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The Electromagnetic Properties of Re-Entrant Dielectric Honeycombs

F. C. Smith, F. Scarpa, and B. Chambers

Abstract—**Dielectric honeycombs are cellular materials often used in applications that require structural and electromagnetic characteristics, e.g., in LO (low observable) and radome components. A re-entrant (or auxetic) honeycomb is a cellular material with structural properties that are superior to those of a conventional honeycomb. By employing the finite-difference time-domain (FDTD) technique with periodic boundary conditions, the electromagnetic properties of re-entrant honeycombs are determined and compared to those of a conventional honeycomb. Re-entrant honeycombs are shown to have substantially superior electromagnetic properties. Measured permittivity data are used to substantiate the conclusions based on predicted FDTD data. The use of re-entrant honeycombs, rather than conventional honeycombs, in LO and radome applications can yield improved structural and electromagnetic performance.**

Index Terms—**Dielectric materials, FDTD, microwave measurements.**

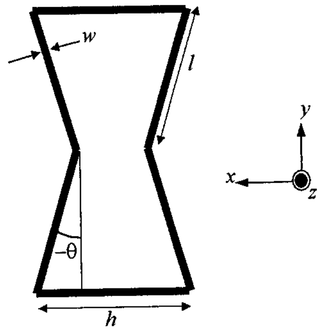

Fig. 1. Honeycomb unit cell. The geometry is defined by h/l, w/l and θ. The two ratios are termed respectively α and β (β describes the relative density of the honeycomb). In a practical honeycomb, the two horizontal facets are often twice the thickness of the remaining four facets.

I. INTRODUCTION

DIELECTRIC honeycombs are a class of cellular material with advantageous structural properties [1]; they also have electromagnetic properties which can be exploited in structural LO (low observable) [2] and electromagnetic window applications.

The unit cell of a typical honeycomb is two-dimensional; along the unit cell's axis geometry, permittivity and permeability are invariant. The geometric and material properties of a honeycomb cause the material to be uniaxially anisotropic at microwave frequencies and below [3]. The dimension of the unit cell—the factor which governs the upper frequency limit of homogenization—normally varies between 2 and 10 mm [4].

The unit cell of a conventional dielectric honeycomb is constructed from six walls (or facets) of equal length arranged such that all the internal angles are equal. The facet arrangement of a conventional honeycomb limits the structural and electromagnetic properties which can be obtained from an array of honeycomb unit cells [5], [6]. From the electromagnetic viewpoint, a conventional honeycomb is always uniaxially anisotropic and the only effect of changing the honeycomb's unit cell properties is simultaneously to scale all three permittivity components [5].

An auxetic (i.e., negative Poisson's ratio) honeycomb is a cellular structure which has superior mechanical properties compared to a conventional honeycomb. It has a re-entrant unit cell structure (i.e., the facet lengths and internal angles are unequal) which produces a stiffening effect that enhances the honeycomb's structural properties. The re-entrant geometry increases the indentation resistance and buckling loading of a honeycomb [7]. Combinations of unit cell geometric parameters also allow significant increases in the dynamic stiffness of a honeycomb sandwich plate [8]. A consequence of structural stiffening is a lowering of the acoustic cut-off frequency—a property which facilitates the absorption of acoustic waves [9]. The TSM (transverse shear modulus) of a honeycomb is one of the most important parameters in the bending behavior of sandwich structures (the TSM is a measure of the honeycomb's resistance to shear forces). Auxetic honeycombs exhibit an increase in TSM of 2.5–3.4 times that of a conventional honeycomb [10].

In this letter, consideration is given to the electromagnetic properties of re-entrant honeycombs.

II. HONEYCOMB UNIT CELL

Fig. 1 shows the unit cell of a honeycomb material whose geometry is re-entrant and whose properties are defined completely by θ, α, β, $\varepsilon_{\mathrm{base}}$ and w. It is assumed here that the unit cell is constructed from a material which is nonmagnetic ($\mu_r = 1$), homogeneous and isotropic. The complex permittivity of the unit cell material is $\varepsilon_{\mathrm{base}}$. The unit cell of a conventional honeycomb is defined as $\alpha = 1$ (equal length facets) and $\theta = +\pi/6$ radians (equal internal angles) [1]. As a consequence of the honeycomb manufacturing process, the unit cell of a practical conventional honeycomb may contain two double thickness facets. The presence of double thickness facets affects the honeycomb's effective permittivity; however, the underlying

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electromagnetic features of the permittivity tensor remain unaltered.

III. ELECTROMAGNETIC PROPERTIES OF RE-ENTRANT DIELECTRIC HONEYCOMBS

The effective permittivity $\bar{\bar{\varepsilon}}$ of a honeycomb has the form [3]

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}. \tag{1}$$

In a conventional honeycomb, α and θ are fixed and the effective permittivity is governed only by β (a measure of the amount of material in the unit cell) and $\varepsilon_{\text{base}}$. Provided β and $\varepsilon_{\text{base}}$ are not large, the values of ε_x, ε_y and ε_z are linearly related to β and $\varepsilon_{\text{base}}$ [5]. The effect on a conventional honeycomb of increasing β or $\varepsilon_{\text{base}}$ is simultaneously to increase ε_x, ε_y and ε_z; ε_x, ε_y and ε_z cannot be varied independently. In a re-entrant honeycomb, θ and α are not fixed. From a structural design viewpoint, θ and α can be chosen to enhance a honeycomb's structural performance [7]. From the electromagnetic design viewpoint, control of θ and α facilitates a much greater control over the honeycomb's effective permittivity than is possible in a conventional honeycomb.

A finite-difference time-domain (FDTD) based method utilizing periodic boundary conditions has been developed to predict the effective permittivity of a dielectric honeycomb [5]. The reflection coefficient is predicted of an infinite planar layer of dielectric honeycomb subject to planewave illumination from three mutually orthogonal directions. The reflection data are then inverted to obtain data on the honeycomb's effective permittivity. The data inversion process is based on a conventional planewave transmission line model of the honeycomb.

The FDTD method has been used to predict the effective permittivity tensor of a typical re-entrant honeycomb. The effective permittivity data are shown in Fig. 2 for two values of β. These data illustrate the two fundamental ways in which the effective permittivity is affected by the unit cell parameters θ, α and β. The parameter w defines the physical dimensions of the unit cell; w therefore dictates the frequency at which the honeycomb fails to homogenize. The dominant effect seen in Fig. 2 of the density parameter β is to induce a scaling of all three permittivity components: this is also true of a conventional honeycomb [5]. β can therefore be manipulated to scale the permittivity values; however, little control is possible over the relative differences between ε_x, ε_y and ε_z. The value of $\varepsilon_{\text{base}}$ used in Fig. 2 is $4 - j0$. The effects of changes to $\varepsilon_{\text{base}}$ and β on effective permittivity are similar: changes in $\varepsilon_{\text{base}}$ induce only a scaling of ε_x, ε_y and ε_z (e.g., further use of the FDTD model has shown the data corresponding to $\beta = 0.2$ in Fig. 2 can be reproduced approximately with $\beta = 0.12$ if $\varepsilon_{\text{base}}$ is increased to $7.93 - j0$).

The data in Fig. 2 indicate that the dominant electromagnetic effect of the geometry parameters θ and α is to determine the relative differences between ε_x, ε_y and ε_z. Thus, compared with a conventional honeycomb, a broader range of effective permittivity values can be obtained from a re-entrant geometry. Moreover, values of θ and α exist which cause a re-entrant honeycomb to be biaxially anisotropic.

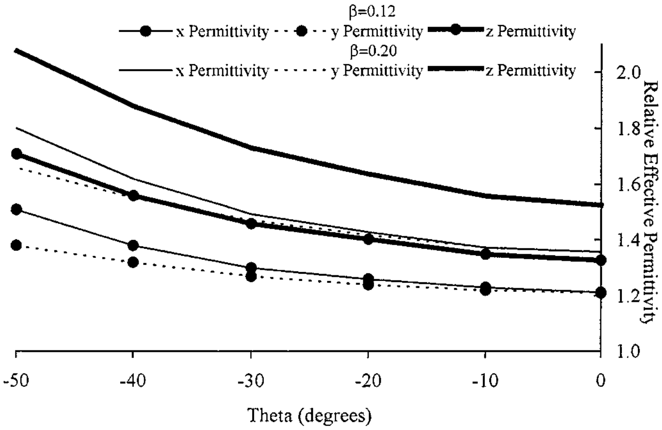

Fig. 2. Predicted effective permittivity of a re-entrant honeycomb for various values of θ and β($\alpha = 2.4$ and $\varepsilon_{\text{base}} = 4 - j0$).

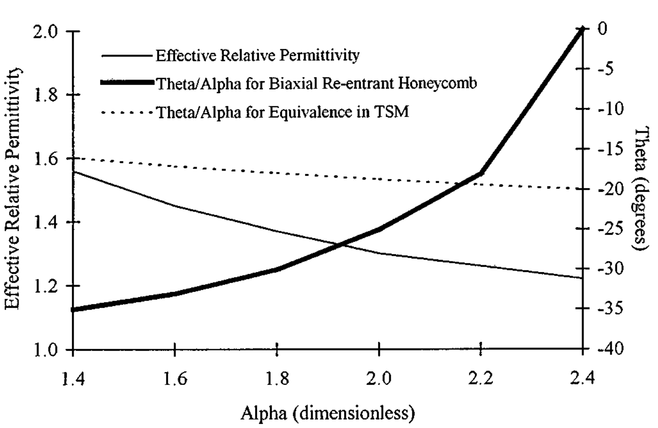

Fig. 3. Predicted values of θ and α necessary to produce a biaxial re-entrant honeycomb, and the effective permittivity of the biaxial honeycomb (ε_x and ε_y, assuming $\beta = 0.12$ and $\varepsilon_{\text{base}} = 4 - j0$). The values of θ and α which yield a re-entrant honeycomb with the same TSM as a conventional honeycomb are shown. The TSM of a biaxial re-entrant honeycomb is superior to the TSM of a conventional honeycomb below the equivalence line (i.e., for $\alpha < 2.18$).

A biaxial characteristic can be beneficial in a number of honeycomb applications. The A sandwich, consisting of a low permittivity core placed between two higher permittivity skins, is widely used in lightweight radar- transparent structures. In principle, radome errors can be quantified and corrections incorporated into the radar control software; in practice, however, the presence of a uniaxial anisotropy in the radome layers can make such corrections difficult and expensive to achieve. A biaxial radome characteristic greatly simplifies the correction process. A biaxial honeycomb characteristic can also be beneficial when honeycombs are used as part of a LO treatment. Most LO applications require absorption which is polarization-independent, the design and manufacture of a honeycomb absorber is simplified if two or more components of the honeycomb's effective permittivity are equal.

The values of θ and α in Fig. 2, which give rise to biaxial properties, are $\alpha = 2.4$ and $\theta = 0°$. The TSM of a honeycomb with $\alpha = 2.4$ and $\theta = 0°$ is poor (approximately 23% lower than a conventional honeycomb [10]); however, biaxial properties can also be achieved with negative values of θ with corresponding improvements in structural integrity. Fig. 3 shows the predicted θ/α pairs necessary to obtain a biaxial characteristic. Fig. 3 also shows the corresponding effective permittivity and a comparison between the TSM of conventional and biaxial honeycombs (after [1] and [7]). The parameters β and $\varepsilon_{\text{base}}$ affect a honeycomb's effective permittivity and absolute value of TSM; however, they do not affect the values of θ and α which give rise to a biaxial re-entrant honeycomb, or the equivalence values of θ and α in Fig. 3.

IV. MEASUREMENT OF THE EFFECTIVE PERMITTIVITY OF A RE-ENTRANT DIELECTRIC HONEYCOMB

Measurements have been performed on a re-entrant honeycomb in order to verify the predicted data in Figs. 2 and 3. The test honeycomb has the properties $\alpha = 1.102$, $\theta = 0.163$ rads, $\beta = 0.334$ and $w = 2.3$ mm and is constructed from cast epoxy resin with $\varepsilon_{\text{base}}$ (measured at 3.3 GHz) $= 3.15 - j0.14$. The properties of the test honeycomb were chosen to simplify sample manufacture; the properties were not chosen for their electromagnetic or mechanical characteristics. Measurements of ε_x, ε_y, and ε_z have been performed in rectangular waveguide between 2.60 and 3.95 GHz using the technique outlined in [11]. Three samples were measured corresponding to the three mutually orthogonal honeycomb orientations. The sample thicknesses were 12.1 mm (ε_x and ε_y data) and 24.2 mm (ε_z data). Provided the honeycomb is nonmagnetic, and provided the honeycomb's axes (these are illustrated in Fig. 1) are aligned with the principal axes of the waveguide, conventional parameter measurement techniques (e.g., [11]) can be used to determine experimentally the permittivity tensor. The measured effective permittivity tensor data are shown in Fig. 4, along with the predicted effective permittivity data from the planewave FDTD model. The measured and predicted data for ε_x and ε_y are in close agreement. The measured ε_z data exhibit an increase at the higher end of the measurement band; a characteristic that is consistent with failure of the composite to homogenize [5].

V. CONCLUSIONS

The electromagnetic properties of a re-entrant honeycomb have been reported. The electromagnetic benefits of re-entrant honeycombs, compared to conventional honeycombs, rest on the fact that much greater control over the values of ε_x, ε_y and $\varepsilon_z$is possible. Furthermore, the special case $\varepsilon_x = \varepsilon_y$ (transverse isotropy) coincides with mechanical properties which are superior to those of a conventional honeycomb. This special case is important in applications related to structural LO treatments and electromagnetic windows.

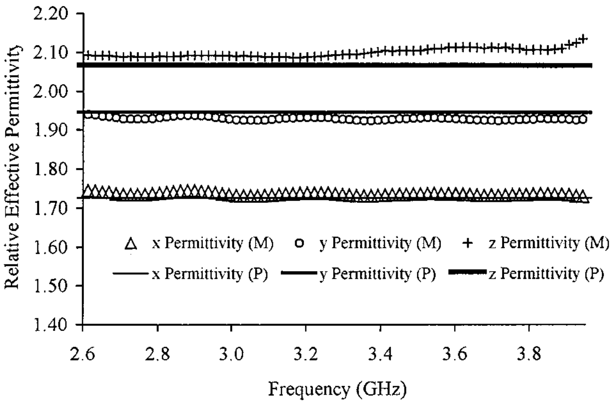

Fig. 4. Measured (M) and predicted (P) real part of effective permittivity of a re-entrant honeycomb constructed from cast epoxy resin. The measured parameters of the honeycomb's unit cell are: $\theta = 9.345$ degrees, $\alpha = 1.102$, $\beta = 0.334$, $w = 2.58$ mm and $\varepsilon_{\text{base}} = 3.15 - j0.14$ (at 3.3 GHz).

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