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A 2D Process-based Morphodynamic Model for Flooding by Non-cohesive Dyke Breach

Mingfu Guan¹; Nigel G. Wright²; P. Andrew Sleigh³

Abstract: Inundation models based on the Shallow Water Equations (SWE) have been shown to perform well for a wide variety of situations even at the limit of their theoretical applicability and, arguably, somewhat beyond. One of these situations is the catastrophic event of floods induced by dyke breach and consequent dyke erosion. The dyke collapse is often not sudden - as assumed by many flood simulations in which the dyke boundary is treated as a "dam-break". The dyke erosion is a gradual and complex process that delays the onset of the flood, affecting the hydrograph of the flow. To simulate correct temporal passage of a flood, it is important to understand the rate at which these dykes collapse. In this paper an overtopping flood event combined with dyke erosion is simulated. The model is built upon the 2D Shallow Water Equations together with sediment-flow interactions and incorporates a sediment transport equation. The model is solved using a second-order Godunov-type finite volume method that is accurate and robust. For breach formation, the lateral erosion collapse due to slope instabilities has a significant impact and must be considered, in this paper a simple mathematical approach in two dimensions is proposed to evaluate the stability of lateral bed slope. Several experimental tests are used for validating the morphodynamic model. It is verified that the simulated results agree well with measured data, and that the model predicts such flow phenomena effectively. The validated model is applied to predict a flood event caused by dyke breach with an initial trapezoidal shape due to flow overtopping. The predicted results for the flood event indicate that the 2D process-based morphodynamic model is capable of simulating the spatial and temporal changes of the flood event, including predicting the outflow hydrograph with good agreement, as well as the erosion of the dyke and subsequent deposition process.

Keywords: dyke breach; flow overtopping; morphodynamic model; sediment transport

Introduction

Inundation modeling is significant in flood risk management and disaster prevention and mitigation. A key example of inundation, the catastrophic event of floods induced by the breaching of a dyke is rather complicated to predict, not only because it is related to flood water propagation, but also to sediment transport which is still not well understood. In recent years, several small-scale experimental studies and field observations have been investigated to further understand the dyke breach process caused by flow-overtopping (Chinnarasri et al. 2003, Coleman et al. 2002, Froehlich 2008, Morris et al. 2007). Such laboratory experiments provide insight into the continuous breach growth process. Based on this understanding, numerical models are increasingly attractive and have emerged in large numbers because they are cost-effective and the simulations are not restricted by the spatial-scale of flood events.

Traditionally dyke collapse is assumed to be a "sudden dam-break" of the whole structure or a constant breach size. However, such treatments are unrealistic in reality and the "sudden collapse" hypothesis is too conservative. In fact, the dyke breach induced by flow-overtopping is a progressive process of water flow-sediment transport interaction. This progressive rather than sudden erosion delays the onset of the flood, changing the outflow hydrograph. Dam breach models have been classified into different groups by researchers (Singh 1996, Wu et al. 2011), with each kind identified as having advantages and disadvantages as a result of its assumptions or simplifications. The first type of model is the so-called parametric or empirical model which assumes the dyke breach enlarges progressively at a constant

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37 downcutting rate (Froehlich 1995, Froehlich 2008, Pierce et al. 2010, Wahl 1998, Walder and Oconnor 1997). This
38 method estimates the peak outflow or breach width by using statistically derived regression equations with a large
39 number of historical parameters for dams and reservoirs. As such, parametric models are very sensitive to the
40 parameters related to the constant downcutting rate and neglect the flow eroding capacity during the dyke breach
41 process. It is thus probable that unrealistic results could occur under certain flow conditions and material properties.
42 More realistic physically-based models have been developed in recent years(Franca and Almeida 2004, Macchione
43 2008), but many of these make significant simplifications, for example they assume the breach has a certain shape,
44 neglect some characteristics of the dyke or simply transpose the classical sediment transport equation to describe the
45 breach evolution, all of which limit their application to real cases. More recently 1D and 2D morphodynamic models
46 have been presented based on shallow water theory focusing on embankment breach process and bank erosion issues
47 (Cao et al. 2011, Faeh 2007, Pontillo et al. 2010, Roelvink et al. 2009, Spinewine et al. 2002, Volz et al. 2012). Two-
48 layer models and two-phase models for high concentration sediment-laden flow (Greco et al. 2012, Zech et al. 2008) are
49 also becoming increasingly attractive. However, due to the complexities of dyke breach processes, the detailed
50 breaching models also present some difficulties in application, e.g. the choice of appropriate sediment entrainment
51 function and transport capacity function, as well as how to better model the lateral bed erosion etc. For the dyke breach,
52 bed slope avalanching is certainly a crucial process. In recent years several bank failure operators have been presented
53 in order handle the issues of bank erosion and the dyke breach growth (Spinewine et al. 2002, Swartenbroekx et al.
54 2010, Volz et al. 2012). Spinewine et al. (2002) suggested, based on experimental evidence, that the critical failure
55 angles should be different above and below the water surface; following this Swartenbroekx et al. (2010) and Volz et al.
56 (2012) developed two-dimensional bank failure operators based on triangular mesh and dual-mesh approaches,
57 respectively.

58 In this paper, we present a 2D layer-based hydro-morphodynamic model focusing on predicting the flood process
59 caused by a complex dyke breach. An advanced second-order TVD-WAF scheme is proposed to solve the model system
60 numerically and the model is validated by several experimental cases. Further, an easy-to-implement 2D bed slope
61 avalanching model applicable to rectangular meshes is proposed in order to evaluate the stability of bed slope. This is
62 tested by comparing results against two theoretical bed slope failure cases. Due to the irregularity of topography caused
63 by morphological change, the method proposed by (Guan et al. 2013) is used to handle the wetting and drying problem.
64 The model is then applied to an experiment-scale partially breached dyke case.

65 **Morphodynamic model**

66 *Model assumptions*

67 Based on an understanding of the physical processes of sheet flow, a layer-based concept divides the whole flow region
68 into an active bed layer; a mixed flow-sediment sheet flow layer and an upper water flow layer (Fig.1). The framework
69 for the layer-based model system considered here consists of:

- 70 • a hydrodynamic module governed by the Shallow Water equations with sediment effects;
- 71 • a sediment transport module controlling the sediment mass conservation;
- 72 • and a bed deformation module for updating the bed elevation under the erosion and deposition of sediment

73 Flow-sediment interaction is a rather complex process and understanding is still in its infancy; thus it is impossible
74 to include a complete picture of the hydraulic and sedimentary effects accurately in any model. The present model is no
75 exception. Consequently in this work the following assumptions are adopted; (1) the sediment material is considered as
76 non-cohesive for all of the cases studied; (2) the collision effects of sediment particle-particle are ignored; (3) the time

77 scale of bed change is much larger than that of flow movement, thus the flow is calculated assuming a “fixed” bed at
 78 each time step.

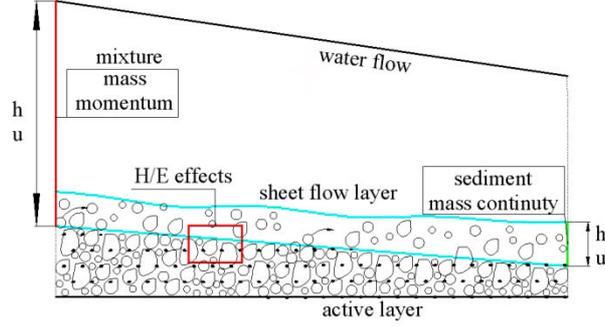


Fig.1. Schematic drawing of the conceptual model in the longitudinal direction

79
80
81

Governing equations

83 The hydrodynamic model is governed by 2D Shallow Water equations including the mass and momentum exchange
 84 between flow and bed. The sediment transport model is governed by the mass conservation of sediment (Li and Duffy
 85 2011, Simpson and Castellort 2006, Xia et al. 2010). Thus the following equations are used to describe the whole
 86 system:

$$\frac{\partial \eta}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0 \quad (1)$$

87

$$\frac{\partial \rho hu}{\partial t} + \frac{\partial}{\partial x} \rho (hu^2 + \frac{1}{2}gh^2) + \frac{\partial \rho huv}{\partial y} = \rho gh(S_{ox} - S_{fx}) \quad (2a)$$

88

$$\frac{\partial \rho hv}{\partial t} + \frac{\partial \rho huv}{\partial x} + \frac{\partial}{\partial y} \rho (hv^2 + \frac{1}{2}gh^2) = \rho gh(S_{oy} - S_{fy}) \quad (2b)$$

89

$$\frac{\partial h_b C_b}{\partial t} + \frac{\partial h_b u_b C_b}{\partial x} + \frac{\partial h_b v_b C_b}{\partial y} = -\frac{(q_b - q_{b*})}{L} \quad (3)$$

90

91 where η =water surface elevation (m); h =flow depth (m); u, v =average flow velocity in x and y direction (m/s); $h_b, u_b, v_b,$
 92 C_b are depth (m), velocity in x direction (m/s), velocity in y direction (m/s) and volumetric concentration (dimensionless)
 93 in sheet flow layer; q_b =real transport rate (m^2/s); q_{b*} = transport capacity (m^2/s); L =non-equilibrium adaptation
 94 coefficient of sediment transport (m); ρ =density of sediment and water mixture (m^3/s), $\rho = \rho_w(I-C) + \rho_s C$; C =volumetric
 95 concentration in flow depth (dimensionless) ρ_s, ρ_w =density of sediment and water respectively (m^3/s). S_{ox}, S_{oy} are the
 96 bed slopes in x and y direction expressed by $S_{ox} = -\frac{\partial z_b}{\partial x}, S_{oy} = -\frac{\partial z_b}{\partial y}$; S_{fx}, S_{fy} are the frictional slopes in x and y direction
 97 calculated by $S_{fx} = \frac{n^2 u \sqrt{u^2 + v^2}}{h^{4/3}}$; $S_{fy} = \frac{n^2 v \sqrt{u^2 + v^2}}{h^{4/3}}$. As the mass flux of sediment transport has, say $huC = h_b u_b C_b \rightarrow$
 98 $h_b C_b = \frac{u}{u_b} hC = \beta hC$ in x direction, the Eq.(3) can be approximately converted to the expression below by expanding
 99 the Eq.(3):

$$\frac{\partial hC}{\partial t} + \frac{1}{\beta} \frac{\partial huC}{\partial x} + \frac{1}{\beta} \frac{\partial hvC}{\partial y} = -\frac{1}{\beta} \frac{(q_b - q_{b*})}{L} \quad (4)$$

100

101 where $\beta = u/u_b$ is the flow-to-sediment velocity ratio. Also, the relationship $\rho = \rho_w(I-C) + \rho_s C$ is substituted into Eqs.(2)
 102 which is then re-formulated. The converted momentum conservation equation is then approximately rewritten as

$$\frac{\partial hu}{\partial t} + \frac{\partial}{\partial x} (hu^2 + \frac{1}{2}gh^2) + \frac{\partial}{\partial y} huv = gh(S_{ox} - S_{fx}) + \frac{\Delta \rho u}{\rho} \frac{\partial z_b}{\partial t} \left(\frac{1-p}{\beta} - C \right) - \frac{\Delta \rho gh^2}{2\rho} \frac{\partial C}{\partial x} - S_A \quad (5a)$$

103

$$\frac{\partial hu}{\partial t} + \frac{\partial}{\partial x} huv + \frac{\partial}{\partial y} (hv^2 + \frac{1}{2}gh^2) = gh(S_{oy} - S_{fy}) + \frac{\Delta \rho v}{\rho} \frac{\partial z_b}{\partial t} \left(\frac{1-p}{\beta} - C \right) - \frac{\Delta \rho gh^2}{2\rho} \frac{\partial C}{\partial y} - S_B \quad (5b)$$

104

105 The morphological evolution is calculated according to the relation of the sediment transport rate and the transport
 106 capacity as

107
$$(1 - p) \frac{\partial z_b}{\partial t} = \frac{(q_b - q_{b*})}{L} \quad (6)$$

108 where p =sediment material porosity (dimensionless); z_b =bed elevation (m); $\Delta\rho=\rho_s-\rho_w$; S_A , S_B are the additional terms
109 related to the velocity ratio β which is expressed by

110
$$S_{A,B} = \frac{\Delta\rho V}{\rho} \left(1 - \frac{1}{\beta}\right) \left[\left(C \frac{\partial hu}{\partial x} + C \frac{\partial hv}{\partial y} \right) - \left(hu \frac{\partial C}{\partial x} + hv \frac{\partial C}{\partial y} \right) \right]$$

111 where $U=u$ for S_A ; $U=v$ for S_B ; The last three source terms of Eqs.(5a-b) represent the interaction effects of sediment and
112 water flow and momentum transfer due to sediment exchange.

113 **Empirical relationships**

114 *Threshold for incipient motion*

115 The threshold of sediment incipient motion is closely related to the dimensionless sediment particle size. The
116 relationship proposed by Soulsby (Soulsby 1997) is applied in this paper.

117
$$\theta_c = \frac{0.30}{1+1.2d^*} + 0.055[1 - \exp(-0.02d^*)] \quad (7)$$

118 in which, $d^*=d[(s-1)g/v^2]^{1/3}$ represents the dimensionless sediment particle size. With consideration of bed slope effects,
119 the critical dimensionless bed shear stress is calculated by

$$\theta_{cr} = k_1 \theta_c \quad (8)$$

120 where θ_{cr} is the critical dimensionless bed shear stress for sediment incipient motion; k_1 is the coefficient corresponding
121 to bed slope effects. Based on the investigation of Smart and Jäggi (Smart and Jäggi 1983), k_1 is determined according
122 to the relation of flow direction and bed slope S as

$$k_1 = \begin{cases} \cos(\arctan |S_{ox}|)(1 - |S_{ox}|/\tan \varphi) & u \cdot S_{ox} < 0 \\ \cos(\arctan |S_{ox}|)(1 + |S_{ox}|/\tan \varphi) & u \cdot S_{ox} > 0 \end{cases}$$

123 where φ is the angle of repose; u , S_{ox} are the velocity and the bed slope in x direction; similar equations can be derived
124 for the y direction.

125 *The flow-to-sediment velocity ratio*

126 The sheet flow velocity has been studied by the derivation of empirical relationships based on experiments (Greimann
127 et al. 2008, Hu and Hui 1996, van Rijn 1984). In this paper, the Eqn. by (Greimann et al. 2008) is used to estimate the
128 approximate velocity ratio. In terms of high bed shear stress with $\theta \geq 20\theta_{cr}$, the flow-to-sediment velocity ratio $\beta=1$ is
129 assumed. Thus

130
$$\beta = \begin{cases} \frac{u}{u_b} = \frac{u}{u_*} \frac{\sqrt{\theta_{cr}}}{1.1(\theta/\theta_{cr})^{0.17}[1 - \exp(-5\theta/\theta_{cr})]} & \theta/\theta_{cr} < 20 \\ 1 & \theta/\theta_{cr} \geq 20 \end{cases} \quad (9)$$

131 where θ is the real dimensionless bed shear stress.

132 *Non-equilibrium adaptation coefficient L*

133 The non-equilibrium adaptation length L means the ability of sediment particles movement in water flows. The
134 coefficient L has been investigated by many researchers (Armanini and Di Silvio 1988, Greimann et al. 2008, Wu 2004),
135 following which, the relationship $L = h\sqrt{u^2 + v^2}/\gamma\omega$ is used, but the coefficient γ is regarded as the ratio of the
136 near-bed concentration and the volumetric concentration in flow with a maximum of $(1-p)$. Thus,

137
$$L = \frac{h\sqrt{u^2 + v^2}}{\gamma\omega} \text{ with } \gamma = \min\left(\frac{C_b}{C}, \frac{1-p}{C}\right) = \min\left(\alpha \frac{h}{h_b}, \frac{1-p}{C}\right)$$

138 in which, $h_b = \mu \theta d_{50}$, μ is a dimensionless coefficient ranging from 6 to 12 related to sediment material that depends on
 139 sediment setting velocity (Pugh and Wilson 1999, Sumer et al. 1996); ω is the effective settling velocity of sediment
 140 determined by Soulsby's equation (Soulsby 1997); α is the sediment-to-flow velocity ratio determined by Eq.(9).

141 *Sediment transport rate*

142 The commonly-used relationship, Meyer-Peter & Müller equation (*MPM*) (Meyer-Peter and Müller 1948), is adopted.
 143 However, the *MPM* equation is derived for bed load transport based on the experiment data for bed slope from 0.0004
 144 to 0.02 and dimensionless bed shear stress smaller than 0.25. Therefore, in this study, *MPM* is applied only for gentle
 145 bed slope of <0.03 and a calibrated coefficient ψ is incorporated to modify the equation. The modified sediment
 146 transport rate is (M_MPM) expressed by:

147
$$q_{b*} = \psi 8(\theta - \theta_{cr})^{1.5} \sqrt{(s-1)gd_{50}^3} \quad 0 \leq S_o < 0.03 \quad (M_MPM)$$

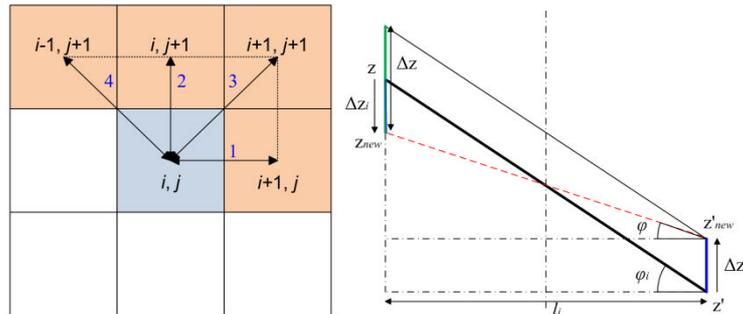
148 where ψ is a calibrated coefficient. With respect to bed slopes of ≥ 0.03 , Smart and Jäggi (Smart and Jäggi 1983)
 149 expanded the database obtained by *MPM* for the steep slope range up to 0.03-0.20. They performed flume experiments
 150 to estimate the maximum transport capacity of mountain streams given by the equation below. However, for the case of
 151 bed slope >0.2 , the bed slope S is modified to be 0.2 in the equation to avoid the calculated transport rate being un-
 152 physically large due to surpassing the limitation range of bed slope. The slightly modified equation (M_SJ) can be
 153 written by:

154
$$q_{b*} = 4 \left(\frac{d_{90}}{d_{30}} \right)^{0.2} \frac{h^{1/6}}{n\sqrt{g}} \min(S_o, 0.2)^{0.6} \theta^{0.5} (\theta - \theta_{cr}) \sqrt{(s-1)gd_{50}^3} S_o \geq 0.03 \quad (M_SJ)$$

155 in which, $d_{90}/d_{30}=1.02$ for uniform sediment particles.

156 ***Two-dimensional bed slope avalanching model***

157 As discussed in the introduction, Swartenbroekx et al. (2010) and Volz et al. (2012) have developed two-dimensional
 158 bank failure operators using different critical angles above and below the water; however, both of these approaches are
 159 presented for triangular meshes and these equations are not applicable to Cartesian cells. In this section, an easy-to-
 160 implement 2D bed slope avalanching model is proposed for application on rectangular meshes. The principle of this
 161 method is that: if the bed slope φ_i of a non-cohesive bed becomes steeper than the critical angle of failure φ , the bed
 162 avalanching will then occur to form a new bedform with a slope approximately equal to the critical angle of repose. In
 163 short, the process of avalanching is simulated by enforcing $|\varphi_i| \leq \varphi$, while maintaining mass conservation of sediment
 164 material. As shown in Fig.2. for the discretisation of rectangular cells in two dimensions, there are eight cells
 165 surrounding each cell. Taking, say cell (i, j) ; bed slope avalanching may occur in four directions (direction 1, 2, 3 and 4)
 166 will be used for updating the elevation in cell (i, j) . Correspondingly, the avalanching approach is divided into four steps
 167 in the four directions, and each update of bed level at cell (i, j) is based on the calculation in the previous step.



168
 169 Fig.2. Schematic diagram of proposed bed level updating; (a) the re-form process in two dimensions; (b) the updating of two adjacent
 170 computational cells in i direction

171

172 We take the re-forming process of sediment in i ($i=1, 2, 3, 4$) direction as an example to derive the updating
 173 equation as follows. When $\varphi_i > \varphi$, the new angle of bed slope is approximately equal to the angle of repose by reducing
 174 the higher cell elevation and elevating the lower cell elevation. This is depicted in Fig.1 for the case of $\varphi_i > \varphi$, in which
 175 case Δz_i is calculated using:

$$176 \quad \Delta z_i = \frac{\Delta z}{2} \approx \frac{l_i(\tan \varphi_i - \tan \varphi)}{2} \quad (10)$$

177 where l_i = the length of two cells in i direction; $l_1=dx$; $l_2=dy$; $l_3=l_4=\sqrt{dx^2 + dy^2}$. As the bed slope angle φ_i in i ($i=1, 2,$
 178 $3, 4$) direction might be negative or positive, the equation above is rewritten with consideration of the positive and
 179 negative of φ_i by

$$180 \quad \Delta z_i = \begin{cases} \frac{\Delta z}{2} \approx \text{sign}(\varphi_i) \frac{l_i(\tan|\varphi_i| - \tan \varphi)}{2} & |\varphi_i| > \varphi \\ 0 & |\varphi_i| \leq \varphi \end{cases} \text{ where } \text{sign}(a) = \begin{cases} 1 & a > 0 \\ 0 & a = 0 \\ -1 & a < 0 \end{cases} \quad (11)$$

181 Thus, the modified 2D bed slope avalanching equation is finally given by

$$182 \quad \begin{cases} z_{new(i,j)} = z_{i,j} + \sum_{i=1}^4 \Delta z_i \\ z_{new(i,j+1)} = z_{i,j+1} - \Delta z_1 \\ z_{new(i+1,j)} = z_{i+1,j} - \Delta z_2 \\ z_{new(i+1,j+1)} = z_{i+1,j+1} - \Delta z_3 \\ z_{new(i-1,j+1)} = z_{i-1,j+1} - \Delta z_4 \end{cases} \quad (12)$$

183 Since avalanching between two cells may induce new avalanching at neighbouring cells, the sweeping process is
 184 repeated using Eq.(12) until no further avalanching occurs. The re-forming process is however time-consuming which
 185 considerably increases the computational time. In general, the time step of bed slope avalanching depends on the
 186 sediment material properties closely and it is difficult to estimate it. In this study, to increase simulation efficiency, the
 187 stability analysis is implemented at a larger time step based on a sensitivity test which shows an insignificant influence
 188 of it on the predicted results. Additionally, different values are used for the critical angles (φ_{dc} for dry bed and φ_{wc} for
 189 wet bed) and the re-formation bed slope angles (φ_{dr} for dry bed and φ_{wr} for wet bed) above and below the water as
 190 supported by (Spinewine et al. 2002). Here, the wet and dry conditions are evaluated according to the simulated value
 191 of water depth at each time step. Correspondingly, the estimated critical and re-formation bed slope angles are assigned
 192 for the two different conditions.

193 Numerical Solution

194 Eq.(1), Eq.(2) and Eq.(3) constitute a shallow water non-linear system. In compact form, the governing equations can be
 195 expressed by

$$196 \quad \frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \mathbf{S} \quad (13)$$

$$197 \quad \mathbf{U} = \begin{bmatrix} \eta \\ hu \\ hv \\ hC \end{bmatrix}, \mathbf{F} = \begin{bmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \\ huv \\ \frac{1}{\beta}huC \end{bmatrix}, \mathbf{G} = \begin{bmatrix} hu \\ huv \\ hv^2 + \frac{1}{2}gh^2 \\ \frac{1}{\beta}hvC \end{bmatrix}, \mathbf{S} = \begin{bmatrix} 0 \\ gh(S_{ox} - S_{fx}) + \frac{\Delta \rho u}{\rho} \frac{\partial z_b}{\partial t} \left(\frac{1-p}{\beta} - C \right) - \frac{\Delta \rho gh^2}{2\rho} \frac{\partial C}{\partial x} - S_A \\ gh(S_{oy} - S_{fy}) + \frac{\Delta \rho v}{\rho} \frac{\partial z_b}{\partial t} \left(\frac{1-p}{\beta} - C \right) - \frac{\Delta \rho gh^2}{2\rho} \frac{\partial C}{\partial y} - S_B \\ -\frac{1}{\beta} \frac{(q_b - q_{b*})}{L} \end{bmatrix}$$

198 To solve the system (13), a HLL based scheme has been used. An excellent description of this approach is given by
 199 Toro (Toro 2001), so a detailed description is omitted here. However, the proposed model system incorporates an extra
 200 governing equation for sediment transport. To incorporate this in to the HLL Riemann solver, the flux at the interface of
 201 two adjacent cells is obtained by the use of a middle contact discontinuity waves S_* . Through the assessment of S_* , the

202 sediment flux is determined based on the concentration at the right cell or left cell. In the following, a brief description
 203 is given explaining how this interface flux is calculated for the coupled flow and sediment model. Firstly, the first three
 204 flux terms can be expressed by the basic HLL scheme expression as follows:

$$\mathbf{E}_{LR\ 1,2,3}^* = \begin{cases} \mathbf{E}_L & \text{if } S_L \geq 0 \\ \mathbf{E}_R & \text{if } S_R \leq 0 \\ \mathbf{E}^* & \text{otherwise} \end{cases} \quad (14)$$

205 where $\mathbf{E}_L = \mathbf{E}(\mathbf{U}_L)$, $\mathbf{E}_R = \mathbf{E}(\mathbf{U}_R)$ are the flux and conservative variable vectors at the left and right side of each cell
 206 interface. \mathbf{E}^* is the numerical flux in the star region, calculated in two dimensions by

$$\mathbf{E}^* \cdot \mathbf{n} = \frac{S_R \mathbf{E}_L \cdot \mathbf{n} - S_L \mathbf{E}_R \cdot \mathbf{n} + S_R S_L (\mathbf{U}_R - \mathbf{U}_L)}{S_R - S_L}$$

208 in which, $\mathbf{n} = [n_x, n_y]^T$; the S_L and S_R denote two wave speeds which must be selected carefully to avoid any entropy
 209 violation. The so-called ‘‘two expansion’’ approach (Toro 1992) was adopted here including dry-bed options to estimate
 210 S_L and S_R . They are expressed by

$$S_L = \begin{cases} \min(\mathbf{q}_L \cdot \mathbf{n} - \sqrt{gh_L}, u^* - \sqrt{gh^*}) & \text{if } h_L > 0 \\ \mathbf{q}_R \cdot \mathbf{n} - 2\sqrt{gh_R} & \text{if } h_L = 0 \end{cases}; S_R = \begin{cases} \min(\mathbf{q}_R \cdot \mathbf{n} + \sqrt{gh_R}, u^* - \sqrt{gh^*}) & \text{if } h_R > 0 \\ \mathbf{q}_L \cdot \mathbf{n} + 2\sqrt{gh_L} & \text{if } h_R = 0 \end{cases}$$

212 where $u^* = \frac{1}{2}(\mathbf{q}_L + \mathbf{q}_R) \cdot \mathbf{n} + \sqrt{gh_L} - \sqrt{gh_R}$, $\sqrt{gh^*} = \frac{1}{2}(\sqrt{gh_L} + \sqrt{gh_R}) + \frac{1}{4}(\mathbf{q}_L - \mathbf{q}_R) \cdot \mathbf{n}$; $\mathbf{q} = [u, v]$. The middle
 213 wave speed S_* is calculated by the following form as recommended by Toro (2001).

$$S_* = \frac{S_L h_R (\mathbf{q}_R \cdot \mathbf{n} - S_R) - S_R h_L (\mathbf{q}_L \cdot \mathbf{n} - S_L)}{h_R (\mathbf{q}_R \cdot \mathbf{n} - S_R) - h_L (\mathbf{q}_L \cdot \mathbf{n} - S_L)}$$

215 To calculate the intercell numerical fluxes, a weighted average flux (WAF) of total variation diminishing (TVD)
 216 method is employed with a flux limiter function. The TVD-WAF scheme is second-order accurate in space and time by
 217 solving the conventional Riemann problem associated with the first-order Godunov scheme. A detailed description can
 218 be found in (Toro, 2001). Taking the calculation of flux in the x direction as an example, this is calculated using:

$$\mathbf{F}_{i+1/2,(1,2,3)}^* = \frac{1}{2}(\mathbf{F}_i + \mathbf{F}_{i+1}) - \frac{1}{2} \sum_{k=1}^N \text{sign}(c_k) \Phi_{i+1/2}^k \Delta \mathbf{F}_{i+1/2}^k \quad (15)$$

220 in which, $\mathbf{F}_i = \mathbf{F}(\mathbf{U}_i)$, $\mathbf{F}_{i+1} = \mathbf{F}(\mathbf{U}_{i+1})$ are the flux and conservative variable vectors at the left and right sides of each cell
 221 interface; c_k is the Courant number for wave k , $c_k = \Delta t S_k / \Delta x$; S_k is the speed of wave k and N is the number of waves in
 222 the solution of the Riemann problem, $N=2$ in conjunction with HLL approximate Riemann solver. $\Delta \mathbf{F}_{i+1/2}^{(k)} = \mathbf{F}_{i+1/2}^{(k+1)} -$
 223 $\mathbf{F}_{i+1/2}^{(k)}$, which is the flux jump across wave k ; $\mathbf{F}_{i+1/2}^{(k)}$ is the value of the flux vector in the interval k ; herein
 224 $\mathbf{F}_{i+1/2}^{(1)} = \mathbf{F}(\mathbf{U}_L)$, $\mathbf{F}_{i+1/2}^{(2)} = \mathbf{F}(\mathbf{U}^*)$, and $\mathbf{F}_{i+1/2}^{(3)} = \mathbf{F}(\mathbf{U}_R)$ which are estimated by virtue of the HLL approximate Riemann
 225 solver, $\Phi(r)$ is the WAF limiter function. The WAF limiter used here is expressed through the well-known conventional
 226 flux limiter term $\phi(r)$ was the *min-mod* limiter:

$$\Phi(r) = 1 - (1 - |c|)\phi(r) \text{ with } \phi(r) = \max[0, \min(1, r)] \text{ (min-mod limiter)}$$

228 where $r^{(k)}$ is the ratio of the upwind change to the local change in scalar quantity q . It can be written by:

$$r^{(k)} = \begin{cases} \Delta q_{i-1/2}^{(k)} / \Delta q_{i+1/2}^{(k)} = (q_i^{(k)} - q_{i-1}^{(k)}) / (q_{i+1}^{(k)} - q_i^{(k)}) & \text{if } c_k > 0 \\ \Delta q_{i-3/2}^{(k)} / \Delta q_{i+1/2}^{(k)} = (q_{i+2}^{(k)} - q_{i+1}^{(k)}) / (q_{i+1}^{(k)} - q_i^{(k)}) & \text{if } c_k < 0 \end{cases}$$

230 For the x split 2D Shallow Water equations we choose $q = \eta$ for the left wave $S_L (k=1)$ and the right wave $S_R (k=2)$.
 231 Based on the solution of the previous three flux terms, the fourth flux term-sediment flux $F_{i+1/2,4}$ at the interface of two
 232 adjacent cells is determined by the relationship of the middle waves S_* and zero, calculated by

$$F_{i+1/2,4}^* = \begin{cases} F_{i+1/2,1}^* C_L & S_* \geq 0 \\ F_{i+1/2,1}^* C_R & S_* < 0 \end{cases} \quad (16)$$

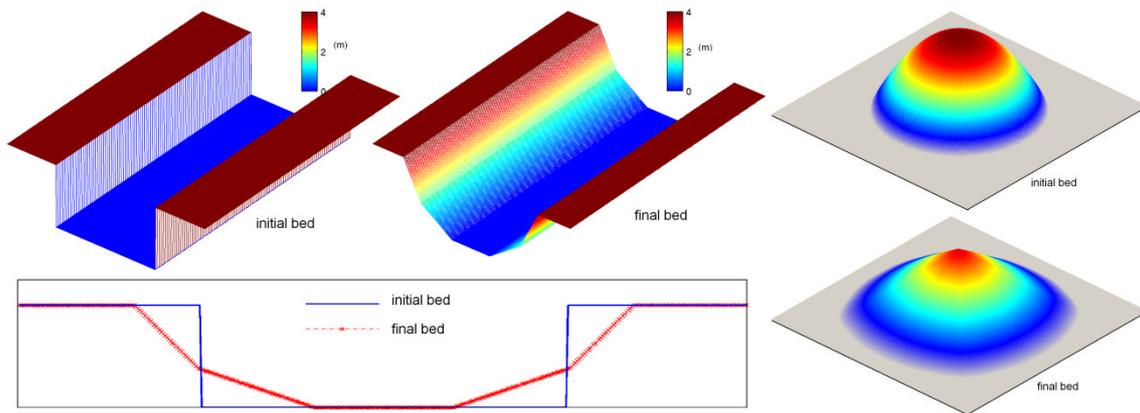
233 where C_L and C_R are the volumetric sediment concentration in left and right cells; $F_{i+1/2,1}$ is the first flux component

234 calculated by Eq.(15). Furthermore, the source term and wetting/drying are treated by using the method published in
 235 (Guan et al. 2013). The numerical scheme is explicit, so for stability the Courant-Friedrichs-Lewy ($0 < CFL < 1$) stability
 236 condition must be applied to limit the time step Δt .

237 **Validation of morphodynamic model**

238 *Unstable bed failure*

239 Two theoretical tests were undertaken to validate the proposed bed slope avalanching model. The first is to test bank
 240 failure in a square channel with vertical banks. The inlet and outlet of the channel are assumed to be glass walls.
 241 Initially, the bank elevation is 4m and the static water level in the channel is 1.5m. The critical failure angles of wet bed
 242 and dry bed are considered as 61° and 31° respectively, and the reformation angle of both are 60° and 30° respectively.
 243 Fig.3a illustrates the topography of channel after applying the bed slope avalanching model. It is shown that the final
 244 stable bank slopes above and below the water are equal to the corresponding critical angles. Test 2 examines the spatial
 245 behaviour of the avalanching model. A half-oval dune was placed in the centre of a completely dry bed (Fig.3b). The
 246 failure angles were assumed to be 40° . In the simulation, the unstable side walls were flattened towards the circle and an
 247 approximately symmetrical cone-shaped configuration formed. The applicability of the geometrical approach was
 248 demonstrated in both tests and mass continuity was maintained despite the large mass movements.

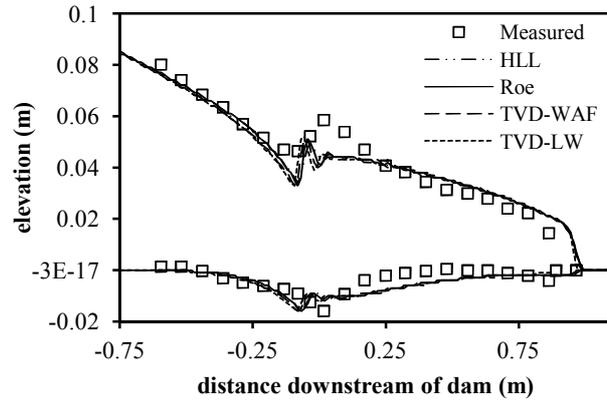


249 Fig.3. Tests for bed slope avalanching; left: vertical bed with water pool; right: circle dune
 250
 251

252 *Unsteady dam-break flow over movable bed*

253 Experiments of a dam-break flow over movable bed were conducted using PVC pellets and sand as the bed material in
 254 the laboratory of UCL in Belgium. Details of these can be found in (Fraccarollo and Capart 2002). The purpose of these
 255 tests, which are a 1D case but implemented in 2D, was to elucidate the applicability of the morphodynamic model in an
 256 unsteady outburst flow. The case of cylindrical PVC pellets was chosen; the parameters of PVC particles include: the
 257 equivalent spherical diameter is 3.5mm , the density is 1540kg/m^3 , the sediment material porosity $p=0.47$, and the
 258 settling velocity is about 18cm/s according to the experiment. The experiments were implemented in a horizontal
 259 prismatic flume with a rectangular cross section of $2.5\text{m} \times 0.1\text{m} \times 0.25\text{m}$. This case is simulated by applying first-order
 260 HLL solver, first-order Roe solver, second-order TVD-WAF scheme and TVD-Lax-Wendroff in order to evaluate the
 261 effect of the solver's accuracy on the results. Fig.4 shows that the numerical model simulates the temporal evolution of
 262 erodible bed effectively; and that the water level and bed scour is predicted with good agreement although there are
 263 some slight differences in terms of bed scour depth and water level. Moreover, the comparison indicates that the
 264 scheme's formal numerical accuracy does not significantly influence the model results. Following this result the

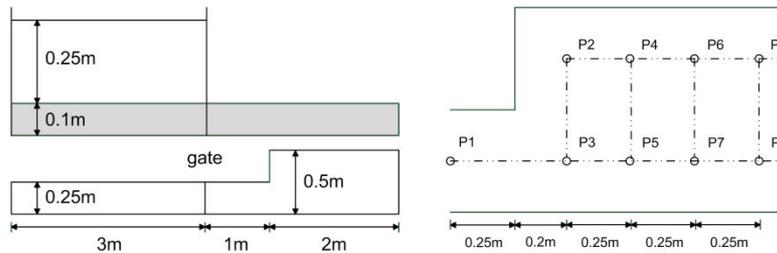
265 attractive second-order TVD-WAF scheme is adopted in all cases discussed in the rest of this paper. This was chosen as
 266 it has been well tested, is robust and is easy to implement (Guan et al. 2013).



267
 268 Fig.4. Comparison between measured data and simulated results at $t=10t^0$ ($t^0=0.101s$) for dam-break flow over movable bed test
 269

270 ***Dam-break flow in an erodible channel with a sudden enlargement***

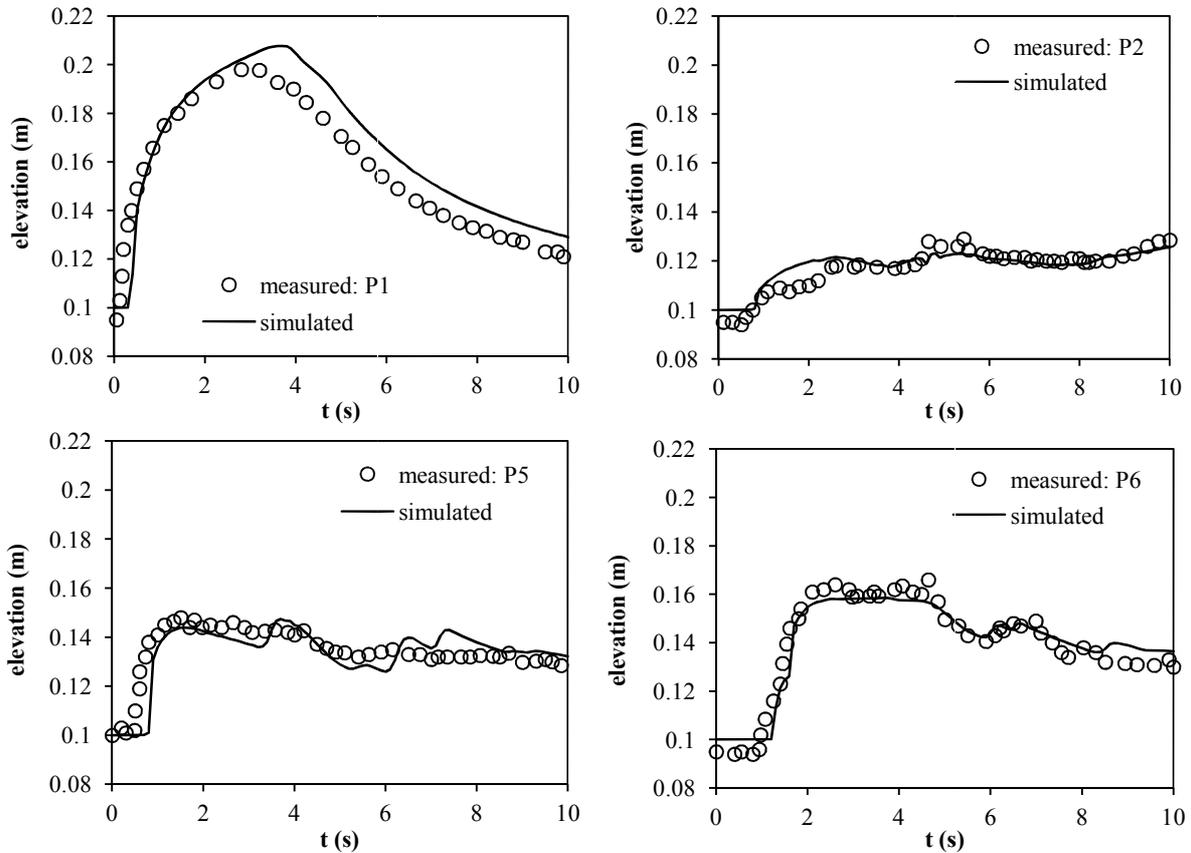
271 To verify the capability of the morphodynamic model in a two-dimensional domain, the experimental test investigated
 272 in the laboratory of UCL in Belgium (Goutiere et al. 2011) is reproduced here. The initial experimental setup is shown
 273 in Fig.4. The initial water depths before and after the gate are 0.25m and 0m respectively; the erodible bed layer is set at
 274 0.1m thick and consists of fully saturated sand with a uniformity index of $d_{84}/d_{16}=1.96$ and a median diameter of
 275 1.72mm. The density of sand is $2.63 \times 10^3 kg/m^3$, and the sediment porosity is 39%. The downstream outlet is an open
 276 free outfall and the sediment bed is maintained at the initial elevation by a vertical plate presenting the same height as
 277 the bed layer. For the simulation, the model domain was discretised by 300×100 uniform cells. As suggested by the
 278 experimental work the Manning's coefficient is set to be equal to 0.023, the calibration coefficient ψ is equal to 1.0 and
 279 the dimensionless coefficient μ is set to be 9.0.



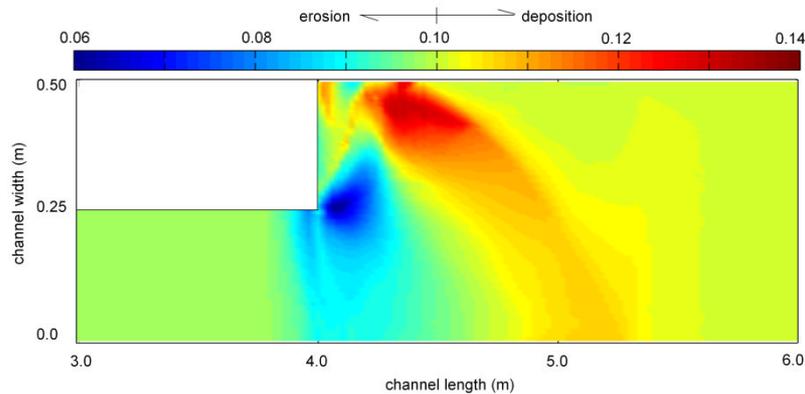
280
 281 Fig.5. Sketch of a dam-break flow experiment over a mobile bed

282 In order to validate the performance of the model a comparison of the simulated water level and the measured data
 283 against the time at the measured gauges of P1, P2, P5 and P6 is shown in Fig.6. This indicates that the simulated water
 284 level agrees with the measured data quite well. The flow-sediment interaction process is completed and reaches a
 285 stationary state after around 50s. This simulated final bed topography is shown in Fig.7. Compared to the experimental
 286 images of the final bed in (Goutiere et al. 2011), the simulated bed shows two similar main characteristic patterns.
 287 Firstly, an eroded hole is generated at the enlargement location area where the most severe erosion occurs; the second
 288 characteristic area is located near to the left side-wall (facing flow direction) behind the expansion outlet where a
 289 deposited mound is predicted. These agree well with the distribution erosion and deposition observed in the experiment.
 290 To further elucidate the erodible bed change, the simulated bed profiles at the two cross sections (CS1: 4.2m, CS2: 4.5m
 291 downstream from the inlet) are compared with the measured data in Fig.8. In addition, we also illustrate the simulated

292 results by the SWE-Exner model by Soares-Frazão and Zech (Soares-Frazão and Zech 2011) to show the improvement
 293 that the present model provides. At the CS2 ($x=4.5m$), the simulated bed profile by the present model achieves fairly
 294 good agreement with the measured cross section, this is much better than the profile simulated by the SWE-Exner
 295 model. At CS1($x=4.2m$), however, the simulated bed profile is over estimated by the present model in terms of
 296 quantitative assessment; there the scour hole moves faster than the experimental observation and the maximum height
 297 of the deposited mound is slightly larger than the measured bed. While the trend of the erosion and deposition at the
 298 cross section is predicted with a similar shape with the measured bed profile, which is qualitatively better than that
 299 simulated by the SWE-Exner model. Possible reasons for the quantitative discrepancy are: firstly, the sediment particle
 300 ($1.72mm$) is very coarse for the water depth (maximum $0.25m$), so the particle-particle collision effects, which are
 301 neglected by the present model, may be significant; secondly, the effect of the secondary flow at the expansion outlet
 302 probably plays a significant role in this particular case.



304 Fig. 6. Comparisons between simulated and measured water levels at P1, P2, P5 and P6
 305



306 Fig. 7. Simulated final bed topography
 307

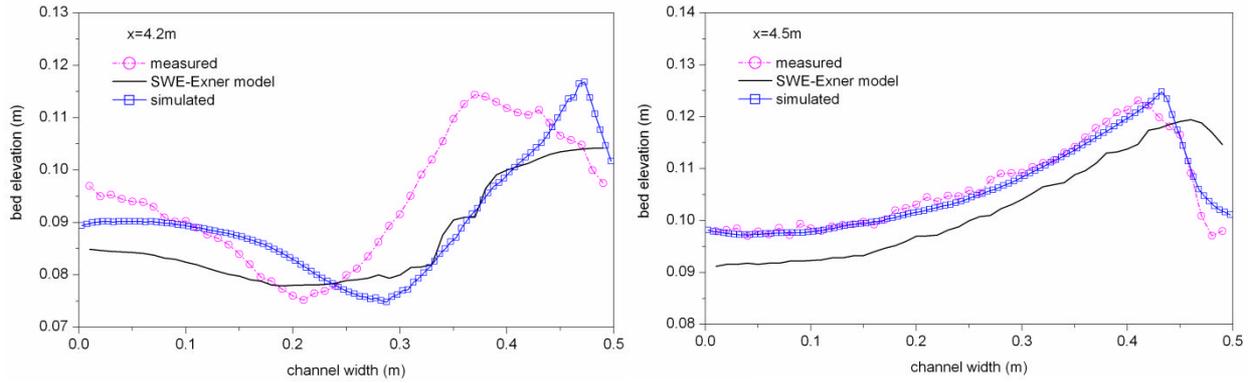


Fig.8. Comparisons between simulated bed profile and measured bed profile at $x=4.2m$ and $x=4.5m$

308
309

310

311 **Dam erosion due to flow overtopping**

312 Dam erosion due to flow overtopping is a complex flow process involving outburst flow, transition from supercritical
 313 flow to subcritical flow and eventually steady flow. This test is presented to verify that the 2D morphodynamic model
 314 can predict the erosion and deposition under complex hydraulic conditions. The experiment denoted Run2 conducted by
 315 (Chinnarasri et al. 2003) is reproduced here. A dyke was located in the middle of a flume of $35m \times 1m \times 1m$ being $0.8m$ in
 316 height, $1m$ wide with a crest width of $0.3m$. The upstream and downstream slope of the dam was $1V:3H$ and $1V:2.5H$,
 317 respectively. The dyke is composed of sand with a median diameter of $1.13mm$, and the density of $2.65 \times 10^3 kg/m^3$. The
 318 initial reservoir level is $0.83m$ and the downstream water level is $0.03m$; the inflow discharge has a constant value of
 319 $1.23 \times 10^{-3} m^3/s$; the bed material porosity is taken as 0.35 ; the Manning coefficient n is determined as 0.018 ; the
 320 calibration coefficient $\psi=1.5$ and the dimensionless coefficient $\mu=9.0$ for this case.

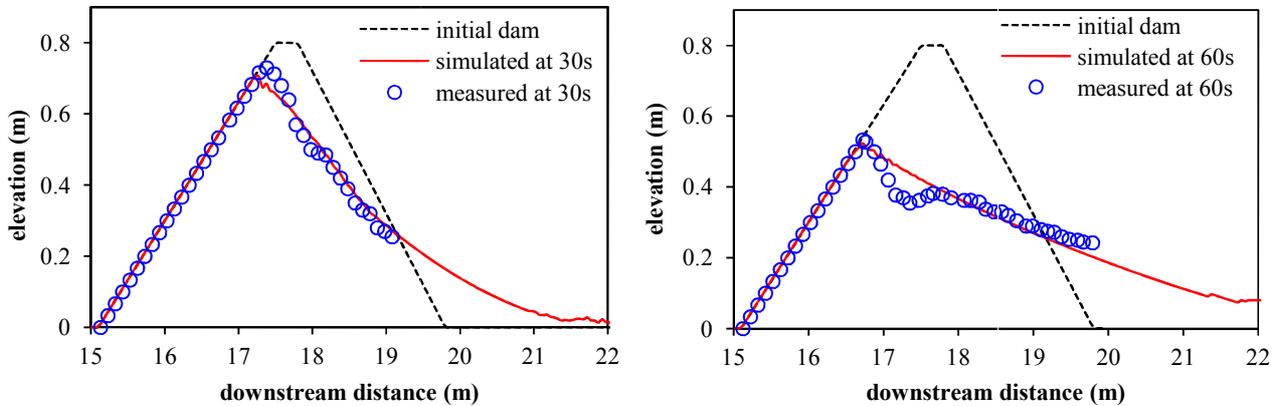


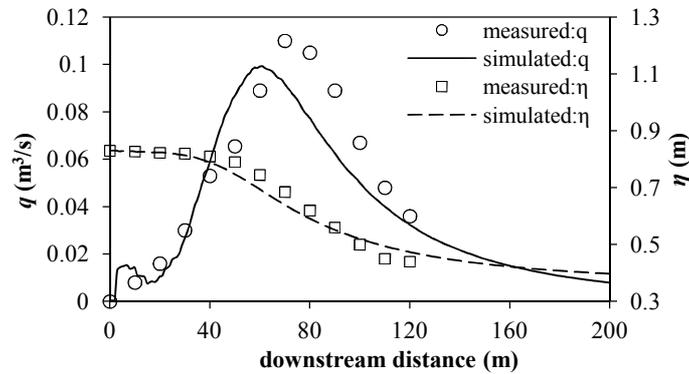
Fig.9. Comparison between simulated dam profile and measured dam profile at $t=30s$ and $t=60s$ for full dyke breach test

321

322

323 For the simulation, the area is discretised by 700×10 cells ($dx=0.05m$, $dy=0.1m$); the sediment transport rate is
 324 calculated by the equation of M_{SJ} and M_{MPM} according to the extent of the bed slope. Fig.9 illustrates the
 325 comparison between the simulated dyke bed profiles and measured data at $t=30s$ and $t=60s$. At $30s$ the comparison
 326 shows fairly good agreement. A reasonably good agreement is also achieved at $t=60s$, but a larger discrepancy is
 327 observed at the top of the dam. A scour hole occurs in the observation, yet this area is smooth in the numerical result.
 328 Fig.10 illustrates the comparisons between the simulated results and the measured data: the water level and the outflow
 329 discharge. The agreements are again reasonably good, but it also be seen that the simulated water level is slightly lower
 330 than the measured data before $90s$ and that the arrival time of the peak discharge is slightly earlier. The inaccuracies
 331 indicated by Fig.9 and Fig.10 are most likely caused by the choice of empirical parameters in the present model, e.g.
 332 Manning's coefficient, bed slope effects, empirical sediment transport function etc. Most of the empirical functions are

333 derived based on experimental data and as such are unlikely to be completely applicable to all the complex flow
 334 conditions. We have performed a simulation on a finer mesh, and found that the mesh size is not a major reason causing
 335 the inaccuracies.



336
 337 Fig.10. Comparisons between simulated results and measured data for full dyke breach test
 338

336
 337
 338

339 **Simulation of a Dyke Breach from a Partial Overtopping Flow**

340 In this section, a flood event caused by a partially breached dyke is reproduced by the validated morphodynamic model
 341 to simulate the spatial and temporal evolution of the dyke breach. The predicted outflow hydrograph and the change of
 342 water level in the reservoir are compared with measured data.

343 **Experimental conditions**

344 The experiment conducted by UCL (Spinewine et al. 2004) was simulated. A sand dyke of 2.4m long and 0.47m high
 345 was built at 11.8m along a 36.2m×3.6m flume; two fixed blocks were placed on the left and right sides of the dyke; the
 346 upstream and downstream slopes of sand dyke were 1:2 and 1:3 respectively, and a 10cm sand layer was laid
 347 downstream of the dam. The sediment material was composed of sand with a median diameter $d_{50}=1.80mm$, specific
 348 gravity of $s=2.615$ and a loose bed porosity $p=0.42$ after compaction. An upstream reservoir contained water for the
 349 experiment, which was held by a gate which was then gradually opened so the water filled the region upstream of the
 350 dyke until water level was at 0.45m. A small trapezoidal breach was dug on the top middle of dyke to initiate the flow
 351 overtopping at this point. Subsequently the breach enlarged with the flow gradually with increasing time. The two
 352 blocks besides the sand dike are treated as the part of the sand dyke with the restriction that in the simulation they are
 353 not erodible.

354 **Measured data**

355 The measured data (Spinewine et al. 2004) used is:

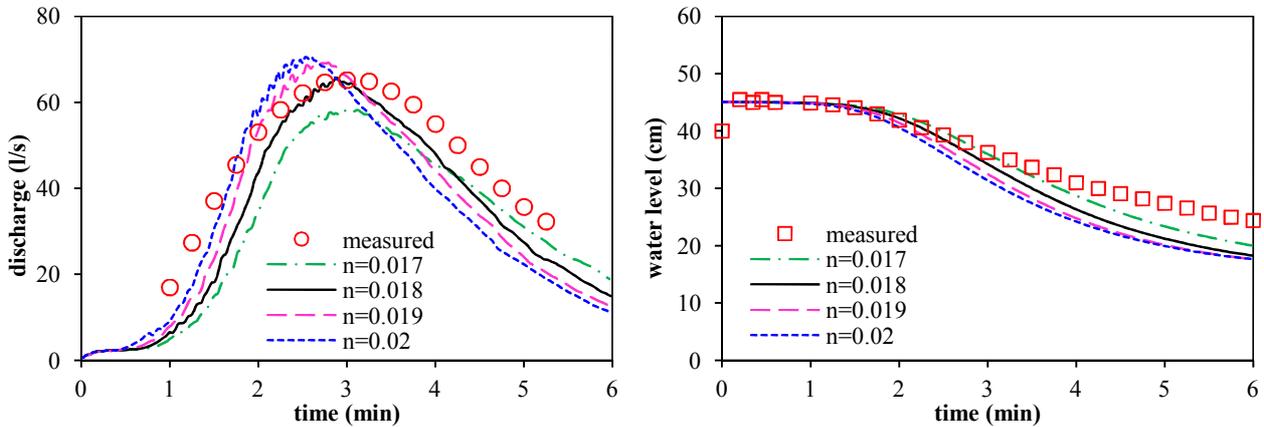
- 356 (1) the water level change with time in the upstream reservoir;
- 357 (2) the outflow discharge against time;
- 358 (3) full digital terrain models (DTMs) of the breach topography interpolated from laser-observed transverse profiles.

359 The outflow discharge was estimated by using the measured water level, thus the estimated outflow hydrographs
 360 show a significant uncertainty range as shown in (Spinewine et al. 2004, Van Emelen et al. 2011); the estimated
 361 discharge Q_2 is used in the following.

362 **Predicted hydrograph**

363 The whole dyke and channel are discretised with $dx=0.035m$ and $dy=0.03m$ and the coefficient values $\psi=1.5$ and $\mu=9.0$
 364 were chosen. For this kind of flood event the outflow peak discharge is a vital hydraulic parameter that needs to be

365 predicted. Manning's coefficient n has a direct influence on the bed shear stress and thus strongly influences the flow-
 366 induced sediment transport; therefore, four different Manning's coefficients ($n=0.017, 0.018, 0.019, 0.02$) are used for
 367 evaluating and analysing its sensitivity in the modelling of the dyke breach process. Fig.10 illustrates the comparisons
 368 between the predicted results and the measured data, showing both the outflow hydrograph (Fig.11a) and the water level
 369 in the reservoir (Fig.11b). It can be seen that the Manning's coefficient changes the peak value and the time of
 370 occurrence of the peak outflow discharge, consequently the water level in the reservoir is also affected. More
 371 specifically, the larger the Manning's coefficient the more water flow from the reservoir, thus the outflow peak
 372 discharge becomes larger and occurs at an earlier time. The reason for this is primarily because increasing Manning's
 373 coefficient increases the calculated bed shear stress, so the dyke is eroded more severely and thereby the breach process
 374 is accelerated. Some small oscillations occur at the simulated outflow hydrograph, in particular at the peak stage. These
 375 occur because the lateral bed avalanching erodes the sediment material of the breach, which raises the elevation of the
 376 breach temporarily and locally blocks the flow; then as further erosion occurs based on the previous updating of the bed.
 377 Overall, the present model predicts the outflow hydrograph and the temporal change of water level in the reservoir
 378 effectively with good agreement to measured data.



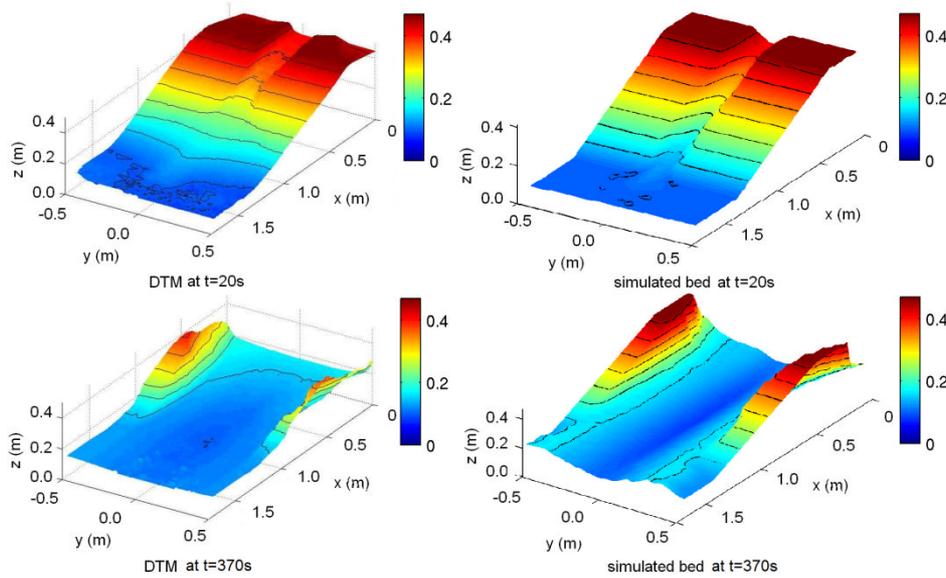
379
 380 Fig.11. Comparisons between predicted result and measured data for partial dyke breach test, (a) water level; (b) outflow discharge

381
 382 **Simulated dyke breach**

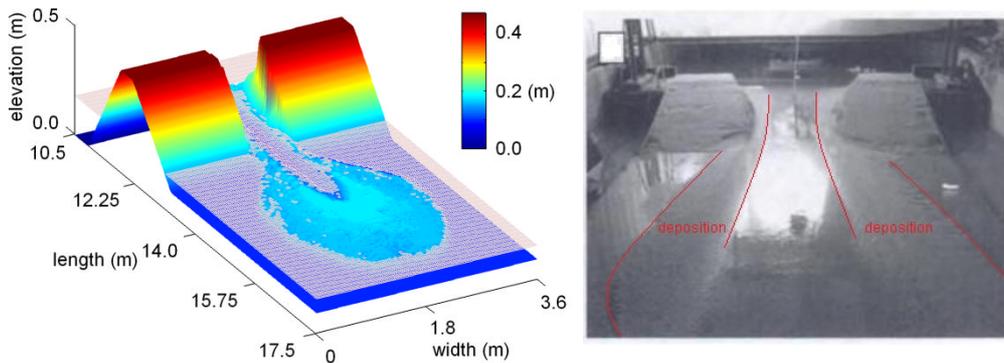
383 As mentioned above, the DTMs (Spinewine et al. 2004, Van Emelen et al. 2011) are compared with the simulated dyke
 384 terrain to assess the capability of the present model and the bed slope avalanching model to predict the breach size. The
 385 DTMs themselves suffer from a lack of accuracy in certain regions because of air/water refraction issues and water
 386 covered land when the measurements were taken. For the simulation, $n=0.018$ is chosen because the model reproduced
 387 the peak discharge accurately at this value. Fig.12 displays the digital terrain measurements compared to the simulated
 388 dyke breach at the initial stage $t=20s$ and the final stage $t=370s$. The breaching process is reasonably well reproduced.
 389 The numerical model predicts slightly more severe erosion at the downstream toe of the dyke at $t=20s$; it is clear that
 390 more deposition is indicated there by the digital terrain data. At $t=370s$, more severe erosion can be observed in the
 391 middle area of the dyke, whilst less lateral erosion occurs at each side of the breach. The sediment transport model
 392 appears to overestimate the vertical erosion, while the bed slope avalanching model slightly underestimates the lateral
 393 erosion presenting a narrower breach. Fig.13 shows the simulated spatial distribution of bed and water in the stretched
 394 ordinates, as well as the experimental data at the final equilibrium stage. It can be seen that the present model
 395 reproduces the characteristic erosion, deposition and wet/dry areas well; the eroded sediment from the breach primarily
 396 deposits behind the dyke and a secondary channel is formed along the centreline. In summary, the present model can

397 reproduce the dyke breach process effectively, while also presenting some shortcomings which need to be addressed
398 when being applied in practice. These shortcomings are:

- 399 (1) It is difficult to estimate the empirical parameters for sediment transport which could cause some differences;
400 thus appropriate calibration parameters are necessary for predicting the dyke breach.
401 (2) The bed slope avalanching occurs based on the consideration of the relationship between the bed slope and the
402 critical angles; hence it does not simulate the lateral random dyke collapse.
403 (3) The failure time step for the lateral erosion which depends closely on the sediment material properties is
404 difficult to estimate; a sensitivity test was carried out with updating using two different failure time steps. It
405 was seen that this did not influence the final breach size, while the arrival time of the peak discharge has a
406 slight difference.



407
408 Fig.12. DTM (Van Emelen et al. 2011) and simulated dyke breach due to flow overtopping at t=20s and 370s

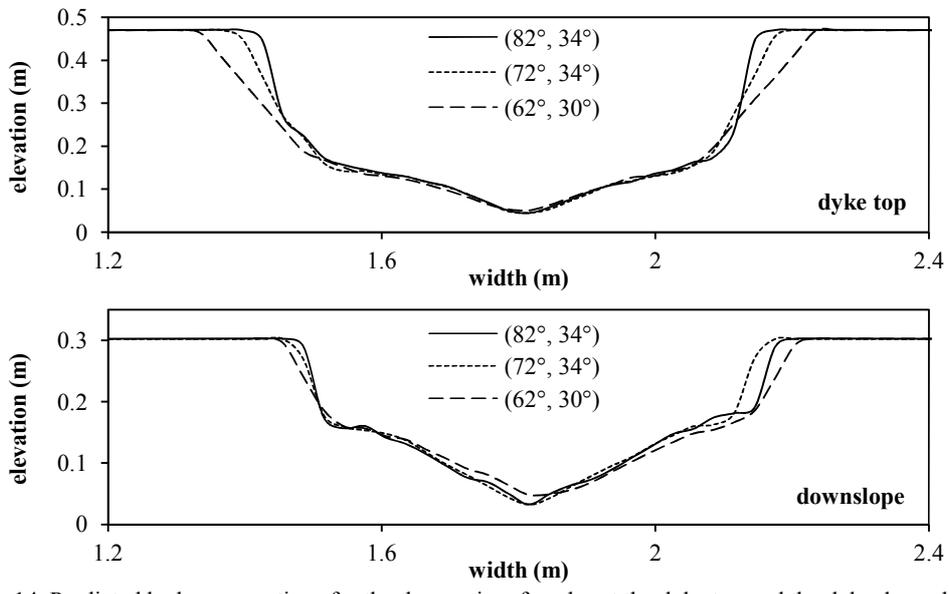


409
410 Fig.13. Simulated final dyke breach and water surface, and observed image by (Spinewine et al. 2004)

412 ***The role of bed slope avalanching***

413 As mentioned above, the main purpose of the bed slope avalanching model in this case is to simulate the lateral erosion
414 of the dyke. We can postulate that the dyke breach will stay constant in a horizontal direction and the erosion can only
415 occur in vertical direction if no bed slope avalanching is implemented. Although the different critical angles and re-
416 formation angles above and below the water are suggested by (Spinewine et al. 2004), the values of these angles are still
417 ambiguous which directly influences the breach size and the outflow hydrograph. Also for the dyke breach process, it

418 has been investigated by (Pickert et al. 2011) that the apparent cohesion represented by the pore-water pressure
 419 influences the stability of the breach slide slopes and thereby the whole breach process. To further investigate the
 420 effects of these angles, three runs with three different pairs of angles are implemented: run1 (82° , 34°) means the
 421 critical angles above and below the water are 82° and 34° respectively, it is similar for run2 (72° , 34°), and run3 (62° ,
 422 30°); the re-formation angles are equal to the critical angles minus 2° . For the breach cross-section profiles,
 423 comparisons for the three runs at the dyke top and the downstream slope of the dyke are given in Fig.14. It is shown that
 424 the breach width is influenced by the angles, as expected. More specifically, the smaller the critical angles, the wider the
 425 breach size, whilst the side slope of the breach is steeper for the larger critical angles. This is because the bed slope
 426 avalanching occurs at an earlier time for the smaller critical angles, and correspondingly more lateral erosion occurs. In
 427 summary, through the above analysis, we emphasise the crucial role of the critical angles in predicting the dyke breach
 428 evolution is shown.



430
 431 Fig.14. Predicted bed cross-sections for the three pairs of angles at the dyke top and the dyke downslope
 432

433 **Conclusions**

434 Dyke breaching is a complex process and the traditional “sudden dam-break” assumption is too conservative to
 435 represent it adequately. On the other hand to estimate breach evolution and outflow discharge by empirical or simplified
 436 physical models involves many unknown factors. This paper proposes a layer-based two-dimensional hydro-
 437 morphodynamic model to predict the complex dyke breach processes. Also, a 2D bed slope avalanching model is
 438 proposed in order to calculate the lateral erosion and also maintain the stability of unstable sloped bed. The model is
 439 solved numerically with a second-order TVD-WAF/HLL which is both accurate and robust. The model is validated by
 440 several experimental benchmark tests, presenting good agreement with the measured data in terms of both
 441 hydrodynamic and morphodynamic aspects. Finally, the validated model is applied to predict a dyke breach process
 442 caused by partial flow overtopping with an initial trapezoidal shape. The complex flow-sediment process is reproduced
 443 by the model with good agreement. In short, the advantages of the 2D morphodynamic model together with the bed
 444 slope avalanching model involve:

- 445 • The key hydraulic components, the water level and the outflow hydrograph, especially peak discharge, can be
 446 predicted fairly well.

- 447 • The spatial and temporal evolutions of the dyke breach are also well reproduced, including the dyke breach
448 shape and size, as well as the distribution of erosion and deposition in the downstream area.

449 The disadvantages of this approach, however, lie in the empirical parameters involved in both morphodynamic
450 model and bed slope avalanching model. Appropriately calibrated parameters are important for the numerical results.
451 This study is primarily focused on the small-scale flood events with flow-sediment interactions. In reality, the hydraulic
452 and bed conditions are much more complex. Therefore, applications of the model in large-scale flood events will be
453 investigated in subsequent research.

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456 University of Leeds for their financial support for his PhD studies.

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