How to share it out: The value of information in teams*

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Abstract

We study the role of information exchange, leadership and coordination in team or partnership structures. For this purpose, we view individuals jointly engaging in productive processes—a 'team'—as endowed with individual and privately held information on the joint production process. Once individual information is shared, team members decide individually on the effort they exert in the joint production process. This effort, however, is not contractible; only the joint output (or profit) of the team can be observed. Our central question is whether or not incentives can be provided to a team in this environment such that team members communicate their private information and exert efficient productive efforts on the basis of this communication. Our main result shows that there exists a simple ranking-based contract which implements both desiderata in a wide set of situations.

JEL: *C7, D7, D8, L2.*
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"Forming a business partnership is the next best thing to getting married." *The Manufacturing Jeweler*, March 1897

1 Introduction

Consider a patient who, following an accident, needs the urgent and intensive attention of a team of several experts (e.g., anesthetist, orthopedist, nurses etc). An initial diagnosis yields private information to each expert which needs to be pooled before surgery. During surgery itself, the individual, specialized expert efforts are not necessarily observe- or verifiable, especially when situations are considered where expertise does not overlap or procedures need to be performed in isolation. Hence, individual incentives may exist to provide sub-optimal efforts, in particular if coordinated with an initially misstated individual opinion on the patient's medical needs.[1] The joint outcome produced is the health status of the patient which is rewarded by (insurance company) payments which are shared among the team members. Our paper derives a rule for sharing this reward among team members in a way that ensures both truthful information exchange and the exertion of efficient unobservable efforts.

As another application, consider the global hedge fund industry which currently manages assets worth in excess of \$2.63 trillion (Hedge Fund Research, 2014). Despite this staggering amount, the economics literature has to date paid little attention to the incentives which motivate the individu-

[1] Popular press accounts of recent medical scandals exhibiting a mixed moral hazard and adverse selection flavor include Abelson & Creswell (2012) who document the systematic use of unnecessary procedures at HCA, the largest for-profit hospital chain in the United States. Armstrong et al. (2014) discuss regularly scheduled 'emergencies-by-appointment' in which patients were given appointments for emergency room treatment at New York Mount Sinai Hospital's catheterization lab. "The cath lab extols its volume; the most recent annual report described the growth in its number of procedures as 'remarkable,' 'substantial,' 'significant' and 'tremendous.' Hospital records show that the lab's compensation system for doctors incentivizes more procedures." Mehtsun et al. (2013) identify 9,744 US surgical 'never events'—serious, largely preventable professional mistakes which should never happen—between 1990 and 2010 resulting in malpractice payments of \$1.3 billion.

als who operate these funds.[2] We close this gap by developing a framework which represents the strategic environment in which fund managers or, more generally, teams of privately informed partners, do their work.

The organizational structure of partnerships (or, synonymously throughout the paper, teams) seems to fit the needs of the asset management industry well. Most hedge funds and investment firms in this industry are privately run as general/limited partnerships. 'Investment clubs' are partnerships in which a small number of members pool their resources to make joint investments. Other examples of investment partnerships, often without limited liability, include 'single family offices,' i.e., private companies that manage investments and trusts for a single, usually very wealthy, family. The main reason why the partnership structure is attractive in the mentioned cases is that the partners have 'skin in the game.' Hence, regulatory oversight is usually minimal because the few heavy-weight partners involved are generally trusted to exert due care in their investment decisions.[3]

The principal elements of our model are private information, unobservable efforts and team structure.[4] As organizations can be seen to exist precisely in order to resolve or process informational problems (Coase, 1937), the introduction of asymmetric information into what is otherwise a classical team production problem seems to be natural. To fix ideas, consider a situation in which one of the team members receives a private signal.[5] Our main result shows that a team remuneration scheme based on a ranking of partners'

[2] The incentives given to these individuals are not trivial: according to Vardi (2013), the top 40 hedge fund managers and traders earned a combined \$16.7 billion in 2012.

[3] The same partnership structure as inherent in our examples from the financial industry is also the predominant form of organizational governance in many other industries including accounting, law, architecture and others (Greenwood & Empson, 2003). We provide a discussion of further applications and examples in the concluding section of this paper.

[4] In team or partnership structures, partners share the profit among themselves. Thus, any incentive mechanism is subject to the constraint to balance the team's budget. Note that many other bilateral or multilateral contractual situations are also subject to the same (implicit) budget restriction (Spulber, 2009, p57, p97).

[5] In our basic model, we only consider a single team member with private information. We generalize this environment later, as part of our extensions, to an arbitrary number of privately informed team members. In terms of our motivating stories, the single informed 'leader' may be a medical specialist with a team of uninformed nurses or a hedge fund manager who obtains private information on market conditions.

efforts exists which can overcome this 'communications dilemma' and implement both efficient information sharing and subsequent efficient efforts. Hence, our mechanism indicates how partner incentives can be structured in order to avoid perverse incentives.

The paper's findings can be summarized as follows. Our main result shows that a profit sharing rule exists which subdivides realized team output unevenly among all team members in symmetric equilibrium. This rule ensures the communication of relevant private information by one team member—whom we call the 'team leader'—and subsequent efficient effort exertion by all team members (including the leader) although efforts are not assumed to be contractible. Moreover, the proposed profit sharing rule allocates the entire realized output among the team members and thus balances its budget in and out of equilibrium. The derived sharing rule depends on some statistic of exerted efforts on which team remuneration can be based, for instance, the precision of a contractible ranking of partners' efforts interpreted as a contest among team members.[6] This result is derived for a general environment only restricted by concave output (as a function of the sum of efforts) and convex effort costs.

The main element that our analysis adds to the literature and which allows for a positive solution to the combined problem of Holmström (1982) and Hermalin (1998) is the noisy ranking of team members' efforts which is the main novelty in the sharing rule that we describe above. This relative performance information seems to be regularly collected and naturally available as part of incentive schemes in many organizations (Lazear & Shaw, 2007). Moreover, since the required effort information is ordinal rather than cardinal, collecting these statistics represents a weaker informational requirement than embodied in standard piece-rate based contracts.

Given the classic results of Holmström (1982) for moral hazard in teams and Hermalin (1998) for the adverse selection leadership case, our positive result may be surprising because, in the combined problem, the profit shar-

[6] With respect to our first example involving medical experts, recent evaluations of (relative) performance pay in the medical professions include Ogunyemi et al. (2009), Bardach et al. (2013) and Himmelstein & Woolhandler (2014). Bonus payments based on competitions are also widespread in the financial services industries.

ing rule needs to address complex, twofold incentives. First, the privately informed leader is able to misrepresent her private information about joint productivity in order to deceive the other team members into providing inefficiently high (or low) efforts while planning to capitalize on this response through a low (or high) effort herself. The second incentive problem that the sharing rule must address is that, because of uncontractible efforts, the other team members may be tempted to 'free ride' by exerting inefficiently low efforts even though the leader provided correct information. Intuitively, the presented sharing rule can dissuade the leader from this behavior by making sure that, even for misrepresented private information, efficient effort provision given that report remains a best response for both the leader and the other team members. Since our profit sharing rule is explicitly constructed to guarantee this, a pair of misleading report and subsequent inefficient effort is not profitable. Consequently, as the leader has incentives to report her information truthfully, the other team members may base their response on this report which allows for a jointly efficient set of efforts. Our sharing rule is able to overcome this second problem through incorporating an appropriately structured contest among all team members which ensures that free riding incentives are counterbalanced with individual winning probabilities based on some statistic of players' efforts.

We subsequently are able to generalize this main result in three directions: *i*) for the case where the leader receives only a 'noisy signal' of the true productivity parameter, *ii*) the case of 'information pooling' in which any number of team members need to contribute their private information in order to make efficient production possible, and *iii*) the case of 'leading by example' in which the leader can exert either contractible or non-contractible upfront efforts. In all three extensions the precise formulation of the required sharing rule changes but our principal result, that full efficiency is implementable, is robust to these model variations.

In summary we present a general solution to the communication and coordination problem couched in a classic joint production problem among symmetric team members. We interpret our result as underpinning the emergence of a profit sharing rule as a function of private information which is, in our model, a required factor of the joint production process. If information or knowledge is dispersed, our model implements efficient cooperation between team members who voluntarily share their private information. This captures the process in which a team or partnership can integrate the specialist knowledge of its members as a precondition for subsequently overcoming the free rider problem.

The plan for the remainder of the paper is to first provide a short overview of the two main literatures unified by this paper followed by the model definition and the characterization of efficient efforts in section 2. Section 3 then presents our main result: the derivation of a general profit sharing rule which ensures the communication of private information through a single team leader and subsequent efficient effort provision by all team members. Section 4 proceeds to illustrate several extensions of the main model, i.e., full characterizations of the efficiency inducing sharing rule *i*) in the presence of stochastic signals, *ii*) when information is dispersed among multiple team members (not just the leader), and *iii*) for the case of 'leading by example' in which the leader exerts upfront observable efforts. Finally, this section discusses the conditions under which our sharing rule satisfies limited liability. In the concluding section, we offer a discussion of a further set of applications and examples centering on the ideas of leadership and coordination. The proofs of the main results can be found in appendix A. A proof ensuring the existence of the equilibrium we derive under a broad class of specifications can be found in appendix B, together with an example illustrating equilibrium existence in further cases.

Related Literature

The present paper combines the two distinct literatures on information-based leadership and moral hazard in teams into a unified contracting framework. Although questions and problems of leadership arise in many situations, remarkably few economic studies exist of the concept itself. The relevant leadership literature consists mainly of Hermalin (1998), Komai et al. (2007), Komai & Stegeman (2010), and Zhou (2011). The emphasis is here on a privately informed player who is communicating her information to others and participates in team production.[7] Hermalin (1998) is the first paper to study information-based leadership; he defines a leader as a team member who induces voluntary following by credibly transmitting private information. Through an observable sacrifice, an informed player can convince her partners to exert higher subsequent efforts than without. Similarly, if the leader's upfront investment is tangible, then 'leading by example' can mitigate the adverse selection problem. However, by using a fixed sharing rule of team output among the players, Hermalin (1998) solely concentrates on information revelation, he does not consider the free-riding problem. Komai et al. (2007) study partially revealing announcements by a leader and show that, in some circumstances, it is better to concentrate information with a single player rather than making it transparent among all players, thus providing a justification for the existence of information-based leadership. Komai & Stegeman (2010) broaden the study of leading-by-example games to binary participation choice and nonlinear utility functions. Zhou (2011) extends this information-based leadership framework to the study of organizational hierarchies. Recent and comprehensive surveys are Ahlquist & Levi (2011) and Hermalin (2012). Contrasting with this literature, we develop a ranking-based compensation scheme in which the prize structure depends on the level of team output and the leader's announcement of the state of the world. This mechanism encourages the leader to truthfully reveal her private information while at the same time eliminating the free-riding incentive of all team members.

A related and structurally similar setup is used in the classical literature on moral hazard in teams as discussed, for instance, in Alchian & Demsetz (1972) and Holmström (1982). Most of this literature focuses solely on mitigating the free-riding incentives under the constraints of budget balance.

[7] Other theoretical models on leadership typically adopt a principal-manager-agent approach in which the manager takes aspects of the role of leaders in our model. See, for example, Rotemberg & Saloner (1993), Blanes i Vidal & Möller (2007), and Bolton et al. (2012). There, the manager's major function is to choose a mission/project and coordinate the subordinates. In our setting, if the leader does not participate in production, a lump sum payment to the leader and a tournament for the other team members as in Gershkov et al. (2009) implements efficiency.

Examples include Legros & Matthews (1993), Battaglini (2006), Kvaløy & Olsen (2006), and Bonatti & Hörner (2011).[8] We extend the pure moral hazard problem studied by Gershkov et al. (2009) by introducing incomplete information. Their contract is based on a partial but verifiable ranking of agents' efforts and implements efficiency. This literature is distinct from the Principal-Agent framework because of the absence of a principal in our model and the implied requirement for the budget to balance.

There has been intensive interest in combined adverse selection and moral hazard problems. See Guesnerie et al. (1989) for a comprehensive review of the early literature. In a repeated setting, Rahman (2012) characterizes an optimal contract if the monitor's observations are private and costly. Gershkov & Perry (2012) characterize optimal contracts in a dynamic principal-agent setting with moral hazard and adverse selection (persistent as well as repeated).

2 The model

There is a set $\mathcal{N}$ of $n \geq 2$ symmetric, risk-neutral players. Each player $i \in \mathcal{N}$ exerts efforts $e_i \in [0, \infty)$ which need not, in principle, be verifiable. Effort costs $c(e_i)$ are assumed to be strictly convex with $c(0) = 0$ and $c'(0) = 0$. Efforts generate increasing and concave team output of $y(\alpha, e_1 + \cdots + e_n)$ which depends on the sum of players' efforts and the realization of some random variable $\alpha \sim H_{[a,b]}$, with $0 < a < b \leq \infty$ and $y(\cdot, \cdot)$ is twice continuously differentiable with $y_2(\alpha, 0) > 0$ for any $\alpha \in [a, b]$.[9] We assume moreover that the team output or production function $y(\alpha, e_1 + \cdots + e_n)$ is

[8] Using a mechanism design approach, Blanes i Vidal & Möller (2013) study the trade-off between two privately informed partners' selection of a joint project ('adaption') and the impact of communicating this private information on subsequent team efforts ('motivation'). They find that first best cannot be implemented whenever the relative payoff influence of adaption is low compared to that of motivating the colleague.

[9] Throughout the paper, h_i, $h \in \{y, f, s\}$, denotes the partial derivative of h with respect to the i^{th} argument. The second derivative with respect to the same i^{th} argument is denoted $h_{i,i}$ and the second order mixed partial derivative with respect to the i^{th} and j^{th} arguments is written $h_{i,j}$. As usual, h' denotes the first derivative of a function with a single argument.

supermodular, that is, exhibits positive cross derivatives between e_i and α. Assume that the signal α is privately observed by player 1, the team leader, while all other team members only know the distribution of α. Throughout, we denote the observed signal by α^* and the reported signal by α'. We call the actual realization of output y^*.

We assume that, in addition to the information about the generated output, there is a tournament that specifies a ranking of the agents according to their exerted efforts. We assume that the ranking is noisy and depends only on the agents' exerted efforts. The outcome of the tournament is observable and verifiable. We employ the following notation: $f^J(e_i, e_{-i})$ is the probability that player i is ranked jth. We assume that these functions f are symmetric with respect to the identity of the players. Because probabilities are additive we have, for any $i \in \mathcal{N}$, e_i and e_{-i},

$$\sum_{J=1}^{n} f^J(e_i, e_{-i}) = 1. \tag{1}$$

In addition to differentiability of $f(e_i, e_{-i})$ with respect to all arguments we assume that, for any e_{-i}, $f^1(e_i, e_{-i})$ increases with e_i, that is, the probability to be ranked first increases with own effort. A team contract specifies the shares of team output of each player. Budget balancing requires that these shares sum to one across players.

2.1 Efficiency benchmark

We start by defining the socially efficient level of team efforts. Efficient efforts are defined as the set of efforts which maximize social welfare as chosen by a benevolent planner (who knows α^* and can dictate agents' efforts)

$$\max_{e} \; y(\alpha^*, e_1 + \cdots + e_n) - \sum_{i=1}^{n} c(e_i). \tag{2}$$

Hence, symmetric first-best efforts $e^*(\alpha^*) = e_1^*(\alpha^*) = \cdots = e_n^*(\alpha^*)$ are defined through

$$y_2(\alpha^*, ne(\alpha^*)) = c'(e(\alpha^*)). \tag{3}$$

Note that supermodularity of the output function implies that $e^*(\alpha^*)$ is increasing. Therefore, the tournament does not play any role in the efficient outcome, but can be used as an information device for implementing the efficient effort choice.

2.2 Dual incentive problem

In our setup, the leader has private information on the value of the group's productivity parameter α^*. Although this information is valuable to everyone, a problem arises if the players share team output in some fixed way because the leader may have an incentive to lie: Intuitively, the leader may find it individually beneficial to claim that the group is in a 'high-productivity' state through some report $\alpha' > \alpha^*$ to induce all the other team members to exert high efforts, even if she plans to put in less. The other team members, anticipating this, may then disregard the leader's report. Thus, in this framework, an efficient team contract, while keeping the budget balanced, has to solve a double incentive problem: *i*) eliciting true information from the leader and *ii*) encouraging efficient effort from both the leader and the other team members.

3 Results

In this section we present the incentive mechanism and our results for the case where information is isolated in the sense that only the team leader, called player 1, has private information. This setup is later generalized to dispersed information where each player receives a private signal on team productivity.

The designer suggests the following mechanism consisting of a ranking-based sharing rule which divides the total generated output y^* and a dynamic structure. At the first stage, after the leader learns her private information and all players observe the proposed sharing rule, they either accept or disagree to participate in the mechanism. If the contract is rejected by at least one agent, the game ends. Conditional on acceptance of all agents, the privately informed player reports her information publicly. At the second stage, all players exert efforts and, after the realization of both output and the ranking of the tournament, the generated team output is shared according to the proposed sharing rule.[10]

This sharing rule depends on the report of the team leader α' and the realized output y^*. We denote by $s^l(y^*, \alpha')$ the share of the agent who was ranked l^{th} according to the tournament, when the realized output is y^* and the report of the leader is α'.[11] Budget balancedness implies that, for any y^* and α',

$$\sum_{l=1}^{n} s^l(y^*, \alpha') = 1. \tag{4}$$

We now show that, for the leader, reporting $\alpha' = \alpha^*$ is part of a (Perfect) Bayesian Nash equilibrium strategy of the game defined by the above mechanism and that, subsequently, exerting the efficient effort choices $e^*(\alpha^*)$ constitutes a Nash equilibrium for all players. The expected utility of player $i \in \mathcal{N}$ from choosing effort level e_i after observing report α' in the true state of the world α^* while the other players choose their equilibrium effort given the reported state, $e^*(\alpha')$, is $u_i(e_i, \mathbf{e}^*_{-i}(\alpha'), \alpha^*) =$

$$\mathbb{E}_{\alpha^*}\left[y(\alpha^*, \sigma(e_i, \mathbf{e}^*_{-i}(\alpha'))) \left(\sum_{l=1}^{n} f^l(e_i, \mathbf{e}^*_{-i}(\alpha')) s^l(y^*, \alpha') \right) \middle| \alpha' \right] - c(e_i) \tag{5}$$

in which $(s^1(\cdot,\cdot), \ldots, s^n(\cdot,\cdot))$ is the output- and report-dependent sharing rule. Competitors' report-dependent efforts are

$$\mathbf{e}^*_{-i}(\alpha') = (\underbrace{e^*(\alpha'), ..., e^*(\alpha')}_{n-1 \text{ times}}),$$

output depends on $\sigma(e_i, \mathbf{e}^*_{-i}(\alpha')) = e_i + (n-1)e^*(\alpha')$ and expectations are over α^* conditional on the reported α'.

[10] This is not the only mechanism that implements efficient efforts. In particular, the direct mechanism, in which the team leader reports her signal privately to the designer and the designer sends effort recommendations to all agents using a similar sharing rule, implements efficiency as well.

[11] Since besides the private information of player 1, all agents are homogenous we show that a symmetric sharing rule can implement efficient efforts. In case of heterogenous agents, identity dependent sharing rules should be used. This is also true for the case in which the leader should be remunerated on the basis of a different sharing rule than the uninformed team members.

Our first result states that ex post efficient efforts by all players, $e^*(\alpha^*)$, can always be obtained as an equilibrium of our game.

Proposition 1. *Efficient, symmetric efforts for all players defined in* (3) *can be implemented through the winner's share* $s^1(y^*, \alpha') =$

$$\frac{1}{n} + \frac{n-1}{ny^* f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))}\left[c'(e^*(\alpha')) - \frac{y_2(\alpha^*(y^*, \alpha'), ne^*(\alpha')))}{n}\right] \quad (6)$$

in which $\alpha^*(y^*, \alpha')$ *is the solution to* $y^* = y(\alpha^*, ne^*(\alpha'))$ *and the losers' share* $s^j(y^*, \alpha') = \frac{1-s^1(y^*,\alpha')}{n-1}$ *for all* $j \neq 1$.

All main proofs can be found in appendix A. We postpone the discussion of equilibrium existence to appendix B.

The idea of the proof of proposition 1 is to construct a sharing rule which encourages the team leader to exert the efficient effort level $e^*(\alpha')$ given her own report α' even if the report does not correspond to the true state of the world $\alpha' \neq \alpha^*$. This sharing rule, in addition to solving the moral hazard problem between all agents, provides the correct incentives for the team leader to report the correct state of the world at the first stage.

The interpretation of how the mechanism works is as follows. Focus first on the second stage of the game, i.e., after some signal α' has been revealed by player 1 at the first stage. Individual efforts have two effects: first, they enlarge the total output available for all players to share. Since the costs of these efforts are born individually, however, there is the usual free-riding incentive in teams. The contest designed around the appropriately chosen reward system (6) introduces, however, a second effect in which increasing the own effort increases also the chance of winning while simultaneously decreasing the other players' chances. By trading off the first against the second effect, the mechanism can provide players with incentives to exert report-contingent efficient efforts $e^*(\alpha')$.[12]

Given equilibrium behavior at stage two, the designed mechanism ensures that the privately informed player 1 finds it disadvantageous to choose

[12] This tradeoff has been previously reported by Gershkov et al. (2009) in a game of complete information.

a pair consisting of a misreport α' at stage one and an inefficient, signal-contingent effort choice at the second stage of the game. A pair consisting of a low misreport (enticing low efforts $e^*(\alpha')$ of the uninformed players in equilibrium), together with higher than efficient effort is undesirable under the reward structure (6) because individual, convex effort cost is too high relative to the appropriately chosen winner's share of (lower) total output. Similarly, a high misreport (enticing high efforts $e^*(\alpha')$ of the uninformed players in equilibrium) together with low own efforts (and costs) to win a larger prize is discouraged because a well designed losing prize decreases in realized output. Since, therefore, the informed player 1 has appropriate incentives to truthfully report her signal, the uninformed team members can rely on a truthful report in equilibrium and exert efficient efforts.

Remark 1. *We can separate the effects of moral hazard and adverse selection on our sharing rule. Recall that in Gershkov et al. (2009), for the case of commonly known state of the world α^*, the report-independent winner's share that implements efficiency is*

$$s^1(y^*, \alpha^*) = \frac{1}{n} + \frac{(n-1)^2 y_2(\alpha^*, ne^*(\alpha^*))}{n^2 f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) y(\alpha^*, ne^*(\alpha^*))} \tag{7}$$

with each of the losers receiving $s^j(y^*, \alpha^*) = \frac{1-s^1(y^*,\alpha^*)}{n-1}$.

Remember, that our new sharing rule (6) *provides efficient incentives to the uninformed agents when they believe that the informed agent reported the right state of the world. Inserting $\alpha' = \alpha^*$ into sharing rule* (6) *and recalling that efficiency implies both $y_2(\alpha^*, ne^*(\alpha^*)) = c'(e^*(\alpha^*))$ and $\alpha^*(y^*, \alpha^*) = \alpha^*$, we get $s^1(y^*, \alpha^*) =$*

$$\begin{aligned} &\frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) y(\alpha^*, ne^*(\alpha^*))}\right. \\ &\quad \left. - \frac{y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) y(\alpha^*, ne^*(\alpha^*)) n}\right] = \frac{1}{n} \\ &\qquad + \frac{n-1}{ny(\alpha^*, ne^*(\alpha^*))}\left[\frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*))} - \frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) n}\right]. \end{aligned} \tag{8}$$

which delivers (7) *immediately. Therefore, the shares of the uninformed agents along the equilibrium path are the same as in Gershkov et al. (2009).*

Hence, the 'correction' of the sharing rule to take care of adverse selection can be expressed as

$$\frac{n-1}{n}\left[\frac{c'(e^*(\alpha'))}{y^* f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))} - \frac{y_2(\alpha^*(y^*,\alpha'), ne^*(\alpha'))}{ny^* f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))} - \frac{(n-1)y_2(\alpha^*, ne^*(\alpha^*))}{nf_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*))y(\alpha^*, ne^*(\alpha^*))}\right] \tag{9}$$

which we interpret as the (off-equilibrium) value of information to the winner.[13] *This correction makes the shares report-dependent.*

Notice that the loser's share $\frac{1-s^1(y^*,\alpha')}{n-1}$ is given by

$$\frac{1}{n} - \frac{1}{ny^* f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))}\left[c'(e^*(\alpha')) - \frac{y_2(\alpha^*(y^*,\alpha'), ne^*(\alpha')))}{n}\right]. \tag{10}$$

Therefore, the difference between the winner's and the loser's compensations is given by

$$\left[c'(e^*(\alpha')) - \frac{y_2(\alpha^*(y^*,\alpha'), ne^*(\alpha')))}{n}\right]\frac{1}{f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))} \tag{11}$$

where the expression in the square brackets is the private marginal disutility from the effort exertion. This element aligns incentives of the agents with the socially efficient objective which, in turn, generates the correct incentives for the agents to report information and to exert socially efficient efforts.

From (6), we get an immediate comparative statics result with respect to the precision of the success function $f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))$.

Corollary 1. *The share of the winner $s^1(y^*,\alpha')$ decreases on the equilibrium path with the precision of the ranking $f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))$.*

This is intuitive (and proved formally in the appendix), since high ranking precision increases the incentives for the agents. Therefore, if $f_1^1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))$ increases, agents want to exert higher efforts. To restore their incentives, the share of the winner should be adjusted/decreased.

[13] This is different from the notion of value of information in Hermalin (1998, footnote 12) who shows that second best team welfare under the true signal exceeds team welfare under the expected signal. (The same would be true in our model.)

Example 1: We illustrate our efficiency result from proposition 1 in a simple example with n players, Tullock ranking technology $f^1(e_i, e_{-i}) = \frac{e_i^r}{\sum_j e_j^r}$,[14] linear production $y(\alpha^*, \sum_i e_i) = \alpha^* \sum_i e_i$, and quadratic effort cost $e_i^2/2$. Note that in this example, the efficient effort level $e^*(\alpha^*) = \alpha^*$. This is implemented through the following ranking-based sharing rule:

$$\begin{aligned} s^1(y^*, \alpha') &= \frac{1}{n} - \frac{n-1}{n^3 f_1^1(\alpha', \alpha')\alpha'} + \frac{(n-1)\alpha'}{ny^* f_1^1(\alpha', \alpha')} = \frac{1}{n} - \frac{1}{nr} + \frac{n\alpha'^2}{y^* r}, \\ s^{j\neq 1}(y^*, \alpha') &= \frac{1}{n} + \frac{1}{n^3 f_1^1(\alpha', \alpha')\alpha'} - \frac{\alpha'}{ny^* f_1^1(\alpha', \alpha')} = \frac{1}{n} + \frac{1}{n(n-1)r} - \frac{n\alpha'^2}{(n-1)y^* r} \end{aligned}$$

in which $s^1(\cdot)$ of the final team output is awarded to the first-ranked player while $s^{j\neq 1}(\cdot)$ is awarded to all other players. In this example, limited liability—defined as non-negative shares of output for all players—is satisfied on the equilibrium path if $r \geq 1$.$\triangleleft$

Remark 2. *An advantage of the contest approach is that it requires only ordinal information on agents' efforts which is arguably easier to collect than information on the precise effort realizations. Nevertheless, the fact that we are able to implement efficient efforts implies that a noisy ordinal ranking of efforts is a sufficient statistic in the sense of Holmström (1982, Section 3) for the cardinal effort information employed in standard contracts.*

Remark 3. *Assume for the moment that it is commonly known that the leader reports truthfully. Then we know from Holmström (1982) that there exists no sharing rule that simultaneously ensures efficient efforts and balances its budget. Hence, without the additional information on the noisy ranking incorporated in our contest we cannot obtain efficient effort exertion. In other words, if a sharing rule cannot condition on effort information (including some noisy ranking of efforts), it cannot induce both truthful reporting and efficient effort exertion. Similarly, Gershkov et al. (2009) show that a sharing rule which takes into account ranking information but does not vary with output cannot generally implement efficiency.*

[14] For completeness, we define $f^1(0, \ldots, 0) = 1/n$; the implied discontinuity at point $(0, \ldots, 0)$ plays no role in this example.

4 Extensions and robustness

4.1 Noisy signals

In this section we illustrate that perfect information of the team leader is not crucial for efficient incentive provision. That is, one may fear that the positive result of proposition 1 follows from the fact that given the observed output y^* and knowing the equilibrium effort $e^*(\alpha')$, the designer may learn the exact private information of the team leader and 'punish' her in case of misreporting. Here we show that it is not the case. We consider an output or production function of the form $y(\alpha^*; \varepsilon; \mathbf{e}) = y(\alpha^* + \varepsilon, \sum_{i=1}^n e_i)$ where $\varepsilon \sim G[\underline{\varepsilon}, \bar{\varepsilon}]$ with $\mathbb{E}(\varepsilon) = 0$ and density g. That is, we assume that the team leader observes the production parameter with some noise. Moreover, we assume that the uncertainty is realized only after the effort exertion. The ex post efficient efforts then solve

$$\max_{e_1,\ldots,e_n} \mathbb{E}_\varepsilon \left[y\left(\alpha^* + \varepsilon, \sum_{i=1}^n e_i\right)\right] - \left(\sum_{i=1}^n c(e_i)\right). \tag{12}$$

Given our assumptions on the production function, the ex post efficient effort level, $e^*(\alpha^*)$ is given by

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} y_2\left(\alpha^* + \varepsilon, ne^*(\alpha^*)\right) g(\varepsilon) d\varepsilon = c'\left(e^*(\alpha^*)\right). \tag{13}$$

The dynamic game structure in the case of a noisy signal is similar to the deterministic case: Player 1 reports α' and, at the second stage and given this report, all players simultaneously choose efforts. We would like to find a sharing rule assigning output shares to the first-, second-, third-ranked players etc.

$$\left(s^1(y^*, \alpha'), \frac{1 - s^1(y^*, \alpha')}{n-1}, \ldots, \frac{1 - s^1(y^*, \alpha')}{n-1}\right) \tag{14}$$

such that, for any observed α^* and reported α', player 1 will choose the report-contingent efficient effort $e_1 = e^*(\alpha')$ and it is a best response for every other player to also choose the report-contingent efficient efforts $e^*(\alpha')$.

The expected utility of player 1 if she observes α^*, reports α' and exerts effort e_1, when the other players choose the report-contingent efficient effort $e^*(\alpha')$ is given by $u_1(e_1, \mathbf{e}^*_{-1}(\alpha'), \alpha^*) =$

$$\mathbb{E}_\varepsilon \left[y\left(\alpha^* + \varepsilon, \sigma\left(e_1, \mathbf{e}^*_{-1}(\alpha')\right)\right) \left(\sum_{h=1}^{n} f^h(e_1, \mathbf{e}^*_{-1}(\alpha')) s^h(y^*, \alpha') \right) \right] - c(e_1). \quad (15)$$

The next result characterizes the sharing rule that induces truth telling by the team leader and efficient effort exertion by all team members.

Proposition 2. *Then the sharing rule* $s^1(y^*, \alpha') =$

$$\frac{1}{y^* f_1^1\left(e^*\left(\alpha'\right), \mathbf{e}^*_{-i}\left(\alpha'\right)\right)} \frac{n-1}{n} \left(c'\left(e^*\left(\alpha'\right)\right) - \frac{y_2\left(\alpha^*\left(y^*, \alpha'\right), ne^*\left(\alpha'\right)\right)}{n} \right) + \frac{1}{n} \quad \textit{(16)}$$

in which $\alpha^*\left(y^*, \alpha'\right)$ *is the solution to* $y^* = y\left(\alpha^*, ne^*\left(\alpha'\right)\right)$ *and* $s^2(y^*, \alpha') = \ldots = s^n(y^*, \alpha') = \frac{1 - s^1(y^*, \alpha')}{n-1}$ *implements the first best outcome.*

Example 2: We continue with our example 1 by replacing the production function with $y(\alpha^*, \varepsilon, \mathbf{e}) = (\alpha^* + \varepsilon) \sum_i e_i$, in which ε is realized after the effort stage. Due to the linear structure of this example the noise washes out and the same sharing rule with winning share

$$s^1(y^*, \alpha') = \frac{1}{n} - \frac{1}{nr} + \frac{n\alpha'^2}{y^* r} \quad (17)$$

and losing share $s^{j \neq 1}(y^*, \alpha') = \frac{1 - s^1(y^*, \alpha')}{n-1}$ implements full efficiency.$\triangleleft$

This section illustrates that our main efficiency result in proposition 1 does not depend critically on the quality of information the leader has. On the contrary, we show that any expectation-zero noise term can be accommodated by our efficient sharing rule (16) without affecting the intuition of our positive result.

4.2 Information aggregation

In this section we assume that each of the n players obtains an individual signal α_i^* in the otherwise unchanged environment from section 3, so that output now takes the form $y^*(\alpha^*, e) = y\left(\sum_i \alpha_i^*, e_1 + \cdots + e_n\right)$. These signals $(\alpha_1^*, \ldots, \alpha_n^*)$ are drawn from the commonly know distribution Z with support $[a, b]^n$. The main difference to the case discussed there is that now not only player 1 but also each other player must have the appropriate incentives to report their complementary signals truthfully.

We apply the revelation principle and restrict attention to direct mechanisms in which the agents report their private information to the mechanism and the mechanism provides all agents with effort recommendations. The revelation principle implies that, in looking for an efficient mechanism, we can focus on those sharing rules under which the agents find it optimal to report their signals truthfully and to follow their recommendation.

For simplicity, we denote by $\mathcal{A}^* = \alpha_1^* + \cdots + \alpha_n^*$ the true productivity and by $\mathcal{A}_i' = \alpha_1^* + \cdots + \alpha_{i-1}^* + \alpha_i' + \alpha_{i+1}^* + \cdots + \alpha_n^*$ the aggregated productivity in the case where all agents but i report truthfully while agent i reports α_i'.

The vector of ex post efficient efforts corresponding to (3) is now defined as the set of efforts which maximize social welfare as chosen by a benevolent planner (who knows $\mathcal{A}^* = \alpha_1^* + \cdots + \alpha_n^*$), i.e.,

$$\underset{e}{max}\ \ y\left(\mathcal{A}^*, e_1 + \cdots + e_n\right) - \sum_i c(e_i). \tag{18}$$

Hence, symmetric ex post efficient efforts $e^*(\mathcal{A}^*) = e_1^*(\mathcal{A}^*) = \cdots = e_n^*(\mathcal{A}^*)$ are defined through

$$y_2(\mathcal{A}^*, e^*(\mathcal{A}^*)) = c'(e^*(\mathcal{A}^*)). \tag{19}$$

To implement efficiency, the designer recommends player efforts $e^*(\mathcal{A}')$, in which $\mathcal{A}' = \sum_i \alpha'$. Below we show that under sharing rule (6), adjusted for the present information structure, it is indeed in the players' interest to follow this recommendation and report their signals truthfully, $\alpha_i' = \alpha_i^*$.

At the second stage, we define player i's expected utility from action pair (α_i', e_i) given equilibrium behavior $e^*(\mathcal{A}')$ of everyone else as

$$\begin{aligned} &u_i((\alpha_i', e_i), \mathbf{e}_{-i}^*(\mathcal{A}'), \mathcal{A}^*) \\ &= \mathbb{E}_{\mathcal{A}^*}\left[y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}_{-i}^*(\mathcal{A}')\right)\right) \left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}_{-i}^*(\mathcal{A}')) s^h(y^*, \mathcal{A}') \right) \middle| \mathcal{A}' \right] - c(e_i) \end{aligned} \tag{20}$$

where observed output is $y^* = y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)$. For a given report α'_i we obtain player i's first-order condition with respect to efforts e_i as

$$\begin{aligned}
&\frac{\partial u_i((\alpha'_i, e_i), \mathbf{e}^*_{-i}(\mathcal{A}'), \mathcal{A}^*)}{\partial e_i} \qquad (21)\\
&= y_2\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h(y^*, \mathcal{A}')\right)\\
&\quad + y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h_1(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h(y^*, \mathcal{A}')\right)\\
&\quad + y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right) y_2\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h_1(y^*, \mathcal{A}')\right)\\
&\quad - c'(e_i)
\end{aligned}$$

which is the same as condition (35) derived for player 1 in the proof of proposition 1, with the only difference that $\mathcal{A}^*$ and $\mathcal{A}'$ are now sums. Therefore, exactly as in the previous case, even after misreporting, all agents will follow the designer's recommendation and choose efforts $e^*(\mathcal{A}')$.

At the first stage, therefore, each player i reports α'_i such as to maximize expected utility (20) which, in symmetric equilibrium, implies that she chooses

$$\max_{\alpha'_i}\ u_i((\alpha'_i, e^*(\mathcal{A}'_i)), \mathbf{e}^*_{-i}(\mathcal{A}'_i), \mathcal{A}^*) = \frac{y\left(\mathcal{A}^*, ne^*(\mathcal{A}'_i)\right)}{n} - c(e^*(\mathcal{A}'_i)) \qquad (22)$$

which, from (19) is maximized at the report $\alpha'_i = \alpha^*_i$ implying that player i reports truthfully. With this changed interpretation, the sharing rule (6) derived in proposition 1, with α' replaced by $\mathcal{A}'$, implements ex post efficient efforts as defined in (19) also in the setup with information aggregation. This fact allows us to state the following result.

Proposition 3. *The efficient outcome is implementable in the setup with information aggregation, i.e., in the case in which all team members receive individual signals α^*_i on joint team productivity $\mathcal{A}^*$.*

Example 3: We continue our example by replacing the production function with $y(\mathcal{A}^*, \mathbf{e}) = \mathcal{A}^* \sum_i e_i$, in which α^*_i is player i's private information about team productivity and the true team productivity is measured by $\mathcal{A}^* = \sum_i \alpha_i^*$. We replace the first stage of the game with a stage in which each player privately reports their private α_i' to the designer who subsequently sends the effort recommendation $e^*(\mathcal{A}') = \mathcal{A}' = \sum_i \alpha'$ to the team members. Compared to the sharing rule of the previous example (17), the similar rule with winning share

$$s^1(y^*, \mathcal{A}') = \frac{1}{n} - \frac{1}{nr} + \frac{n\mathcal{A}'^2}{y^* r} \tag{23}$$

and losing shares $s^{j\neq 1}(y^*, \mathcal{A}') = \frac{1-s^1(y^*,\mathcal{A}')}{n-1}$ implements the first best outcome.$\triangleleft$

4.3 Leading by Example

In this section we change the structure of the interaction and allow for the leader to choose her effort before the other players. This effort is assumed to be observable by her team partners. We show that in such a case, there exists a simpler ranking-based sharing rule which implements the efficient outcome. This sharing rule will only depend on observed output.

Here, we consider the following sequential game: at the first stage, the leader chooses effort e_1. Then, at the second stage, all other players $j \neq 1$ observe e_1 and choose their own efforts $e_j(e_1(\cdot))$. Following this, a noisy ranking of all players' efforts realizes. The winner receives fraction s^1 of final team output and each of the losers receives share $\frac{1-s^1}{n-1}$.

In this environment, the sharing rule may be conditioned on the observed output y^* alone because the leader's effort e_1 is observed by all other players before they choose their own efforts. Thus, the leader's effort serves as a signal of the team's productivity parameter α^*. Moreover, and this is crucial, this time structure limits the strategic possibilities of the leader. While in the original game—in which everyone chooses efforts simultaneously—the leader was able to deviate in both her report α' and the chosen effort (so multidimensional deviations had to be taken into account), one of these channels is shut here. In the current structure, a misreport is more costly to the leader, as she cannot report α' and subsequently choose an effort which is inconsistent with this report.

Proposition 4. *Assume the sequential game described above. The sharing rule consisting of*

$$s^1(y^*) = \frac{1}{n} + \frac{n-1}{n}\frac{c'(e^*(\breve{\alpha}(y^*))) - \frac{y_2(\breve{\alpha}(y^*), ne^*(\breve{\alpha}(y^*)))}{n}}{y^* f_1^1(e^*(\breve{\alpha}(y^*)), \mathbf{e}^*_{-i}(\breve{\alpha}(y^*)))} \tag{24}$$

and $s^j(y^*) = \frac{1-s^1(y^*)}{n-1}$ *for all* $j \neq 1$ *in which* $\breve{\alpha}(y^*)$ *is the solution to* $y^* = y(\alpha, ne_1(\alpha))$*, implements efficient efforts.*

Example 4: We continue our example with $y = \alpha^* \sum_i e_i$, $c(e_i) = \frac{1}{2}e_i^2$, and $f^1(e_i, e_{-i}) = e_i^r / \sum e_j^r$ by replacing the simultaneous game with the sequential structure described above. In this sequential game, a tournament with shares

$$s^1 = \frac{(n-1)+r}{nr}, \qquad s^j = \frac{r-1}{nr} \text{ for } j \neq 1 \tag{25}$$

implements efficiency. To see this, note that on observing the effort choice of player 1, e_1, players $j \neq 1$ believe that the productivity parameter of the team is $\alpha^* = e_1$. Given sharing rule (25), it is a best response for the uninformed players to follow their leader by choosing exactly $e_j = e_1$. At the first stage, anticipating that the uninformed players are going to follow suit by choosing $e_j = e_1$, player 1's best strategy is to choose effort $e_1 = \alpha^*$, thus communicating the true state of world and implementing efficiency.

Note that sharing rule (25) is independent of output. This is due to the fact that this example uses linear production function. As shown in proposition 4, however, this independence is not obtained in the general production case.◁

In a setup with linear production and quadratic effort costs, Hermalin (1998, p1192) finds that leading by example is superior to a range of other mechanisms. Nevertheless, leading by example fails to achieve full efficiency because the usual moral hazard problem remains. The reason for this failure is the fixed sharing rule Hermalin (1998) uses throughout the paper. We show that a well-designed tournament, by orchestrating competition among the players, removes the free-riding incentives while ensuring truthful information revelation.

Remark 3 explains that fixed shares can generally not provide incentives for the efficient provision of efforts. This remains true with upfront exertion of observable efforts by a privately informed leader.

We assumed so far that only final output is contractible. If the leader's upfront effort is contractible, however, then the next proposition states that there exists a sharing rule that provides the correct incentives for all agents which conditions only the leader's observed effort, e_1.

Proposition 5. *Assume the sequential game described above. If the leader's effort e_1 is contractible, then the sharing rule consisting of*

$$s^1(e_1) = \frac{1}{n} + \frac{n-1}{n} \frac{c'(e_1) - \frac{y_2(\alpha^{**}(e_1), ne_1)}{n}}{y(\alpha^{**}(e_1), ne_1) f_1^1(e_1, e_1)} \tag{26}$$

in which $\alpha^{**}(e_1)$ is the solution to $e^*(\alpha^{**}) = e_1$, together with equally split losers' prizes, implements efficient efforts.

4.4 Limited liability along the equilibrium path

For this section, we restrict attention to symmetric, ratio-based contest success functions. We define a success function $\hat{f}^1(x_i)$ as *ratio-based* if it only depends on the vector of ratios of a player's effort over each of her opponents' efforts $x_i = \left(\frac{e_i}{e_1}, \ldots, \frac{e_i}{e_{i-1}}, \frac{e_i}{e_{i+1}}, \ldots, \frac{e_i}{e_n}\right)$.[15] We take symmetry to imply that, for any two players $l \neq m$ and for any two vectors of efforts, $(e_1, \ldots, e_n)$ and $(\tilde{e}_1, \ldots, \tilde{e}_n)$ with $e_k = \tilde{e}_k$ for $k \notin \{l, m\}$ and $e_l = \tilde{e}_m$ and $e_m = \tilde{e}_l$, we have

$$\hat{f}^1(x_l) = \hat{f}^1(\tilde{x}_m). \tag{27}$$

The Tullock success function is an example of such a symmetric, ratio-based function. When agents exert identical equilibrium efforts $e^*(\alpha^*)$, then the above implies that for any $i \in \{1, \ldots, n\}$, $l, m \neq i$, we have that $\frac{\partial \hat{f}^1(\mathbf{1})}{\partial x_{il}} = \frac{\partial \hat{f}^1(\mathbf{1})}{\partial x_{im}}$ where $\mathbf{1}$ is the $n-1$ dimensional vector with 1 at every position. The

[15] In order to avoid technical complications with unbounded ratios, we require efforts to be positive for the purposes of this limited liability discussion. In other words $e_i \in [\delta, \infty)$ in which $\delta > 0$ can be arbitrarily close to zero.

relationship between the ratio-based success function and the original effort-based success function is such that

$$\hat{f}^1\left(\frac{e_i}{e_1},\frac{e_i}{e_2},\ldots,\frac{e_i}{e_{i-1}},\frac{e_i}{e_{i+1}},\ldots,\frac{e_i}{e_n}\right)=f^1\left(e_i,e_{-i}\right) \tag{28}$$

with derivative

$$\begin{aligned}&\frac{d}{de_i}\hat{f}^1\left(\frac{e_i}{e_1},\frac{e_i}{e_2},\ldots,\frac{e_i}{e_{i-1}},\frac{e_i}{e_{i+1}},\ldots,\frac{e_i}{e_n}\right)\\&=\sum_{j\neq i}\frac{1}{e_j}\frac{\partial}{\partial x_{ij}}\hat{f}^1\left(x_{i1},\ldots,x_{ii-1},x_{ii+1},\ldots,x_{in}\right)\\&=f_1^1\left(e_i,e_{-i}\right).\end{aligned} \tag{29}$$

In symmetric equilibrium we therefore have

$$\frac{d}{de_i}\hat{f}^1\left(\mathbf{1}\right)\ =\sum_{j\neq i}\frac{1}{e^*\left(\alpha'\right)}\frac{\partial}{\partial x_{ij}}\hat{f}^1\left(\mathbf{1}\right)=f_1^1\left(e^*\left(\alpha'\right),\mathbf{e}^*_{-\mathbf{i}}\left(\alpha'\right)\right). \tag{30}$$

In our base model, efficient efforts are defined by $y_2(\alpha^*,ne(\alpha^*))=c'(e^*(\alpha^*))$. Substituting the latter with the prior in sharing rule (6) gives us

$$s^1(y^*,\alpha^*)\ =\frac{1}{n}+\frac{(n-1)\,y_2(\alpha^*(y^*,\alpha^*),ne^*(\alpha^*))}{ny(\alpha^*,ne^*(\alpha^*))f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))}\left[\frac{n-1}{n}\right]>\frac{1}{n} \tag{31}$$

implying that the equilibrium winner's share is always greater than a loser's share.

Winning share (31) satisfies limited liability along the equilibrium path if, in addition to the inequality already shown in (31), $s^1(y^*,\alpha^*)\leq 1$. This is the case if

$$f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))\geq\frac{n-1}{n}\frac{y_2(\alpha^*(y^*,\alpha^*),ne^*(\alpha^*))}{y(\alpha^*,ne^*)}. \tag{32}$$

The last inequality implies that

$$f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))\geq\frac{1}{e^*}\frac{n-1}{n^2}\frac{ne^*y_2(\alpha^*(y^*,\alpha^*),ne^*(\alpha^*))}{y(\alpha^*,ne^*)} \tag{33}$$

in which the last fraction on the right-hand side is smaller than 1 because of concavity of y. Thus, (30) together with concavity of production implies that, for any symmetric, ratio-based success function, we obtain as sufficient condition for limited liability to be satisfied that

$$\hat{f}_1^1(\mathbf{1}) \geq \frac{1}{n^2}. \tag{34}$$

Losers receive positive shares in equilibrium if the ranking technology is sufficiently accurate. This argument cannot demonstrate, however, that the satisfaction of limited liability is also possible outside of equilibrium.

5 Concluding remarks

In addition to the motivation offered in the introduction, our model can be interpreted as providing a solution to the coordination problem. This problem refers to situations in which the interests of the individuals involved coincide. It would be therefore jointly optimal if they would individually all select this jointly most beneficial option. Nevertheless, in many such situations, it is not in the individual's self-interest to choose the action which implements the greater good.

Leadership is but one means by which social groups attempt to solve the coordination problem and can take many forms in general public life.[16] The present paper analyzes the question of what constitutes the coordinative essence of leadership in team structures where (some of) the team members are privately informed about some aspect of the profitability of a joint project. Such proprietary information arises naturally if, for example, some team member occupies a role in a predefined organizational structure by virtue of which she acquires and disseminates information.

An effective coordinating scheme then needs to implement *i)* the communication of private information and *ii)* the efficient effort provision by all team members although these efforts may not be directly observed. In the environment we consider, this gives rise to the nested and triple problem

[16] The information-based foundation that we offer for leadership is, of course, not the only possible explanation. Alternatives include delegation, sharing of responsibility, inclusion of stakeholders and others.

of *adverse selection* because of the leader's private information, *moral hazard* because individual efforts are unobservable, and *balanced budget* because of the team structure which renders the classic principal-agent and budget breaker solutions inapplicable.

Examples which emphasize the coordination aspect and feature the properties outlined above can be found in military history. During the First World War, for instance, officers in most armies used a 'trench whistle' to communicate isolated timing information to a team. Its high pitched sound was used to coordinate large scale attacks. At the officers' blow of their whistles, the soldiers would go 'over the top' of the trenches and attack the enemy. Note that signalling the attack at the wrong time may result in this story in over or under exertion of team member efforts relative to the efficient level which may benefit or harm the standing of the whistling leader.

A final example of carefully designed incentive structures in partnerships is the 19th century American whaling industry beautifully described in Hilt (2006). The author describes how managing partners provided appropriate incentives to the whaler's captains and crews on their entirely unobservable multi-year expeditions. During the 1830s, *part of the industry* changed its structure from the previously unincorporated partnerships to corporative ownership. "This represented a significant departure from the traditional reliance on concentrated ownership to resolve incentive conflicts in the industry, and it failed: none of the whaling corporations survived beyond the 1840s, and few experienced much financial success, at a time the American whaling industry as a whole continued to expand." (Hilt, 2006, p198)

Appendix A: Omitted proofs

Proof of proposition 1. For a given report α', given that every other player chooses $e^*(\alpha')$, the team-leader's first-order condition with respect to effort choice is

$$\frac{\partial u_1(e_1, \mathbf{e}^*_{-1}(\alpha'), \alpha^*)}{\partial e_1} = y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n} f^l(e_1, \mathbf{e}^*_{-1}(\alpha')) s^l(y^*, \alpha') \right)$$
$$+ y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n} f^l_1(e_1, \mathbf{e}^*_{-1}(\alpha')) s^l(y^*, \alpha') \right)$$
$$+ y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n} f^l(e_1, \mathbf{e}^*_{-1}(\alpha')) s^l_1(y^*, \alpha') \right)$$
$$- c'(e_1) \tag{35}$$

which equals

$$y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n} f^l(e_1, \mathbf{e}^*_{-1}(\alpha')) s^l(y^*, \alpha') \right) - c'(e_1)$$
$$+ y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n-1} f^l_1(e_1, \mathbf{e}^*_{-1}(\alpha')) \left(s^l(y^*, \alpha') - s^n(y^*, \alpha') \right) \right)$$
$$+ y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{l=1}^{n} f^l(e_1, \mathbf{e}^*_{-1}(\alpha')) s^l_1(y^*, \alpha') \right) \tag{36}$$

where $f^l_1(e_1, \mathbf{e}_{-1}) = \frac{\partial}{\partial e_1} f^l(e_1, \mathbf{e}_{-1})$. The equality holds because of (1), balanced budget, and $\sum_l f^l_1(e_1, \mathbf{e}^*_{-1}(\alpha')) = \sum_l s^l_1(y^*, \alpha') = 0$. In case of equal effort levels of all the agents, $e_1 = e^*(\alpha')$, $f^l(e_1 = e^*(\alpha'), e_{-1} = e^*(\alpha')) = 1/n$ for all $l = 1, \ldots, n$. Then, setting the first-order condition with respect to effort choice zero gives

$$\begin{aligned} 0 &= \frac{\partial u_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha'), \alpha^*)}{\partial e_1} = \frac{y_2(\alpha^*, ne^*(\alpha'))}{n} \\ &+ y^* \left(\sum_{l=1}^{n-1} f^l_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha')) \left(s^l(y^*, \alpha') - s^n(y^*, \alpha') \right) \right) - c'(e^*(\alpha')) \end{aligned} \tag{37}$$

with output in equilibrium $y^* = y(\alpha^*, ne^*(\alpha'))$. Therefore, we get that

$$\begin{aligned} &\sum_{l=1}^{n-1} f^l_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha')) \left(s^l(y^*, \alpha') - s^n(y^*, \alpha') \right) \\ &= \frac{c'(e^*(\alpha')) - y_2(\alpha^*, ne^*(\alpha'))/n}{y^*}. \end{aligned} \tag{38}$$

Under the simple prize structure $s^1(y^*, \alpha') \geq s^2(y^*, \alpha') = \cdots = s^n(y^*, \alpha')$ (which implies that $s^n(y^*, \alpha') = \frac{1-s^1(y^*,\alpha')}{n-1}$), this equals

$$f_1^1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha')) \left(s^1(y^*, \alpha') - s^n(y^*, \alpha')\right) = \frac{c'(e^*(\alpha')) - y_2(\alpha^*, ne^*(\alpha'))/n}{y^*} \iff$$

$$s^1(y^*, \alpha') = \frac{1}{n} + \frac{n-1}{ny^*}\left[\frac{c'(e^*(\alpha'))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha'))} - \frac{y_2(\alpha^*(y^*, \alpha'), ne^*(\alpha')))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha'))n}\right] \tag{39}$$

where $\alpha^*(y^*, \alpha')$ is the solution to

$$y^* = y(\alpha^*, ne^*(\alpha'))) \tag{40}$$

Note that a similar argument implies that also all the uninformed agents for any report of the team leader α' prefer to exert the report-contingent efficient effort level $e^*(\alpha')$. Therefore, we only have to show that, at the first stage, the team leader prefers to announce the true signal. At the reporting stage, a player 1 who exerts efforts $e^*(\alpha')$ reports α' such as to maximize expected utility (5) which implies that she chooses

$$\underset{\alpha'}{max}\ u_1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha'), \alpha^*) \ = \frac{y\left(\alpha^*, ne^*(\alpha')\right)}{n} - c(e^*(\alpha')) \tag{41}$$

which, from (3) is maximized at the report $\alpha' = \alpha^*$ implying that player 1 reports truthfully. All team members will find it optimal to accept the contract because each player will get $1/n$ of the generated efficient social surplus which must be positive because $y_2(\alpha^*, 0) > 0$ and $c'(0) = 0$. Therefore, the players' ex post efficient efforts $e^*(\alpha^*)$ are implementable. □

Proof of corollary 1. Recall that the winner's share is

$$s^1(y^*, \alpha') = \frac{1}{n} + \frac{n-1}{ny^*}\left[\frac{c'(e^*(\alpha'))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-i}^*(\alpha'))} - \frac{y_2(\alpha^*(y^*, \alpha'), ne^*(\alpha'))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-i}^*(\alpha'))n}\right]. \tag{42}$$

Given the truth telling behavior of the team leader, we can rewrite the equilibrium winner's share as follows

$$
\begin{aligned}
s^1(y^*,\alpha^*) &= \frac{1}{n}+\frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*}-\frac{y_2(\alpha^*(y^*,\alpha^*),ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*n}\right]\\
&= \frac{1}{n}+\frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*}-\frac{y_2(\alpha^*,ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*n}\right]\\
&= \frac{1}{n}+\frac{n-1}{n}\left[\frac{y_2(\alpha^*,ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*}-\frac{y_2(\alpha^*,ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*),\mathbf{e}^*_{-i}(\alpha^*))y^*n}\right]\\
&= \frac{1}{n}+\left(\frac{n-1}{n}\right)^2\frac{1}{f_1^1\left(e^*(\alpha^*),\mathbf{e}^*_{-\mathbf{i}}(\alpha^*)\right)}\frac{y_2(\alpha^*,ne^*(\alpha^*))}{y(\alpha^*,ne^*(\alpha^*))}
\end{aligned}
\tag{43}
$$

where the second line follows since $\alpha^* = \alpha^*(y^*,\alpha^*)$ and the third line follows since in the efficient allocation we have $c'(e^*(\alpha^*)) = y_2(\alpha^*, ne^*(\alpha^*))$. □

Proof of proposition 2. The derivative of the team leader's expected utility with respect to her effort is given by

$$
\begin{aligned}
&\int_{\underline{\varepsilon}}^{\bar{\varepsilon}}\Bigg[y_2\big(\alpha^*\\
&+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\big)\Bigg\{f^1\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)s^1\left(y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right),\alpha'\right)\\
&+\left(1-f^1\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right)\frac{1-s^1\left(y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right),\alpha'\right)}{n-1}\Bigg\}+y\big(\alpha^*\\
&+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\big)f_1^1\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\{s^1\left(y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right),\alpha'\right)\\
&-\frac{1-s^1\left(y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right),\alpha'\right)}{n-1}\}\\
&+y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right)y_2\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right)\\
&\times\times s_1^1\left(y\left(\alpha^*+\varepsilon;\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right),\alpha'\right)\Bigg\{f^1\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\\
&-\frac{1-f^1\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)}{n-1}\Bigg\}\Bigg]g(\varepsilon)\,d\varepsilon-c'(e_1).
\end{aligned}
\tag{44}
$$

We will show that for the stated sharing rule, the first order condition is satisfied. Recall that $f^1\left(e^*(\alpha'),\mathbf{e}^*_{-1}(\alpha')\right) = 1/n$. Therefore, the first-order condition boils down to

$$
\begin{aligned}
&\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[\frac{y_2\left(\alpha^* + \varepsilon; ne^*\left(\alpha'\right)\right)}{n} + y\left(\alpha^* + \varepsilon; ne^*\left(\alpha'\right)\right) f_1^1\left(e_1, \mathbf{e}_{-1}^*\left(\alpha'\right)\right) \times \right. \\
&\left. \times \left(n\frac{s^1\left(y\left(\alpha^* + \varepsilon; ne^*\left(\alpha'\right)\right), \alpha'\right)}{n-1} - \frac{1}{n-1} \right) \right] g\left(\varepsilon\right) d\varepsilon \\
&- c'(e^*\left(\alpha'\right)) = 0.
\end{aligned} \tag{45}
$$

Inserting the rule from (16) completes the proof. We still have to show that it is optimal for the leader to report truthfully her signal. For the leader,

$$
\max_{\alpha'} \; u_1(e_1, e^*(\alpha'), \alpha^*) \; = \max_{\alpha'} \frac{\mathbb{E}y\left(\alpha^* + \varepsilon, ne^*\left(\alpha'\right)\right)}{n} - c\left(e^*\left(\alpha'\right)\right) \tag{46}
$$

is solved at $\alpha' = \alpha^*$ by efficiency condition (13). $\square$

Proof of proposition 4. We show that this sharing rule *i)* induces the leader to choose $e_1 = e^*\left(\alpha^*\right)$, and *ii)* all the other agents to follow the leader and to choose also e_1. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents follow the described strategy. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$
\begin{aligned}
y\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right) &\left[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) s^1\left(y\right) \right. \\
&\left. + \left(1 - f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right) \frac{1 - s^1\left(y\right)}{n-1} \right] - c(e).
\end{aligned} \tag{47}
$$

The derivative of the last expression with respect to e is given by

$$
\begin{aligned}
&y_2\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right) \left[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) s^1\left(y\right) \right. \\
&\left. + \left(1 - f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right) \frac{1 - s^1\left(y\right)}{n-1} \right] - c'(e) \\
&+ y\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right) \left[f_1^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) \left(\frac{ns^1\left(y\right)}{n-1} - \frac{1}{n-1} \right) \right. \\
&+ s^{1'}\left(y\right) \left(f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) \right. \\
&\left. \left. - \frac{\left(1 - f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)}{n-1} \right) y_2\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right) \right].
\end{aligned} \tag{48}
$$

For $e = e_1$ to be an equilibrium, it must be that the last derivative at point $e = e_1$ is 0. Since $f^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) = 1/n$ the previous expression can be rewritten as

$$\begin{aligned} c'(e^*(\alpha^*)) - \frac{y_2(\alpha^*, ne^*(\alpha^*))}{n} \\ = y(\alpha^*, ne^*(\alpha^*)) \left[f_1^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) \left(\frac{ns^1(y)}{n-1} - \frac{1}{n-1} \right) \right] \end{aligned} \tag{49}$$

Inserting

$$s^1\left(y\right) = \frac{1}{n} + \frac{n-1}{n} \frac{c'(e^*\left(\breve{\alpha}\left(y^*\right)\right)) - \frac{y_2(\breve{\alpha}(y^*), ne^*(\breve{\alpha}(y^*)))}{n}}{y^* f_1^1(e^*\left(\breve{\alpha}\left(y^*\right)\right), \mathbf{e_{-j}}^*\left(\breve{\alpha}\left(y^*\right)\right))} \tag{50}$$

and noticing that in the equilibrium $\breve{\alpha}\left(y^*\right) = \alpha^*$ gives the required condition.

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α^* is given by

$$y\left(\alpha^*, ne_1\right) \frac{1}{n} - c(e_1). \tag{51}$$

This is maximized at

$$c'\left(e_1\right) = y_2\left(\alpha^*, ne_1\right) \tag{52}$$

which is the efficient effort level, given the state of the world α^*. □

Proof of proposition 5. We show that this sharing rule *i)* induces the leader to choose $e_1 = e^*\left(\alpha^*\right)$, and *ii)* all the other agents to follow the leader and to choose also e_1. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents follow the described strategy. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$\begin{aligned} y\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right) \Big[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) s^1 \\ + \left(1 - f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right) \frac{1-s^1}{n-1} \Big] - c(e). \end{aligned} \tag{53}$$

Since s^1 is independent of the realized output, agent j cannot affect it. The derivative of the last expression with respect to e is given by

$$\begin{aligned} & y_2\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\Bigg[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)s^1 \\ & +\left(1-f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)\frac{1-s^1}{n-1}\Bigg] \\ & +y\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f_1^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1}{n-1}-\frac{1}{n-1}\right)\right] \\ & -c'(e). \end{aligned} \tag{54}$$

For $e = e_1$ to be an equilibrium, it must be that the derivative is 0 at this point. Since $f^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) = 1/n$ and $c'(e^*\left(\alpha^*\right)) = y_2(\alpha^*, ne(\alpha^*))$, the previous expression can be rewritten as

$$\begin{aligned} & c'\left(e^*\left(\alpha^*\right)\right)-\frac{y_2\left(\alpha^*, ne^*\left(\alpha^*\right)\right)}{n} \\ & \quad = y\left(\alpha^*, ne^*\left(\alpha^*\right)\right)\left[f_1^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1}{n-1}-\frac{1}{n-1}\right)\right] \end{aligned} \tag{55}$$

which implies

$$s^1\left(e_1\right) = \frac{1}{n}+\frac{n-1}{n}\frac{c'\left(e_1\right)-\frac{y_2(\alpha^*, ne_1)}{n}}{y\left(\alpha^{**}, ne_1\right)f_1^1(e_1, \mathbf{e_1})}. \tag{56}$$

Inserting

$$s^1\left(e_1\right) = \frac{1}{n}+\frac{n-1}{n}\frac{c'\left(e_1\right)-\frac{y_2(\alpha^{**}(e_1), ne_1)}{n}}{y\left(\alpha^{**}\left(e_1\right), ne_1\right)f_1^1(e_1, \mathbf{e_1})} \tag{57}$$

and noticing that in the equilibrium $\alpha^{**}\left(e_1\right) = \alpha^*$ gives the required condition.

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α^* is given by

$$y\left(\alpha^*, ne_1\right)\frac{1}{n}-c(e_1) \tag{58}$$

which is maximized at

$$c'\left(e_1\right) = y_2\left(\alpha^*, ne_1\right) \tag{59}$$

which is the efficient effort level, given the state of the world α^*. □

Appendix B: Equilibrium existence

We now examine in which cases the efficient efforts implemented through sharing rule (6) constitute an equilibrium. Since equilibrium existence depends on the detailed specification of the curvature of the ranking technology, the production function and costs, we switch into a particular class in which we demonstrate that the exertion of efficient efforts constitutes a global utility maximum under our proposed sharing rule (6).

This is not the only case in which equilibria exist in our model. In order to illustrate this, we add an example of a commonly used model setup in which our candidate equilibrium exists. This example falls outside the class investigated in the following proposition.

Proposition 6. *We restrict attention to the class of problems consisting of output* $y(\alpha^*, e_1+(n-1)e^*(\alpha')) = \alpha^*\bar{w}(e_1+(n-1)e^*(\alpha'))$*, cost* $c(e_1) = (e_1^x)/x$*, for* $x > 1$*,* $\bar{w} > 0$ *and generalized Tullock contest success technology with precision parameter* r*. Moreover, we restrict permissible* α *to the compact range* $[a, na]$ *for* $a > 0$*. A sufficient condition for efficient effort provision by every player and truthful type reporting by player 1 to be an equilibrium in this class is that* $x = r$*.*

Proof of proposition 6. We start with the second stage effort choice problem given any report α'. Consider the objective

$$\begin{aligned} &u_1\left(e_1, \mathbf{e}^*_{-1}(\alpha'), \alpha^*\right) \\ &\quad = y(\alpha^*, e_1)\left(\frac{(1-f^1(e_1))(1-s^1(e_1))}{n-1} + f^1(e_1)s^1(e_1)\right) - c(e_1) \end{aligned} \tag{60}$$

where

$$s^1(e_1) = \frac{(n-1)\left(\frac{c'^*(\alpha')}{f_1^1(e^*(\alpha'))} - \frac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf_1^1(e^*(\alpha'))}\right)}{ny(\alpha^*, e_1)} + \frac{1}{n}. \tag{61}$$

We use shorthand notation $y(\alpha^*, \hat{e}) = y(\alpha^*, \hat{e} + (n-1)e^*(\alpha'))$, $f^1(\hat{e}) = f^1(\hat{e}, \mathbf{e_{-i}}^*(\alpha'))$ with $\hat{e} \in \{e_1, e^*(\alpha')\}$ and similarly for all other expressions.

Then $\dfrac{\partial u_1}{\partial e_1} =$

$$y(\alpha^*, e_1)\Bigg[\frac{(1-f^1(e_1))\left(\frac{(n-1)y_2(\alpha^*,e_1)\mu}{ny(\alpha^*,e_1)^2}+\frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha^*,e_1)}\right)}{n-1} \tag{62}$$

$$+ f^1(e_1)\Bigg(-\frac{(n-1)y_2(\alpha^*,e_1)\mu}{ny(\alpha^*,e_1)^2}-\frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha^*,e_1)}\Bigg)$$

$$+ f_1^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha^*,e_1)}+\frac{1}{n}\right)-\frac{f_1^1(e_1)\left(-\frac{(n-1)\mu}{ny(\alpha^*,e_1)}-\frac{1}{n}+1\right)}{n-1}\Bigg]$$

$$+y_2(\alpha^*,e_1)\left\{f^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha^*,e_1)}+\frac{1}{n}\right)+\frac{(1-f^1(e_1))\left(-\frac{(n-1)\mu}{ny(\alpha^*,e_1)}-\frac{1}{n}+1\right)}{n-1}\right\}$$

$$-c'(e_1)$$

in which $\mu = \dfrac{c'^*(\alpha')}{f_1^1(e^*(\alpha'))} - \dfrac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf_1^1(e^*(\alpha'))}$. (62) simplifies to

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{y_2(\alpha^*,e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1),e^*(\alpha'))}{n}\right) \\ &\quad - \tilde{\alpha}'(e_1)\frac{y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}.\end{aligned} \tag{63}$$

Inserting

$$\tilde{\alpha}'(e_1) = \frac{y_2(\alpha^*,e_1)}{y_1(\tilde{\alpha}(e_1),e^*(\alpha'))} \tag{64}$$

we obtain

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{y_2(\alpha^*,e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1),e^*(\alpha'))}{n}\right) \\ &\quad - \frac{y_2(\alpha^*,e_1)}{y_1(\tilde{\alpha}(e_1),e^*(\alpha'))}\frac{y_{1,2}(\tilde{\alpha}(e_1),e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}.\end{aligned} \tag{65}$$

For the linear case $y(\alpha^*,\hat{e}) = \alpha^* w(\hat{e}+(n-1)e^*(\alpha')) = \alpha^* w(\hat{e})$ using again the shortened notation for the function $w(\cdot)$ in the last step we get

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha^* w'(e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{\alpha^* w(e_1)w'^*(\alpha'))}{nw(e^*(\alpha'))}\right) \\ &\quad - \frac{\alpha^* w'(e_1)}{w(e^*(\alpha')}\frac{w'^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1)-1\right\}\end{aligned} \tag{66}$$

in which we substituted the linear adjustment

$$\tilde{\alpha}'(e_1) = \frac{\alpha^* w'(e_1)}{w(e^*(\alpha'))}. \tag{67}$$

Using linear $w(\hat{e}) = \bar{w}(\hat{e} + (n-1)e^*(\alpha'))$ and monomial cost $c(\hat{e}) = \hat{e}^x/x$, the foc equals

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = \frac{\alpha^*\bar{w}}{n} - e_1^{x-1} + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha^*\bar{w}(e_1 + e^*(\alpha')(n-1))}{n^2 e^*(\alpha')}\right) \\ - \frac{\alpha^*}{ne^*(\alpha')}\frac{\bar{w}}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1) - 1\right\}.\end{aligned} \tag{68}$$

We use $ke^*(\alpha')$ in order to allow for any possible effort deviation. Then substituting $e_1 = ke^*(\alpha')$, the report-contingent efficient $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$ and Tullock technology into the ratio of success function slopes gives

$$\begin{aligned}\frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))} &= \left(\frac{(n-1)re_1^{r-1}e^*(\alpha')^r}{(e_1^r + (n-1)e^*(\alpha')^r)^2}\right) / \left(\frac{(n-1)r}{e^*(\alpha')n^2}\right) \\ &= \frac{e^*(\alpha')n^2(e_1 e^*(\alpha'))^r}{e_1\left(e_1^r + (n-1)e^*(\alpha')^r\right)^2} \\ &= \frac{n^2\left(k(\alpha'\bar{w})^{\frac{2}{x-1}}\right)^r}{k\left(\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r + (n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r\right)^2} \\ &= \frac{n^2 k^{r-1}}{(k^r + n - 1)^2}.\end{aligned} \tag{69}$$

Making the same substitutions in the remainder of (68) step by step gives, for $e_1 = ke^*(\alpha')$

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = &\frac{\alpha^*\bar{w}}{n} - (e^*(\alpha')k)^{x-1} \\ &+ \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha^*\bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha')n^2}\right) \\ &- \frac{\alpha^*\bar{w}(nf^1(e^*(\alpha')k) - 1)}{e^*(\alpha')n^3 f_1^1(e^*(\alpha'))},\end{aligned} \tag{70}$$

inserting Tullock technology gives

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = & \frac{\alpha^* \bar{w}}{n} - (e^*(\alpha')k)^{x-1} \\ & + \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))} \left(e^*(\alpha')^{x-1} - \frac{\alpha^* \bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha')n^2} \right) \\ & - \frac{\alpha^* \bar{w} \left(\frac{n}{(n-1)e^*(\alpha')^r (e^*(\alpha')k)^{-r} + 1} - 1 \right)}{(n-1)nr},\end{aligned} \tag{71}$$

and finally inserting $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$ gives

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = & \frac{\alpha^* \bar{w}}{n} - \left(k(\alpha'\bar{w})^{\frac{1}{x-1}} \right)^{x-1} + \frac{n^2 k^{r-1}}{(k^r + n - 1)^2} \left(\left((\alpha'\bar{w})^{\frac{1}{x-1}} \right)^{x-1} \right. \\ & \left. - \frac{\alpha^* \bar{w} (\alpha'\bar{w})^{-\frac{1}{x-1}} \left(k(\alpha'\bar{w})^{\frac{1}{x-1}} + (n-1)(\alpha'\bar{w})^{\frac{1}{x-1}} \right)}{n^2} \right) \\ & - \frac{\alpha^* \bar{w} \left(\frac{n}{(n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}} \right)^r \left(k(\alpha'\bar{w})^{\frac{1}{x-1}} \right)^{-r} + 1} - 1 \right)}{(n-1)nr}\end{aligned} \tag{72}$$

which simplifies into

$$\frac{\partial u_1}{\partial e_1} = \bar{w} \left(\frac{\alpha^*}{n} - \alpha' k^{x-1} - \frac{\alpha^* (k^r - 1)}{nr (k^r + n - 1)} + \frac{k^{r-1} (\alpha' n^2 - \alpha^*(k + n - 1))}{(k^r + n - 1)^2} \right). \tag{73}$$

As a special case, we substitute $x = r$ and get

$$\frac{\partial u_1}{\partial e_1} = \frac{n}{\alpha^*} k^{r-1} \left(\frac{\alpha' n^2 - \alpha^*(k + n - 1)}{(k^r + n - 1)^2} - \alpha' \right) + 1 - \frac{k^r - 1}{r (k^r + n - 1)}. \tag{74}$$

We need to find a condition which ensures that this is positive for $k < 1$ and negative for $k > 1$.

1. $k < 1$: We need to ensure that

$$\frac{n}{\alpha^*} k^{r-1} \left(\frac{\alpha' n^2 - \alpha^*(k + n - 1)}{(k^r + n - 1)^2} - \alpha' \right) + 1 > \frac{k^r - 1}{r (k^r + n - 1)} \tag{75}$$

the right-hand side of which is negative whenever $k < 1$. In order for the left-hand side to be positive, we need

$$\frac{\alpha^* k^{1-r}}{n} + \frac{\alpha' n^2 - \alpha^*(k+n-1)}{(k^r+n-1)^2} > \alpha' \tag{76}$$

which is implied by

$$\alpha' \underbrace{\left(\frac{n^2}{(k^r+n-1)^2} - 1\right)}_{=A} + \alpha^* \underbrace{\left(\frac{k^{1-r}}{n} - \frac{1}{k^r+n-1}\right)}_{=B} > 0. \tag{77}$$

$A > 0$ for all $k < 1$ and $r > 0$ and $B > 0$ if $k < 1$ and

$$r > \frac{\log\left(\frac{k(n-1)}{n-k}\right)}{\log(k)} \geq \frac{n}{n-1} \tag{78}$$

where the final right-hand side term is the limit of the increasing log-ratio as $k \to 1$.

We showed that for $x = r$, it is true that

$$\frac{\alpha^*}{n} - \alpha' k^{x-1} - \frac{\alpha^*(k^r-1)}{nr(k^r+n-1)} + \frac{k^{r-1}(\alpha' n^2 - \alpha^*(k+n-1))}{(k^r+n-1)^2} > 0. \tag{79}$$

However, since the derivative of the left-hand side of the last inequality with respect to x is $-\alpha' k^{x-1} \ln k$ which is positive for any $k < 1$, the last inequality holds for any $x \geq r$.

2. $k > 1$: We start from (73) and want to show that

$$\frac{k^{r-1}(\alpha' n^2 - \alpha^*(k+n-1))}{(k^r+n-1)^2} - \alpha' k^{x-1} < \frac{\alpha^*(k^r-1)}{nr(k^r+n-1)} - \frac{\alpha^*}{n}$$

implied by

$$\frac{n}{\alpha^* k}\left(\frac{k^r(\alpha' n^2 - \alpha^*(k+n-1))}{(k^r+n-1)^2} - \alpha' k^x\right) + 1 < \frac{1}{\frac{nr}{k^r-1} + r} \tag{80}$$

the right-hand side of which is positive. Thus, we need to show that

$$\frac{n}{\alpha^* k}\left(\frac{k^r\left(\alpha' n^2-\alpha^*(k+n-1)\right)}{\left(k^r+n-1\right)^2}-\alpha' k^x\right)+1<0 \tag{81}$$

or, equivalently, that

$$\frac{\alpha^* k^{1-r}}{\alpha' n}+\frac{\alpha' n^2-\alpha^*(k+n-1)}{\alpha'\left(k^r+n-1\right)^2}<k^{x-r} \tag{82}$$

which is implied by

$$\frac{n^2}{\left(k^r+n-1\right)^2}-\frac{\alpha^*(k+n-1)}{\alpha'\left(k^r+n-1\right)^2}<\frac{k^{-r}\left(\alpha' n k^x-\alpha^* k\right)}{\alpha' n} \tag{83}$$

which gives

$$\frac{\alpha^* k-\alpha' n k^x}{n k^r}<\frac{\alpha^*(k+n-1)-\alpha' n^2}{\left(k^r+n-1\right)^2}. \tag{84}$$

We restrict possible $\alpha \in [a, b = sa]$, with $s > n$, and—since (84) is linear in α on both sides—obtain two subcases:

(a) Highest misreport $\alpha^* = a$, $\alpha' = b$: resulting in

$$\frac{a k^{-r}\left(k-s n k^x\right)}{n}<\frac{a\left(k-s n^2+n-1\right)}{\left(k^r+n-1\right)^2} \tag{85}$$

which holds for x sufficiently higher than r. For instance, for $x = r$, we obtain

$$\frac{k}{k^r}-1<s n-1+\frac{n(k-1+n-s n^2)}{(k^r-1+n)^2} \tag{86}$$

in which the left-hand side is negative and the right-hand side is positive for $k > 1$ because

$$\begin{aligned}\frac{sn\left(k^r+n-1\right)^2+n\left(k-1+n-sn^2\right)-\left(k^r-1+n\right)^2}{\left(k^r-1+n\right)^2} &> 0 \Longleftrightarrow \\ ns\left(k^r+n-1\right)^2-\left(k^r+n-1\right)^2+n\left(k+n^2(-s)+n-1\right) &> 0\end{aligned} \tag{87}$$

which equals

$$(ns-1)\left(k^{r}+n-1\right)^{2}>n^{3}s-n(k+n-1). \quad (88)$$

Recall that the left-hand side equals the right-hand side at $k = 1$ by construction. The left-hand side derivative is $2rk^{r-1}(ns-1)\left(k^{r}+n-1\right)>0$ and the rhs derivative is $-n<0$. Hence, for $k>1$, (86) holds.

(b) Lowest misreport $\alpha^{*}=b$, $\alpha'=a$: resulting in

$$\frac{ak^{-r}\left(sk-nk^{x}\right)}{n}<\frac{a(k-1+n)s-an^{2}}{\left(k^{r}+n-1\right)^{2}} \quad (89)$$

which also holds for x sufficiently higher than r. For instance, for $x=r$, we obtain

$$\frac{k}{k^{r}}-1<\frac{n}{s}-\frac{n\left(n^{2}-s(k+n-1)\right)}{s\left(k^{r}+n-1\right)^{2}}-1 \quad (90)$$

in which the left-hand side is negative for $k>1$ and the right-hand side is positive if

$$\frac{n}{s}-1>\frac{n\left(n^{2}-s(k+n-1)\right)}{s\left(k^{r}+n-1\right)^{2}}. \quad (91)$$

We can rewrite the last inequality as follows

$$\begin{aligned}\frac{n-s}{s} &> \frac{n\left(n^{2}-s(k+n-1)\right)}{s\left(k^{r}+n-1\right)^{2}} \Longleftrightarrow \\ (n-s)\left(k^{r}+n-1\right)^{2} &> n^{3}-sn(k+n-1).\end{aligned} \quad (92)$$

We have equality for $k=1$. The derivative of the right-hand side of the last inequality is $-sn$ which is negative for $s>0$, while the derivative of the left-hand side is $(n-s)\left(k^{r}+n-1\right)2rk^{r-1}$ which is positive for $n>s$. Hence, for $k>1$, (90) holds.

Given player 1's choice of $e_1 = e^*(\alpha')$ at the second stage,[17] we now move on to the reporting stage where she chooses α' in order to maximize utility

$$\begin{aligned} & \max_{\alpha'} y(\alpha^*, ne(\alpha')) \left(\sum_{l=1}^{n} f^l(\mathbf{e}^*(\alpha'))s^l(y^*, \alpha')\right) - c(e(\alpha')) \\ = & \max_{\alpha'} u_1(\mathbf{e}^*(\alpha'), \alpha^*) = y(\alpha^*, ne(\alpha'))\tfrac{1}{n} - c(e(\alpha')) \end{aligned} \tag{93}$$

because $f^l(\mathbf{e}^*(\alpha')) = 1/n$ for every l and $\sum_l s^l(y^*, \alpha') = 1$. This yields the first-order condition

$$y_2(\alpha^*, ne(\alpha')) = c'(e(\alpha')) \tag{94}$$

which equals the social planner's efficiency condition. Therefore, if the solution to the planner's problem is unique, then player 1 shares the same objective and will choose to truthfully report $\alpha' = \alpha^*$. □

The following example shows that it is easy to find instances violating the sufficient conditions of proposition 6 while still exhibiting the equilibrium identified in proposition 1.

Example 5: Consider the following two-players example outside of the class for which we show existence in proposition 6: (i) square-root team production $y(\alpha^*, e_1, e_2) = \alpha^*\bar{w}\sqrt{e_1 + e_2}$ and (ii) 'exponential difference' contest success function defined for two players as

$$f^1(e_1, e_2) = \frac{1}{1 + \exp\left(r(e_2 - e_1)\right)}, \text{ for } r > 0. \tag{95}$$

All other specifications are as in proposition 6. Assume that player two behaves according to our equilibrium prescription, i.e., $e_2 = e^*(\alpha') = 8^{\frac{1}{1-2x}}(\alpha'\bar{w})^{\frac{2}{2x-1}}$ (from the solution to the planner's problem). In this example setup, we obtain player one's objective as $u_1(e_1, e^*_{-1}(\alpha'), \alpha^*) =$

$$\frac{\alpha^*\bar{w}(e_1 + e^*(\alpha'))\left(s^1(y^*, \alpha')\left(\exp(r(e_1 - e^*(\alpha'))) - 1\right) + 1\right)}{\exp(r(e_1 - e^*(\alpha'))) + 1} \tag{96}$$

in which the equivalent of the sharing rule (6) is

$$s^1(y^*, \alpha') = \frac{1}{2}\left(1 - \frac{\alpha^*\bar{w}e_1 + \alpha^*\bar{w}e^*(\alpha') - 4e^*(\alpha')^x}{\alpha^*\bar{w}e_1e^*(\alpha')r + \alpha^*\bar{w}e^*(\alpha')^2 r}\right). \tag{97}$$

[17] Since the effort choice problem of the uninformed players is identical to that of the leader, this argument directly implies that also $e_j = e^*(\alpha')$ for every $j > 1$.

Consider parameter values $x = 2$, $r = 2.5$, $\alpha \in [1, 50]$. A plot of player one's objective against $e^*(\alpha')$ by player two in figure 1 shows no profitable deviations.

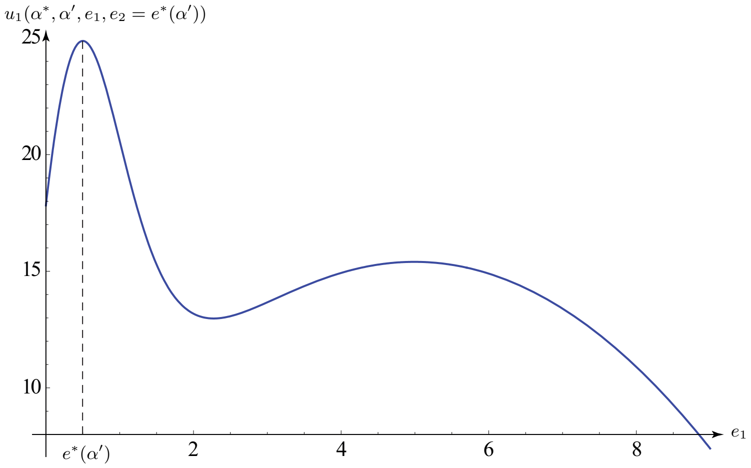

Figure 1: Possible deviations from $e^*(\alpha')$ for $\alpha \in [1, 50]$; the objective possess no other maxima.

Hence, this example illustrates that situations can be found which exhibit the equilibrium behavior derived in proposition 1 and lie outside of the class defined by the sufficient conditions presented in proposition 6.◁

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