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# Rotor Eddy-Current Loss in Permanent Magnet Brushless Machines

Hiroaki Toda, Zhenping Xia, Jiabin Wang, *Senior Member, IEEE*, Kais Atallah, and David Howe

**Abstract**—This paper presents an analysis of the rotor eddy-current loss in modular and conventional topologies of permanent magnet brushless machine. The loss is evaluated both analytically and by time-stepped finite-element analysis, and it is shown that it can be significant in both machine topologies. It is also shown that the loss can be reduced significantly by segmenting the magnets.

**Index Terms**—Eddy-current, modular machines, permanent magnet machines, rotor loss.

## I. INTRODUCTION

**D**UE to their high efficiency and high power density and their excellent dynamic performance, permanent magnet (PM) brushless machines are being used in an ever-increasing range of applications. Recently, a relatively new topology of PM brushless ac machine, often referred to as “modular” [1], [2], has emerged which offers a number of significant advantages over conventional PM brushless machines, particularly for automotive and aerospace applications.

The stator winding of a modular PM machine differs from that of conventional brushless ac machines in that the coils which belong to one phase are concentrated and wound on alternative or consecutive teeth so that there is no overlapping of phase windings. This is not only a distinct manufacturing advantage [3], [4], but is also conducive to a high packing factor, and, hence, a high efficiency, and to reducing the likelihood of an inter-phase fault. It also results in a small number of slots for a given number of poles, e.g., 24 slots for a 22-pole machine as compared to 36 slots for a 24-pole conventional brushless machine, as illustrated in Fig. 1. Modular machines also yield a fractional number of slots per pole, with the smallest common multiple between the slot number and the pole number being very large. Consequently, the cogging torque can be extremely small without the use of skew [2].

The rotor eddy-current loss in conventional PM brushless ac machines is usually considered to be negligible, since high-order time harmonics in the stator current and space harmonics in the winding distribution are generally small. However, in modular PM machines, the stator magneto-motive force (mmf) distribution contains a richer set of space harmonics, and the fundamental stator mmf has fewer poles than the permanent magnet rotor, the torque being produced by the interaction of

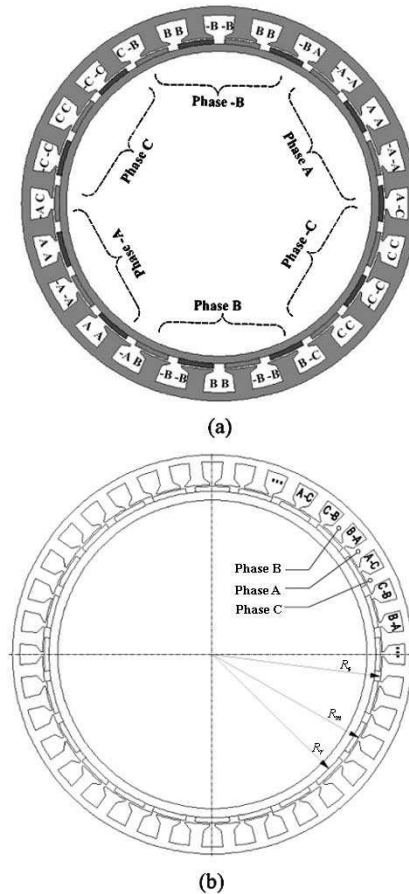


Fig. 1. Schematic of brushless PM machines. (a) Modular 24 slot/22-pole, (b) Conventional 36 slot/24-pole.

a high-order stator mmf harmonic with the permanent magnets. Consequently, both lower and higher order space harmonic mmfs, the rotational speeds of which differ from that of the rotor magnets, may induce a significant eddy-current loss in the magnets, and result in excessive heating. Furthermore, the presence of stator slot openings causes a variation of the magnetic field in the magnets, this component of rotor eddy-current loss being dependent on the width of the slot openings and the pole/slot number combination.

This paper presents an analysis of the rotor eddy-current loss of both modular and conventional topologies of PM brushless machine. The analytical predictions are validated by results derived from time-stepped finite-element (FE) analysis.

## II. ANALYTICAL MODEL

An analytical technique for predicting the eddy-current loss in fault-tolerant permanent magnet machines, in which only al-

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TABLE I  
CHARACTERISTIC PARAMETERS OF STATOR HARMONICS IN  
MODULAR MACHINE

Order	Magnitude (pu)	$\left(\frac{R_r}{R_s}\right)^{np_s}$	Speed (pu)	Frequency (Hz)	Skin depth (mm)
1	0.02	0.96	-11	340.8	22.3
5	0.1	0.82	11/5	170.4	31.5
7	0.17	0.758	-11/7	511.2	18.2
11	1	0.647	1	0	-
13	1	0.598	-11/13	681.6	15.7
17	0.17	0.51	11/17	170.4	31.5
19	0.1	0.47	-11/19	852	14.0

TABLE II  
CHARACTERISTIC PARAMETERS OF STATOR HARMONICS IN  
CONVENTIONAL MACHINE

Order	Magnitude (pu)	$\left(\frac{R_r}{R_s}\right)^{np_s}$	Speed (pu)	Frequency (Hz)	Skin depth (mm)
1	1	0.622	1	0	-
5	0.64	0.093	-1/5	2044.8	9.09
7	0.458	0.036	1/7	2044.8	9.09
11	0.08	0.0054	-1/11	4089.9	6.42
13	0.064	0.002	1/13	4089.6	6.42
17	0.188	0.0003	-1/17	6134.4	5.24
19	0.17	0.0001	1/19	6134.4	5.24

ternate teeth carry a coil, has been reported in [5]. This technique is applied to analyze the eddy-current loss in three-phase modular and conventional topologies of PM brushless machine. The stator and rotor cores are assumed to be infinitely permeable, and the eddy-currents are assumed to be resistance-limited. Hence, the skin depth at the inducing frequencies is assumed to be greater than the pole-arc and the radial thickness of the magnets. The stator winding is represented by an equivalent current sheet, which for a 3-phase machine is given by

$$J(\theta, t) = \begin{cases} \sum_{n=-\infty}^{\infty} \frac{3}{2} J_n \cos(np_s \theta + (np_s - p_r)\omega t), & n = (3k + m) \\ \sum_{n=-\infty}^{\infty} -\frac{3}{2} J_n \cos(np_s \theta + (np_s + p_r)\omega t), & n = (3k - m) \\ 0 & n \neq 3k \pm m \end{cases}$$

where  $k = 1, 2, \dots, n$  is the space harmonic order,  $p_s$  is the fundamental number of pole pairs associated with stator winding,  $p_r$  is the number of rotor magnet pole-pairs,  $\omega$  is the rotor angular velocity, and  $J_n = (2N_s I_m / \pi R_s) K_{wn}$ , where  $N_s, I_m$ , and  $R_s$  are the number of series turns per phase, the peak phase current and the stator bore inner radius, respectively, and  $K_{wn}$  is the winding factor. The value ( $\pm 1$ ) for  $m$  is dependent on the winding configuration. For the 24-slot/22-pole modular machine  $m = -1$ , while for the 36-slot/24-pole conventional machine  $m = 1$ . Tables I and II show the magnitude, the ratio  $(R_r/R_s)^{np_s}$ , the speed and frequency of the low order space harmonics as seen by the rotor, and their associated skin depths in the permanent magnets at the rated speed of 1700 rpm for the modular and conventional machines, respectively. It can be seen that for the modular machine the values of  $(R_r/R_s)^{np_s}$ , which is a good measure of the level of penetration of the space harmonic field into the permanent magnets, viz. a high value indicates a high penetration and vice-versa, is relatively high.

It can also be seen that the skin depth of the permanent magnet material associated with the different space harmonics

is comparable to or greater than the radial and circumferential dimensions of the permanent magnet segments, viz. 5 mm and 16.9 mm respectively, for the machines under consideration. Therefore, the induced eddy-currents are effectively resistance-limited. For the conventional brushless machine, however, the values of  $(R_r/R_s)^{np_s}$ , are significantly lower, while the frequencies associated with the space harmonics are significantly higher. It should also be noted that for the modular machine only the 11th harmonic interacts with the permanent magnets to produce useful torque.

The eddy-current loss in one permanent magnet segment can be analytically derived [2], and is given by:  $P = \sum_{n=1}^{\infty} (P_{cn} + P_{an})$  (W/m), where  $P_{cn}$  and  $P_{an}$  are the loss components associated with  $(J_n/np_s)^2$  and  $(J_n/np_s)^4$  respectively, and they are given by

$$P_{cn} = \frac{9\mu_o^2 \alpha}{8\rho} \frac{J_n^2}{n^2 p_s^2} (np_s \mp p_r)^2 \omega^2 \times \left[ \left(\frac{R_m}{R_s}\right)^{2np_s} \frac{R_s^2 R_m^2}{(2np_s + 2)} \left(1 - \left(\frac{R_r}{R_m}\right)^{2np_s + 2}\right) + \left(\frac{R_r}{R_s}\right)^{2np_s} R_s^2 R_r^2 F_n + \left(\frac{R_r}{R_s}\right)^{2np_s} \times R_s^2 (R_m^2 - R_r^2) \right] / \left[ 1 - \left(\frac{R_r}{R_m}\right)^{2np_s} \right]^2$$

in which

$$F_n = \begin{cases} \frac{[(R_m/R_r)^{-2np_s + 2} - 1]}{-2np_s + 2}, & \text{for } np_s \neq 1 \\ \ln(R_m/R_r), & \text{for } np_s = 1 \end{cases}$$

and

$$P_{an} = -\frac{9\mu_o^2}{\alpha \rho} \frac{J_n^4}{n^4 p_s^4} (np_s \mp p_r)^2 \omega^2 \times \left[ \left(\frac{R_m}{R_s}\right)^{np_s} \frac{R_s R_m^2}{(np_s + 2)} \left(1 - \left(\frac{R_r}{R_m}\right)^{np_s + 2}\right) + \left(\frac{R_r}{R_s}\right)^{np_s} R_s R_r^2 G_n \right]^2 \times \frac{\sin^2(np_s \frac{\alpha}{2})}{(R_m^2 - R_r^2) [1 - (R_r/R_s)^{2np_s}]^2}$$

in which

$$G_n = \begin{cases} \frac{[(R_m/R_r)^{-np_s + 2} - 1]}{-np_s + 2}, & \text{for } np_s \neq 2 \\ \ln(R_m/R_r), & \text{for } np_s = 2 \end{cases}$$

where “-” applies to  $n = 3k + m$ , and “+” to  $n = 3k - m$ .  $R_r$  and  $R_m$  are the inner and outer radii of the rotor magnets, respectively,  $\alpha$  is the mechanical pole-arc of a magnet segment, and  $\rho$  is the electrical resistivity of the magnet material. It will be noted that for a rotor comprising a multipole magnetized cylindrical magnet, i.e.,  $\alpha = 2\pi$ , the term  $P_{an}$  is zero.

### III. RESULTS AND DISCUSSION

Table III lists the leading design parameters of the modular and conventional machines under consideration. They have the same torque density and current rating. The eddy-current loss in the magnets was calculated under various conditions when both machines were running at 1700 rpm, by both time-stepped FE analysis and the foregoing analytical model.

TABLE III  
MACHINE SPECIFICATIONS AND PARAMETERS

Topology	Modular	Conventional
Pole number	22	24
Slot number	24	36
Magnet resistivity ( $\mu\Omega\text{-cm}$ )	70	70
Peak current to produce 105Nm torque (A)	500	500
Air-gap (mm)	1.6	1.6

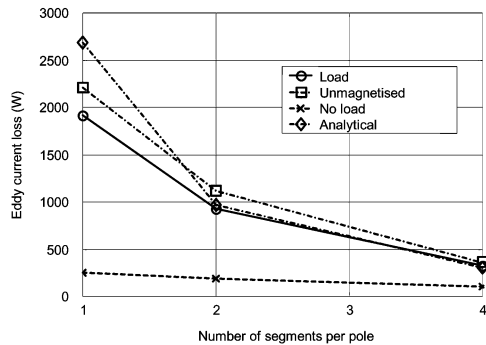


Fig. 2. Influence of number of magnet segments per pole on eddy-current loss for modular machine.

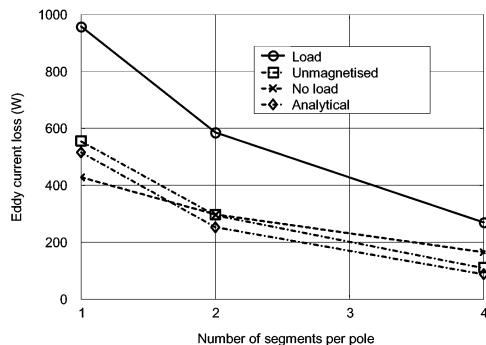


Fig. 3. Influence of number of magnet segments per pole on eddy-current loss for conventional machine.

Figs. 2 and 3 show the influence of the number of magnet segments per pole on the eddy-current loss of the modular and conventional machines, respectively, when both are supplied with sinusoidal phase current waveforms. Since the analytical prediction does not account for the effect of the stator slot openings, its results can only be compared directly with those from FE calculations with the magnets unmagnetized. Conversely, the FE results under a no-load (or open-circuit) condition represent the eddy-current loss due only to the slot openings. As will be seen for both machines, circumferential segmentation of the magnets is effective in reducing the eddy-current loss. It can also be seen that there is good agreement between results from the analytical model and FE predictions of the eddy-current loss associated with the stator mmf space harmonics. However, it will be noted that while for the modular machine the component of eddy-current loss which is associated with the variation of the magnet working point due to stator slotting is relatively small, it is significant for the conventional machine. It should also be noted that due to the influence of skin effect and saturation, which are neglected in the analytical model, the total eddy-current loss at

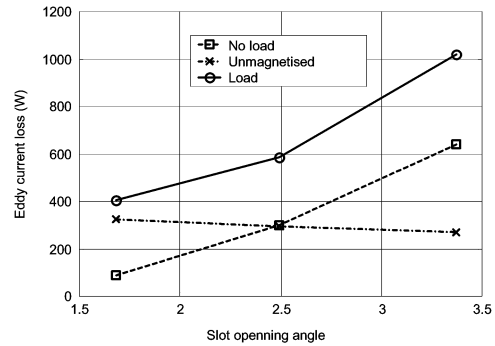


Fig. 4. Influence of slot opening angle on eddy-current losses for the 36-slot/24-pole machine. (Number of magnet segments per pole: 2).

rated load does not equal the sum of the losses calculated separately on no-load and with the magnets unmagnetized.

Fig. 4 shows the variation of the eddy-current loss in the permanent magnets of the conventional machine with the slot opening angle. It can be seen that the slot opening has a significant effect on the loss component associated with the variation of the permanent magnet working point due to slotting. Since the frequency of the variation is proportional to the number of slots, the eddy-current loss due to slotting in the conventional machine is more significant than that in the modular machine. Therefore, in addition to the influence of the width of the slot openings on the cogging torque and synchronous inductance, its effects on the eddy-current loss in the permanent magnets may have to be considered during the design stage.

#### IV. CONCLUSION

The rotor eddy-current loss in modular and conventional topologies of PM machines has been analyzed both analytically and by FE calculations. It has been shown that a significant eddy-current loss can be generated in both machines, and that segmenting the magnets is effective in reducing this component of loss. Analytical predictions have been shown to be in good agreement with FE results for the modular machine. However, for the conventional machine, the difference between the analytical predictions and FE results is greater, which is attributed to the fact that the eddy-current loss due to the stator slot openings is more significant, and is not considered in the analytical model.

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