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Rational Model Data Smoothers and Identification Algorithms

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February 12, 1996

Research Report No. 618

Abstract

This study presents a new algorithm for nonlinear rational model identification. The new algorithm consists of a two-step procedure: a nonlinear rational function smoother is initially designed and used to smooth the data, system identification is then performed based on the smoothed signal. By using the smoothed signal instead of the raw data, the severe noise problems which arise in rational model identification are avoided. The new approach significantly simplifies the procedure for dynamic nonlinear rational model identification, compared with earlier estimators, and provides unbiased estimates with the same degree of accuracy.

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This study presents a new algorithm for nonlinear rational model identification. The new algorithm consists of a two-step procedure: a nonlinear rational function smoother is initially designed and used to smooth the data, system identification is then performed based on the smoothed signal. By using the smoothed signal instead of the raw data, the severe noise problems which arise in rational model identification are avoided. The new approach significantly simplifies the procedure for dynamic nonlinear rational model identification, compared with earlier estimators, and provides unbiased estimates with the same degree of accuracy.

1 Introduction

Nonlinear rational models, defined as the ratio of two nonlinear polynomials, have received considerable attention in the past few years. Compared to polynomial models, nonlinear rational models are able to approximate mathematical functions to the same degree of accuracy with a smaller number of terms. The form of rational functions also allow the representations of curves which approach asymptotes whereas ordinary polynomials generally do not have this property (Ratkowsky 1987, Ponton 1993). The properties of static rational function models also carry over to dynamic nonlinear rational models (Billings and Chen, 1989a). Despite the advantages of rational function models, the identification of these models is very difficult because the rational model is nonlinear in the input, output, noise and, especially, the parameters. The Marquardt nonlinear least squares (Marquardt, 1963) and the prediction error algorithm (Billings and Chen, 1989a) can be applied, but both these are computationally expensive. An alternative approach is to multiply out the rational model to form a linear-in-the-parameters expression. But multiplying out the model induces a severe noise problem which induces bias even for additive white noise. Billings and Zhu (1991, 1994a), Zhu and Billings (1993) have shown how this problem will produce biased estimates if least squares algorithms are used directly, and demonstrated that the bias can be removed by reformulating the least squares solution.

The present study introduces a new algorithm for nonlinear rational model identification which consists of two steps. A nonlinear rational function smoother is estimated with no *a prior* information based on past, present and future raw data samples about the current time step. Data smoothing is then performed and tests are introduced to determine when the noise has been acceptably suppressed. Structure detection and parameter estimation can then be applied to the smoothed data to yield unbiased estimates of dynamic nonlinear rational models in a manner which avoids all the complexity associated with previous estimators such as the Rational Model Estimator (RME).

The paper is organized as follows: The noise problem induced by the rational model and a new solution are outlined in Section 2. In Section 3.1 a new nonlinear rational function smoother is derived and a procedure for data smoothing is discussed in Section 3.2. Parameter estimation and model structure detection based on the smoothed data are discussed in Section 3.3. Numerical simulation examples are presented in Section 4 to test the performane of the new approach.

2 The noise problem in dynamic rational model identification and a new solution

Consider the dynamic nonlinear rational model given by:

$$
\begin{aligned}
y(k) &= \frac{F_1[y(k-1)\ldots y(k-n_{ny}), u(k-1)\ldots u(k-n_{nu}), e(k-1)\ldots e(k-n_{ne})]}{F_2[y(k-1)\ldots y(k-n_{dy}), u(k-1)\ldots u(k-n_{du}), e(k-1)\ldots e(k-n_{de})]} + e(k) \\
&= \frac{\sum_{j=1}^{nnum} \varphi_{nj}(k)\theta_{nj}}{\sum_{j=1}^{nden} \varphi_{dj}(k)\theta_{dj}} + e(k) \qquad (1)
\end{aligned}
$$

where $u(k)$ and $y(k)$ denote the input and output at time $k(k = 0, 1, \ldots)$ respectively, $\{e(k)\}$ is the unobservable independent noise sequence with zero mean and finite variance σ^2, $F_1(\bullet)$ and $F_2(\bullet)$ are nonlinear polynomials, $n_\bullet$ denotes the order, $\varphi_\bullet(k)$ and $\theta_\bullet$ denote the regressor and the parameters respectively.

Expresson (1) is nonlinear in the parameters and the ordinary least squares estimation algorithm can not be used directly. Multiplying out the model to form a linear in the parameters expression, yields

$$
Y(k) = F_1(\bullet) - y(k)[F_2(\bullet) - \varphi_{d1}(k)\theta_{d1}] + F_2(\bullet)e(k)
$$

$$
\begin{aligned}
&= \sum_{j=1}^{nnum} \varphi_{nj}(k)\theta_{nj} - \sum_{j=2}^{nden} \varphi_{dj}(k)y(k)\theta_{dj} + \xi(k) \\
&= \phi(k)\Theta + \xi(k) \qquad (2)
\end{aligned}
$$

where:

$$Y(k) = y(k)\varphi_{d1}(k)\theta_{d1}\,|_{\theta_{d1}=1}$$

$$\xi(k) = F_2(\bullet)e(k)$$

$$\Theta = [\theta_{n1} \ldots \theta_{nnnum}\;\; \theta_{d2} \ldots \theta_{dnden}]^T$$

$$\phi(k) = [\varphi_{n1}(k) \ldots \varphi_{nnnum}(k),\;\; -\varphi_{d2}(k)y(k) \ldots - \varphi_{dnden}(k)y(k)]$$

These expressions show that multiplying out the model induces a severe noise problem, the regressor $\phi(k)$ correlates with the current residue $\xi(k)$, and this correlation will result in biased estimates if the ordinary least squares algorithm is used directly. Notice that unlike the case for linear or polynomial nonlinear models the bias is induced even when $\xi(k)$ is zero mean and white. To overcome the problem, Billings and Zhu (1991) reformulated the least squares algorithm by taking the correlation into consideration (Billings and Zhu 1991, 1994a, Zhu and Billings 1993). The disadvantage of this method however was the need for an iterative procedure which involved essentially estimating and removing the bias terms induced by the current residue $\xi(k)$.

Consider a new solution to the noise problem.

From eqn (2)

$$
\begin{aligned}
Y(k) &= \sum_{j=1}^{nnum} \varphi_{nj}(k)\theta_{nj} - \sum_{j=2}^{nden} \varphi_{dj}(k)y(k)\theta_{dj} + \sum_{j=1}^{nden} \varphi_{dj}(k)\theta_{dj}e(k) \\
&= \sum_{j=1}^{nnum} \varphi_{nj}(k)\theta_{nj} - \sum_{j=2}^{nden} \varphi_{dj}(k)[\overline{y}(k) + e(k)]\theta_{dj} + \sum_{j=1}^{nden} \varphi_{dj}(k)\theta_{dj}e(k)
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{j=1}^{nnum} \varphi_{nj}(k)\theta_{nj} - \sum_{j=2}^{nden} \varphi_{dj}(k)\overline{y}(k)\theta_{dj} + \varphi_{d1}(k)e(k) \\
&= \phi_{free}(k)\Theta + \xi_1(k) \qquad (3)
\end{aligned}
$$

where

$$\phi_{free}(k) = [\varphi_{n1}(k)\ldots\varphi_{nnnum}(k),\ -\varphi_{d2}(k)\overline{y}(k)\ldots - \varphi_{dnden}(k)\overline{y}(k)]$$

$$\xi_1(k) = \varphi_{d1}(k)e(k)$$

$$\overline{y}(k) = y(k) - e(k) = \frac{F_1(\bullet)}{F_2(\bullet)}$$

Removing the $\xi_1(k)$ to the left side of equation (3), yields

$$\overline{Y}(k) = \phi_{free}(k)\Theta \qquad (4)$$

where

$$\overline{Y}(k) = \varphi_{d1}(k)\overline{y}(k)$$

$\overline{y}(k)$ is the current time noise $e(k)$ free part of $y(k)$. If $\overline{y}(k)$ can be extracted from the noisy output $y(k)$ beforehand and the parameter estimator is formulated based on the regressive equation (4), the noise problem arising from multiplying out the model will be solved automatically. The above consideration motivates the development in the next section where a new nonlinear rational function signal smoother is designed and used to extract $\overline{y}(k)$ from $y(k)$ before the identification stage.

3 Nonlinear rational model signal smoothers

3.1 The nonlinear rational smoother

One way of eliminating or reducing unwanted noise in a signal is by filtering or smoothing. Several kinds of filter or smoother are available, for example, the linear autoregressive (AR) filter, linear moving average (MA) filter, linear finite-duration impulse response (FIR) filter and the nonlinear polynomial filter. However, the aforementioned algorithms are in general insufficient to process the signal produced from a nonlinear rational model, and if used the dynamic characteristics of the original signal may be distorted. Recently nonlinear polynomial smoothers (Aguirre and Billings 1995) have been shown to be much more efficient than filters. This suggests that nonlinear rational smoothers will be more appropriate than rational filters for the noise problem above because they use past, present and future information. The above consideration motivates the search for a non-causal nonlinear rational smoother which can be estimated and then used as a signal preprocessor. Consider

$$y_s(k) = \frac{f_1[y(k+n_{sny})\dots y(k-n_{sny}), u(k+n_{snu})\dots u(k-n_{snu}), e(k-1)\dots e(k-n_{sne})]}{f_2[y(k+n_{sdy})\dots y(k-n_{sdy}), u(k+n_{sdu})\dots u(k-n_{sdu}), e(k-1)\dots e(k-n_{sde})]}$$

where $y_s(k)$ denotes the smoothed output and $n_\bullet$ denotes order. Notice that the terms on the rhs of the model above include both positive and negtive lags and span a time window which includes past and future terms about the current sample instant. Because $e(k-i) = y(k-i) - y_s(k-i)$, the employed smoother is of the form

$$\begin{aligned} y_s(k) &= \frac{f_{sn}[y(k+n_{sy})\dots y(k-n_{sy}), u(k+n_{su})\dots u(k-n_{su}), y_s(k-1)\dots y_s(k-n_{se})]}{f_{sd}[y(k+n_{sy})\dots y(k-n_{sy}), u(k+n_{su})\dots u(k-n_{su}), y_s(k-1)\dots y_s(k-n_{se})]} \\ &= \frac{\sum_{j=1}^{n_s} \varphi_{snj}(k)\theta_{snj}}{\sum_{j=1}^{n_s} \varphi_{sdj}(k)\theta_{sdj}} \end{aligned} \tag{5}$$

where

$$n_{sy} = \max\{n_{sny}, n_{sdy}, n_{sne}, n_{sde}\}$$

$$n_{su} = \max\{n_{snu}, n_{sdu}\}$$

$$n_{se} = \max\{n_{sne}, n_{sde}\}$$

$$n_s = n_{sy} + n_{su} + n_{se}$$

Multiplying out the model to form a linear-in-the-parameters expression, yields

$$\begin{aligned} Y_s(k) &= \sum_{j=1}^{n_s} \varphi_{snj}(k)\theta_{snj} - \sum_{j=2}^{ns} \varphi_{sdj}(k)y_s(k)\theta_{sdj} \\ &= \phi_s(k)\Theta_s \end{aligned} \tag{6}$$

where

$$Y_s(k) = \varphi_{sd1}(k)\theta_{sd1}y_s(k)|_{\theta_{d1s}=1}$$

$$\Theta_s = [\theta_{sn1} \dots \theta_{snn_s}, \theta_{sd2} \dots \theta_{sdn_s}]^T$$

$$\phi_s(k) = [\varphi_{sn1}(k) \dots \varphi_{snn_s}(k), \ \varphi_{sd2}(k)y_s(k) \dots \varphi_{sdn_s}(k)y_s(k)]$$

It should be realized that both sides of equation (6) contain $y_s(k)$. If the initial value of $y_s(k)$ is not close to $\overline{y}(k)$, the smoother which is estimated based on eqn (6) will amplify the error. This problem can be overcome by estimating the smoother based on the following equation

$$Y(k) = \phi_s(k)\Theta_s + \xi_s(k) \tag{7}$$

where

$$\xi_s(k) = \varphi_{sd1}(k)e(k) = \varphi_{sd1}(k)[y(k) - y_s(k)] = Y(k) - Y_s(k)$$

The coeficients of the nonlinear rational smoother can then be obtained by minimizing the following cost function:

$$J_1(N, \Theta_s) = \frac{1}{N}\sum_{k=1}^{N}[Y(k) - \phi_s(k)\Theta_s]^2 \tag{8}$$

In theory, when the sample number N goes to infinity the estimate of Θ_s obtained by minimizing eqn (8) will approach the true value. In practice, however, only finite samples are available and the estimates might be far from the true values when the data are very noisy. Recently, regularization was introduced to improve the estimation accuracy (Hoerl and Kennard 1970, Barron and Xiao 1991, Bishop 1991, Orr 1995) and this approach can be used to advantage in the rational smoother design. The regularized solution of Θ_s is defined as the value which minimises

$$J_2(N, \lambda_N, \Theta_s) = \frac{1}{N}\sum_{k=1}^{N}[Y(k) - \phi_s(k)\Theta_s]^2 + \Theta_s^T\lambda_N\Theta_s \tag{9}$$

where λ_N =diag$\{\lambda_N^i,\ i = 1, 2\ldots 2n_s - 1\}$, $\lambda_N^i > 0$ is a scalar regularization parameter. Optimizing $J_2(\bullet)$ with respect to the unknown smoother parameters Θ_s, yields

$$\hat{\Theta}_s = [\frac{1}{N}\sum_{k=1}^{N}\phi_s^T(k)\phi_s(k) + \lambda_N]^{-1}\frac{1}{N}\sum_{k=1}^{N}\phi_s^T(k)Y(k) \tag{10}$$

Obviously, when a constant is used as the regularization parameter, the estimates will be biased. Taking the two aspects of unbiased estimates and data overfitting into consideration, the regularization parameter should be selected to satisfy

i. λ_N^i decreases with increasing N, and approaches zero when N goes to infinity.

ii. λ_N^i decreases at a reasonable rate so that its effect is not vanishing for a finite N.

The first condition guarantees that the estimates are unbiased and the second condition ensures that the regularization improves estimation accuracy.

Once the rational smoother parameter estimates have been determined, the smoothed signal can then be obtained from

$$y_s(k) = \frac{\sum_{j=1}^{n_s} \varphi_{snj}(k)\hat{\theta}_{snj}(k)}{\sum_{j=1}^{n_s} \varphi_{sdj}(k)\hat{\theta}_{sdj}(k)} \tag{11}$$

3.2 Signal smoothing

In order to produce a smoothed signal $y_s(k)$ which approaches the desired signal $\overline{y}(k)$ the signal has to be passed through the smoother eqn (11) several times. This procedure can be summarized as follows

i. Define the structure of rational function smoother including n_{su}, n_{sy}, n_{se} and the degree of nonlinearity n_{sl}.

ii. Initially set $y_s(k) = y(k)$ and estimate the smoother. Smooth the data to produce $y_s(k)$.

iii. Estimate a new smoother using the smoothed data, and then produce the new smoothed data set $y_s(k)$.

iv. Test the validity of the estimated smoother. Two tests based on higher order cross-correlation functions between the output and residuals and between the output, residuals and input recently introduced (Billings and Zhu 1994b) are used. These two tests are

$$cor_{\epsilon\xi_s^2}(\tau) = \frac{\sum_{k=1}^{N}(\epsilon(k)-\overline{\epsilon})(\xi_s^2(k-\tau)-\overline{\xi_s^2})}{\left[\left(\sum_{k=1}^{N}(\epsilon(k)-\overline{\epsilon})^2\right)\left(\sum_{k=1}^{N}(\xi_s^2(k)-\overline{\xi_s^2})^2\right)\right]^{0.5}} \tag{12}$$

and

$$cor_{\epsilon u^2}(\tau) = \frac{\sum_{k=1}^{N}(\epsilon(k)-\overline{\epsilon})(u^2(k-\tau)-\overline{u^2})}{\left[\left(\sum_{k=1}^{N}(\epsilon(k)-\overline{\epsilon})^2\right)\left(\sum_{k=1}^{N}(u^2(k)-\overline{u^2})^2\right)\right]^{0.5}} \tag{13}$$

where

$$\epsilon(k) = y(k)\xi_s(k)$$

$$\bar{\epsilon} = \frac{1}{N}\sum_{k=1}^{N}\epsilon(k)$$

$$\overline{u^2} = \frac{1}{N}\sum_{k=1}^{N}u^2(k)$$

$$\overline{\xi_s^2} = \frac{1}{N}\sum_{k=1}^{N}\xi_s^2(k)$$

If the residue satisfies

$$cor_{\epsilon\xi_s^2}(\tau) = \begin{cases} k_1 > 0 & \text{if } \tau = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$cor_{\epsilon u^2}(\tau) = 0 \quad \text{for any } \tau.$$

the estimated smoother is considered to be adequate to process the data and $y_s(k) = \overline{y}(k)$, otherwise steps iii-iv are repeated.

In practice, the 95% confidence limits at approximately $1.96/\sqrt{N}$ are used to determine if the smoothed data is valid.

Remark

In system identification, under the parsimonious condition the final model is often required to be as concise as possible. Therefore model structure detection is considered as an important step. Compared with the final model, the structure of the smoother is less important because the smoother is only an intermediate result which is used to preprocess the data. In addition, structure detection can not easily be carried out before the residue satisfies the validity tests, thus structure detection is not important for data smoothering. This does not however mean that the structure of the smoother can be arbitrarily large. In practice, small values of n_{sy}, n_{su}, n_{se} should initially be selected and tried.

3.3 Parameter estimation and model structure detection

Once an estimate of $\overline{y}(k)$ has been obtained using smoothed data, the identification including parameter estimation and model structure detection of the rational model can be carried out based on eqn (4) using the identification algorithms developed for nonlinear polynomial models. This includes the orthogonal least squares estimator and associated error reduction ratio for structure detection. Details of identifying nonlinear polynomial models are not repeated here, see for example Billings and Chen (1989b,1989c).

A comparison of the above identification algorithm with RME based algorithms (Billings and Zhu 1991, 1994a, Zhu and Billings 1993), shows that the new algorithm significantly simplifies the parameter estimation and model structure detection procedures of the nonlinear rational model. This simplification arises because the severe bias problem which is normally induced by the $e(k)$ term in the rational model of eqn (1) has effectively been removed by the rational data smoother.

4 Numerical simulations

The identification algorithm based on smoothed data will be demonstrated using two examples. In both examples, 500 data samples were used, the input was a uniformly distributed random excitation with amplitute ± 1 and the noise was a normally distributed random disturbance with zero mean and variance 0.01.

Example 1

Consider a dynamic nonlinear rational model given by

$$y(k) = \frac{0.5y^3(k-1) - 0.1u^2(k-1)u(k-2) + u(k-2) + e(k-2)}{1 + u^2(k-1) + 0.7u^3(k-1)} + e(k) \quad (14)$$

Multiplying out the rational model to form a linear-in-the-parameters form, yields

$$\begin{aligned} y(k) &= 0.5y^3(k-1) - 0.1u^2(k-1)u(k-2) + u(k-2) + e(k-2) \\ &\quad -u^2(k-1)y(k) - 0.7u^3(k-1)y(k) + \xi(k) \end{aligned} \quad (15)$$

where

$$\xi(k) = [1 + u^2(k-1) + 0.7u^3(k-1)]e(k)$$

Directly applying the ordinary least squares algorithm to estimate the parameters in eqn (15) (assuming the model structure is known *a priori*), produced the parameter estimates shown in Table 1. The estimates are far from the true values even using the correct model structure. This is because the ordinary least squares estimates are biased due to the correlation between $\xi(k)$ and the regressors $u^2(k-1)y(k)$ and $u^3(k-1)y(k)$ in eqn (15).

numerator polynomial	true value	estimate
$y^3(k-1)$	0.5	0.2922
$u^2(k-1)u(k-2)$	-0.1	-0.8034
$u(k-2)$	1	0.9277
$e(k-2)$	1	0.6596
denominator polynomial	true value	estimate
$u^2(k-1)$	1	-0.5363
$u^3(k-1)$	0.7	0.1738

Table 1: Ordinary least squares parameter estimates for Example 1

Eqn (15) can also be written as

$$\begin{aligned} y(k) &= 0.5y^3(k-1) - 0.1u^2(k-1)u(k-2) + u(k-2) + e(k-2) \\ &\quad -u^2(k-1)\overline{y}(k) - 0.7u^3(k-1)\overline{y}(k) + e(k) \end{aligned} \quad (16)$$

where

$$\overline{y}(k) = y(k) - e(k)$$

Applying the new algorithm to this problem a rational function smoother was initially designed to smoothed the data prior to identification. The procedures summarized in Section 3.2 were applied where the smoother structure was defined as follows: lags $n_{su} = 2$, $n_{sy} = 2$ and $n_{se} = 2$ and the degree of nonlinearity $n_{sl} = 3$. After 15 iterations, the validation tests in Step (iv) were satisfied. The smoothed signal $y_s(k)$, and the error $\overline{y}(k) - y_s(k)$ after 15 iterations were shown in Fig.1 (only 100 samples are shown).

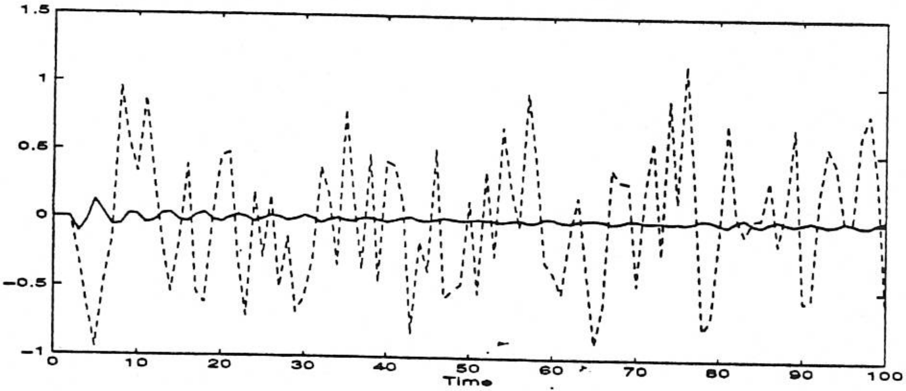

Figure 1: The smoothed signal and error of Example 1
(dashed line—$y_s(k)$ solid line—$\overline{y}(k) - y_s(k)$)

Based on the smoothed data, parameter estimation and structure detection were applied to produce the estimated model in Table 2. No assumptions regarding the terms in the model were made. The initial search space was defined by setting $n_{ny} = n_{dy} = 2$, $n_{nu} = n_{du} = 2$ and $n_{ne} = n_{de} = 2$ and the degree of nonlinearity $n_l = 3$. These values define a model with 168 possible terms. The final model in Table 2 was obtained by using the polynomial model orthogonal estimator to select significant terms and estimate the unknown parameters. The model validity tests are shown in Fig.2.

Comparing the results in Tables 1 and 2 shows the vast improvement that has been made by using the smoothed data.

numerator polynomial	true value	estimate
$y^3(k-1)$	0.5	0.4780
$u^2(k-1)u(k-2)$	-0.1	-0.1070
$u(k-2)$	1	0.9835
$e(k-2)$	1	0.8995
denominator polynomial	true value	estimate
$u^2(k-1)$	1	0.8594
$u^3(k-1)$	0.7	0.6042

Table 2: Parameter estimates based on the smoothed data for Example 1

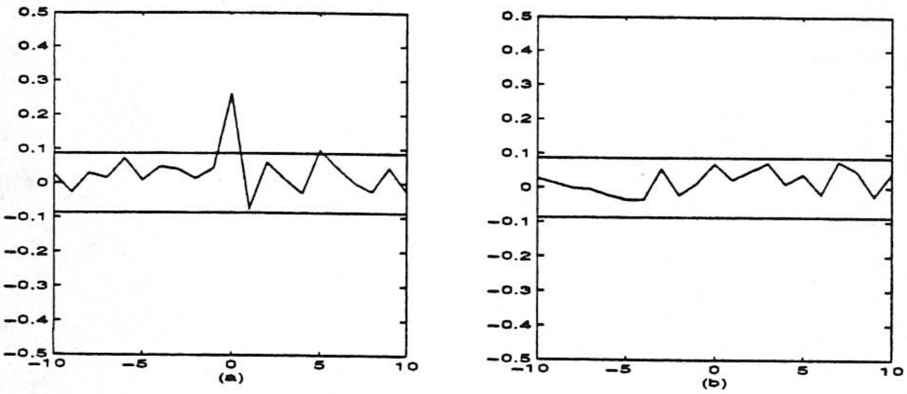

Figure 2: Model validity tests of Example 1 (a —$cor_{\epsilon\xi^2}$ b—$cor_{\epsilon u^2}$)

Example 2

$$y(k) = \frac{y^2(k-1) + u(k-1) + e(k-1)}{1 + y^2(k-1) + y(k-1)u(k-1)} + e(k)$$

Multiplying out the rational model to form linear-in-the-parameters expressions, yields

$$\begin{aligned} y(k) &= y^2(k-1) + u(k-1) + e(k-1) \\ &\quad -y^2(k-1)y(k) - y(k-1)u(k-1)y(k) + \xi(k) \qquad (17) \\ &= y^2(k-1) + u(k-1) + e(k-1) \\ &\quad -y^2(k-1)\overline{y}(k) - y(k-1)u(k-1)\overline{y}(k) + e(k) \qquad (18) \end{aligned}$$

where

$$\xi(k) = [1 + y^2(k-1) + y(k-1)u(k-1)]e(k)$$

$$\overline{y}(k) = y(k) - e(k)$$

The smoother structure was defined as follows: lags $n_{su} = 2$, $n_{sy} = 2$ and $n_{se} = 2$ and the degree of nonlinearity $n_{sl} = 2$. The procedures summarized in Section 3.2 were then applied. After 15 iterations, the validation tests were satisfied. The smoothed data $y_s(k)$, and the error $\overline{y}(k) - y_s(k)$ are shown in Fig.3 (only 100 samples are shown). Based on the smoothed data, the final model is listed in Table 3. The initial model structure was defined as $n_{ny} = n_{dy} = 2$, $n_{nu} = n_{du} = 2$ and $n_{ne} = n_{de} = 2$ and the degree of nonlinearity $n_l = 2$. These values define a model with 56 possible terms. The orthogonal estimator was used to both select the significant model terms and to estimate the unknown parameters. The model validity tests are shown in Fig.4.

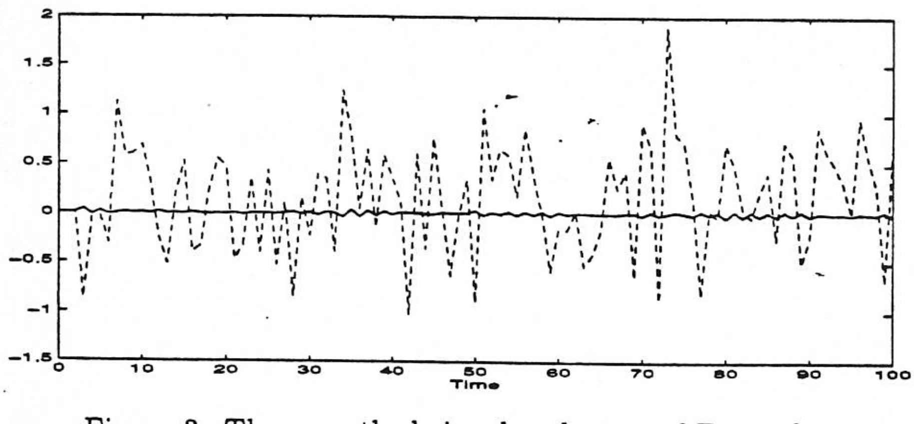

Figure 3: The smoothed signal and error of Example 2
(dashed line—$y_s(k)$ solid line—$\overline{y}(k) - y_s(k)$)

Directly applying the ordinary least squares algorithm to eqn (17), the parameter estimates using a correct model structure are shown in Table 4. Comparing the results in Tables 3 and 4 again shows the vast improvement that is achieved by using the smoothed data.

numerator polynomial	true value	estimate
$y^2(k-1)$	1	0.9679
$u(k-1)$	1	0.9800
$e(k-1)$	1	1.0501
denominator polynomial	true value	estimate
$y^2(k-1)$	1	0.9215
$y(k-1)u(k-1)$	1	0.9479

Table 3: Parameter estimates based on the smoothed data for Example 2

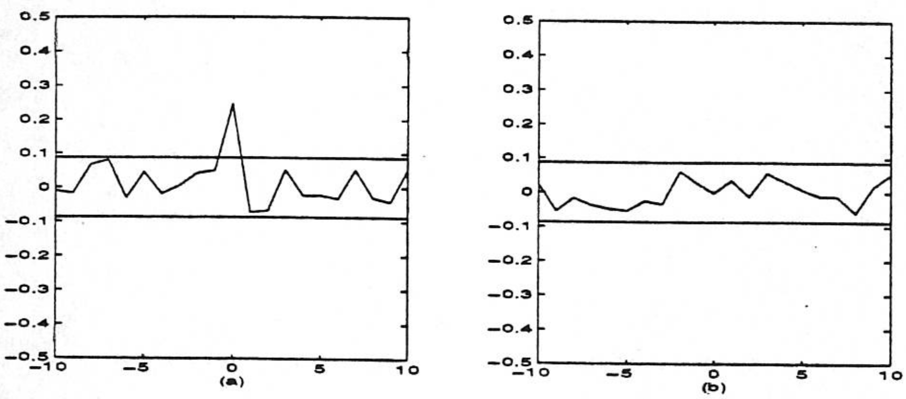

Figure 4: Model validity tests for Example 2 (a —$cor_{\epsilon\xi^2}$ b—$cor_{\epsilon u^2}$)

5 Conclusion

A new rational model smoothing algorithm has been introduced. The smoother is estimated without any *a priori* information from the raw data and is based on past, present and future sample values about the current time step. It has been shown that preprocessing the data using the smoother alleviates the severe noise problem in rational model estimation and allows all the well developed structure detection and parameter estimation algorithms of polynomial models to be applied to rational models. This provides, for the first time, the ability to fit complex nonlinear rational models, which have excellent approximation properties, using polynomial model algorithms.

numerator polynomial	true value	estimate
$y^2(k-1)$	1	0.4111
$u(k-1)$	1	0.8366
$e(k-1)$	1	0.8201
denominator polynomial	true value	estimate
$y^2(k-1)$	1	-0.0012
$y(k-1)u(k-1)$	1	0.0443

Table 4: Ordinary least squares parameter estimates for Example 2

6 Acknowledgement

SAB gratefully acknowledges that part of this work was supported by EPSRC.

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