Monograph:
Tsang, K.M. and Billings, S.A. (1992) Identification of Systems from Non-Uniformly Sampled Data. Research Report. ACSE Research Report 452 . Department of Automatic Systems Control and Engineering

Identification of systems from non-uniformly sampled data

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Research Report No 452

May 1992

Abstract

The identification of continuous time models from non-uniformly sampled data records is investigated and a new identification algorithm based on the state variable filter approach is derived. It is shown that the orthogonal least squares estimator can be adapted for the identification of continuous time models from non-uniformly sampled data records and instrumental variables are introduced to reduce the bias in stochastic system identification. Multiplying the filtered variables obtained from the state variable filter with higher powers of the noise free output signal prior to the estimation is shown to enhance the parameter estimates. Simulated examples are included to illustrate the methods.

1. Introduction

The problem of discrete time model identification has been thoroughly studied, and numerous parameter estimation methods have been developed for discrete time models based on uniformly sampled data records [1,2,3,4,5]. The problem of identification of systems with non-uniform sampling using digital computers has however received little attention. In closed loop control systems, the output response of a control system is usually very steady and high frequency system response components are rare. The sampling rate for a control system can therefore be significantly reduced in order to save computing power or release the computer to perform some other urgent tasks. A fast sampling rate is only required during the transition period, that is when there is a change in the input demand or if there are any external disturbances. Provided the slowest sampling rate does not violate the Nyquist sampling frequency, aliasing will be avoided.

Discrete time modelling of a dynamical process is highly dependent upon the sampling time of the system and for non-uniform sampling the discrete model is not well defined. The original plant dynamics are of course independent of the sampling time of the data because the continuous time model or the differential equation that describes the dynamics of the system under investigation remain unchanged. Hence if a continuous time model can be successfully recovered from non-uniformly sampled data records discrete time models for any sampling frequencies can be derived and classical or modern control laws can easily be derived based on either the continuous or discretized model.

In recent years, a lot of attention has been directed to the identification of continuous time systems and a comprehensive review of these developments can be found in [6], [7] and [8]. Some of these parameter estimation methods involve orthogonal functions such as block-pulse functions and numerical integration [9,10,11] and they all rely on integration rather than derivative operations. The main drawback with integration is the initial condition problem. Since the initial states of a system are usually neither known or zero they have to be identified together with the system parameters. An alternative approach is to use state variable filters [12,13] where "measures" of higher order derivatives are obtained from the state variable filters which can in turn be used for the identification of the system parameters. All the estimation methods cited above are based upon uniform sampling or in some cases special transducers or components are used to extract measures of the higher order derivatives for system identification.

In the present study a new estimation method is presented for the identification of systems from non-uniformly sampled data records. The new algorithm involves procedures for implementing state variable filters to recover measures of the higher order derivatives [12,13] of the input and output signals coupled with an orthogonal least squares estimator. This provides information regarding the structure, or which terms to include in the model, as well as parameter estimates from non-uniformly sampled data. A new instrumental variables version of the orthogonal estimator is also introduced to overcome bias which would otherwise be induced by noisy data. Simulation studies are included to illustrate the concepts.

2. Application of filters in continuous time model identification

Consider a single input single output continuous time system that can be described by the linear differential equation

$$A(D)y(t) = B(D)u(t) \tag{1}$$

where $u(t)$ is the input to the system, $y(t)$ is the output response, D is the differential operator, and A and B are polynomials in D.

$$\begin{aligned} A(D) &= a_0 D^n + a_1 D^{n-1} + \dots + 1 \\ B(D) &= b_1 D^{n-1} + \dots + b_n \\ D^n &= \frac{d^n}{dt^n} \\ n &= \textit{order of the dynamical system} \end{aligned} \tag{2}$$

If the input, output and the higher order derivatives are available, a set of simultaneous equations can then be set up for the identification of the unknown system parameters in eqn.(2). Unfortunately, higher order derivatives of the input and output are practically very difficult to obtain and this creates a fundamental problem when using eqn.(1) for system identification. One way around this problem is to replace the higher order derivatives with corresponding numerical approximations. However it has been found that derivative approximations are noise accentuating and the effects of noise in parameter estimation are undesirable. An alternative approach is to use state variable filters [12,13]. The state variable filter can therefore be included in the current identification process to provide approximations of the higher order derivatives. Consider the application of an all pole filter $1/F(D)$ on eqn.(1) to give

$$\frac{A(D)}{F(D)} y(t) = \frac{B(D)}{F(D)} u(t) \tag{3}$$

where

$$F(D) = f_0 D^m + f_1 D^{m-1} + \dots + f_m$$

and m is the order of the filter. Equation (3) can be written as

$$A(D) y^F(t) = B(D) u^F(t) \tag{4}$$

where $F(D) y^F(t) = y(t)$ and $F(D) u^F(t) = u(t)$ are the filtered output and input respectively. From eqn.(4), a regression model can be formed as

$$y^F(t) = [\ 1 - A(D)\ ]\ y^F(t) + B(D) u^F(t) \tag{5}$$

Hence if the filtered input, output and the associated higher order derivatives can be obtained, a set of simultaneous equations can be set up and the unknown parameters for A and B can be identified. Initially just consider the filtering operation applied to the input signal $u(t)$ and define

$$\begin{aligned} x_1(t) &= u^F(t) \\ x_2(t) &= D x_1(t) = D u^F(t) \\ &\vdots \\ x_m(t) &= D x_{m-1}(t) = D^{m-1} u^F(t) \end{aligned} \tag{6}$$

So that from eqn.(3) the filtered input signal $x_1(t) = u^F(t)$ and the associated higher order derivatives are related by

$$D x_m(t) = \frac{1}{f_0} \left(- f_1 x_m(t) - f_2 x_{m-1}(t) - \dots - f_m x_1(t) + u(t) \right) \tag{7}$$

Combine eqns.(6) and (7) and form the block diagram shown in Fig.1 where the filtered input signal and the higher order derivatives can easily be obtained from the output of individual integrators. The block diagram of Fig.1. can be realized using the basic summation and integration circuits common in analogue computers. Similarly, an identical filter can be applied to the output signal $y(t)$ to produce the filtered output signal and associated higher order derivatives. If the order of the filter m is higher than the order of the dynamical system n, there will be sufficient filtered higher order derivatives for the estimation of the unknown parameters A and B in eqn.(5).

3. Digital implementation of the state variable filters

If the filtered input, output and the associated higher order derivatives of a dynamical system Fig.1 are sampled with either a uniform or non-uniform sampling frequency a set of simultaneous equations can be set up for the estimation of the unknown polynomials A and B. The disadvantage of using the analogue filter shown in Fig.1 is that the filter can easily become saturated, and modifications of the filter characteristics is not easy. Also additional transducers are required to obtain measurements of the filtered input, output and the higher order derivatives. To allow for the maximum flexibility in implementing the state variable filters, the filters can be implemented on a digital computer so that characteristic of the filter can if necessary easily be modified by adjusting the coefficients of the filter function and very high order filters can be implemented. The matching of the input and output filters will be trivial and the problem of saturation is alleviated. The number of transducers required is also significantly reduced because only the system input and output signals are required to generate the filtered input, output and the derivatives. The filters can be implemented using the Euler or Runge-Kutta integration methods. The Euler method is preferable because of the efficiency and ease of implementation both of which are extremely important in on-line system identification and control.

Consider the filtering operations carried out by eqns.(6) and (7) and define

$$X(t) = [\ x_1(t)\ \ x_2(t)\ \ \dots\ \ x_m(t)\ ]^T \tag{8}$$

$$H(X(t),u(t),t) = \begin{bmatrix} x_2(t) \\ x_3(t) \\ \vdots \\ x_m(t) \\ \frac{1}{f_0}\left(-f_1 x_m(t) - f_2 x_{m-1}(t) - \dots - f_m x_1(t) + u(t) \right) \end{bmatrix} \tag{9}$$

where $u(t)$ is the input to the filter. The Euler or Runge-Kutta method must be used to solve

$$\frac{dX(t)}{dt} = H(X(t),u(t),t) \tag{10}$$

for $X(t)$. The numerical solution provided by the Euler approach [14] is given by

$$X(t_j) = X(t_{j-1}) + (t_j - t_{j-1})\, H(\ X(t_{j-1}),\ u(t_{j-1}),\ t_{j-1}\ ) \tag{11}$$

whereas a 4-th order Runge-Kutta method [14] yields

$$X(t_j) = X(t_{j-1}) + \frac{(t_j - t_{j-1})}{6} \left[K_1 + 2K_2 + 2K_3 + K_4 \right] \tag{12}$$

where

$$K_1 = H\left(X(t_{j-1}),\ u(t_{j-1}),\ t_{j-1} \right)$$

$$K_2 = H\left(X(t_{j-1}) + \frac{(t_j - t_{j-1})K_1}{2},\ u\left(\frac{t_j + t_{j-1}}{2}\right),\ \frac{t_j + t_{j-1}}{2} \right)$$

$$K_3 = H\left(X(t_{j-1}) + \frac{(t_j - t_{j-1})K_2}{2},\ u\left(\frac{t_j + t_{j-1}}{2}\right),\ \frac{t_j + t_{j-1}}{2} \right)$$

$$K_4 = H\left(X(t_{j-1}) + (t_j - t_{j-1})K_3,\ u(t_j),\ \frac{t_j - t_{j-1}}{2} \right)$$

Application of the filtering operations given by eqns.(11) or (12) to the system input and output signals produces the filtered input, output and the higher order derivatives which can be used for the identification of the unknown system parameters. Notice that eqns.(11) and (12) produce reasonable estimates of the filtered signal and the higher order derivatives for any step sizes or sampling periods $t_j - t_{j-1}$ satisfying the Nyquist sampling frequency.

4. The orthogonal least squares estimator

The orthogonal least squares estimation algorithm [15,16,17] has been shown to be a very efficient procedure for identifying unknown linear and nonlinear systems. The strength of the algorithm lies mainly in the fact that it provides information regarding which terms are significant in the model. This is often vital especially in the identification of nonlinear systems. The orthogonal least squares is also numerically superior to the ordinary least squares method. It would therefore appear to be appropriate to extend the orthogonal estimator so that it can be used for the identification of continuous time models from non-uniformly sampled data.

Consider the linear-in-the-parameter model

$$y^F(t) = \sum_{i=1}^{M} \theta_i p_i(t) + \epsilon(t) \tag{13}$$

where $y^F(t)$ represents the filtered system output; θ_i, $i=1,\ldots,M$ represents the M real unknown parameters associated with the variables $p_i(t)$, $p_i(t)$ are the filtered input, output and the higher order derivatives, and $\epsilon(t)$ is the modelling error. The orthogonal least squares algorithm involves a procedure of transforming eqn.(13) into an equivalent orthogonal equation

$$y^F(t) = \sum_{i=1}^{M} g_i w_i(t) + \epsilon(t) \tag{14}$$

such that

$$\frac{1}{N}\sum_{j=1}^{N} w_k(t_j) w_i(t_j) = 0 \qquad \forall \quad k \neq i \tag{15}$$

where N denotes the number of data records involved in the analysis. The orthogonalisation of the data records is performed by defining [15,16]

$$\begin{aligned} w_1(t_j) &= p_1(t_j) \\ w_i(t_j) &= p_i(t_j) - \sum_{k=1}^{i-1} \alpha_{ki} w_k(t_j) \; ; \; i=1,\ldots,M \\ \alpha_{ki} &= \frac{\frac{1}{N}\sum_{j=1}^{N} w_k(t_j) p_i(t_j)}{\frac{1}{N}\sum_{j=1}^{N} w_k^2(t_j)} \quad ; \; \begin{cases} k=1,\ldots,i-1 \\ i=1,\ldots,M \end{cases} \end{aligned} \tag{16}$$

The orthogonal parameter estimates $\hat{g}_i$ can then be obtained according to the formula

$$\hat{g}_i = \frac{\frac{1}{N}\sum_{j=1}^{N} y^F(t_j) w_i(t_j)}{\frac{1}{N}\sum_{j=1}^{N} w_i^2(t_j)} \quad ; \quad i=1,\ldots,M \tag{17}$$

and the original system parameters estimates $\hat{\theta}_i$ can be recovered from eqns.(16) and (17) as

$$\begin{aligned} \hat{\theta}_M &= \hat{g}_M \\ \hat{\theta}_k &= \hat{g}_k - \sum_{i=k+1}^{M} \alpha_{ki} \hat{\theta}_i \; ; \; k=M-1,\ldots,1 \end{aligned} \tag{18}$$

A by-product of the orthogonal estimation algorithm is the error reduction ratio which is defined as [15,16]

$$\epsilon RR_i = \frac{\frac{1}{N}\sum_{j=1}^{N} \hat{g}_i^2 w_i^2(t_j)}{\frac{1}{N}\sum_{j=1}^{N} \left(y^F(t_j)\right)^2} \times 100 \quad ; \quad i=1,\dots,M \tag{19}$$

This can serve as an indicator of how important a particular dependent variable is and how close the estimated model is to the original system dynamics. If the sum of the error reduction ratio is close to 100 the fitted model should be an adequate representation of the system under investigation.

5. Stochastic system identification

Linear stochastic systems can generally be described by the differential equation

$$A(D)z(t) = B(D)u(t) + C(D)\zeta(t) \tag{20}$$

where $z(t)$ is the noisy output, $u(t)$ is the input and $\zeta(t)$ is uncorrelated white noise

$$C(D) = c_0 D^n + c_1 D^{n-1} + \dots + 1 \tag{21}$$

Applying the state variable filter $1/F(D)$ to eqn.(20) gives

$$A(D)z^F(t) = B(D)u^F(t) + \frac{C(D)}{F(D)}\zeta(t) \tag{22}$$

or

$$z^F(t) = [\ 1 - A(D)\ ]\ z^F(t) + B(D)u^F(t) + C^*(D)\zeta(t) \tag{23}$$

where $C^*(D) = C(D)/F(D)$. From eqn.(23), an unbiased estimate of the system can be obtained if $u^F(t)$, $z^F(t)$, $\zeta(t)$ and the associated higher order derivatives are available. Since $\zeta(t)$ and the higher order derivatives are unmeasurable, estimation algorithms which exclude the noise from the estimation are desirable. The instrumental variable method [18] is one possible way of eliminating noise from the estimation. This is achieved by introducing instrumental variables which are highly correlated with the system output but uncorrelated with the noise to produce unbiased estimates of A and B.

5.1 Signal to noise ratio enhancement

Consider a noise corrupted process

$$z(t) = y(t) + e(t) \tag{24}$$

where $z(t)$ is the noise corrupted signal, $y(t)$ is the noise free signal and $e(t)$ is a zero mean additive noise which is independent of $y(t)$. Multiplying $z(t)$ with itself and taking the expected value gives

$$E[z^2(t)] = E[y^2(t)] + E[e^2(t)] \tag{25}$$

because $E[y(t)e(t)] = 0$ where $E[.]$ denotes the expected value. The signal to noise ratio for eqn.(25) is defined as

$$[S/N]_0 = \frac{E[y^2(t)]}{E[e^2(t)]} \tag{26}$$

If the signal to noise ratio is high, the signal $z(t)$ becomes more deterministic and the prediction of z(t) should be good. However, if the signal to noise ratio is low, the signal $z(t)$ will be dominated by the noise process and the prediction of the signal will be poor. Hence if the signal to noise ratio of a signal $z(t)$ can be artificially improved, the output prediction of the signal will also be enhanced.

Consider the multiplication of the noise corrupted signal $z(t)$ with the r-th power of the noise free signal $y(t)$ to give

$$y^r(t)z(t) = y^{r+1}(t) + y^r(t)e(t) \quad , \quad r=1,2,3,... \tag{27}$$

Squaring both sides of eqn.(27) and taking expected value gives

$$E[y^{2r}(t)z^2(t)] = E[y^{2(r+1)}(t)] + E[y^{2r}(t)]E[e^2(t)] \tag{28}$$

because $E[y^{2r+1}(t)e(t)] = 0$ and $E[y^{2r}(t)e^2(t)] = E[y^{2r}(t)]E[e^2(t)]$. The signal to noise ratio for the modified signal of eqn.(27) is given as

$$[S/N]_r = \frac{E[y^{2(r+1)}(t)]}{E[y^{2r}(t)]E[e^2(t)]} \tag{29}$$

When the signal $z(t)$ is multiplied by $y^{r-1}(t)$, the signal to noise ratio becomes

$$[S/N]_{r-1} = \frac{E[y^{2r}(t)]}{E[y^{2(r-1)}(t)]E[e^2(t)]} \tag{30}$$

Assuming the process is ergodic such that the expected value can be approximated by the sampled mean

$$E[y^{2(r+1)}(t)] \approx \frac{1}{N}\left(y^{2(r+1)}(t_1) + y^{2(r+1)}(t_2) + \dots + y^{2(r+1)}(t_N)\right) \tag{31}$$

$$E[y^{2(r-1)}(t)] \approx \frac{1}{N}\left(y^{2(r-1)}(t_1) + y^{2(r-1)}(t_2) + \dots + y^{2(r-1)}(t_N)\right) \tag{32}$$

$$E[y^{2r}(t)] \approx \frac{1}{N}\left(y^{2r}(t_1) + y^{2r}(t_2) + \dots + y^{2r}(t_N)\right) \tag{33}$$

where N is the number of data records. Multiplying eqn.(31) with eqn.(32) and comparing with the square of eqn.(33) gives

$$\left\{\frac{1}{N}\left(y^{2(r+1)}(t_1)+\dots+y^{2(r+1)}(t_N)\right)\right\}\left\{\frac{1}{N}\left(y^{2(r-1)}(t_1)+\dots+y^{2(r-1)}(t_N)\right)\right\} \geq \left\{\frac{1}{N}\left(y^{2r}(t_1)+\dots+y^{2r}(t_N)\right)\right\}^2 \tag{34}$$

or

$$E[y^{2(r+1)}(t)]E[y^{2(r-1)}(t)] \geq \left(E[y^{2r}(t)]\right)^2 \tag{35}$$

by the Schwarz inequality. Rearranging eqn.(35) and dividing both sides by $E[e^2(t)]$ gives

$$\frac{E[y^{2(r+1)}(t)]}{E[y^{2r}(t)]E[e^2(t)]} \geq \frac{E[y^{2r}(t)]}{E[y^{2(r-1)}(t)]E[e^2(t)]}$$

or

$$[S/N]_r \geq [S/N]_{r-1} \tag{36}$$

From eqn.(36), the signal to noise ratio of the noisy process can be artificially improved by multiplying by higher orders of the noise free signal The signal to noise ratio is progressively improved as the order of the noise free signal power is increased. If the order of the noise free signal power is sufficiently high, the modified process approaches a deterministic system. Notice that the signal to noise ratio can be improved according to eqn.(36) only if the noise free signal is available. In actual practice, the noise free signal is difficult to obtain but an estimate can be evaluated using any of the well known estimation algorithms such as the least squares estimation algorithm.

5.2 Estimation procedures

Define the noise free model associated with eqn.(22) as

$$A(D)y^F(t) = B(D)u^F(t) \tag{37}$$

d) Multiply the filtered records $u^F(t)$, $Du^F(t)$, ..., $z^F(t)$, $Dz^F(t)$, ... by $(y^F(t))^r$ for a large value of r and re-apply the orthogonal least squares estimator to obtain new estimates for A and B.

e) Repeat c) and d) until convergence.

6. Simulated examples

A linear system S_1 given by

$$0.25D^2y(t) + 0.7Dy(t) + y(t) = 1.25u(t) \tag{43}$$

was excited by a random input signal of bandwidth 4 rad/s. 3000 pairs of non-uniformly sampled input and output data records were collected for the identification of the system parameters. Figure 2(a) shows the input and output and a zoomed plot of the sampled records (Fig. 2(b)) reveals the non-uniform sampling intervals. A sixth order Butterworth filter with a cutoff frequency of 4 rad/s which covered the whole range of frequency of interest was selected as the state variable filter to produce the filtered input, output and the higher order derivatives for system identification. The Euler method was used to implement the Butterworth filter and the filtered input, output and the higher order derivatives shown in Fig.3 were generated after passing the input and output data records to the filter. A second order dynamical model was therefore specified for estimation and the generated $u^F(t)$, $Du^F(t)$, $y^F(t)$, $Dy^F(t)$ and $D^2y^F(t)$ were used for the identification of the unknown parameters. Application of the orthogonal least squares estimator to the filtered data records produced the estimate

$$\begin{aligned} y^F(t) = &-0.2515D^2y^F(t) - 0.7044Dy^F(t) + 0.0055Du^F(t) + 1.25u^F(t) + \varepsilon(t) \\ &\quad (61.9710) \qquad\quad (27.7805) \qquad\quad (0.0025) \qquad (10.2460) \end{aligned} \tag{44}$$

The bracketed value beneath the individual parameters indicates the error reduction ratio of each corresponding candidate term. As the fitted model of eqn.(44) has a sum of error reduction ratios of 100 this suggests that it is an adequate representation of the system under investigation. A comparison with the original model of eqn.(43) indicates that an excellent model has been obtained and that the estimated model is unaffected by the non-uniform sampling intervals.

The identification of a stochastic linear system with a signal to noise ratio of 17dB was used to defined system S_2

$$\begin{aligned} 0.25D^2y(t) + 0.7Dy(t) + y(t) &= 1.25u(t) \\ z(t) &= y(t) + e(t) \end{aligned} \tag{45}$$

where $e(t)$ is a zero mean white noise. 3000 data records were collected and these are illustrated in Fig.4. Sixth order Butterworth filters with a cutoff frequency of 4 rad/s were applied to the input and output data to produce the filtered input, output and the higher order derivatives as shown in Fig.5. Application of the orthogonal least squares estimator to the filtered data records produced the estimate

$$\underset{}{z^F(t)} = \underset{(61.4917)}{-0.2426D^2z^F(t)} - \underset{(27.9009)}{0.6978Dz^F(t)} - \underset{(0.0055)}{0.0080Du^F(t)} + \underset{(10.5702)}{1.2412u^F(t)} + \epsilon(t) \tag{46}$$

A comparison with eqn.(45) reveals that a good estimate has been obtained and the fitted model eqn.(46), captured 99.9683% of the total output power. The model can be further refined by multiplying the filtered variables with high powers of the noise free output obtained from eqn.(46) as described in section 5.2. Initially, a set of instrumental variables$y^F(t)$ were evaluated using the initial model of eqn.(46), the noise free filtered input and associated higher order derivatives, and the Euler integration method. From eqn.(36) an improved estimate can be obtained if the filtered variables are multiplied by higher powers of the noise free output before the estimation as the modified system becomes more deterministic. When the filtered variables were multiplied by $y^F(t)$, the model

$$z^F(t) = \underset{(70.2085)}{-0.24421D^2z^F(t)} - \underset{(13.0575)}{0.6978Dz^F(t)} - \underset{(0.0008)}{0.0040Du^F(t)} + \underset{(16.7262)}{1.2449u^F(t)} + \epsilon(t) \tag{47}$$

resulted after two iterations. The sum of the error reduction ratios was 99.993% indicating an improved estimate has been obtained compared to 99.968% captured by the model of eqn.(46). When the filtered input, output and the higher order derivatives were multiplied by $(y^F(t))^2$, the fitted model became

$$z^F(t) = \underset{(74.7625)}{-0.2451D^2z^F(t)} - \underset{(8.27415)}{0.6982Dz^F(t)} - \underset{(0.000149)}{0.0020Du^F(t)} + \underset{(16.9604)}{1.2474u^F(t)} + \epsilon(t) \tag{48}$$

which captured 99.997% of the total output power. Multiplying the filtered variables with $(y^F(t))^3$ produced the estimate

$$z^F(t) = \underset{(78.391)}{-0.2456D^2z^2(t)} - \underset{(6.13948)}{0.6983Dz^F(t)} - \underset{(2.5412e-5)}{0.00095Du^F(t)} + \underset{(15.468)}{1.2485u^F(t)} + \epsilon(t) \tag{49}$$

capturing 99.9986% of the total output power. A comparison of eqns.(46), (47), (48) and (49) with eqn.(45) indicates that a progressively better estimate has been obtained as the filtered variables are multiplied by higher powers of the noise free output signal. The coupling of the noise free output signal with the filtered variables greatly enhances the final estimate.

A third example system S_3 was derived from system S_2 by reducing the signal to noise ratio to -3dB. Again 3000 data records were collected and these are illustrated in Fig.6. Figure 7 shows the filtered input, output and the associated higher order derivatives when sixth order Butterworth filters were applied to the input and output data records. The initial estimate after applying the orthogonal least squares estimator was

$$\begin{aligned} z^F(t) = -&\underset{(55.8529)}{0.1723D^2z^F(t)} - \underset{(26.665)}{0.6124Dz^F(t)} - \underset{(1.6792)}{0.1108Du^F(t)} + \underset{(13.5815)}{1.1261u^F(t)} + \epsilon(t) \end{aligned} \quad (50)$$

The sum of the error reduction ratios is 97.7787% indicating that around 2.22% of the output power has not been captured by the initial estimate of eqn.(50). Table 1 shows the results of the estimation as the filtered variables are multiplied higher powers of the noise free output. The estimated model progressively improves as the filtered variables are multiplied by higher powers of the noise free output signal. Also the sum of the error reduction ratios progressively increases as higher powers of the noise free output are coupled to the filtered variables.

7. Conclusions

An orthogonal least squares estimation algorithm coupled with a state variable filter has been derived for the identification of stochastic systems with uniform or non-uniform sampling. It has been shown that the application of the state variable filter reduces the effects of noise on the estimation and that coupling the filtered variables with higher powers of the noise free signal can further improve the final estimates.

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coupling variable	a_0 0.25	a_1 0.7	b_1 0	b_2 1.25	$\sum \epsilon RR_i$
$(y^F(t))^0$	0.1723	0.6124	-.1108	1.1261	97.7787
$(y^F(t))^1$	0.1850	0.6255	-.0821	1.1765	99.4739
$(y^F(t))^2$	0.1928	0.6322	-.0643	1.1995	99.7580
$(y^f(t))^3$	0.1979	0.6376	-.0547	1.2126	99.8774
$(y^F(t))^4$	0.2010	0.6404	-.0497	1.2195	99.9323
$(y^F(t))^5$	0.2031	0.6413	-.0470	1.2230	99.9595

Table 1. Parameter estimates for system S_3

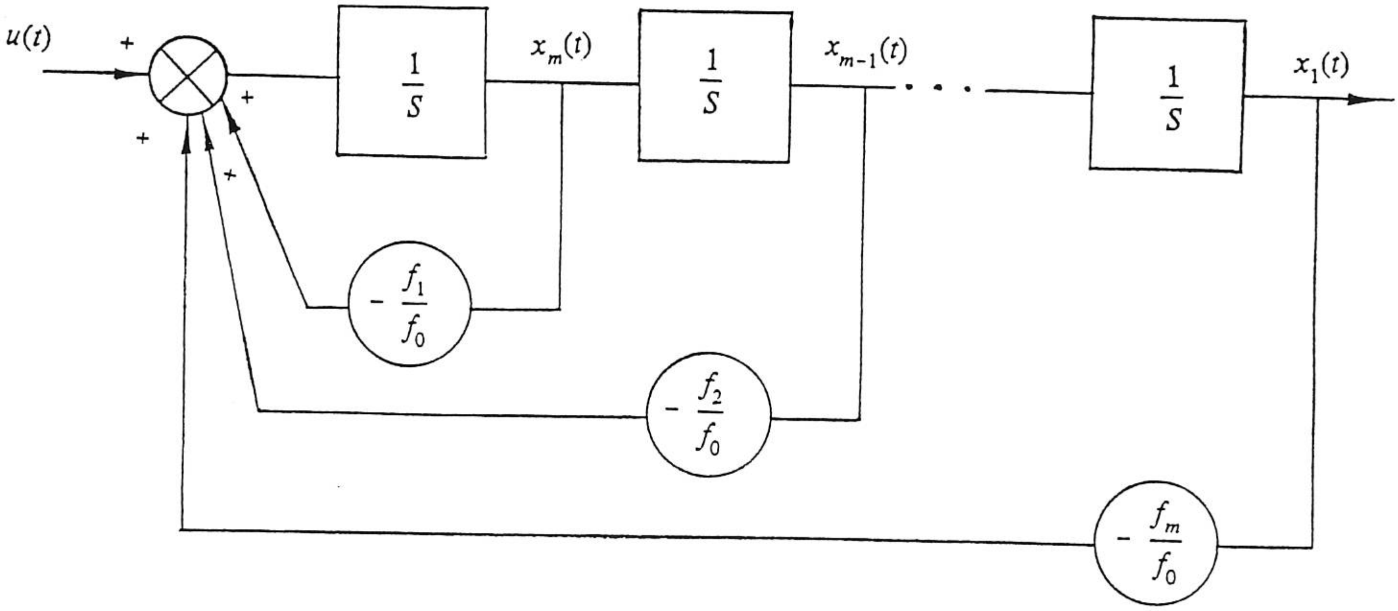
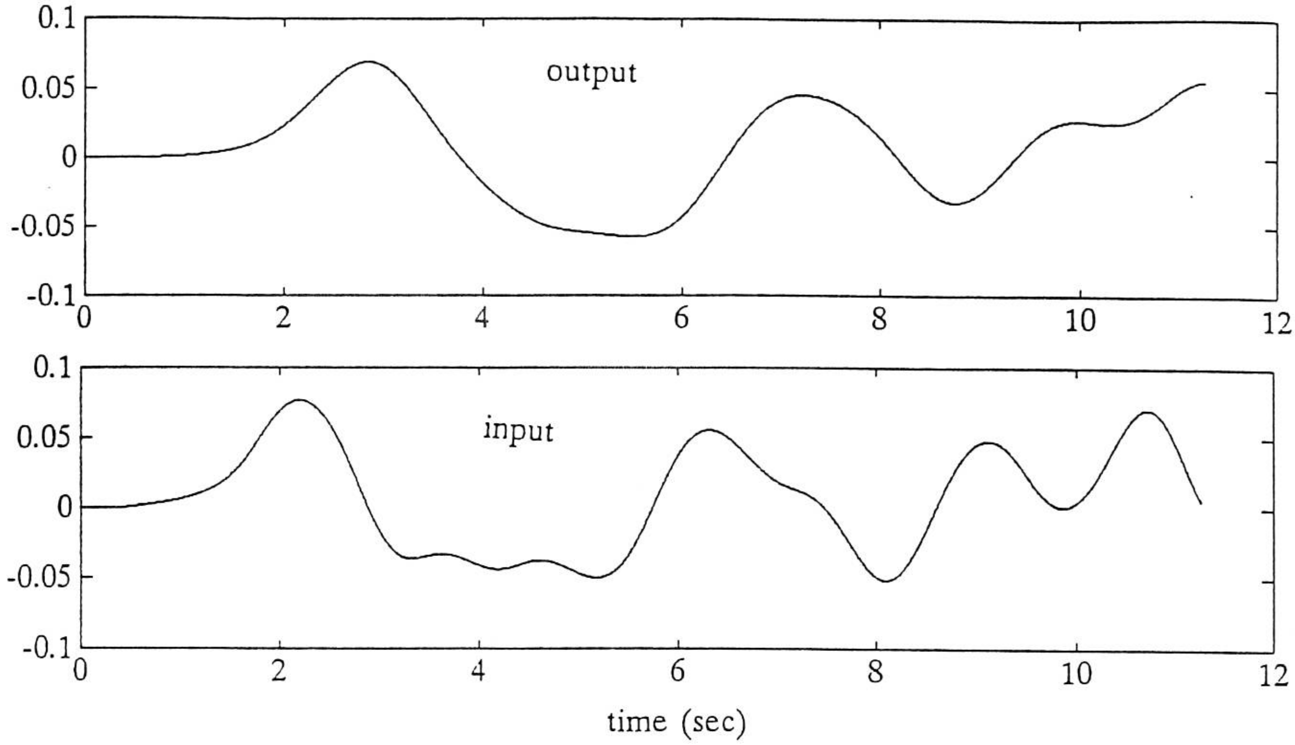
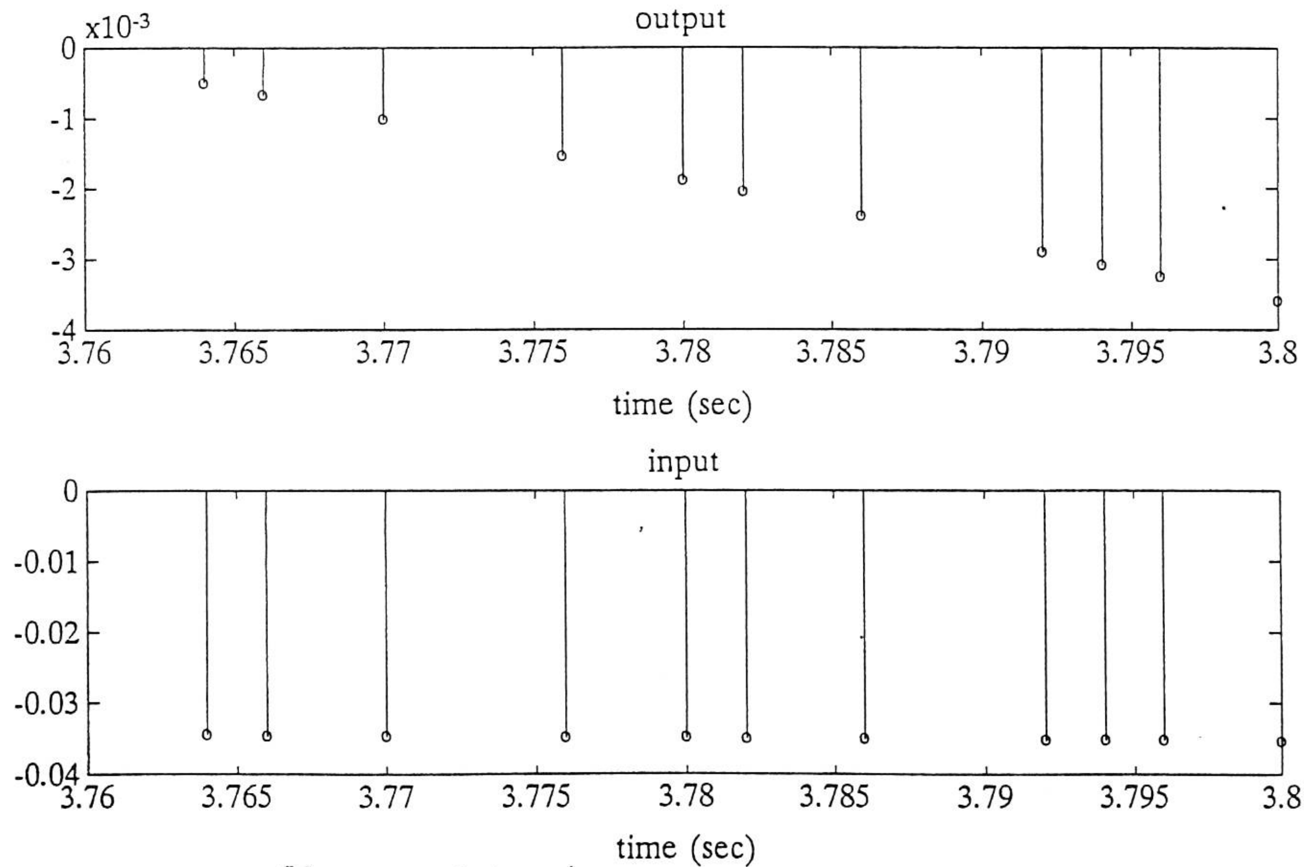


Figure 1. Block diagram of a state variable filter

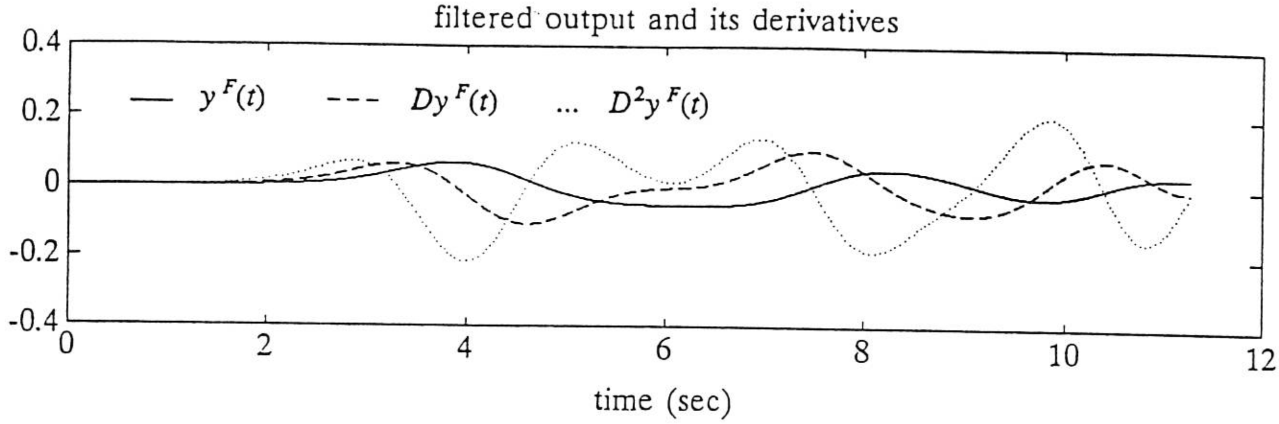


(a) Input and output for system S_1



(b) a zoomed view of the input and output for system S_1

Figure 2. Input and output data records for system S_1



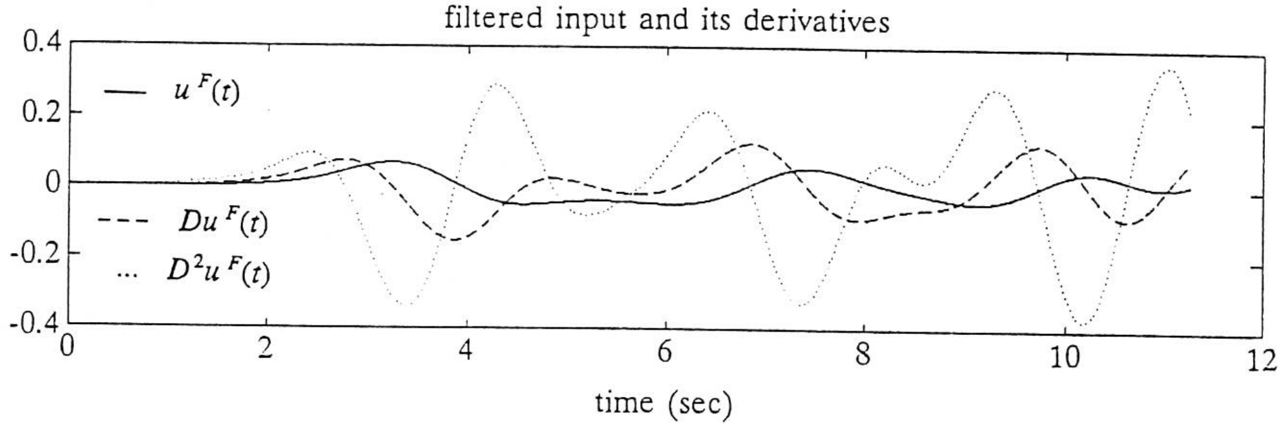
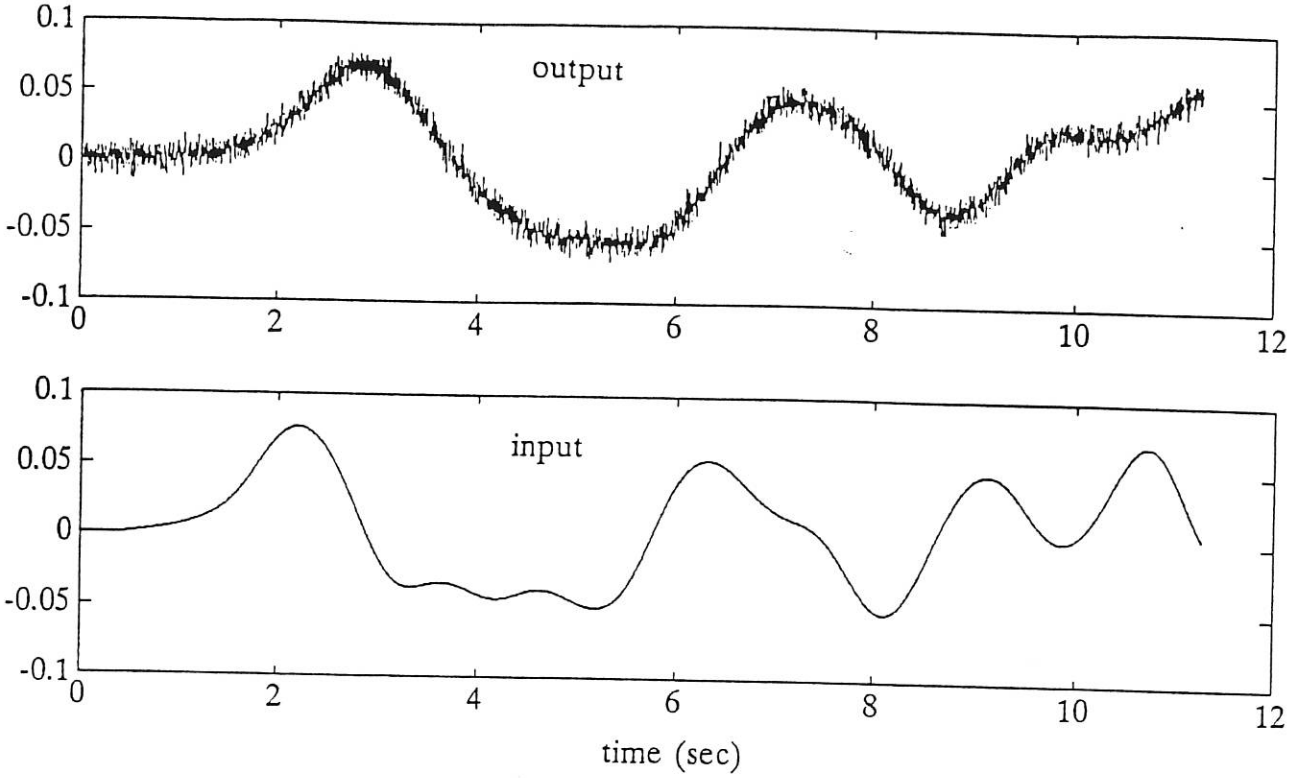
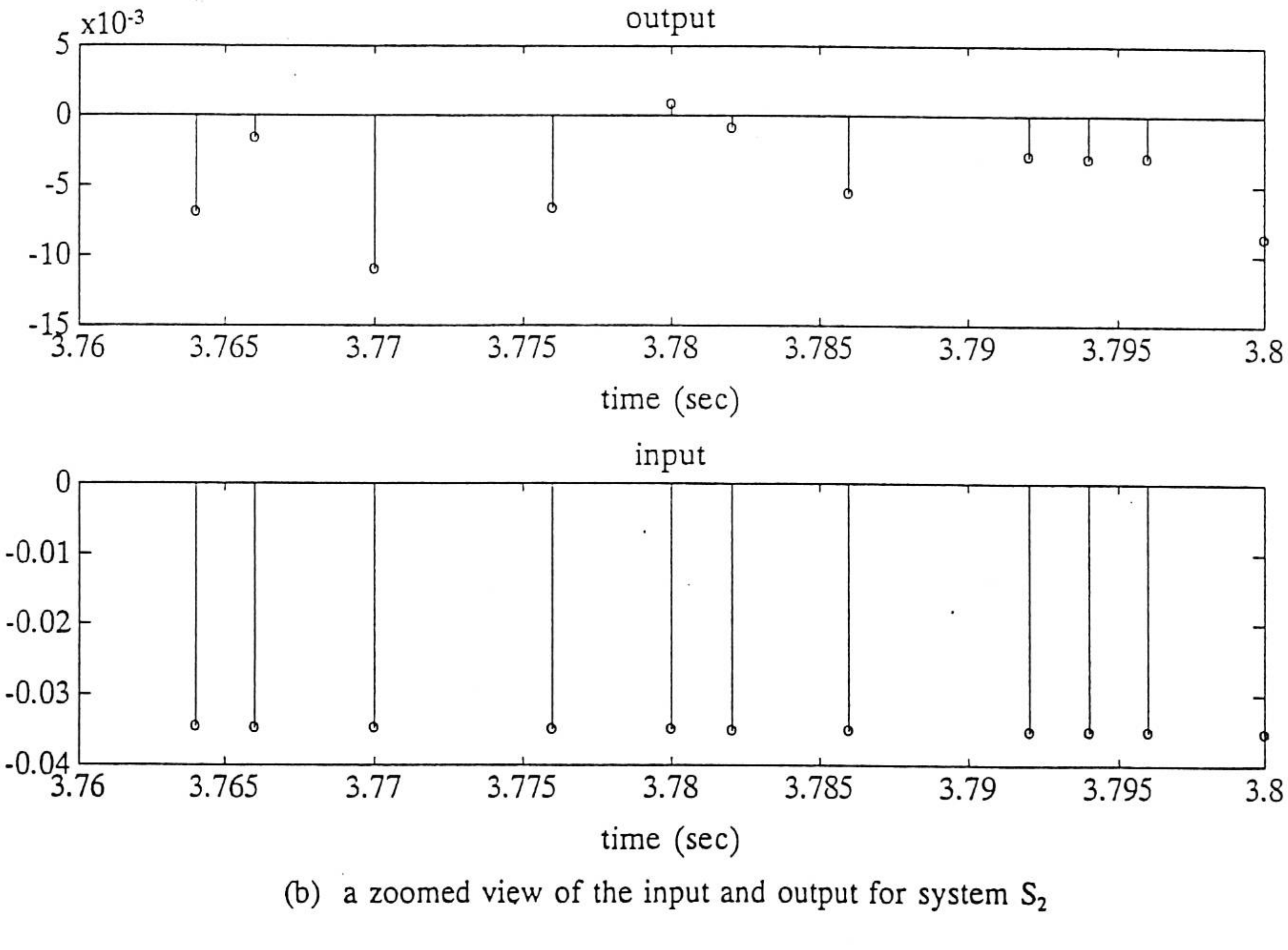


Figure 3. Filtered input, output and the associated higher order derivatives for S_1

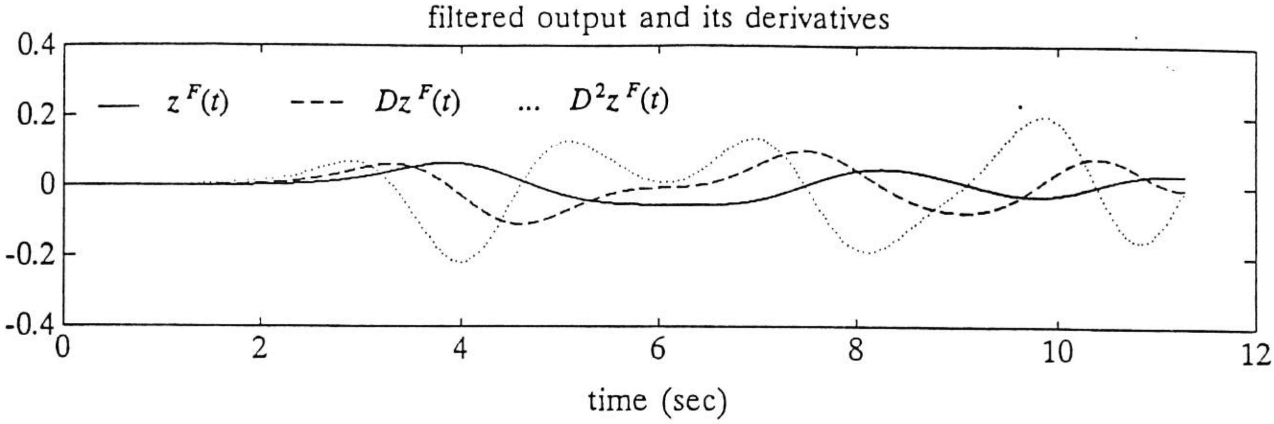


(a) Input and output for system S_2



(b) a zoomed view of the input and output for system S_2

Figure 4. Input and output data records for system S_2



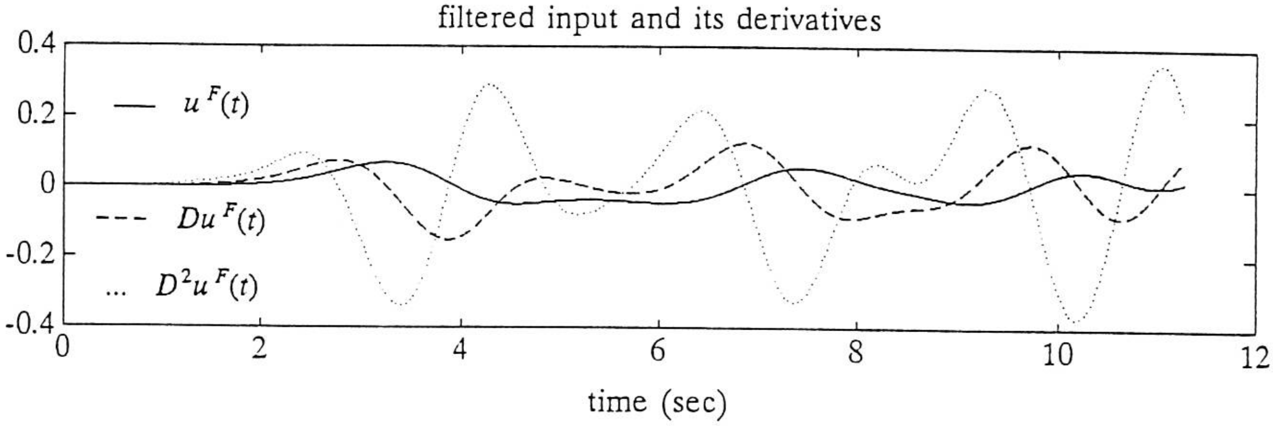
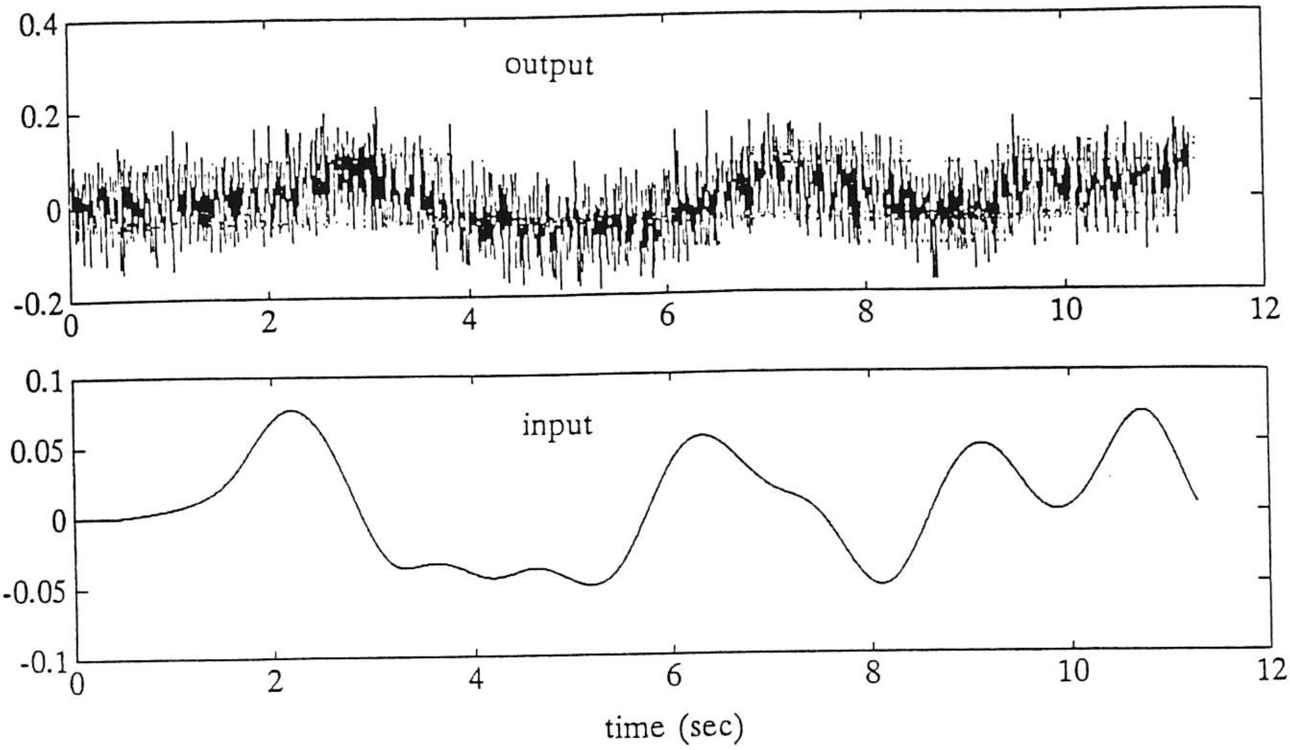
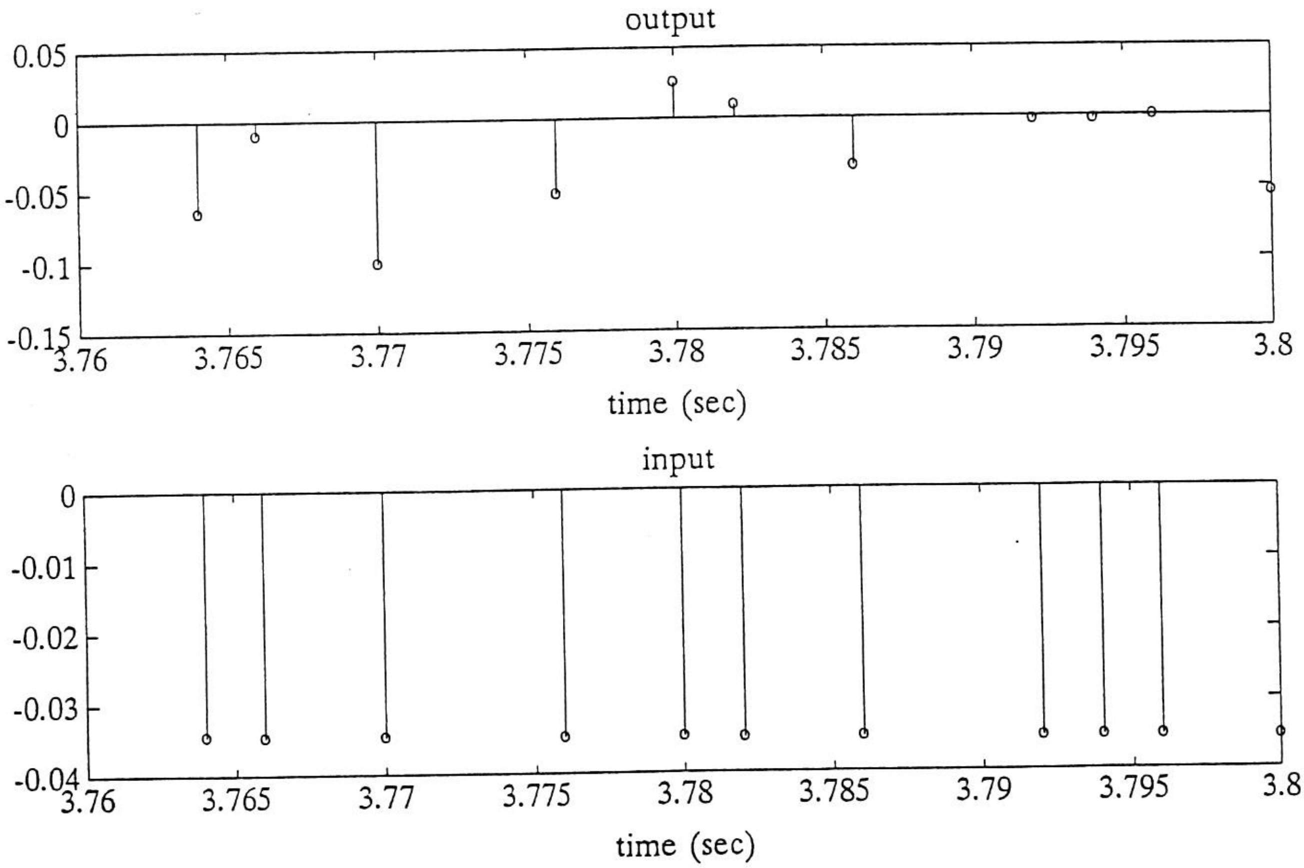


Figure 5. Filtered input, output and the associated higher order derivatives for S_2

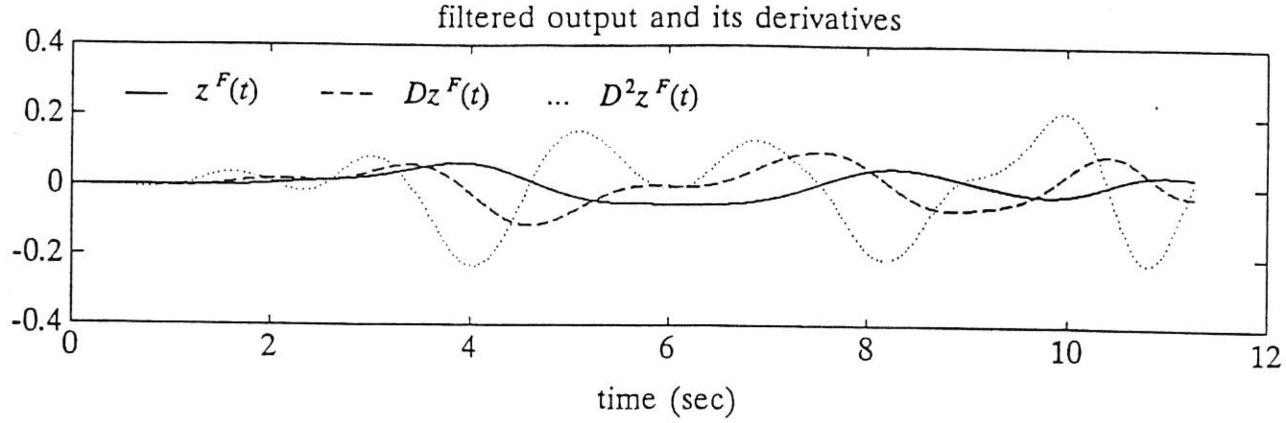


(a) Input and output for system S_3



(b) a zoomed view of the input and output for system S_3

Figure 6. Input and output data records for system S_3



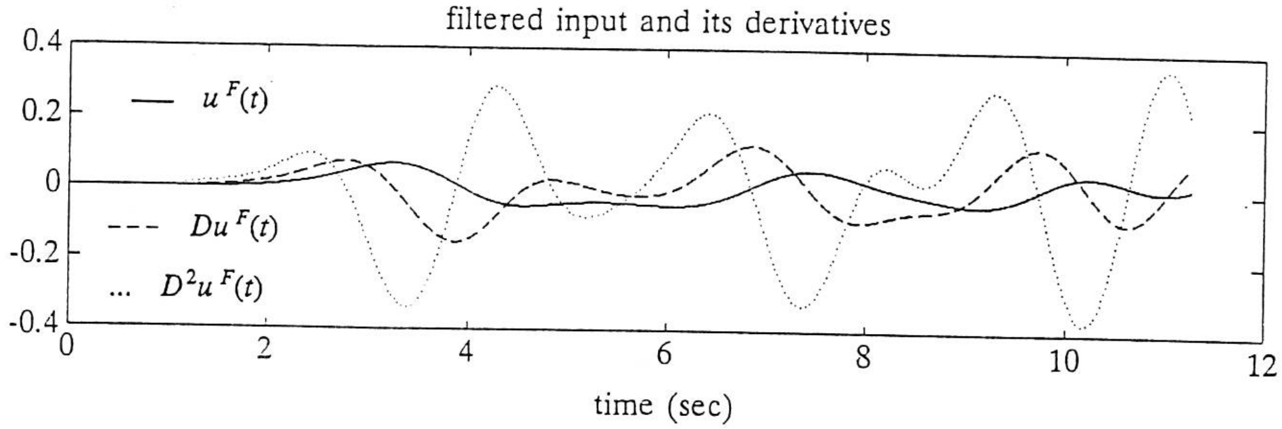


Figure 7. Filtered input, output and the associated higher order derivatives for S_3