Article:

Bhattacharya, P and Bhattacharya, S (2013) Methods for moment manipulation of various traffic sessions in modern small cell environment. International Journal of Research in Wireless Systems (IJRWS), 2 (3). 3. 23 - 34. ISSN 2320-3617

Methods for Moment Manipulation of Various Traffic Sessions in Modern Small Cell Environment

Paramita Bhattacharya and Samya Bhattacharya

Abstract

In this paper, we exercise some important derivation involving moment manipulations to estimate various traffic sessions in modern small cell environment. In the light of some previous work on cellular environment, we derive accurate expressions for a proposed Semi-Markov model that accounts for the smaller population size of the users in modern small cells. The small cells are generally powered by access points (WiFi/mini base stations) and are typically used as mobile clouds, hotspots. We validate the analysis for small cell model with an existing cellular traffic model by increasing the population size of the users to a large value (typical for a macro/micro-cellular environment). Regardless of the use of communication technology and nature of user sessions (voice, video, data etc.), the generic analysis can provide a good insight to the state-of-the-art traffic dimensioning for small cells.

Index Terms

Binomial moment generating function, freed carried traffic, hand-off, probability generating function, semi-markov, two-moments.

I. Introduction

With the rapid growth of mobile users, deployment of smaller cells powered by miniature base stations (BSs), such as access points (APs) or WiFi clouds have become increasingly popular cost-effective choice for the mobile operators around the world. Supporting multimedia sessions in such cells require accurate traffic dimensioning to maintain various quality of service (QoS) requirements. In addition, to maintain seamless connectivity on the move, these small cells require better session hand-off management. Therefore, hand-off traffic plays an important role in state-of-the-art traffic analysis. The researchers in the past have extensively studied the above in the context of cellular networks. Early traffic analyses for cellular networks were carried out with M/M/C loss systems and single-moment approach where fresh (i.e. new) and hand-off calls were assumed to be Poisson arrival processes [1], [2], [3]. In the context of micro-cellular systems where a user undergoes frequent hand-offs, it was shown [4], [5], [6], [7] that the hand-off process does not remain Poisson distributed. As a result, even if the fresh call arrival process can be represented as a Poisson distribution, the aggregated traffic stream is more realistically represented by a General distribution. It was also proposed [7], [8], [9] that in case of General distributed traffic, the two-moment representation of traffic (using mean and variance) is a better approach than a single-moment representation. Rajaratnam and Takawira [7], [8], [9] in their studies pointed out that the hand-off traffic is the main distinguishing features between the PSTN and mobile cellular networks by analysing the traffic using General distributed hand-off traffic. These analyses were applicable for densely populated macro-micro cells. With diminishing cell sizes and thus user population in a cell, fresh call arrivals, or a new session request in the context of modern mobile environment, follow Engset distribution [10] instead of Poisson distribution. The corresponding effects on the traffic streams and user sessions in small cell environment have not been thoroughly investigated in the literature to the best of our knowledge. In [11], [12], authors suggested to follow the models developed in [7], [8], [9] for a less populated area. But the use of the above models leads to difficulties, when the fresh call arrival process is Engset distributed. Firstly, the estimation of traffic offered to the *virtual cell* in their model needs a Semi-Markov analysis with Engset distribution. Secondly, the suggested use of Binomial-Poisson-Pascal (BPP) method in [7] yields inaccurate estimates of the carried traffic in a cell from the traffic offered to that cell when the *peakedness factor* (or *peakedness*) of the offered traffic falls below 0.5 (common in presence of an Engset distributed traffic stream) [13]. Also, the suggested use of Girard's method [9] to estimate the carried traffic in a cell from the traffic offered to that cell is limited to Pure Chance Type-I (PCT-I) traffic; making it inapplicable in the case of Engset distributed fresh call arrival process i.e. Pure Chance Type-II (PCT-II) traffic [10]. Thus, a thorough Semi-Markov analysis for Engset distributed fresh call traffic and the corresponding traffic estimates in modern mobile environment merits investigation.

Therefore, our contributions in this paper are two-fold. Firstly, we propose a Semi-Markov analysis for the Engset distributed fresh traffic sessions and General distributed handoff sessions in a small cell environment. Secondly, we derive two moments (mean and variance) of fresh and handoff traffic sessions and use those estimates to compute corresponding congestion parameters in a small cell. In Section II, we derive various traffic estimates for the models developed in [7], [8], [9] for micro-cellular environment. This forms the background for the proposed work in Section III. Section IV presents some numeric results related to the model verification and validation. Finally, we conclude in Section V.

Paramita Bhattacharya and Samya Bhattacharya are with M/S Untamed Spectrum, Post Box 119, Espoo 02320, Finland, e-mail: {p.bhattacharya, s.bhattacharya}@untamedspectrum.com. Samya Bhattacharya is also with the School of Electronic and Electrical Engineering, University of Leeds, Leeds LS2 9JT, United Kingdom, email: s.bhattacharya@leeds.ac.uk.

II. Related Work

In this section we study the various traffic sessions related to traditional cellular networks. In cellular networks, a channel was considered to be a single call carrying resource [7], [8], [9]. Fixed Channel Allocation (FCA) was considered where the number of simultaneous ongoing calls or sessions is fixed in a cell. A traffic stream was represented by two moments (M,V) denoting the corresponding mean and variance. The hand-off traffic was obtained when a fraction of users from a cell hand-off to an adjacent cell in their two two-cell model (Fig. 1). Initially, it was assumed that the originating cell receives no hand-off traffic. Later, the restriction was removed through an iterative process. To estimate the offered traffic to the target cell, the carried traffic estimate in the previous (originating) cell is needed. It has been shown that the variance of the carried traffic in the originating cell does not match with that of the traffic offered to the target cell, though the corresponding means match. This is because the offered traffic used to be stochastically different from the carried traffic. Thus, a virtual (hypothetical) cell of infinite capacity has been introduced between the two cells. The carried traffic in the virtual cell (also known as *freed carried traffic* [14]) will now be equivalent to the offered traffic in the target cell. Later, the two-cell model was generalised to a multi-cell model where the estimated hand-off traffic using the two-cell model was aggregated with fresh call offered traffic to form a combined traffic stream offered to a cell.

Using various moment manipulation techniques presented in [13], [14], we in this section, present an alternative approach to [7] to formulate and derive the moments of the corresponding traffic streams in their two-cell model. These form the background of our work in the next section involving small cells. A session initiated by a mobile user may suffer blocking at two stages - (a) at session initiation, known as *fresh blocking* or *fresh congestion*, and (b) at the cell boundary, which is called *hand-off blocking* or *hand-off congestion*. Let us consider the i^{th} cell L_i $(i = 1, 2)$ in Fig. 1. Let C be the number of channels

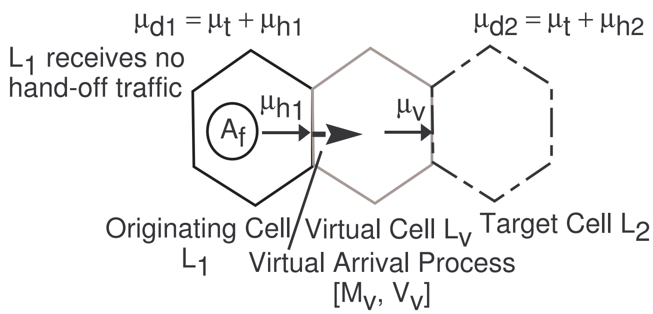


Figure 1: The two-cell model with a virtual cell [7].

assigned to the cell; $\lambda_i = \lambda_f$ be the mean arrival rate of fresh requests to the cell, which is Poisson distributed. A mobile user vacates a channel in a cell due to either session termination or session hand-off to the other cell. Both processes are assumed to be Poisson distributed with mean rate μ_t and μ_{hi}, respectively. Thus, session departure process from a cell is also Poisson distributed with mean rate $\mu_{di} = \mu_t + \mu_{hi}$. It is imperative to state that the channel holding time is Negative Exponential distributed with mean $1/\mu_{di}$. It is noted that some studies modelled the channel holding time as Gamma distribution [15] or Hyper-Erlang distribution [16] or Lognormal distribution [17] or a combination of constant and Negative Exponential distribution [8]. However, considering the urban scenario [18], the channel holding time is modelled as Negative Exponential distribution in this work. Assuming that the originating cell L_1 initially receives no hand-off traffic, the offered traffic to that cell is Poisson distributed (fresh requests only) with mean $A_f = \lambda_f/\mu_{d1}$. The carried traffic (M_{C_1}, V_{C_1}) in the cell is obtained as below.

A. Expressions for the Carried Traffic in L_1

Let there be k ongoing sessions in L_1. Then the probability (p_k) that L_1 is in state k [13], is given by

$$p_k = \frac{\frac{A_f^k}{k!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} = \frac{\frac{A_f^k}{k!}}{e_C(A_f)} \tag{1a}$$

$$p_C = \frac{\frac{A_f^C}{C!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} = B(A_f, C) \tag{1b}$$

where $e_C(A_f) = \sum_{i=0}^{C} \frac{A_f^i}{i!}$ is the incomplete exponential function and p_C is the *Erlang loss function*. Since the carried traffic has only the fresh component, it is sufficient to use *Probability Generating Function* (PGF) [13] alone, which is defined as

$G(z) = \sum_{k=0}^{\infty} p_k z^k$. Using Eqn. (1a), we obtain

$$G(z) = \frac{\sum_{k=0}^{C} z^k \frac{A_f^k}{k!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \tag{2}$$

$$\text{Therefore,} \qquad G'(z) = \frac{\sum_{k=1}^{C} k z^{k-1} \frac{A_f^k}{k!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \tag{3}$$

$$\text{and} \qquad G''(z) = \frac{\sum_{k=2}^{C} k(k-1) z^{k-2} \frac{A_f^k}{k!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \tag{4}$$

We express the mean (M_{C_1}) of the carried traffic from [13] as

$$\begin{aligned}
M_{C_1} &= G'(1) \\
&= \frac{\sum_{k=1}^{C} k \frac{A_f^k}{k!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} && \text{[Using Eqn. (3)]} \\
&= \frac{\sum_{k=1}^{C} \frac{A_f^k}{(k-1)!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f \frac{\sum_{i=0}^{C-1} \frac{A_f^i}{i!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f \frac{\sum_{i=0}^{C} \frac{A_f^i}{i!} - \frac{A_f^C}{C!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f \left(1 - \frac{\frac{A_f^C}{C!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \right) \\
&= A_f [1 - B(A_f, C)] && \text{[Using Eqn. (1b)]}
\end{aligned} \tag{5}$$

We express the variance (V_{C_1}) of the carried traffic from [13] as

$$\begin{aligned}
V_{C_1} &= G''(1) + G'(1) - (G'(1))^2 \\
&= M_{C_1} - [(G'(1))^2 - G''(1)] && \text{[Using Eqn. (5)]}
\end{aligned} \tag{6}$$

Using Eqn. (1b), (3), (4), and (5), we get

$$\begin{aligned}
[(G'(1))^2 - G''(1)] &= A_f^2 (1 - p_C)^2 - \frac{\sum_{k=2}^{C} \frac{A_f^k}{(k-2)!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f^2 (1 - p_C)^2 - A_f^2 \frac{\sum_{i=0}^{C-2} \frac{A_f^i}{i!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f^2 (1 - p_C)^2 - A_f^2 \frac{\sum_{i=0}^{C} \frac{A_f^i}{i!} - \frac{A_f^C}{C!} - \frac{A_f^{C-1}}{(C-1)!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \\
&= A_f^2 (1 - p_C)^2 - A_f^2 \left[1 - \frac{\frac{A_f^C}{C!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} - \frac{C}{A_f} \frac{\frac{A_f^C}{C!}}{\sum_{i=0}^{C} \frac{A_f^i}{i!}} \right] \\
&= A_f^2 (1 - p_C)^2 - A_f^2 \left[1 - p_C - \frac{C}{A_f} p_C \right] \quad \text{[Using Eqn. (1b)]} \\
&= A_f^2 - 2A_f^2 p_C + A_f^2 p_C^2 - A_f^2 + A_f^2 p_C + C A_f p_C \\
&= C A_f p_C - A_f^2 p_C + A_f^2 p_C^2 \\
&= A_f p_C \left[C - A_f \left(1 - p_C \right) \right] \\
&= A_f p_C \left[C - M_{C_1} \right] \\
&= A_f B(A_f, C) \left[C - M_{C_1} \right]
\end{aligned} \tag{7}$$

Substituting Eqn. (7) in Eqn. (6), we obtain

$$V_{C_1} = M_{C_1} - A_f B(A_f, C)(C - M_{C_1}) \tag{8}$$

B. Factorial Moments for the Offered Traffic in L_2

Let the inter-arrival time of the offered traffic be denoted as $A(t)$. From [14], the k^{th} factorial moments ($M_{(k)}$) of the offered traffic is expressed as

$$M_{(k)} = \frac{1}{\mu_d(EA)} \prod_{j=1}^{k-1} \frac{jA^*(j\mu_d)}{1 - A^*(j\mu_d)} \qquad k \in I \tag{9}$$

where μ_d is the mean service rate (Poisson distributed), EA is the mean inter-arrival time. If $A^*(s)$ denotes the L.S.T. of $A^*(t)$, then for Poisson distributed offered traffic, we have

$$A^*(s) = \frac{\lambda_f}{s + \lambda_f} \tag{10}$$

Further, from [14], we get

$$EA = -A^*(0) = -\frac{-\lambda_f}{(s+\lambda_f)^2}\bigg|_{s=0} = \frac{1}{\lambda_f} \tag{11}$$

Substituting EA from Eqn. (11) in Eqn. (9), we obtain

$$\begin{aligned} M_{(k)} &= \frac{1}{\mu_d \frac{1}{\lambda_f}} \prod_{j=1}^{k-1} \frac{\frac{j\lambda_f}{j\mu_d + \lambda_f}}{1 - \frac{\lambda_f}{j\mu_d + \lambda_f}} \\ &= \frac{\lambda_f}{\mu_d} \prod_{j=1}^{k-1} \frac{\lambda_f}{\mu_d} \\ &= \left(\frac{\lambda_f}{\mu_d}\right)^k \\ &= M^k \end{aligned} \tag{12}$$

where $M = \frac{\lambda_f}{\mu_d}$ is the first factorial moment of the Poisson distributed offered traffic.

C. Expressions for the Overflow Traffic in L_1

Let $M_{s,(k)}$ ($s = 1, 2$ and $k = 1, 2, \ldots$) be the k^{th} factorial moment of the s^{th} traffic stream, where $s = 1$ denotes the overflow stream and $s = 2$ denotes the freed carried traffic stream [14]. In our notation, the mean (M_1) of the overflow traffic [14] is written as

$$\frac{1}{M_1} = \frac{1}{M_{1,(1)}} = \sum_{l=0}^{C} \binom{C}{l} \frac{l!}{M_{(l+1)}}$$

$$\begin{aligned} \text{Therefore,} \qquad M_1 &= \frac{1}{\sum_{l=0}^{C} \binom{C}{l} \frac{l!}{M_{(l+1)}}} \\ &= \frac{1}{\sum_{l=0}^{C} \binom{C}{l} \frac{l!}{M^{l+1}}} && \text{[Using Eqn. (12)]} \\ &= \frac{M}{\sum_{l=0}^{C} \frac{C! M^{-l}}{(C-l)!}} \\ &= M \frac{\frac{M^C}{C!}}{\sum_{l=0}^{C} \frac{M^{C-l}}{(C-l)!}} \\ &= M \frac{\frac{M^C}{C!}}{\sum_{i=0}^{C} \frac{M^i}{i!}} \\ &= MB(M, C) && \text{[Using Eqn. (1b)]} \end{aligned} \tag{13}$$

From [13], the variance (V_1) of the overflow traffic can be expressed in terms of the mean (M_1) and the second factorial moment ($M_{1,(2)}$) of the overflow traffic as

$$V_1 = M_{1,(2)} - M_1(M_1 - 1) \tag{14}$$

The second factorial moment ($M_{1,(2)}$) of the overflow traffic from [14] is written as

$$\frac{1}{M_{1,(2)}} = \sum_{l=0}^{C} \binom{C}{l} \frac{(l+1)!}{M_{(l+2)}}$$

$$\begin{aligned} \text{Therefore,} \qquad V_1 + M_1(M_1 - 1) &= \frac{1}{\sum_{l=0}^{C} \binom{C}{l} \frac{(l+1)!}{M_{(l+2)}}} && \text{[Using Eqn. (14)]} \\ &= \frac{1}{\sum_{l=0}^{C} \frac{C!(l+1)!}{l!(C-l)!M^{l+2}}} \\ &= \frac{1}{\frac{C!}{M^{C+2}} \sum_{l=0}^{C} (l+1) \frac{M^{C-l}}{(C-l)!}} \end{aligned}$$

$$
\begin{aligned}
&= \frac{M^2 \frac{M^C}{C!}}{\sum_{k=0}^{C}(C+1-k)\frac{M^k}{k!}} \\
&= \frac{M^2 \frac{M^C}{C!}}{(C+1)\sum_{k=0}^{C}\frac{M^k}{k!} - M\sum_{j=0}^{C-1}\frac{M^j}{j!}} \\
&= \frac{M^2 \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C}\frac{M^k}{k!}}}{(C+1) - M\left[\frac{\sum_{j=0}^{C}\frac{M^j}{j!} - \frac{M^C}{C!}}{\sum_{k=0}^{C}\frac{M^k}{k!}}\right]} \\
&= \frac{M^2 B(M,C)}{C+1-M[1-B(M,C)]} \\
&= \frac{MM_1}{C+1-M+M_1} \qquad \text{[Using Eqn. (13)]}
\end{aligned}
$$

$$
\text{Therefore,} \qquad \frac{V_1}{M_1} + (M_1 - 1) = \frac{M}{C+1-M+M_1}
$$

$$
\therefore V_1 = M_1 - M_1^2 + \frac{MM_1}{C+1-M+M_1} \tag{15}
$$

D. Expressions for the Freed Carried Traffic in L_V

From the conservation of flow, the mean (M_V) of the freed carried traffic can be expressed as

$$
M_V = M_2 = M - M_1 = M[1 - B(M,C)] \qquad \text{[Using Eqn. (13)]} \tag{16}
$$

which is equal to the mean carried traffic. The second factorial moment ($M_{2,(2)}$) of the freed carried traffic from [14] is written as

$$
M_{2,(2)} = M_2 \frac{M_{(2)}}{M_{(1)}} - M_1[V_1 + M_1(M_1-1)] \sum_{l=1}^{C} \binom{C}{l} \frac{l!}{M_{(l+1)}} \sum_{m=1}^{l} \left[\frac{mM_{(m)}}{M_{(m+1)}} + 1\right] \tag{17}
$$

Using the relation between ordinary, central and factorial moments of a distribution [13] we rewrite Eqn. (17) as

$$
\begin{aligned}
V_2 + M_2(M_2 - 1) =& M_2 \frac{M_{(2)}}{M_{(1)}} - M_1[V_1 + M_1(M_1-1)] \sum_{l=1}^{C} \binom{C}{l} \frac{l!}{M_{(l+1)}} \\
&\cdot \sum_{m=1}^{l} \left[\frac{mM_{(m)}}{M_{(m+1)}} + 1\right] \\
=& M_2 M - M_1[V_1 + M_1(M_1-1)] \sum_{l=1}^{C} \frac{C! M^{-(l+1)}}{(C-l)!} \sum_{m=1}^{l} \left(\frac{m}{M} + 1\right) \\
=& M_2 M - M_1[V_1 + M_1(M_1-1)] \sum_{l=1}^{C} \frac{C! M^{-(l+1)}}{(C-l)!} \left[l + \frac{l(l+1)}{2M}\right] \\
=& M_2 M - M_1[V_1 + M_1(M_1-1)] \left[\sum_{l=1}^{C} \frac{C! M^{-(l+1)} l}{(C-l)!}\right. \\
&\left. + \sum_{l=1}^{C} \frac{C! M^{-(l+2)} l(l+1)}{2(C-l)!}\right] \\
=& M_2 M - M_1[V_1 + M_1(M_1-1)] \left[\underbrace{\sum_{k=0}^{C-1} \frac{C! M^{-(C-k+1)}(C-k)}{k!}}_{=A \text{ (say)}}\right. \\
&\left. + \underbrace{\sum_{k=0}^{C-1} \frac{C! M^{-(C-k+2)}(C-k)(C-k+1)}{2k!}}_{=B \text{ (say)}}\right]
\end{aligned} \tag{18}
$$

Now,

$$
\begin{aligned}
A =& \sum_{k=0}^{C-1} \frac{C! M^{-(C-k+1)}(C-k)}{k!} \\
=& CC! \sum_{k=0}^{C-1} \frac{M^{-(C-k+1)}}{k!} - C! \sum_{k=1}^{C-1} \frac{M^{-(C-k+1)}}{(k-1)!} \\
=& \frac{CC!}{M^{C+1}} \sum_{k=0}^{C-1} \frac{M^k}{k!} - \frac{C!}{M^C} \sum_{j=0}^{C-2} \frac{M^j}{j!} \\
=& \frac{CC!}{M^{C+1}} \left[\sum_{k=0}^{C} \frac{M^k}{k!} - \frac{M^C}{C!} \right] - \frac{C!}{M^C} \left[\sum_{j=0}^{C} \frac{M^j}{j!} - \frac{M^C}{C!} - \frac{M^{C-1}}{(C-1)!} \right] \\
=& \frac{C}{M} \frac{\sum_{k=0}^{C} \frac{M^k}{k!}}{\frac{M^C}{C!}} \left[1 - \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} \right] - \frac{\sum_{j=0}^{C} \frac{M^j}{j!}}{\frac{M^C}{C!}} \left[1 - \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} - \frac{C}{M} \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} \right] \\
=& \frac{C}{M} \frac{1}{B(M,C)} [1 - B(M,C)] - \frac{1}{B(M,C)} \left[1 - B(M,C) - \frac{C}{M} B(M,C) \right] \\
=& \frac{C}{M_1} - \frac{CB(M,C)}{M_1} - \frac{1}{B(M,C)} + 1 + \frac{C}{M} \\
=& 1 + \frac{C}{M_1} - \frac{1}{B(M,C)} \quad \text{[Using Eqn. (13)]}
\end{aligned} \tag{19}
$$

and,

$$
\begin{aligned}
B =& \sum_{k=0}^{C-1} \frac{C! M^{-(C-k+2)}(C-k)(C-k+1)}{2k!} \\
=& \frac{C!}{2M^{C+2}} \sum_{k=0}^{C} \frac{M^k}{k!} (C-k)(C-k+1) \\
=& \frac{C!}{2M^{C+2}} \sum_{k=0}^{C} \frac{M^k}{k!} [C^2 + C - 2Ck + k(k-1)] \\
=& \frac{C!}{2M^{C+2}} \left[(C^2 + C) \sum_{k=0}^{C} \frac{M^k}{k!} - 2C \sum_{k=1}^{C} \frac{M^k}{(k-1)!} + \sum_{k=2}^{C} \frac{M^k}{(k-2)!} \right] \\
=& \frac{C!}{2M^{C+2}} \left[(C^2 + C) \sum_{k=0}^{C} \frac{M^k}{k!} - 2CM \sum_{j=0}^{C} \frac{M^j}{j!} + 2CM \frac{M^C}{C!} + M^2 \sum_{j=0}^{C} \frac{M^j}{j!} \right. \\
& \left. - M^2 \frac{M^C}{C!} - M^2 \frac{M^{C-1}}{(C-1)!} \right] \\
=& \frac{1}{2M^2} \frac{\sum_{k=0}^{C} \frac{M^k}{k!}}{\frac{M^C}{C!}} \left[C^2 + C - 2CM + 2CM \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} + M^2 - M^2 \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} \right. \\
& \left. - CM \frac{\frac{M^C}{C!}}{\sum_{k=0}^{C} \frac{M^k}{k!}} \right] \\
=& \frac{1}{2M^2 B(M,C)} [C^2 + C - 2CM + 2CMB(M,C) + M^2 - M^2 B(M,C) \\
& - CMB(M,C)] \\
=& \frac{1}{2MM_1} [C^2 + C - 2CM + CM_1 + M^2 - MM_1] \quad \text{[Using Eqn. (13)]}
\end{aligned} \tag{20}
$$

Substituting A from Eqn. (19) and B from Eqn. (20) in Eqn. (18), we obtain

$$\begin{aligned}
V_2 + M_2(M_2-1) =& M_2 M - M_1[V_1 + M_1(M_1-1)] \bigg[1 + \frac{C}{M_1} - \frac{1}{B(M,C)} \\
& + \frac{1}{2MM_1}\left(C^2 + C - 2CM + CM_1 + M^2 - MM_1\right)\bigg] \\
=& M_2 M - [V_1 + M_1(M_1-1)] \bigg[M_1 + C - M \\
& + \frac{1}{2M}\left(C^2 + C - 2CM + CM_1 + M^2 - MM_1\right)\bigg] \\
& \text{[Using Eqn. (13)]} \\
=& M_2 M - \frac{V_1 + M_1(M_1-1)}{2M}[C^2 - M^2 + C + CM_1 + MM_1] \\
=& M_2 M - \frac{M_1 - M_1^2 + \frac{MM_1}{C+1-M+M_1} + M_1(M_1-1)}{2M} \\
& \cdot [C^2 - M^2 + C + CM_1 + MM_1] \\
& \text{[Using Eqn. (15)]}
\end{aligned}$$

$$=M_2 M - \frac{M_1}{2(C+1-M_2)}[C^2 + C + CM - CM_2 - MM_2]$$

[Using Eqn. (16)]

Therefore,

$$\frac{V_2}{M_2} = 1 - M_2 + M - \frac{M_1}{2M_2(C+1-M_2)}\left[C(C+1-M_2) + M(C-M_2)\right]$$

$$= 1 - \frac{M_1}{2M_2}\left[\frac{2M_2^2}{M_1} - \frac{2MM_2}{M_1} + C + \frac{M(C-M_2)}{C+1-M_2}\right]$$

$$= 1 - \frac{MB(M,C)}{2M(1-B(M,C))}\left[\frac{2M_2(M_2-M)}{M_1} + C + \frac{M(C-M_2)}{C+1-M_2}\right]$$

[Using Eqn. (13)]

$$= 1 - \frac{B(M,C)}{2[1-B(M,C)]}\left[C - 2M_2 + \frac{M(C-M_2)}{C+1-M_2}\right]$$

[Using Eqn. (16)]

$$= 1 - \frac{B(M,C)}{2[1-B(M,C)]}\left[C - M_2 - \left(M_2 - \frac{M(C-M_2)}{C+1-M_2}\right)\right]$$

$$= 1 - \frac{B(M,C)}{2[1-B(M,C)]}\left[C - M_2 + M_1 - \left(M - \frac{M(C-M_2)}{C+1-M_2}\right)\right]$$

[Using Eqn. (16)]

$$= 1 - \frac{B(M,C)}{2[1-B(M,C)]}\left[C - M_2 + M_1 - \frac{MC + M - MM_2 - MC + MM_2}{C+1-M_2}\right]$$

$$= 1 - \frac{B(M,C)}{2[1-B(M,C)]}\left[C - M_2 + M_1 - \frac{M}{C+1-M_2}\right] \tag{21}$$

In the present case, $M = A_f$, $M_1 = A_f B(A_f, C)$, $M_2 = M_V$, and $V_2 = V_V$. Substituting the above quantities in Eqn. (21), we obtain

$$V_V = M_V \left[1 - \frac{B(A_f, C)}{2\,[1 - B(A_f, C)]}\left(C - M_{C_1} + A_f B(A_f, C) - \frac{A_f}{C - M_{C_1} + 1}\right)\right] \tag{22}$$

E. Expressions for the Hand-off Offered Traffic

From [13], the first two ordinary moments (α_1, α_2) of the aggregated offered traffic and the i^{th} $(i = 1, 2, \ldots)$ individual traffic stream (α_1^i, α_2^i) are related as

$$\alpha_1^i = P_i \alpha_1 \tag{23a}$$

$$\alpha_2^i = P_i^2 \alpha_2 + P_i(1 - P_i)\alpha_1 \tag{23b}$$

where P_i is the probability of a session belonging to stream i. In the present case, there are two streams, viz. (i) the stream containing sessions that are completing within a cell and (ii) the stream containing sessions that are handed off to another cell. For the second stream (P_i) is expressed as

$$P_i = \frac{\mu_h}{\mu_h + \mu_t} = \zeta \tag{24}$$

The parameter ζ is called the mobility [1]. Using the relation between the ordinary moments, mean, and variance of a distribution [13], we write

$$\alpha_1 = M_V \quad \text{and} \quad \alpha_2 = V_V + M_V^2 \tag{25a}$$

$$\alpha_1^i = M_h \quad \text{and} \quad \alpha_2^i = V_h + M_h^2 \tag{25b}$$

Using Eqn. (23), (24), and (25), we obtain

$$\begin{aligned} M_h &= P_i M_V \\ &= \frac{\mu_h}{\mu_h + \mu_t} M_V \qquad \text{(26a)} \\ V_h + M_h^2 &= P_i^2 \left(V_V + M_V^2\right) + \left(P_i - P_i^2\right) M_V \\ \text{Therefore,} \qquad V_h &= P_i^2 V_V + P_i^2 M_V^2 + P_i M_V - P_i^2 M_V - M_h^2 \\ &= P_i^2 M_V + M_h^2 + M_h - P_i^2 M_V - M_h^2 \qquad \text{[Using Eqn. (26a)]} \\ &= M_h + P_i^2 \left(V_V - M_V\right) \\ &= M_h + P_i \frac{M_h}{M_V} \left(V_V - M_V\right) \qquad \text{[Using Eqn. (26a)]} \\ &= M_h \left[1 + P_i \left(\frac{V_V}{M_V} - 1\right)\right] \\ &= M_h \left[1 + \frac{\mu_h}{\mu_h + \mu_t} \left(\frac{V_V}{M_V} - 1\right)\right] \qquad \text{(26b)} \end{aligned}$$

III. Case of Finite Population size in Small Cell WiFi / Mobile Cloud Environment

As mentioned earlier, the fresh traffic in a small cell environment follows Engset distribution [10]. Thus, cell L_1 in the two-cell model receives Engset distributed fresh requests in our case instead of Poisson distributed fresh requests in case of [7]. Thus, the user population in L_1 is now finite. Let N be the number of users in L_1, $\lambda_i = \lambda'_f$ be the mean arrival rate of fresh sessions per idle user and μ_d be the mean departure rate from L_1. We develop a joint Semi-Markov model to obtain the variance of the offered traffic to L_2. The Semi-Markov model does not have a closed form solution and needs complex manipulation using Probability Generating Function (PGF) and Binomial Moment Generating Function (BMGF).

A. Carried Traffic in L_1 for the Engset type arrival

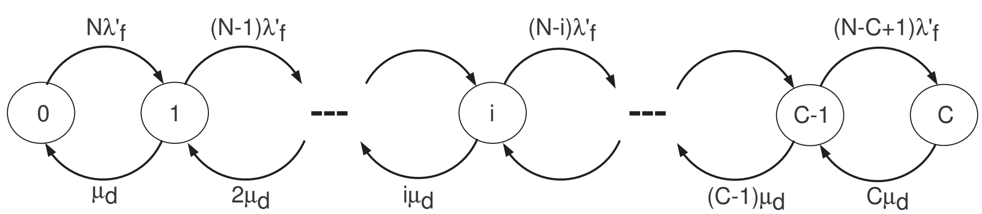


Figure 2: State transitions of the Engset distribution.

The state transition diagram for the Engset distribution is shown in Fig. (2). Let p_i be the probability of L_1 in state i. The cut equations in Fig. 2 yields

$$\begin{aligned} i\mu_d p_i &= (N - i + 1)\lambda'_f p_{i-1} \qquad \text{for} \quad 0 \le i \le C \\ \therefore i p_i &= \frac{\lambda'_f}{\mu_d}(N - i + 1) p_{i-1} \qquad \text{(27)} \end{aligned}$$

The expression for the mean (M_C) of the carried traffic is written as

$$\begin{aligned}
M_C &= \sum_{i=0}^{C} i p_i \\
&= \sum_{i=1}^{C} \frac{\lambda'_f}{\mu_d}(N-i+1)p_{i-1} \\
&= \sum_{i=0}^{C-1} \frac{\lambda'_f}{\mu_d}(N-i)p_i \\
&= \frac{\lambda'_f}{\mu_d} N \sum_{i=0}^{C-1} p_i - \frac{\lambda'_f}{\mu_d} \sum_{i=0}^{C-1} i p_i \\
&= \frac{\lambda'_f}{\mu_d} N(1-p_C) - \frac{\lambda'_f}{\mu_d}\left(\sum_{i=0}^{C} i p_i - C p_C\right) \qquad \because \sum_{i=0}^{C} p_i = 1 \\
&= \frac{\lambda'_f}{\mu_d} N(1-p_C) - \frac{\lambda'_f}{\mu_d}(M_C - C p_C) \\
\therefore M_C\left(1+\frac{\lambda'_f}{\mu_d}\right) &= \frac{\lambda'_f}{\mu_d}\left[N-(N-C)p_C\right] \\
\therefore M_C &= \frac{N\lambda'_f}{\lambda'_f+\mu_d}\left[1-\frac{N-C}{N}p_C\right]
\end{aligned} \tag{28}$$

The expression for the variance (V_C) of the carried traffic is written as

$$\begin{aligned}
V_C &= \sum_{i=0}^{C} \left(i-M_C\right)^2 p_i \\
&= \sum_{i=0}^{C} i^2 p_i - 2M_C \sum_{i=0}^{C} i p_i + M_C^2 \sum_{i=0}^{C} p_i
\end{aligned}$$

$$= \sum_{i=0}^{C} i^2 p_i - 2M_C^2 + M_C^2$$
$$= \sum_{i=0}^{C} i^2 p_i - M_C^2 \tag{29}$$

Now, let $X = \sum_{i=0}^{C} i^2 p_i$.

$$\therefore X = \sum_{i=0}^{C} i \cdot i p_i$$
$$= \sum_{i=1}^{C} i \frac{\lambda'_f}{\mu_d}(N - i + 1)p_{i-1}$$
$$= \sum_{i=0}^{C-1} \frac{\lambda'_f}{\mu_d}(i+1)(N-i)p_i$$
$$= \frac{\lambda'_f}{\mu_d} \sum_{i=0}^{C-1} [Nip_i - i^2 p_i + Np_i - ip_i]$$
$$= \frac{\lambda'_f}{\mu_d} \sum_{i=0}^{C-1} \left[N(M_C - Cp_C) - \sum_{i=0}^{C-1} ip_i + N(1 - p_C) - (M_C - Cp_C) \right]$$
$$\therefore \left(1 + \frac{\lambda'_f}{\mu_d}\right) X = \frac{\lambda'_f}{\mu_d} \left[N + NM_C - M_C + C^2 p_C - NCp_C + Cp_C - Np_C\right]$$
$$= \frac{\lambda'_f}{\mu_d} [N + M_C(N-1) + Cp_C(C-N) + p_C(C-N)]$$
$$= \frac{\lambda'_f}{\mu_d} [N + M_C(N-1) + (C-N)(C+1)p_C]$$
$$\therefore X = \frac{\lambda'_f}{\lambda'_f + \mu_d} [N + M_C(N-1) + (C-N)(C+1)p_C] \tag{30}$$

Hence,

$$V_C = \frac{\lambda'_f}{\lambda'_f + \mu_d} [N + M_C(N-1) + (C-N)(C+1)p_C] - M_C^2 \tag{31}$$

As mentioned before, mean of the offered traffic to L_V is equal to the mean of the carried traffic in L_1. Therefore, we need to find out variance of the traffic in L_V.

B. Semi-Markov Analysis to estimate the variance of the traffic in L_V

The state space Markov diagram for the carried sessions in L_1 and L_V is shown in Fig. 3. Let $p_{n,m}$ denotes the joint probability distribution, when there are n carried sessions in cell L_1 and m sessions are carried to L_V. The PGF of the joint probability distribution is defined as

$$G_n(z) = \sum_{m=0}^{\infty} p_{n,m} z^m \tag{32a}$$
$$\therefore zG_n(z) = \sum_{m=1}^{\infty} p_{n,m-1} z^m \tag{32b}$$

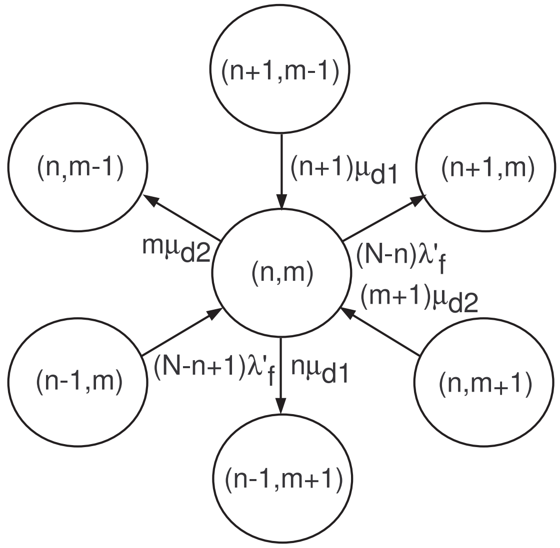


Figure 3: Semi-Markov model for the carried sessions in L_1 and offered sessions to L_V.

From Eqn. (32a), we obtain

$$\begin{aligned} G_n'(z) &= \sum_{m=1}^{\infty} m p_{n,m} z^{m-1} \\ &= \sum_{m=0}^{\infty} (m+1) p_{n,m+1} z^m \end{aligned} \tag{32c}$$

$$\begin{aligned} \therefore zG_n'(z) &= \sum_{m=1}^{\infty} m p_{n,m} z^m \\ &= \sum_{m=0}^{\infty} m p_{n,m} z^m \end{aligned} \tag{32d}$$

Further, the Binomial Moment Generating Function (BMGF) of the joint probability distribution is defined as

$$F_n(x) = G_n(x+1) = \sum_{m=0}^{\infty} \beta_{n,m} x^m \tag{33a}$$

$$\text{where} \quad \beta_{n,m} = \sum_{q=m}^{n} \binom{q}{m} p_q \tag{33b}$$

$$\text{where} \quad p_q = \sum_{k=0}^{\infty} (-1)^{k-q} \binom{k}{q} \tag{33c}$$

From the above Markov state diagram, the B-D equations in terms of joint probability distribution are written as

$$\begin{aligned} \left(N\lambda_f' + m\mu_{d2}\right) p_{0,m} =& \mu_{d1} p_{1,m-1} \\ &+ (m+1)\mu_{d2} p_{0,m+1} && \text{for} \quad n=0 && \text{(34a)} \\ \left[(N-n)\lambda_f' + n\mu_{d1} + m\mu_{d2}\right] p_{n,m} =& (m+1)\mu_{d2} p_{n,m+1} \\ &+ (N-n+1)\lambda_f' p_{n-1,m} \\ &+ (n+1)\mu_{d1} p_{n+1,m-1} && \text{for} \quad 0 < n < C && \text{(34b)} \\ (m\mu_{d2} + C\mu_{d1}) p_{C,m} =& (m+1)\mu_{d2} p_{C,m+1} \\ &+ (N-C+1)\lambda_f' p_{C-1,m} && \text{for} \quad n=C && \text{(34c)} \end{aligned}$$

For $n = 0$, we transform Eqn. (34a) in terms of PGF, using Eqn. (32), as

$$\begin{aligned}
&N\lambda'_f G_0(z) + \mu_{d2} z G'_0(z) &&= \mu_{d1} z G_1(z) + \mu_{d2} G'_0(z) \\
\Rightarrow &N\lambda'_f G_0(z) + \mu_{d2}(z-1) G'_0(z) &&= \mu_{d1} z G_1(z) \\
\Rightarrow &N\lambda'_f G_0(x+1) + \mu_{d2}(x) G'_0(x+1) &&= \mu_{d1}(x+1) G_1(x+1)
\end{aligned} \tag{35a}$$

Similarly, for $0 < n < C$, Eqn. (34b) leads to

$$\begin{aligned}
&(N-n)\lambda'_f G_n(z) + \mu_{d1} n G_n(z) \\
&\quad + \mu_{d2} z G'_n(z) &&= \mu_{d2} G'_n(z) + (N-n+1)\lambda'_f G_{n-1}(z) \\
& &&\quad + (n+1)\mu_{d1} z G_{n+1}(z) \\
\Rightarrow &\mu_{d2}(z-1) G'_n(z) \\
&\quad + \left[(N-n)\lambda'_f + n\mu_{d1}\right] G_n(z) &&= (n+1)\mu_{d1} z G_{n+1}(z) \\
& &&\quad + (N-n+1)\lambda'_f G_{n-1}(z) \\
\Rightarrow &\mu_{d2} x G'_n(x+1) \\
&\quad + \left[(N-n)\lambda'_f + n\mu_{d1}\right] G_n(x+1) &&= (n+1)\mu_{d1}(x+1) G_{n+1}(x+1) \\
& &&\quad + (N-n+1)\lambda'_f G_{n-1}(x+1)
\end{aligned} \tag{35b}$$

Similarly, for $n = C$, Eqn. (34c) leads to

$$\begin{aligned}
&\mu_{d2} z G'_C(z) + C\mu_{d1} G_C(z) &&= \mu_{d2} G'_C(z) + (N-C+1)\lambda'_f G_{C-1}(z) \\
\Rightarrow &\mu_{d2}(z-1) G'_C(z) + C\mu_{d1} G_C(z) &&= (N-C+1)\lambda'_f G_{C-1}(z) \\
\Rightarrow &\mu_{d2}(x) G'_C(x+1) + C\mu_{d1} G_C(x+1) &&= (N-C+1)\lambda'_f G_{C-1}(x+1)
\end{aligned} \tag{35c}$$

Now, transforming the set of Eqn. (35) in terms of BMGF using Eqn. (33), we obtain

$$N\lambda'_f F_0(x) + \mu_{d2} x F'_0(x) = \mu_{d1}(x+1) F_1(x) \qquad \text{for} \quad n = 0 \tag{36a}$$

$$\begin{aligned}
&\mu_{d2} x F'_n(x) \\
&+ \left[(N-n)\lambda'_f + n\mu_{d1}\right] F_n(x) = (n+1)\mu_{d1}(x+1) F_{n+1}(x) \\
&\qquad + (N-n+1)\lambda'_f F_{n-1}(x) \qquad \text{for} \quad 0 < n < C
\end{aligned} \tag{36b}$$

$$\mu_{d2} x F'_C(x) + C\mu_{d1} F_C(x) = (N-C+1)\lambda'_f F_{C-1}(x) \qquad \text{for} \quad n = C \tag{36c}$$

Expressing the set of Eqn. (36) in terms of $\beta_{n,m}$, we obtain

$$\begin{aligned}
&\mu_{d2} x \sum_{m=0}^{\infty} \beta_{0,m} m x^{m-1} + N\lambda'_f \sum_{m=0}^{\infty} \beta_{0,m} x^m \\
&= \mu_{d1}(x+1) \sum_{m=0}^{\infty} \beta_{1,m} x^m \qquad \text{for} \quad n = 0
\end{aligned} \tag{37a}$$

$$\begin{aligned}
&\mu_{d2} x \sum_{m=0}^{\infty} \beta_{n,m} m x^{m-1} + \left[(N-n)\lambda'_f + n\mu_{d1}\right] \sum_{m=0}^{\infty} \beta_{n,m} x^m \\
&= (n+1)\mu_{d1}(x+1) \sum_{m=0}^{\infty} \beta_{n+1,m} x^m \\
&+ (N-n+1)\lambda'_f \sum_{m=0}^{\infty} \beta_{n-1,m} x^m \qquad \text{for} \quad 0 < n < C
\end{aligned} \tag{37b}$$

$$\begin{aligned}
&\mu_{d2} x \sum_{m=0}^{\infty} \beta_{C,m} m x^{m-1} + C\mu_{d1} \sum_{m=0}^{C} \beta_{C,m} x^m \\
&= (N-C+1)\lambda'_f \sum_{m=0}^{\infty} \beta_{C-1,m} x^m \qquad \text{for} \quad n = C
\end{aligned} \tag{37c}$$

For $n = 0$, equating coefficients of x from Eqn. (37a), we get

$$\begin{aligned}
&\mu_{d2}\beta_{0,1} + N\lambda'_f\beta_{0,1} &&= \mu_{d1}\beta_{1,0} + \mu_{d1}\beta_{1,1} \\
&\Rightarrow (N\lambda'_f + \mu_{d2})\beta_{0,1} &&= \mu_{d1}\beta_{1,1} + \mu_{d1}p'_1 \qquad \text{(38a)} \\
\text{where} \quad &\beta_{n,0} &&= p'_n = \underbrace{\frac{\binom{N}{n}\left(\frac{\lambda'_f}{\mu_{d1}}\right)^n}{\sum_{k=0}^{C}\binom{N}{k}\left(\frac{\lambda'_f}{\mu_{d1}}\right)^k}}_{\substack{\text{State probability of the}\\ \text{Engset distribution [10]}}} \qquad \text{(38b)}
\end{aligned}$$

For $0 < n < C$, equating coefficients of x from Eqn. (37b), we get

$$\begin{aligned}
&\mu_{d2}\beta_{n,1} + \left[(N-n)\lambda'_f + n\mu_{d1}\right]\beta_{n,1} &&= (n+1)\mu_{d1}\left(\beta_{n+1,0} + \beta_{n+1,1}\right) \\
& &&\quad + (N-n+1)\lambda'_f\beta_{n-1,1} \\
&\Rightarrow \left[(N-n)\lambda'_f + \mu_{d2} + n\mu_{d1}\right]\beta_{n,1} &&= (n+1)\mu_{d1}(\beta_{n+1,1} + p'_{n+1}) \\
& &&\quad + (N-n+1)\lambda'_f\beta_{n-1,1} \qquad \text{(38c)}
\end{aligned}$$

For $n = C$, equating coefficients of x from Eqn. (37c), we get

$$\begin{aligned}
&\mu_{d2}\beta_{C,1} + C\mu_{d1}\beta_{C,1} &&= (N-C+1)\lambda'_f\beta_{C-1,1} \\
&\Rightarrow (\mu_{d2} + C\mu_{d1})\beta_{C,1} &&= (N-C+1)\lambda'_f\beta_{C-1,1} \qquad \text{(38d)}
\end{aligned}$$

For $n = 0$, equating coefficients of x^2 from Eqn. (37a), we obtain

$$\mu_{d2}2\beta_{0,2} + N\lambda'_f\beta_{0,2} = \mu_{d1}\beta_{1,1} + \mu_{d1}\beta_{1,2} \qquad \text{(39a)}$$

For $0 < n < C$, equating coefficients of x^2 from Eqn. (37b), we obtain

$$\begin{aligned}
\mu_{d2}2\beta_{n,2} + \left[(N-n)\lambda'_f + n\mu_{d1}\right]\beta_{n,2} &= (n+1)\mu_{d1}(\beta_{n+1,1} + \beta_{n+1,2}) \qquad \text{(39b)} \\
&\quad + (N-n+1)\lambda'_f\beta_{n-1,2} \qquad \text{(39c)}
\end{aligned}$$

For $n = C$, equating coefficients of x^2 from Eqn. (37c), we obtain

$$\mu_{d2}2\beta_{C,2} + C\mu_{d1}\beta_{C,2} = (N-C+1)\lambda'_f\beta_{C-1,2} \qquad \text{(39d)}$$

Adding Eqn. (39b), (39c), and (39d), we obtain

$$\begin{aligned}
&\mu_{d2}(2\beta_{0,2} + 2\beta_{1,2} + \ldots + 2\beta_{C,2}) + \mu_{d1}(\beta_{1,2} + 2\beta_{2,2} + \ldots + C\beta_{C,2}) \\
&+ \lambda'_f\left[N\beta_{0,2} + (N-1)\beta_{1,2} + \ldots + (N-C+1)\beta_{C-1,2}\right] \\
&= \mu_{d1}(\beta_{1,1} + 2\beta_{2,1} + \ldots + C\beta_{C,1}) + \mu_{d1}(\beta_{1,2} + 2\beta_{2,2} + \ldots + C\beta_{C,2}) \\
&+ \lambda'_f\left[N\beta_{0,2} + (N-1)\beta_{1,2} + \ldots + (N-C+1)\beta_{C-1,2}\right] \\
\Rightarrow\, &\mu_{d2}2\sum_{n=0}^{C}\beta_{n,2} = \mu_{d1}\sum_{n=0}^{C}n\beta_{n,1} \\
\Rightarrow\, &2\sum_{n=0}^{C}\beta_{n,2} = \frac{\mu_{d1}}{\mu_{d2}}\sum_{n=0}^{C}n\beta_{n,1} \qquad \text{(40)}
\end{aligned}$$

where $\sum_{n=0}^{C}\beta_{n,2}$ is the complete 2[nd] Binomial moments of the offered traffic to L_V. The terms $\beta_{n,1}$ are the partial Binomial moments of the offered traffic to L_V.

From [13], using Eqn. (32), we express the variance of the offered traffic to L_V as

$$\begin{aligned}
V_V &= G''_n(1) + G'_n(1) - \left[G'_n(1)\right]^2 \\
&= G''_n(1) - M_V^2 + M_V \qquad \text{(41)}
\end{aligned}$$

Using Eqn. (32) and (33), we obtain

$$
\begin{aligned}
G_n(z) &= \sum_{n=0}^{C}\sum_{m=0}^{\infty}\beta_{n,m}(z-1)^m \\
&= \sum_{n=0}^{C}\sum_{m=0}^{\infty}\sum_{k=0}^{m}\binom{m}{k}z^k(-1)^{m-k} \\
&= \left[(-1)^m + mz(-1)^{m-1} + \binom{m}{2}z^2(-1)^{m-2} + \dots\right] \\
\therefore G_n'(z) &= \sum_{n=0}^{C}\sum_{m=0}^{\infty}\beta_{n,m}\left[0 + m(-1)^{m-1} + m(m-1)z(-1)^{m-2} + \dots\right] \\
\therefore G_n''(z) &= \sum_{n=0}^{C}\sum_{m=0}^{\infty}\beta_{n,m}\Big[0 + 0 + m(m-1)(-1)^{m-2} + m(m-1)(m-2) \\
&\quad + m(m-1)(m-2)(m-3) + \dots\Big]
\end{aligned} \tag{42}
$$

Now

$$
\begin{aligned}
&\text{For} \quad m = 0\dots 2 \\
&G_n''(1) = \sum_{n=0}^{C} 2\beta_{n,2} \\
&\text{For} \quad m = 0\dots 3 \\
&G_n''(1) = \sum_{n=0}^{C}\left[2\beta_{n,2} + [(-3\cdot 2) + (3\cdot 2)]\beta_{n,3}\right] \\
&\text{For} \quad m = 0\dots 4 \\
&G_n''(1) = \sum_{n=0}^{C}\left[2\beta_{n,2} + (-3\cdot 2 + 3\cdot 2)\beta_{n,2} + (4\cdot 3 - 4\cdot 3\cdot 2 + 4\cdot 3\cdot 2)\beta_{n,4}\right] \\
&\quad\vdots \qquad \vdots \\
&G_n''(1) = \sum_{n=0}^{C} 2\beta_{n,2}
\end{aligned} \tag{43}
$$

Finally, from Eqn. (41), we obtain

$$
\begin{aligned}
V_V &= 2\sum_{n=0}^{C}\beta_{n,2} - M_V^2 + M_V \\
&= \frac{\mu_{d1}}{\mu_{d2}}\sum_{n=0}^{C} n\beta_{n,1} - M_V^2 + M_V \qquad \text{[From Eqn. (40)]}
\end{aligned} \tag{44}
$$

We solve the equation set (38) for $\beta_{n,1}$ as follows. Consider a $C \times 1$ matrix X that contains the $C+1$ solutions in terms of $\beta_{n,1}$. Further, consider a $C \times C$ matrix A and a $C \times 1$ matrix B such that AX=B. We obtain the elements of A and B from Eqn (38) as

$$
A_{n,m} = \left|\begin{array}{ll}
-(N-m)\lambda_f' & \text{for} \quad 0 \le n \le C, m = n-1 \\
(N-n)\lambda_f' + \mu_{d1} + n\mu_{d2} & \text{for} \quad 0 \le n \le C, m = n \\
-(n+1)\mu_{d1} & \text{for} \quad 0 \le n \le C, m = n+1 \\
\mu_{d1} + C\mu_{d2} & \text{for} \quad n = C, m = n
\end{array}\right. \tag{45a}
$$

$$
B_{n,m} = \left|\begin{array}{ll}
(n+1)\mu_{d1}p_{n+1} & \text{for} \quad 0 \le n \le C \\
0 & \text{otherwise}
\end{array}\right. \tag{45b}
$$

Once we compute X, the variance V_V can be easily computed from Eqn. (44). Finally. following the steps in Section (II-E), mean and variance of the hand-off offered traffic to L_2 can easily be computed. Since, various types of congestions are important QoS parameters in traffic dimensioning, we compare various types of congestions [12] estimated using our analysis for small cell model with that of [7] in the next section.

IV. Small Cell Model Validation and Discussion

We validate the proposed model by considering a special case where it reduces to some already existing model. As mentioned in the literature, when the population size increases and theoretically tends to infinity, the Engset distribution behaves as Poisson distribution. In our small cell case, we observed that around $N \geq 150$, the corresponding fresh traffic behaves as Poisson distribution. Thus, we choose N, λ'_f such that some equivalent fresh offered traffic (Poisson distributed) is injected into the proposed model. In such a condition, the congestions estimated using the proposed small cell model are compared with (a) the model proposed by Rajaratnam and Takawira (R-T Model) [7], [8], [9] and (b) the model proposed by Foschini *et al.* (Foschini's Model) [1]. Table I presents the estimated congestion (fresh and hand-off) for the above models. In the first case

Table I: Small Cell Model Validation

(a) Effect of varying number of users on congestion in a small cell.

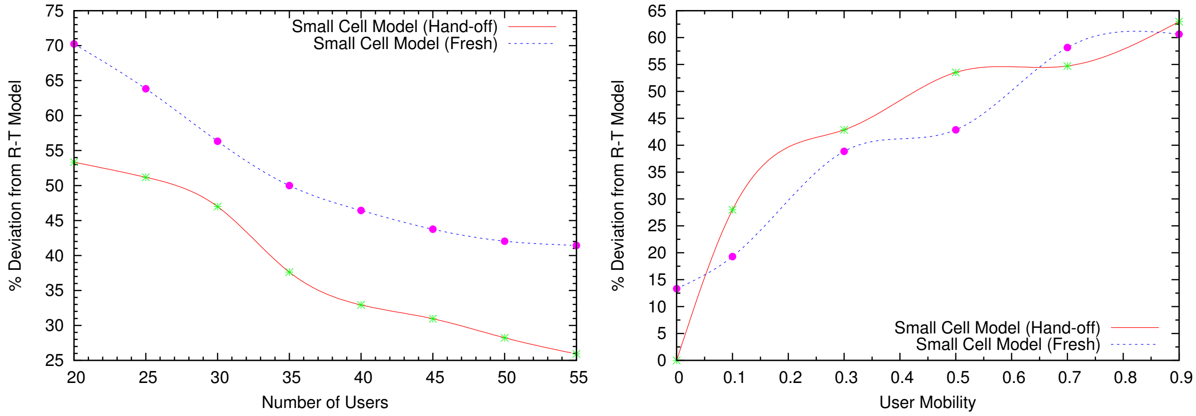
No. Of Users	**Small Cell Model (Hand-off Congestion)**	**Small Cell Model (Fresh Congestion)**	**R-T Model (Hand-off Congestion)**	**R-T Model (Fresh Congestion)**	**Foschini's Model (Fresh / Hand-off Congestion)**
150	0.0001534	0.0001593	0.0001592	0.0001594	0.0001584
190	0.001766	0.001795	0.001808	0.001812	0.001803
230	0.008448	0.008477	0.008404	0.008456	0.008526
270	0.022	0.022	0.022	0.022	0.022
310	0.041	0.041	0.041	0.042	0.042
350	0.062	0.064	0.063	0.064	0.064
390	0.085	0.087	0.085	0.087	0.087
430	0.108	0.11	0.108	0.11	0.11

(b) Effect of varying mobility of users on congestion in a small cell.

User Mobility	**Small Cell Model (Hand-off Congestion)**	**Small Cell Model (Fresh Congestion)**	**R-T Model (Hand-off Congestion)**	**R-T Model (Fresh Congestion)**	**Foschini's Model (Fresh / Hand-off Congestion)**
0	0	0.046	0	0.046	0.046
0.1	0.044	0.044	0.044	0.044	0.044
0.3	0.041	0.041	0.041	0.041	0.041
0.5	0.037	0.037	0.036	0.037	0.037
0.7	0.03	0.03	0.03	0.03	0.03
0.9	0.018	0.018	0.017	0.018	0.018

(Table Ia), the number of users in a small cell is varied from 150 to 430, while other parameters such as user mobility, session duration, session request rate, cell capacity (channels) are kept constant. In the second case (Table Ib), the user mobility in a small cell is varied from 0 (static) to 0.9 (highly mobile), while other parameters such as number of users, session duration, session request rate, cell capacity (channels) are kept constant. Table I shows that all models are in good agreement. This validates the correctness of the small cell model. We further observe in Table Ia that both types of congestions increase with the number of users, which is expected. In Table Ib, we notice that the fresh congestion decreases with mobility. This is expected because the dropped hand-off requests allow greater chances of acceptance to the fresh requests. However, hand-off congestion decreases with user mobility as well, which is unusual. This happens because of the insensitivity of the existing models towards population size. The existing models treat hand-off traffic as fresh traffic from the same cell of large population size. Hence, the phenomena for fresh traffic repeats for hand-off traffic. This reveals the key limitation of the existing models making them inapplicable for small cells.

Fig. 4 shows the percentage of deviation in congestion estimates using the Small Cell Model with that of the R-T Model. Since the variance of the fresh requests, initiated from a finite population size, is smaller compared to that initiated from a large population size, we observe that the R-T Model largely over estimates fresh congestion. The hand-off congestion is also over estimated because a part of the fresh session requests hand-offs to the adjacent cell. We observe in Fig. 4a that the deviation decreases with the increase in number of users. This can be attributed to the fact that with increasing number of population the fresh request distribution (Engset) tends more towards Poisson distribution. Thus, the variance of the fresh offered traffic increases and the difference with the case of large population size decreases. Since, the fresh request is directly affected by the population size, the deviation is more in this case. However, with the increase in user mobility in Fig. 4b, the deviation for both fresh requests and hand-off request increase drastically. This can be explained as follows. In the Small Cell Model, hand-off congestion increases with the mobility, which is not the case for the R-T Model. Thus, the estimates largely differs



(a) Varying number of users in a small cell.

(b) Varying user mobility in a small cell.

Figure 4: Percentage deviation in congestion (Small Cell Model versus R-T Model).

for the hand-off congestion. In case of fresh congestion, deviation occurs from two phenomena. Firstly, the small population size in the Small Cell Model limits the variance of the fresh session requests which, in turn, makes the fresh traffic more predictable. This reduces the fresh congestion. Secondly, the increasing hand-off congestion favours the fresh session requests and reduces the fresh congestion. R-T Model is insensitive to these phenomena.

V. Conclusion

We developed some traffic estimation techniques using various moment manipulations for Semi-Markov model applicable in a small cell scenario. The analysis accounts for the limited population size of the users in modern small cells. It is observed that the distributions of the traffic streams largely vary from the large population scenario in traditional cells. These affect the QoS parameters such as congestions both for fresh session requests and hand-off requests. The deviation between the small cell model and large population model increases with the user mobility while the same decreases with the increase in population size. Thus, it can be concluded that as the cell size diminishes, small cell model becomes more appropriate for dimensioning. Further, if the cell size increases and population size becomes comparable with the large cell case, both small cell models and R-T models yield same results because population size shapes the fresh traffic to be Poisson nature. Thus, for accurate traffic dimensioning in small cells, it is essential to adopt proper modelling and estimation technique.

Acknowlegement

The authors acknowledge Prof. H. M. Gupta and Prof. S. Kar, Department of Electrical Engineering, Indian Institute of Technology Delhi, and Vivek Pal, Untamed Spectrum for their valuable comments and suggestions.

References

[1] G. J. Foschini, B. Gopinath, and Z. Miljanic, "Channel cost of mobility," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 414–424, November 1993.

[2] D. Hong and S. S. Rappaport, "Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures," *IEEE Trans. Veh. Technol.*, vol. VT-35, no. 3, pp. 77–92, August 1986.

[3] D. Macmillan, "Traffic modeling and analysis for cellular mobile networks," in *Proc. 13th ITC*, 1991, pp. 627–632.

[4] E. Chlebus and W. Ludwin, "Is handoff traffic really poissonian?" in *Proc. IEEE ICUPC '95 Conf. Record*, November 1995, pp. 348–353.

[5] H. Zeng and I. Chlamtac, "Handoff traffic distribution in cellular networks," in *Proc. WCNC'99, New Orleans*, September 1999, pp. 413–417.

[6] D. R. Manfield and T. Downs, "Decomposition of traffic in loss systems with renewal inputs," *IEEE Trans. Commun.*, vol. COM-27, pp. 44–58, January 1979.

[7] M. Rajaratnam and F. Takawira, "Hand-off traffic modelling in cellular networks," in *Proc. IEEE Conf. Global Telecommun.* Phoenix, AZ: GLOBECOM'97, 3-8 November 1997, pp. 131–137.

[8] ——, "The two moment performance analysis of cellular mobile networks with and without channel reservation," in *Proc. IEEE ICUPC'98*, Florence, Italy, October 1998, pp. 1157–1161.

[9] ——, "Nonclassical traffic modeling and performance analysis of cellular mobile networks with and without channel reservation," *IEEE Trans. Veh. Technol.*, vol. 49, no. 3, pp. 817–834, May 2000.

[10] V. B. Iversen, *Teletraffic Engineering Handbook*, revised ed. Building 345v, DK-2800 Kgs. Lyngby: COM Center, Technical University of Denmark, May 2006, vol. ITU-D SG 2/16 & ITC Draft 2001-06-20.

[11] M. Rajaratnam and F. Takawira, "Hand-off traffic characterisation in cellular networks under engset new call arrivals and generalised service time," in *Proc. IEEE Conf. Wireless Commun. and Netw.* WCNC'99, 21-24 September 1999, pp. 423–427.

[12] ——, "Two moment analysis of channel reservation in cellular networks," in *Proc. AFRICON'99*. Capetown, South Africa: IEEE, 28 Sept.-1 Oct. 1999, pp. 257–262.

[13] A. Girard, *Routing and Dimensioning in Circuit-Switched Networks*. Addison-Wesley Publication, 1990.

[14] A. Brandt and M. Brandt, “On the moments of overflow and freed carried traffic for the GI/M/C/0 system,” *Methodol. Comput. Appl. Probab.*, vol. 4, pp. 69–82, 2002.
[15] Y. Fang and Y. Zhang, “Call admission control schemes and performance analysis in wireless mobile networks,” *IEEE Trans. Veh. Technol.*, vol. 51, no. 2, pp. 371–381, March 2002.
[16] Y. Fang and I. Chlamtac, “Teletraffic analysis and mobility modeling of pcs networks,” *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1062–1071, July 1999.
[17] A. E. Xhafa and O. K. Tonguz, “Does mixed lognormal channel holding time affect the handover performance of guard channel scheme?” in *Proc. IEEE Conf. Global Telecommun.* San Francisco, CA: GLOBECOM’03, 1-5 December 2003, pp. 3452–3456.
[18] R. Guerin, “Channel occupancy time distribution in a cellular radio system,” *IEEE Trans. Veh. Technol.*, vol. VT-35, no. 3, pp. 89–99, 1987.