag.

To illustrate this, let us consider the flow on route 1 on the network over a period of 300 days from 201 to 500 in a realisation of 1000 days. Let us also consider that another set of 300 observations also picked up from the same realisation from 426 to 725 days. Figure 4 shows the joint probability distribution of flows on route 1 for each of the four departure periods, for each of the two sets of observations as described above.

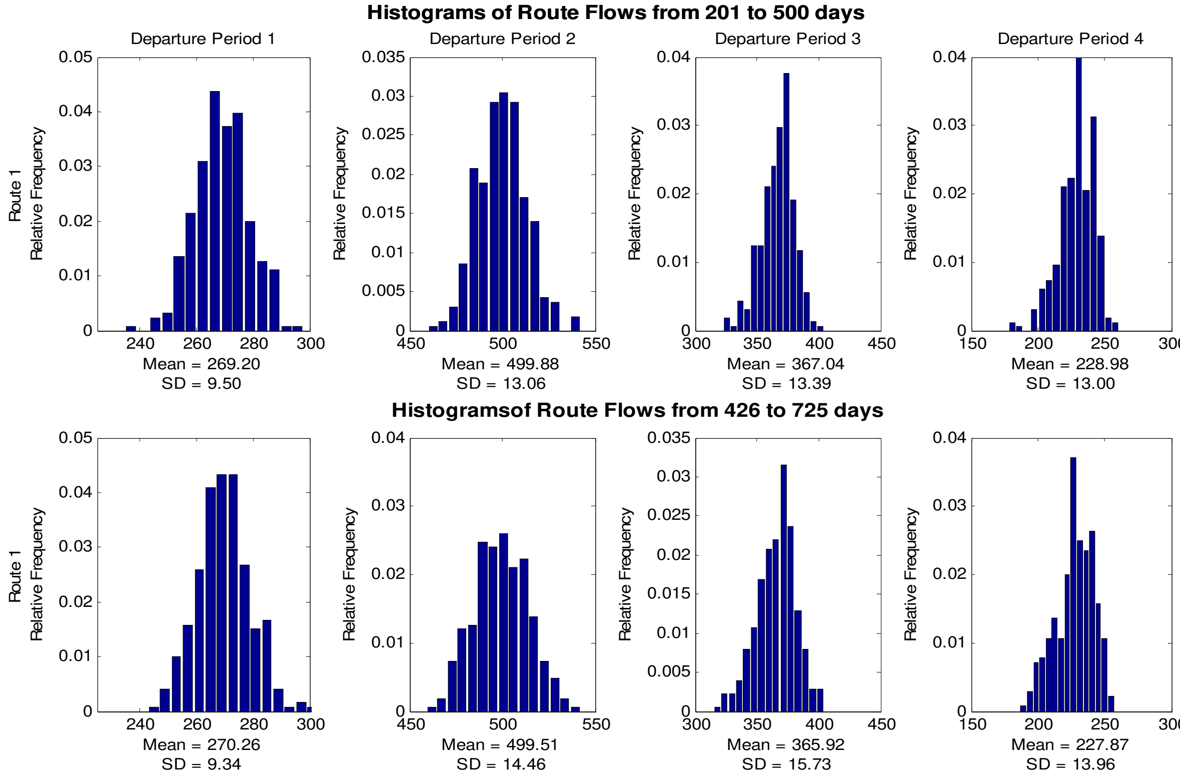

FIGURE 4 Histograms of Flows on Route 1

Visual observation of Figure 4 reveals that the distribution of the flows on route 1 in each departure period is similar in each case for the two sets of the observations. Moreover, in each case the mean and standard deviation of the route flows are nearly identical to each other indicating that the stochastic process being considered is stationary. Histograms of flows on routes 2 and 3 corroborated the earlier comments on route 1 reassuring the stationarity of the process, but due to reasons of brevity they are not included here.

Autocorrelations of Route Flows

In order to further ensure that the stochastic process is stable, we have analysed the autocorrelations of route flows based on a 1000-day long simulation. Autocorrelations are expected to die down with larger lags for a stationary series and indicate that the random variable under consideration is stable about its mean value. Figure 5 shows the autocorrelations in path flows on routes 1,2 and 3 for up to 15 days of lag over a realisation of 1000 days.

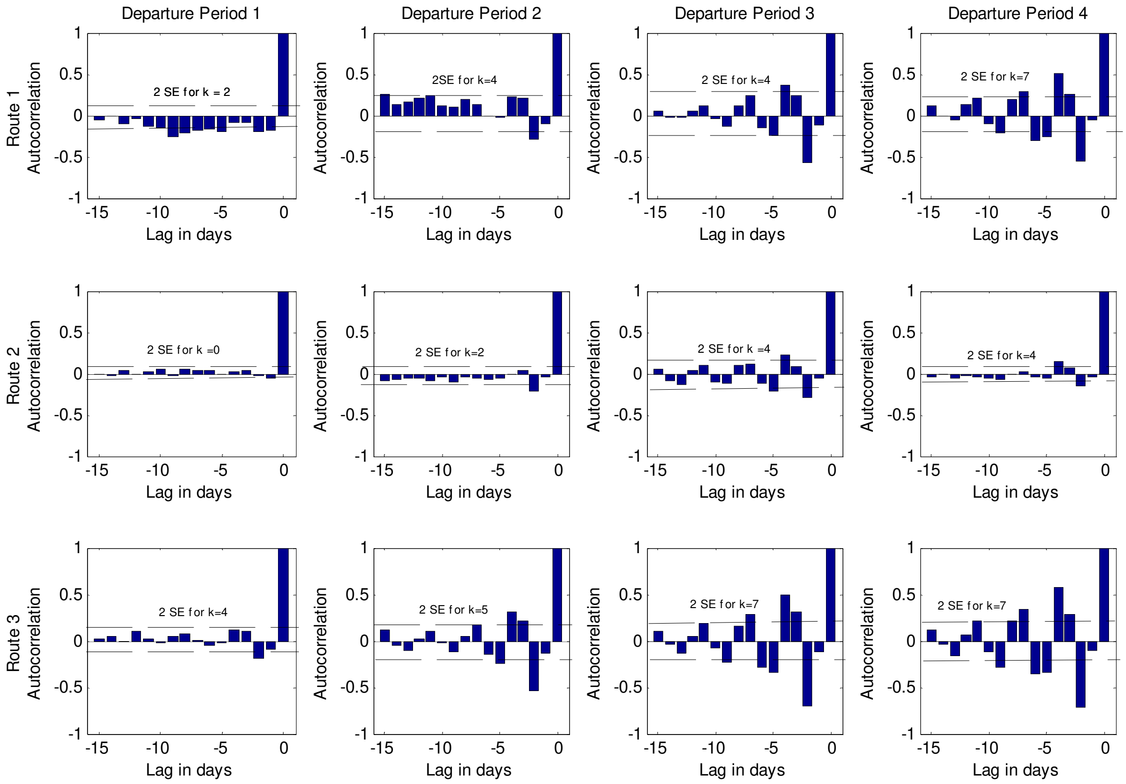

FIGURE 5 Correlogram for Flows on Routes 1,2 and 3

As the correlation of the flows with themselves is unity, the first bar (with '0' lag) reflects the same. From then on, the autocorrelations can be observed to reduce with increasing lags. Figure 5 includes error bars (based on Bartlett's formula for large lag standard error (*15*)) for each of the routes 1,2 and 3, for some lag k>0 beyond which the theoretical autocorrelation function has deemed to have died out. Insignificant autocorrelations compared to standard errors at some lag k>0 indicate that the flows on any route do not depend on the flows on the same route beyond k days during the same departure period. This condition implies that the process is stationary, but it is not sufficient to prove the stationarity. A necessary and sufficient test of stationarity of the time series would be that the determinant of the autocorrelation matrix and all the minors should be greater than zero, thus requiring a large number of conditions to be satisfied, all of which can be brought together by using spectral density functions (*15*).

The autocorrelations in departure periods 3 and 4 appear significant compared to the departure periods 1 and 2. This is due to the effect of carried over traffic from earlier departure periods to the later departure periods. Clearly, the link travel times at any instance are functions of the number of vehicles on the link at that instance, which can be composed of earlier, contemporaneous and later departures from any origin. In addition to the above, the travel times in departure periods 3 and 4 could also be affected by the type of travel time function chosen for the experiment, namely a *linear* function, as we shall explain here. Specifically it is known that linear travel time functions are likely to systematically overestimate the travel times in uncongested conditions (*14*), with the degree of overestimation increasing as the number of vehicles on the link increases (up to some point where congestion starts to form). In the experiments reported above, the effect of this over-estimation is seen to be particularly significant in departure periods 3 and 4, where the carried-over traffic from earlier departure periods contributes substantially to the number of vehicles on the links in periods 3 and 4, resulting in an inflated over-estimation of travel times. This in turn affects the route travel times on any given day, and in turn the route choice of the drivers the following day. Hence, higher autocorrelations in departure periods 3 and 4 are observed than in periods 1 and 2. Had we, on the other hand, adopted travel time functions of a higher order, we would expect the degree of over-estimation of uncongested travel times to be lower, and this may have led to less of a difference in magnitude of autocorrelations between departure periods.

Effect of Varying Perception Error

Quite differently from the above discussion, it is informative to investigate how the autocorrelations reflect a change in dispersion of the perceived costs which is parameterised by the logit choice parameter θ. As described earlier, autocorrelations in Figure 5 were based on a value of θ = 0.1. Figure 6 illustrates the autocorrelations of route flows with θ = 0.01. As θ decreases (in the limit as $\theta \to 0$), the dispersion of the perceived costs increases indicating that the drivers ignore the experienced costs and choose routes at random in which case the route flows on any day do not depend on any other day's flows implying that the autocorrelations will be smaller compared to those in Figure 5. In the limit, the autocorrelations bars will vanish with even lower values of θ.

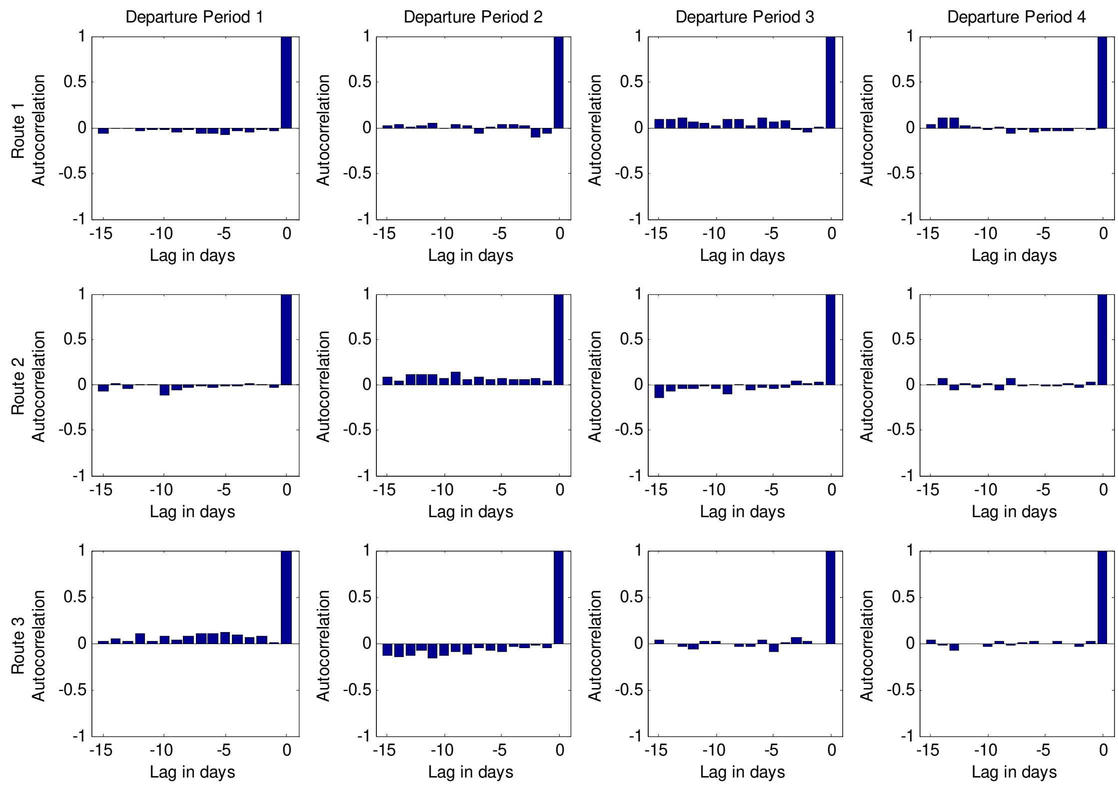

FIGURE 6 Correlogram of Flows on Routes 1,2 and 3 (θ = 0.01)

On the other hand, increasing values of θ will reduce the dispersion of the perceived costs and then the drivers start thinking alike while perceiving the route costs and making route choices. Due to this lack of taste variation, the solutions tend to be 'all-or-nothing' in the limit, giving rise to a kind of deterministic periodic system. In this case, most of the drivers choose the least cost route on any given day, then there is very little probability that they choose the same route on the following day, because they experience high cost of travelling on the previous day. This means that the route flows tend to be negatively correlated as shown in Figure 7. In the limit with higher values of θ, the autocorrelations will be equal to -0.5 at lag k = -1 and -2, indicating a deterministic periodic motion with a period of m = 2. This is true as long as there being another shorter route available to shift to at the end of the day. Otherwise the drivers continue to choose the same route on all days irrespective of the experienced costs (just as in the case of fixed route costs) and then the autocorrelations will be equal to zero for all routes. Due to this reason, in Figure 7, the autocorrelations for route 2 are smaller relative to routes 1 and 2 and are expected to be zero with even higher values of θ.

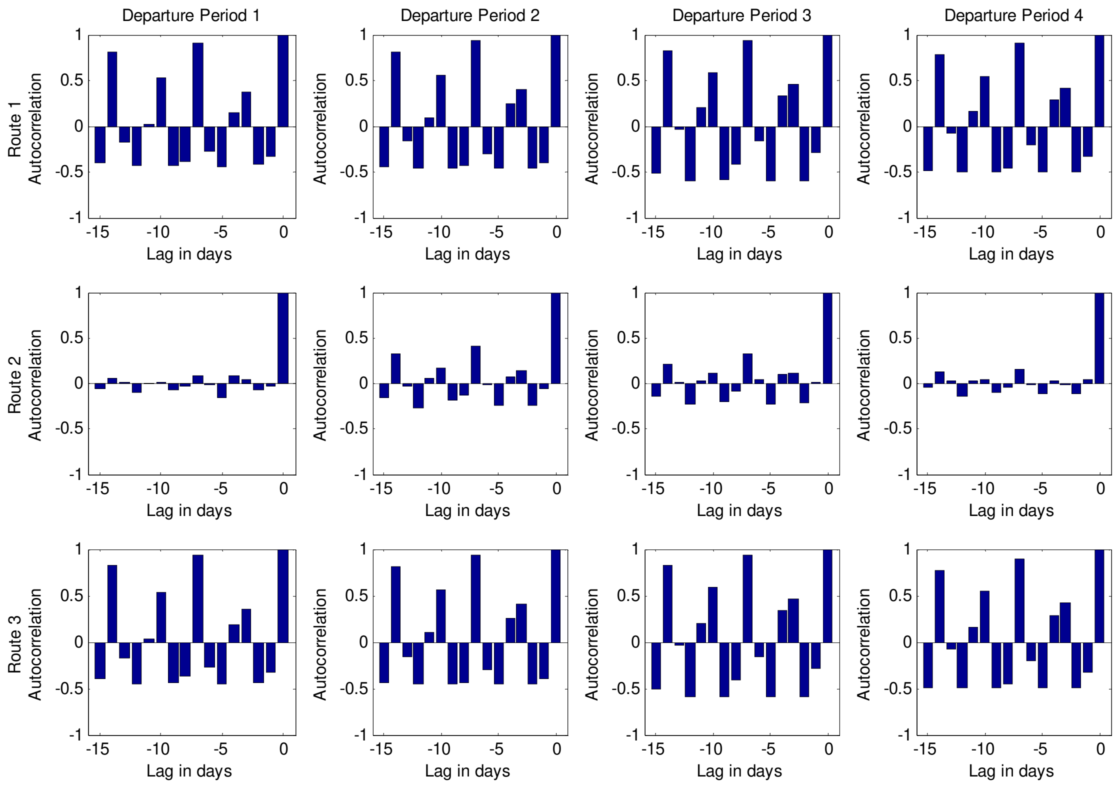

FIGURE 7 Correlogram of Flows on Routes 1,2 and 3 (θ = 0.5)

Link Time Plots

On a separate issue to that of investigating stationarity of the process is the question of the validity of the dynamic network loading model used for the within-day propagation of route flows. In order to verify the model in this respect, Figure 8 shows the link-time plot for routes 1, 2 and 3 in each of the departure periods for one particular day. They indicate that the travel times are fanning out in general, meaning that the congestion builds up as we progress with the dynamic loading of vehicles over the network. This phenomenon is particularly clear on links 1 and 2. On the other hand, parallel travel time lines indicate that the links are uncongested and operate below the capacity, as is the case with most of the links on routes 1, 2 and 3. Figure 8 also indicates that the model results are consistent with FIFO property as we do not have any intersecting link travel time lines. The figure is also indicative of satisfying the FIFO property at the path level.

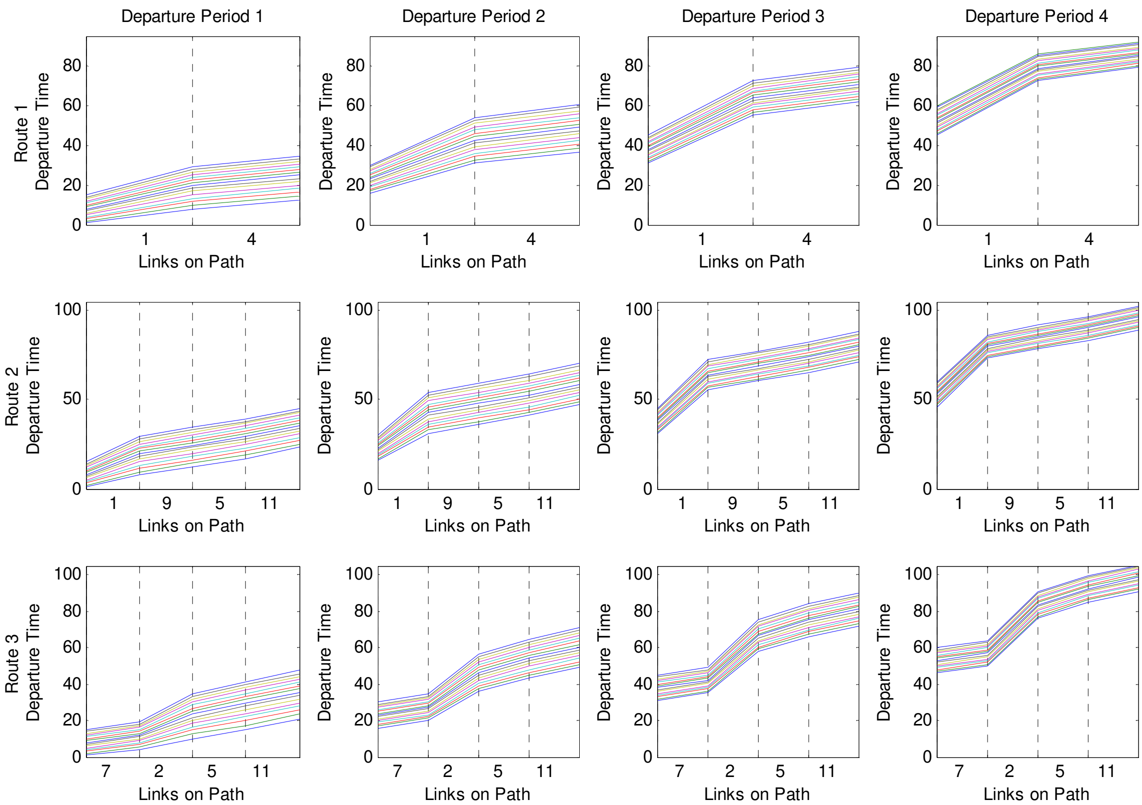

FIGURE 8 Link-Time Plots for Routes 1,2 and 3

Although links 1 and 2 show significant fanning of travel times, the case of link 2 will be interesting to see as it is used by several paths as set out by the link path incidence relationships (Table 2). Figure 9 shows the inflows, travel times and outflows from link 2, spread over the within-day time scale. Link 2 receives its inflows from the combined outflows of links 7 and 8 which shoot up to a maximum of about 30 vehicles/ minute at about 35 minutes after the simulation is started. Then the inflow rates start reducing as indicated by the two troughs between 35 and 60 minutes of elapsed time. The travel time profile steadily increases corresponding to the steady increase in the inflows and flattens once the inflows start dwindling and falls steeply when the inflows cease, and attains free flow time when there are no vehicles on the link any more. It can easily be observed that although the travel time falls steeply, the gradient of the line is greater than -1, thus satisfying the condition for FIFO (*16*). The outflows are computed corresponding to the travel times, and the outflow profile illustrates the dispersion of the outflows over larger periods than the inflow periods again indicating that there is some congestion on the link. Although Figure 9 indicates that the vehicles on link 2 no longer operate under free flow speeds, the link has still some spare capacity as indicated by the outflow profile lying below the exit capacity.

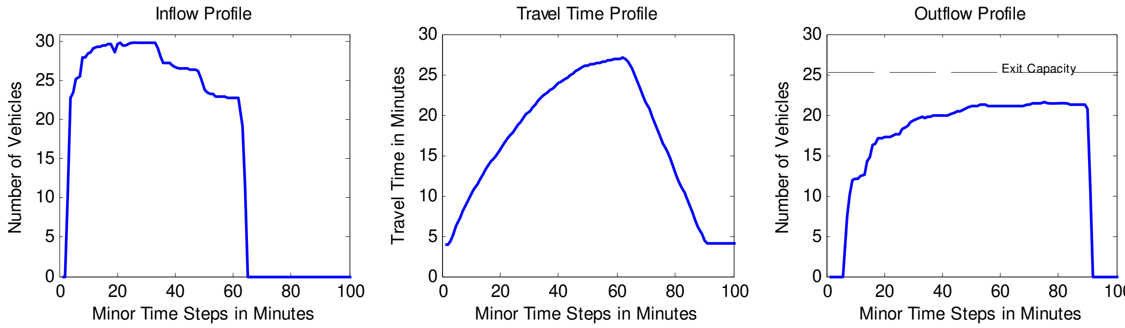

FIGURE 9 Inflow and Outflow Profiles for Link 2

CONCLUSIONS

The technique of simulation modelling provides solutions to complex traffic assignment problems such as the doubly dynamic traffic assignment described in this paper through a fairly simple and transparent process. For transport modellers applying such models, the practical counterpart to deterministic equilibrium is the stationary state of the stochastic processes, and in this paper correlograms were analysed as a way of detecting the stationarity. Properties of link travel time models including FIFO compliance were illustrated. In the future, affirmative tests of stationarity of the stochastic processes such as the ones involving spectral density functions will be investigated. Further useful experiments could also be performed to investigate the impact of alternative travel time functions (e.g. higher order than linear) on the autocorrelation functions.

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