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Parametric analysis of field cancellation in a three–dimensional propagation medium

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Table of contents

Abstract

An analysis of the process of field cancellation in a three-dimensional non-dispersive propagation medium is given. This analysis is presented from the perspective of the design of active noise control (ANC) systems for broadband noise emanating from compact sources. The analysis of cancellation is presented in parametric terms, both in time and in frequency, leading to a three-dimensional description of cancellation that is suitable for determining the geometrical arrangement for a given degree of cancellation and consequently can be used for the design of ANC systems. Such parameters can also be used to determine the performance of the resulting ANC system.

1 Introduction

Active cancellation of unwanted noise is based upon the intentional superposition of acoustic waves such that to result in destructive interference. This uses artificially generated source(s) to emit sound waves that interfere with the unwanted noise so that the destructive interference of the component waves can be used to attenuate unwanted noise. There are a number of factors that influence the cancellation of the unwanted noise, among them are: the artificially generated (secondary) source(s) as related to the unwanted (primary) source, physical separation between the sources and the properties of the medium into which the sources emit waves.

To achieve cancellation at a given point in the propagation medium it is first required that the sources should be coherent. This implies that the secondary source must emit a wave of the same frequency as the primary wave and with a constant phase relative to it. The second requirement in this process is that the waves be in anti-phase at the point so that they interfere with each other destructively. If the first requirement is not met then the waves are uncorrelated and stable cancellation is not possible. If the first requirement is fulfilled but not the second then the interference between the waves may be constructive implying a reinforcement of the primary wave and an overall increase in the level of sound.

In propagating through the medium the waves are affected in amplitude as well as in phase by the properties of the medium. This will determine whether the interference at a specified point in the medium is destructive or constructive. Here, assuming a linear propagation medium, the separation between the sources will play a major role.

These effects are discussed and verified through an analysis and development of the necessary conditions for active noise cancellation of a compact (point) source by a second, artificially generated source based on the assumption that the sources emit waves into a non-dispersive (linear) propagation medium under stationary or slowly varying conditions. In a stochastic environment, e.g. due to the random nature of the noise, and/or when the controller is adaptive such an analysis holds during periods of time when there is no substantial variation in the system; i.e. when the noise varies slowly or remains stationary and the control process realises steady (fixed) transfer characteristics. A time-domain and the corresponding frequency-domain analysis of the process of phase cancellation leading to the basic conditions of cancellation have previously been considered [1 - 3]. However, a rigorous analysis and geometric description of the process of cancellation is not given. The process of field cancellation is considered here from a frequency-domain view point so that the analysis, for a single tone of the component waves, provides a quantitative measure of the degree of cancellation, in terms of amplitude and phase parameters of the waves. In the case of broadband noise this is extended to provide relationships for the degree of cancellation as function of frequency. Moreover, geometry-related conditions involving separation between the sources and location of a point in the medium at which cancellation is required leading to a three-dimensional description of cancellation are obtained.

2 General conditions of field cancellation

In this section a quantitative measure of the degree of cancellation is developed. This is given in terms of the amplitude and phase parameters of the component waves and the general conditions of field cancellation upon which the synthesis of ANC systems can be based.

2.1 *The field cancellation factor*

Let a (primary) source S_p emit a (sound) wave $p(t)$ as function of time t into a non-dispersive (linear) propagation medium. In doing so it will produce a field $p(r, t)$ in the medium, where r is the distance measured from S_p. At a fixed point O in the medium, with distance r_g from the primary source, the primary wave $p(t)$ will give rise to a field $p(r_g, t)$, or simply $p_o(t)$ as function of t only. As depicted in Fig. 1, let a (secondary) source, S_s, be placed at a distance d from S_p in the medium and emit a wave $s(t)$. In propagating through a distance r_h from S_s, the wave $s(t)$ will give rise to $s_o(t)$ at point O. The combination of the component waves from S_p and S_s at point O will result in a (combined or observed) signal $o(t)$. With ω representing the radian frequency, let the power spectral densities of the waves be denoted as follows

$G_{pp}(\omega) =$ auto power spectral density of the primary wave $p(t)$,

$G_{ss}(\omega) =$ auto power spectral density of the secondary wave $s(t)$,

$G_{ppo}(\omega) =$ auto power spectral density of the primary wave $p_o(t)$,

$G_{sso}(\omega) =$ auto power spectral density of the secondary wave $s_o(t)$,

$G_{cco}(\omega)$ = auto power spectral density of the observed wave $o(t)$.

Cancellation at the observation point O (of the primary wave) occurs when the power spectral density of the combined wave is less than that of the primary wave acting alone;

$$G_{cco}(\omega) < G_{ppo}(\omega) \tag{1}$$

For a quantitative description of cancellation the field cancellation factor K is defined as the ratio of the cancelled spectrum, $G_{ppo}(\omega) - G_{cco}(\omega)$, to the primary wave spectrum, $G_{ppo}(\omega)$; i.e.

$$K \triangleq \frac{G_{ppo}(\omega) - G_{cco}(\omega)}{G_{ppo}(\omega)}$$

or

$$K = 1 - \frac{G_{cco}(\omega)}{G_{ppo}(\omega)} \tag{2}$$

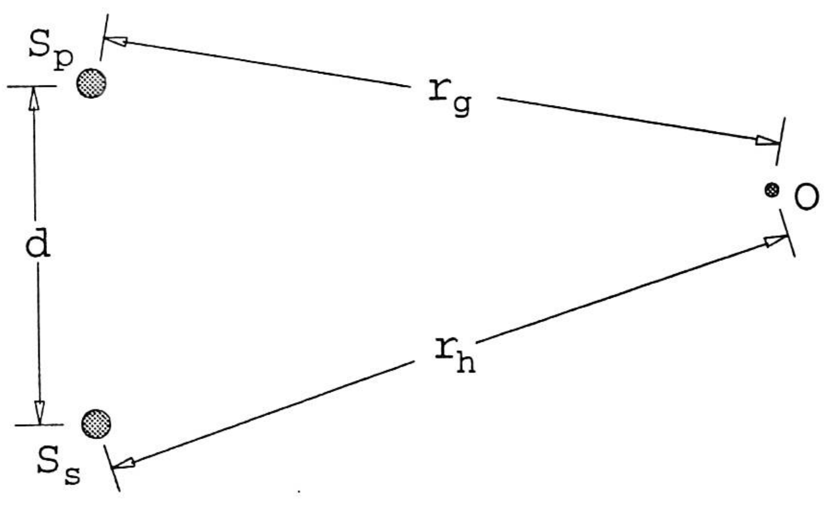


Fig.1: Primary and secondary sources with an observation point O in an acoustic medium.

It follows from equations (1) and (2) that, for cancellation to occur, the field cancellation factor must be between zero and unity;

$$0 < K \leq 1 \tag{3}$$

A zero value for K corresponds to no cancellation and a unity value of K corresponds to complete cancellation.

A frequency-domain description of of Fig. 1 is given in Fig. 2 where $G(j\omega)$ and $H(j\omega)$ represent the transfer characteristics of the medium from S_p and S_s, respectively, through the distances r_g and r_h to the observation point O and $C(j\omega)$ represents a linear frequency-dependent transfer function, with amplitude $H_c(\omega)$ and phase $\theta_c(\omega)$, through which the secondary source signal $s(t)$ is derived from the primary source signal $p(t)$ [3]. Assuming a non-dispersive propagation medium, these functions are given by

$$\begin{aligned} G(j\omega) &= \frac{A}{r_g} e^{-j\omega t_g} \\ H(j\omega) &= \frac{A}{r_h} e^{-j\omega t_h} \\ C(j\omega) &= H_c(\omega)\, e^{j\theta_c(\omega)} \end{aligned} \tag{4}$$

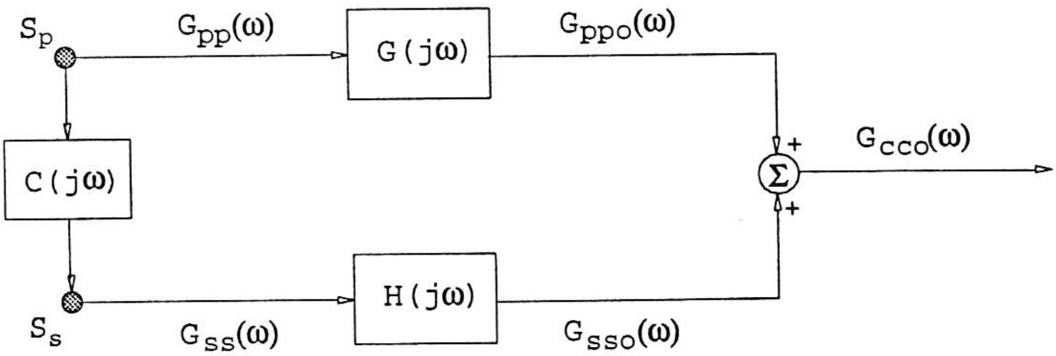


Fig. 2: Transfer function description of acoustic paths.

where A is a constant and t_g and t_h are the times required by the primary and secondary waves in propagating through the distances r_g and r_h from S_p and S_s, respectively, to reach the observation point O;

$$t_g = \frac{r_g}{c} \quad , \quad t_h = \frac{r_h}{c}. \tag{5}$$

where c represents the velocity of sound in the medium.

If the frequency-domain representation of the primary signal $p(t)$ and secondary signal $s(t)$ are respectively denoted by $P(j\omega)$ and $S(j\omega)$ then using equation (4) the auto-power spectral densities can be written as [4, 5]

$$\begin{aligned}
G_{pp}(\omega) &= | P(j\omega) |^2 \\
G_{ppo}(\omega) &= \frac{A^2}{r_g^2} G_{pp}(\omega) \\
G_{ss}(\omega) &= | S(j\omega) |^2 = \alpha_s G_{pp}(\omega) \\
G_{sso}(\omega) &= \alpha_s \frac{A^2}{r_h^2} G_{pp}(\omega) \\
G_{cco}(\omega) &= | G(j\omega) P(j\omega) + H(j\omega) S(j\omega) |^2
\end{aligned} \tag{6}$$

where α_s denotes the ratio of the auto-power spectral densities of the secondary wave to the primary wave, or simply the ratio of the secondary wave power to the primary wave power, before propagation;

$$\alpha_s = \frac{G_{ss}(\omega)}{G_{pp}(\omega)}$$

Note that, for a broadband signal α_s represents a continuous function of frequency.

Substituting for $G_{cco}(\omega)$ from equation (6) into equation (2), using equations (4) and (6) and simplifying yields

$$K = -2\beta\sqrt{\alpha} - \alpha \tag{7}$$

where

$$\alpha = \frac{G_{sso}(\omega)}{G_{ppo}(\omega)}$$

$$\beta = \cos\left[\omega\frac{\Delta r}{c} + \theta_c(\omega)\right] \; ; \quad \Delta r = r_g - r_h \tag{8}$$

This gives an analytical relationship between the cross-power spectral density factor, interpreted as the relative phase, the auto power spectral density ratio, interpreted as the relative amplitudes, and the degree of cancellation given by the cancellation factor K.

In practice, the secondary source is generated through a process of detecting and processing of the primary signal. For a broadband signal this leads to an amplitude and phase relation between the component waves as function of frequency. Such a relation is described in the above by the frequency-dependent parameters α_s and $\theta_c(\omega)$ which in turn relate to a control process realising either a fixed or adaptive controller.

2.2 *Conditions of cancellation*

Substituting for K from equation (7) into equation (3) and simplifying yields

$$\frac{\sqrt{\alpha}}{2} < -\beta \leq \frac{1+\alpha}{2\sqrt{\alpha}} \tag{9}$$

Since α is the ratio of powers of the waves it is therefore a positive real number. This implies that both the left- and right-hand sides of equation (9) are positive. Thus, equation (9) can only hold if the cross-spectral density factor assumes negative values; i.e.

$$\beta < 0$$

This is the phase condition for cancellation.

By definition the cross-spectral density factor β is bounded in magnitude by unity, therefore, equation (9) becomes

$$\frac{\sqrt{\alpha}}{2} < -\beta \leq 1 \qquad (10)$$

Thus the required amplitude condition for cancellation is given by

$$0 < \frac{\sqrt{\alpha}}{2} < -\beta \qquad (11)$$

Squaring the inequality in equation (11) yields the range of permissible power ratio, for cancellation to be feasible, as

$$0 < \alpha < 4\,\beta^2 \qquad (12)$$

Substituting for α from equation (8) into equation (12) yields

$$0 < G_{sso}(\omega) < 4\,\beta^2\,G_{ppo}(\omega) \qquad (13)$$

Note that if $-\beta$ is at its upper limit (unity) then equations (12) and (13) imply that cancellation at the observation point can occur if the spectral density of secondary wave is less than four times the spectral density of the primary wave or, equivalently, if the power associated with the secondary wave does not exceed four times the power associated with the primary wave.

Substituting for β from equation (8) into equation (10) yields

$$\frac{\sqrt{\alpha}}{2} < -\cos\Big[\,\omega\tau + \theta_c(\omega)\,\Big] < 1 \;\;,\quad 0 < \alpha < 4 \qquad (14)$$

where $\tau = \dfrac{\Delta r}{c}$ is the relative time delay $t_g - t_h$ between primary and secondary waves, of the same frequency, reaching the observation point. It follows from equation (14) that for cancellation to occur at the observation point the phase difference $\omega\tau + \theta_c(\omega)$ between same frequency components of the primary and secondary waves must satisfy the relation

$$(2n+1)\pi - \cos^{-1}\frac{\sqrt{\alpha}}{2} < \omega\tau + \theta_c(\omega) < (2n+1)\pi + \cos^{-1}\frac{\sqrt{\alpha}}{2}$$
$$\text{for } n = 0, 1, 2, \cdots \text{ and } 0 < \alpha < 4 \tag{15}$$

This is shown in Fig. 3.

3 Three–dimensional description of cancellation

The cancellation factor, as noted in the previous section, is a function of the power ratio α and the cross-spectral density factor β. These in turn are functions of the frequency ω of the component waves and relative distance from the sources to the observation point. A spatial description of these variables is obtained in this section leading to a geometric description of the pattern of cancellation and reinforcement in the medium.

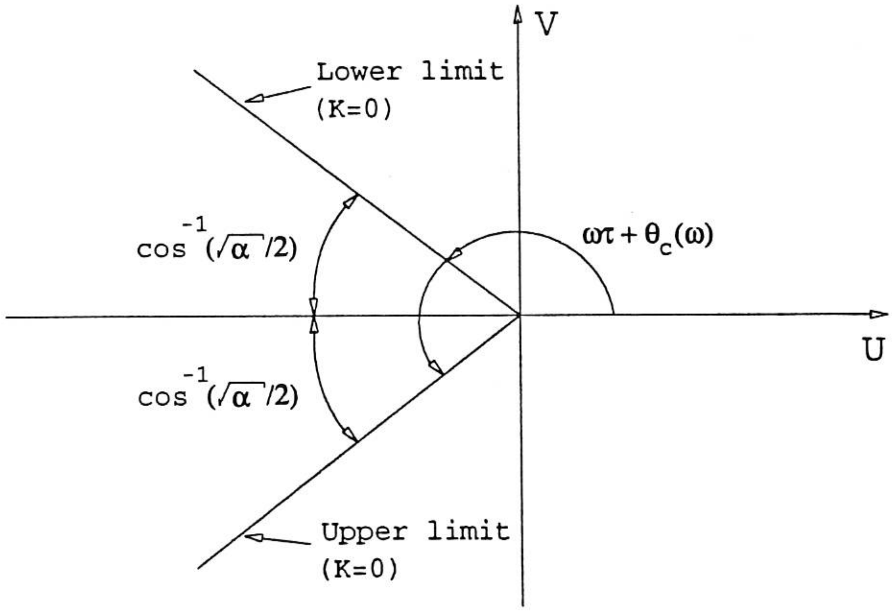


Fig. 3: Range of difference in phase between primary and secondary waves for cancellation.

3.1 *Power ratio and cross–spectral density factor*

Consider Fig. 4 which is a three-dimensional description of Fig. 1 in the *UVW*–space, where points *P*, *S* and *O* respectively represent locations of the primary source, secondary source and observer. The distances r_g and r_h are thus given by

$$r_g = \left[(u - \frac{d}{2})^2 + v^2 + w^2 \right]^{\frac{1}{2}}$$
$$r_h = \left[(u + \frac{d}{2})^2 + v^2 + w^2 \right]^{\frac{1}{2}} \tag{16}$$

3.1.1 Loci of constant power ratios

Substituting for $G_{ppo}(\omega)$ and $G_{sso}(\omega)$ from equation (6) into equation (8), using equation (5), and simplifying yields

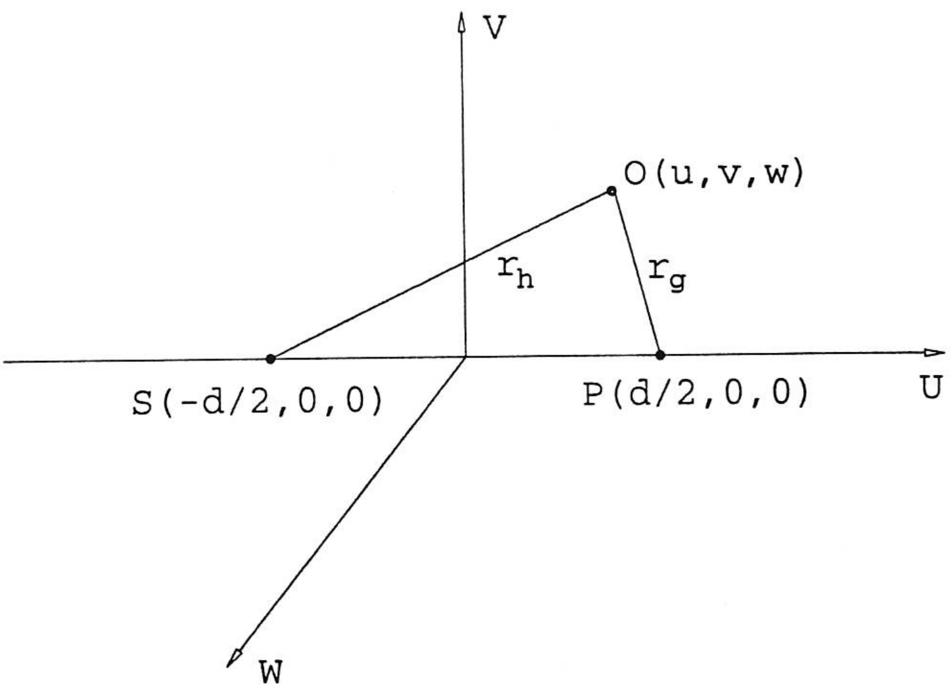


Fig. 4: Primary and secondary sources in three-dimensional coordinates.

$$\alpha = a^2\,\alpha_s \tag{17}$$

where the positive real number a is defined by

$$a = \frac{r_g}{r_h} \tag{18}$$

It follows from equation (17) that, provided α_s is constant, the the power ratio α as function of the location of the observation point in the three-dimensional space is described by the distance ratio a.

Substituting for r_g and r_h from equation (16) into equation (18) and simplifying yields

$$\left[u - \left(\frac{1+a^2}{1-a^2} \right) \frac{d}{2} \right]^2 + v^2 + w^2 = \left[\frac{a\,d}{1-a^2} \right]^2 \tag{19}$$

This, with the distance ratio a as a parameter, describes a family of spheres with centres on the U–axis at a distance $\left(\frac{1+a^2}{1-a^2} \right) \frac{d}{2}$ from the origin and radii of $\frac{a\,d}{|1-a^2|}$. This family is shown in Fig. 5 in the two-dimensional UV–plane, for $d = 4$ units, revolution of which around the U–axis gives the corresponding three-dimensional spherical surfaces. Note that $a = 0$ and $a = \infty$ respectively indicate locations of primary and secondary sources and for $a = 1$ the spherical surface changes to a plane surface coinciding with the VW–plane. Each of the spherical surface represents a certain distance ratio a and, for constant α_s, corresponds to a certain power ratio α. Therefore, the spherical surfaces of equation (19) are referred to as loci of constant distance ratio or, equivalently, loci of constant power ratio.

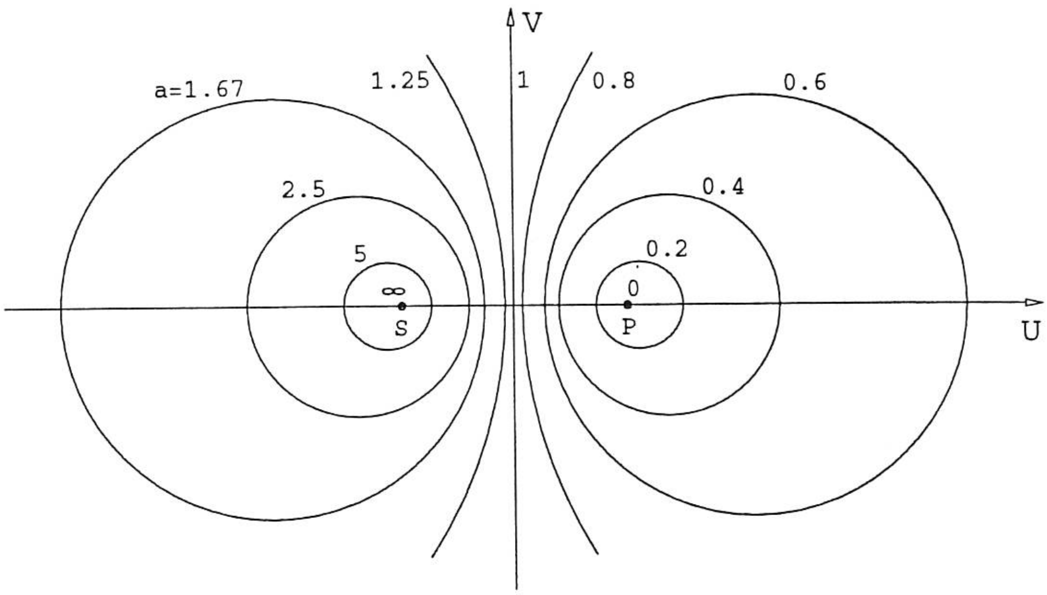


Fig. 5: Loci of constant distance ratio for d=4.

3.1.2 Loci of constant cross–spectral density factors

It follows from equation (8) that for a single tone of the component waves the cross-spectral density factor β is a function of the distance difference Δr only. This implies that a description of β in the three-dimensional space is given by the distance difference Δr. Let this be denoted by a real number b;

$$\Delta r = r_g - r_h = b \ ; \quad |\, b \,| \leq d \tag{20}$$

Substituting for r_g and r_h from equation (16) into equation (20) and simplifying yields

$$\frac{u^2}{\left(\frac{b^2}{4}\right)} - \frac{v^2}{\left(\frac{d^2 - b^2}{4}\right)} - \frac{w^2}{\left(\frac{d^2 - b^2}{4}\right)} = 1 \tag{21}$$

As the distance difference b varies from $-\,d$ to $+\,d$ equation (21) defines a family of hyperbolic surfaces with foci either at point P or point S. This family is shown in Fig. 6 in the two-dimensional UV–plane revolution of which around the U–axis yields the corresponding family of three-dimensional hyperbolic surfaces. Note that for $b = 0$ the hyperbolic surface changes to a plane surface coinciding with the *VW*–plane. Each of the hyperbolic surface represents certain distance difference b or, equivalently, a certain value of the cross-spectral density factor β. Therefore, equation (21) is referred to as loci of constant distance differences or, equivalently, loci of constant cross-spectral density factors. Since the distance difference Δr is directly related to the relative time delay τ equation (21) can also be referred to as the loci of constant time differences τ.

3.1.3 Loci of constant α – constant β

The locus of points for which both the distance ratio a and the distance difference b are constant is described by the intersection of the surfaces in equations (19) and (21). Such an intersection will exist if the condition

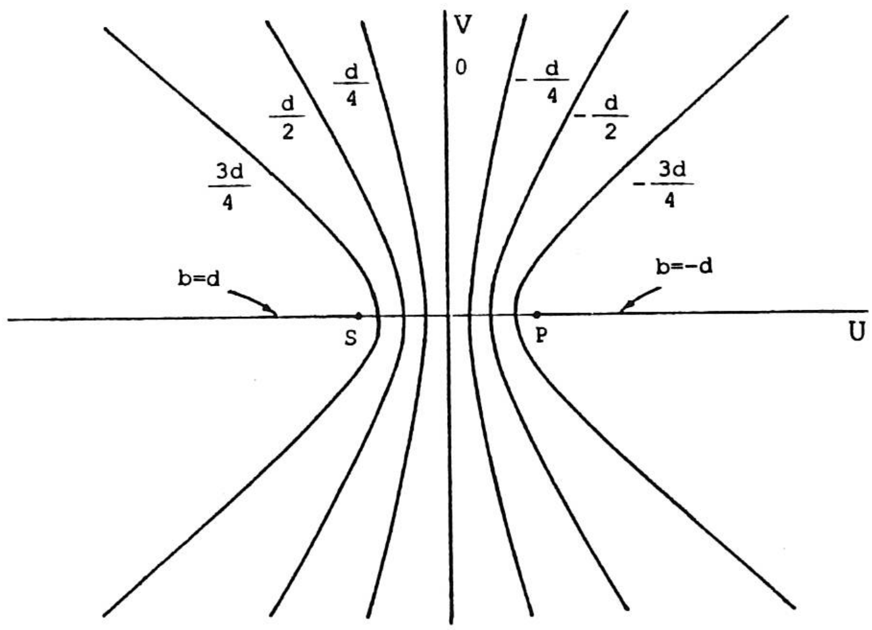


Fig. 6: Loci of constant distance differences.

$$\frac{|b|}{d}\left(\frac{1+a}{1-a}\right) \geq 1 \qquad \text{for } a < 1$$

$$\frac{|b|}{d}\left(\frac{a+1}{a-1}\right) \geq 1 \qquad \text{for } a > 1$$

holds, under which the result will be a circle in the three-dimensional space of Fig. 4, on a plane parallel to the VW–plane, with centre on the U–axis at a distance u_c from the origin, where

$$u_c = \frac{b^2}{2d}\left(\frac{1+a}{1-a}\right).$$

In such a manner the circle is defined by

$$v^2 + w^2 = \frac{d^2}{4}\left[1 - \left(\frac{b}{d}\right)^2\right]\left[\left(\frac{b}{d}\right)^2\left(\frac{1+a}{1-a}\right)^2 - 1\right] \tag{22}$$

Note that the constant a and constant b surfaces corresponding, respectively, to $a = 1$ and $b = 0$ define plane surfaces that coincide with each other and with the VW–plane.

The family of circles defined by equation (22), for sets of constant values of a and b, represent loci on which both the distance ratio a and distance difference b are constant. For single tones of the component waves these loci also represent circles on which both the power ratio α and cross-spectral density factor β are constant. Therefore, they are referred to as loci of constant a – constant b or, equivalently, as loci of constant α – constant β.

3.2 *Cancellation as a function of power ratio*

Treating β as a parameter in equation (7); i.e. restricting the observation point to constant β surfaces, and varying the point so that to coincide with various distance ratios or, equivalently, define various α surfaces will result in the cancellation factor K as a function of α forming the family of curves shown in Fig. 7, where Fig. 7a corresponds to cancellation and Fig. 7b corresponds to reinforcement. Each curve in Fig. 7 shows cancellation as function of the power ratio α on the corresponding constant β surface.

It follows from equation (7) and (12) that on a β surface, with $\beta < 0$, cancellation occurs at points which are between α surfaces corresponding to power ratios of 0 and $4\beta^2$, with maximum cancellation occurring at such points for which the relation

$$\alpha = \beta^2$$

holds, corresponding to which the maximum value of field cancellation factor, K_{max}, is

$$K_{max} = \beta^2$$

Therefore, the maxima of the family of curves in Fig. 7 lie on the straight line

$$K = \alpha \quad \text{for } 0 < \alpha \leq 1$$

Manipulation of equations (7) and (10) reveals that cancellation larger than that corresponding to any K is possible for waves for which the power ratio α is within the limits

$$\left[1 - \sqrt{1-K}\right]^2 \leq \alpha \leq \left[1 + \sqrt{1-K}\right]^2$$

while the magnitude of β should be higher than $|\beta|_{min}$, where $|\beta|_{min}$, from a maximisation of equation (7) for β as a function of α, is

$$|\beta|_{min} = \sqrt{K}$$

It follows from Fig. 7b that on β surfaces corresponding to $\beta > 0$ the cancellation factor K is always negative implying that under such a situation the primary wave is always reinforced, with reinforcement increasing as the power ratio is increased. An example of the pattern of cancellation and reinforcement on a hyperbolic surface of

(a)

(b)

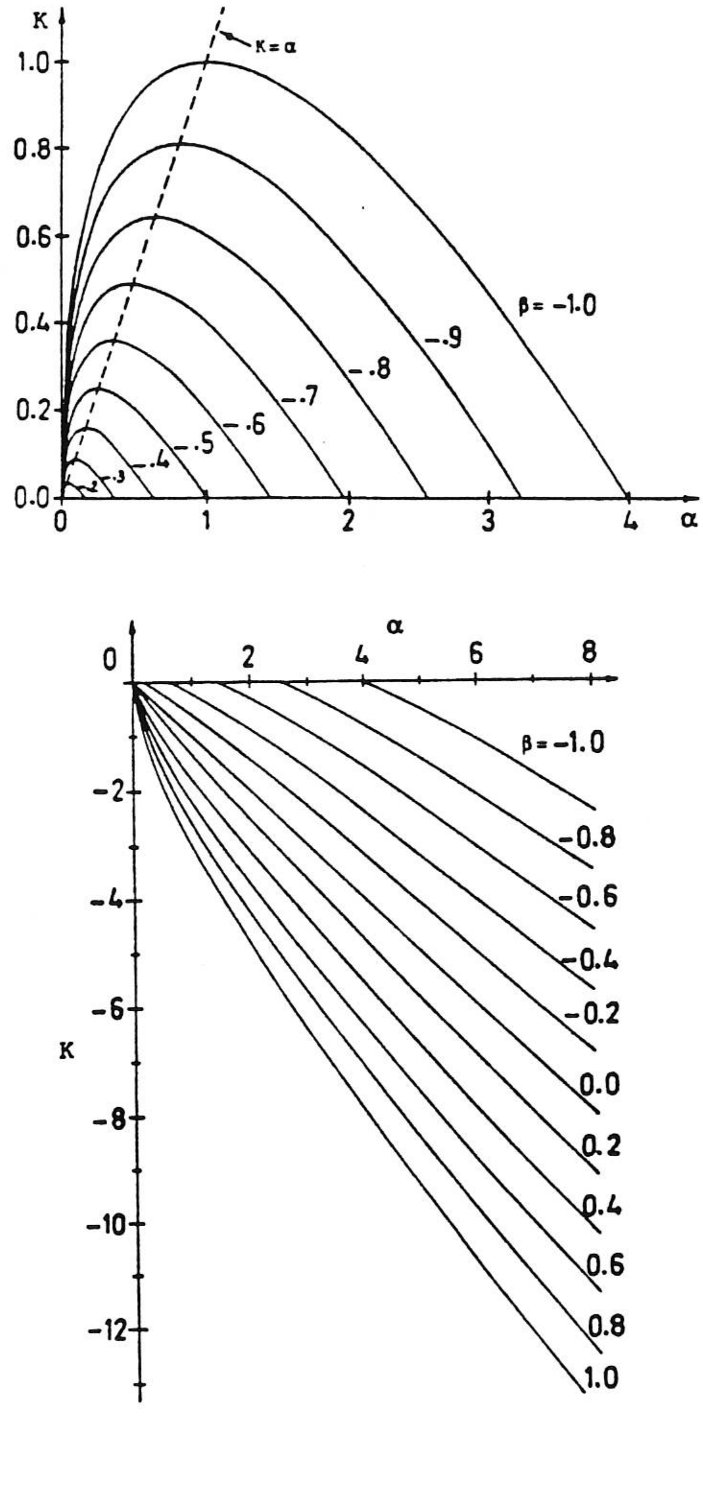


Fig. 7: Field cancellation factor as function of power ratio; (a) K>0, (b) K<0.

$b = -0.7$ *metres* is shown in two dimensions in Fig. 8 for a tone of 100 Hz with $\alpha_s = 1$, $\theta_c(\omega) = 180^\circ$ and $d = 1$ *metres*, revolution of which around the U-axis results the corresponding three-dimensional picture. The surface corresponds to $\beta = -0.2737$ on which the range of permissible power ratios for cancellation is $0 < \alpha > 0.2996$; i.e. points between spheres of $a = 0$ and $a = 0.2996$. This is indicated by the dashed-line portions of the surface which includes the base of the surface from $u = 0.35$ to $u = 1.638$ *metres*. Maximum cancellation occurs at points on the surface (a circular strip) for which $u = 0.7825$ *metres* with $\alpha = K = 0.0749$. The solid-line portion of the curve corresponds to region of reinforcement on which reinforcement is minimum at points with $u = 1.638$ *metres* and increases as the point moves away from the origin.

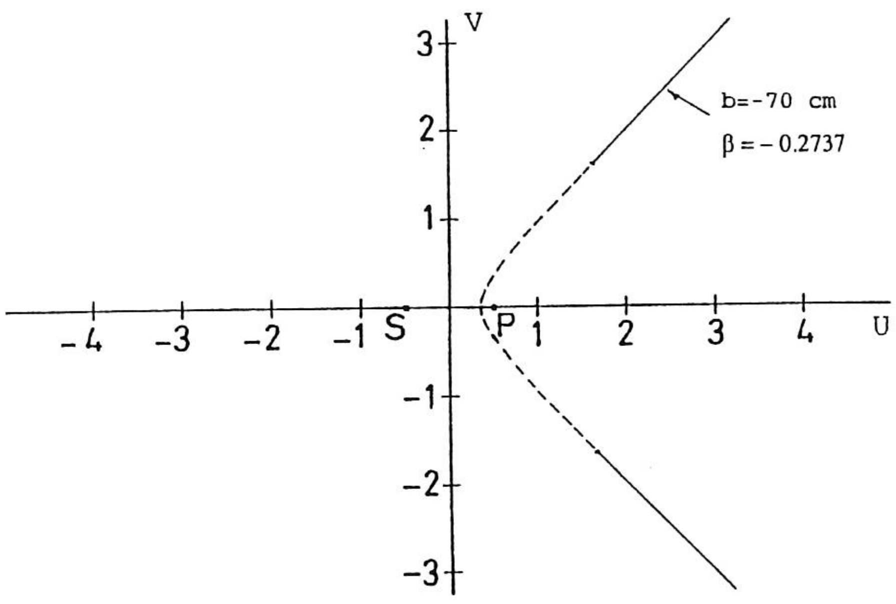


Fig. 8: Cancellation and reinforcement pattern on a b=-70 cm surface.

3.3 *Cancellation as a function of distance difference*

Treating α as a parameter in equation (7); i.e. restricting the observation point to constant α surfaces, and varying the point so that to coincide with various Δr surfaces or, equivalently, define various β surfaces, will result in the cancellation factor K as function of β forming the family of straight lines shown in Fig. 9. As β varies between -1 and $+1$ the cancellation factor K remains less than or equal unity. For cancellation to occur at points on an α surface the corresponding power ratio α should be within 0 and 4;

$$0 < \alpha < 4 \tag{23}$$

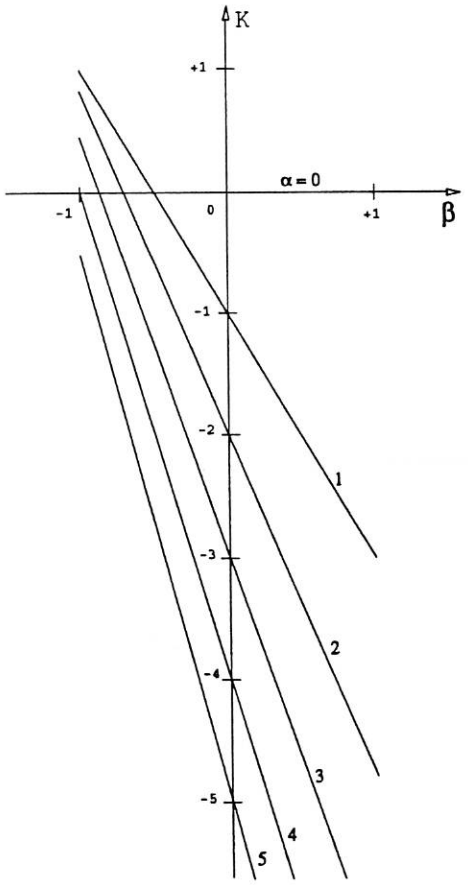


Fig. 9: Field cancellation factor as function of cross-spectral density factor.

while the corresponding value of β should remain within the range

$$-1 \leq \beta < -\frac{\sqrt{\alpha}}{2} \tag{24}$$

It follows from the above that on a constant α surface satisfying equation (23) the process of interference will correspond to a combined pattern of cancellation and reinforcement in a manner that maximum cancellation will be obtained at points for which $\beta = -1$. On either side of these points cancellation decreases linearly as the observation point varies in the direction of increasing β. There will be neither cancellation nor reinforcement ($K = 0$) at points for which $\beta = -\frac{\sqrt{\alpha}}{2}$. Beyond these points, in the direction of increasing β, the situation will correspond to reinforcing the primary wave. Maximum reinforcement of the wave will occur at points for which β is $+1$. An example of the pattern of cancellation and reinforcement on an α surface of $a = 0.8$ is shown in two dimensions in Fig. 10 for a tone of 500 Hz with $\alpha_s = 1$, $\theta_c(\omega) = 180^{\circ}$ and $d = 1$ *metre*, revolution of which around the U-axis yields the corresponding three-dimensional picture. The dashed-line and solid-line portions of the surface, respectively, indicate regions of cancellation and reinforcement. Points (circular strip) of limit of cancellation ($K = 0$), marked as A, A', B, B', F and F' correspond to points common to the surface and β surfaces of $b = -0.1347$, -0.5453 and -0.8147 *metres*. Maximum cancellation (with $K = +0.8704$) occurs at points M and M' which are common to this surface and β surface of $b = -0.68$ *metres*. Maximum reinforcement (with $K = -1.6896$) occurs at points L and L' which are common to this surface and β surface of $b = -0.34$ *metres*.

On a constant α surface for which α is outside the limits given in equation (23) the primary wave is reinforced at all points of the surface. The reinforcement varies linearly with β; reinforcement being minimum at points for which $\beta = -1$ and maximum at points for which $\beta = +1$.

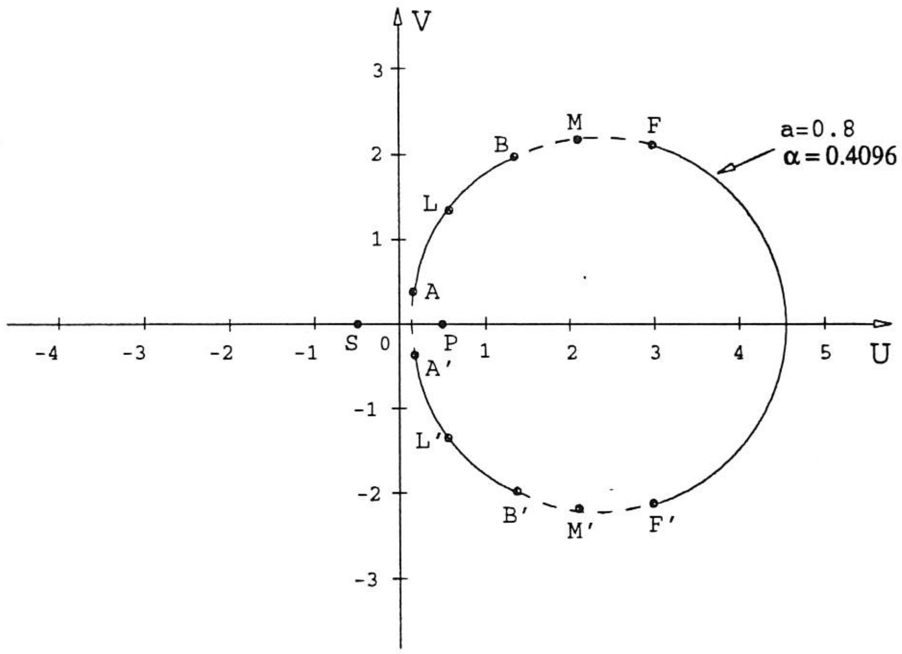


Fig. 10: Cancellation and reinforcement pattern on an a=0.8 surface.

Using equation (8) and the relation between velocity of sound, signal wavelength, and signal frequency into equation (15) and simplifying yields

$$\left[(2n+1) - \frac{1}{\pi}\,\theta_c(\omega) - \frac{1}{\pi}\cos^{-1}\frac{\sqrt{\alpha}}{2} \right] \frac{\lambda}{2} < \Delta r <$$

$$\left[(2n+1) - \frac{1}{\pi}\,\theta_c(\omega) + \frac{1}{\pi}\cos^{-1}\frac{\sqrt{\alpha}}{2} \right] \frac{\lambda}{2} \qquad (25)$$

$$\text{for } n = 0, 1, 2, \cdots \text{ and } 0 < \alpha < 4$$

This implies that, on a constant α surface, the range of cancellation is directly proportional to the signal wavelength or, in other words, inversely proportional to the frequency of the component waves.

If the range of cancellation given in equation (25) is to cover all points of the constant α surface then it should be greater than the maximum value $|\, b \,|$ can have, i.e.

$$\text{Range of cancellation} \geq d \tag{26}$$

or, from equation (25) for $n = 0$, the following should hold

$$\left| \frac{\omega d}{c} + \theta_c(\omega) - \pi \right| < \cos^{-1} \frac{\sqrt{\alpha}}{2} \ , \quad 0 < \alpha < 4 \tag{27}$$

As a special case, if $\theta_c(\omega) = 180\,^{\circ}$ then equation (27) yields

$$d < \frac{c}{\omega} \cos^{-1} \frac{\sqrt{\alpha}}{2} \ , \quad 0 < \alpha < 4$$

For example, to ensure cancellation of a 100 Hz tone at all points of a constant α surface of $\alpha = 0.4096$ the source separation should not exceed 0.6737 *metres* whereas for a frequency of 500 Hz the upper limit of d will be 0.1347 *metres.*

Equations (7) and (8) lead to

$$K = -\,2\,\sqrt{\alpha}\,\cos\left[\frac{\Delta r}{\lambda/2}\,\pi + \theta_c(\omega)\right] - \alpha \tag{28}$$

from which it follows that, on a constant α surface $(0 < \alpha < 4)$, cancellation is a cosinusoidal function of the distance difference Δr with a period equal to the signal wavelength. The cancellation factor has an average value of $-\,\alpha$ implying that for non-zero α the range of cancellation is always less than the range of reinforcement. Cancellation reaches its maximum at such Δr surfaces for which $\beta = -\,1$;

$$\cos\left[\frac{\Delta r}{\lambda/2}\,\pi + \theta_c(\omega)\right] = -\,1$$

or

$$\Delta r = \left[(\,2n + 1\,) - \frac{1}{\pi}\,\theta_c(\omega)\right]\frac{\lambda}{2} \quad ; n = 0,\ 1,\ 2,\ \cdots \tag{29}$$

Therefore, on a constant α surface ($0 < \alpha < 4$), maximum cancellation occurs on such Δr surfaces for which the distance difference is directly proportional to half the wavelength, specifically, if $\theta_c(\omega)$ is zero then for maximum cancellation Δr will be an odd multiple of half the wavelength, whereas if $\theta_c(\omega) = 180^\circ$ then maximum cancellation occurs on those β surfaces for which Δr is an even multiple of half the wavelength.

From equation (28) points of limit of cancellation ($K = 0$), on either side of the Δr surface defined by equation (29) occur at surfaces Δr_1 and Δr_2, where

$$\Delta r_1 = \left[2n + 1 - \frac{1}{\pi}\,\theta_c(\omega) \right] \frac{\lambda}{2} - \frac{\lambda}{2\pi} \cos^{-1} \frac{\sqrt{\alpha}}{2} \qquad \text{for } n = 0,\ 1,\ 2,\ \cdots$$

$$\Delta r_2 = \left[2n + 1 - \frac{1}{\pi}\,\theta_c(\omega) \right] \frac{\lambda}{2} + \frac{\lambda}{2\pi} \cos^{-1} \frac{\sqrt{\alpha}}{2} \qquad \text{for } n = 0,\ 1,\ 2,\ \cdots$$

Therefore, on an α surface ($0 < \alpha < 4$), the width (range) of a cancellation zone, R_c and the width (range) of a reinforcement zone, R_r, are given by

$$\begin{aligned} R_c &= \Delta r_2 - \Delta r_1 = \frac{\lambda}{\pi} \cos^{-1} \frac{\sqrt{\alpha}}{2} \\ R_r &= \lambda \left[1 - \frac{1}{\pi} \cos^{-1} \frac{\sqrt{\alpha}}{2} \right] \end{aligned} \tag{30}$$

Note, in equation (30), that, if α is not zero, R_c is always less than R_r. For a zero power ratio both, R_c and R_r, are equal to half the signal wavelength. R_c and R_r as function of the power ratio α are shown in Fig. 11.

From equation (30) the range of cancellation R_c is given by the change in distance difference, $|\Delta b| = |\Delta r_1 - \Delta r_2|$, between two successive β surfaces corresponding to $K = 0$ (or $\beta = -\frac{\sqrt{\alpha}}{2}$) provided maximum cancellation ($\beta = -1$) occurs between the two surfaces. Likewise R_r is given by the change in distance difference, $|\Delta b|$, between two successive β surfaces corresponding to $K = 0$ provided maximum reinforcement ($\beta = 1$)

occurs between the two surfaces. This in relation to Fig. 9 means that to measure R_c, β should vary from $-\frac{\sqrt{\alpha}}{2}$ to -1 and further to $-\frac{\sqrt{\alpha}}{2}$, and, likewise, to measure R_r, β should vary from $-\frac{\sqrt{\alpha}}{2}$ to $+1$ and further to $-\frac{\sqrt{\alpha}}{2}$.

3.4 *Cancellation as a function of frequency*

To study cancellation as function of frequency both the power ratio α and distance difference Δr should be kept constant. This in terms of a three-dimensional description means that the observer should be restricted to points defining constant a – constant Δr circles of equation (22).

Substituting for β from equation (8) into equation (7) yields

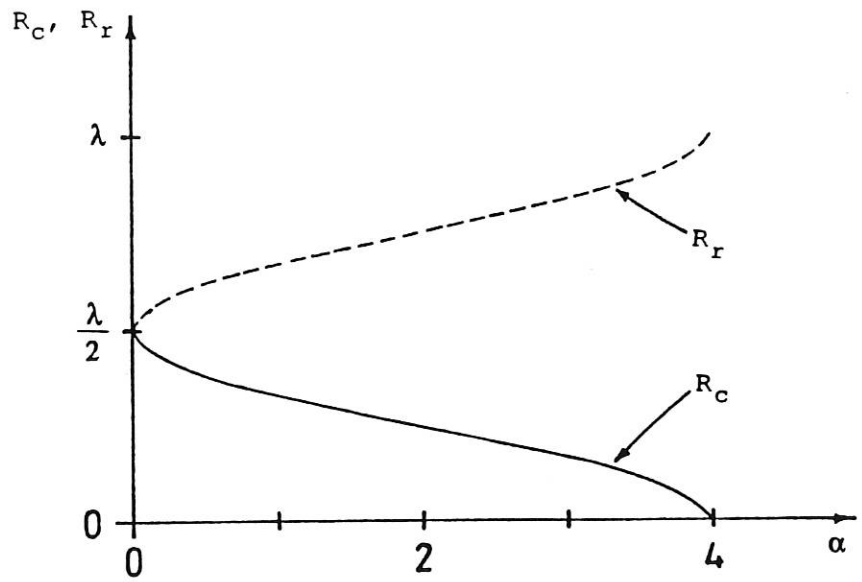


Fig. 11: Ranges of cancellation and reinforcement zones for a single tone of component waves as function of power ratio.

$$K = -2\,\sqrt{\alpha}\,\cos\left[\,\theta(\omega)\,\right] - \alpha \tag{31}$$

where

$$\theta(\omega) = \frac{\Delta r}{c}\,\omega + \theta_c(\omega)$$

This implies that cancellation of a broadband noise varies as a cosine function of $\theta(\omega)$. If the power ratio α is within 0 and 4 over a broad frequency range then this will lead to a cancellation/reinforcement pattern. However, if the power ratio exceeds 4 then the situation will correspond to reinforcing the primary wave. At observation points for which the power ratio is within 0 and 4, maximum cancellation occurs at frequencies ω_m for which

$$\cos\left[\,\theta(\omega_m)\,\right] = -1$$

or

$$\theta(\omega_m) = (\,2n+1\,)\,\pi \quad ; \; n = 0,\,1,\,2,\,\cdots$$

On either side of ω_m cancellation will reach its lowest limit ($K = 0$) at frequencies ω_l, where

$$\theta(\omega_l) = (\,2n+1\,)\,\pi \pm \cos^{-1}\frac{\sqrt{\alpha}}{2} \quad , \; 0 < \alpha < 4 \quad ; \; n = 0,\,1,\,2,\,\cdots$$

As a special case if $\theta_c(\omega) = 180^{\circ}$ in equation (31) then K is given as a cosinusoidal function of the frequency ω with period of $\frac{2\pi\, c}{\Delta r}$ and an average value of $-\,\alpha$ for which the frequencies ω_m at which maximum cancellation occurs are

$$\omega_m = (\,2n\,)\,\frac{\pi\, c}{|\,\Delta r\,|} \quad ; \, n = 0,\,1,\,2,\,\cdots \tag{32}$$

In the vicinity of these frequencies cancellation will decrease as cosine of $\frac{\Delta r\,\omega}{c}$ and will eventually reach its lowest limit at frequencies ω_l, where

$$\omega_l = \left[(2n)\,\pi \pm \cos^{-1} \frac{\sqrt{\alpha}}{2} \right] \frac{c}{|\Delta r|} \quad , \quad 0 < \alpha < 4 \;\; ; \quad n = 0, 1, 2, \cdots \tag{33}$$

If, in this particular case, cancellation is required to occur at all frequencies of interest then ω_l (for $n = 0$) is to be at least equal to the maximum source frequency ω_s or

$$|\Delta r| \leq \frac{c}{\omega_s} \cos^{-1} \frac{\sqrt{\alpha}}{2} \quad , \quad 0 < \alpha < 4$$

for example, to cancel a broadband noise of 0 to 500 Hz with $\alpha_s = 1$ at observation points for which $0 < \alpha < 4$ the distance difference $|\Delta r|$ should not exceed 0.113 *metres*.

From equation (33) the band of frequency over which cancellation occurs is given by

$$B_c = \frac{2\,c}{|\Delta r|} \cos^{-1} \frac{\sqrt{\alpha}}{2} \quad , \quad 0 < \alpha < 4$$

and the band of frequency over which reinforcement occurs is given by

$$B_r = \frac{2\,c}{|\Delta r|} \left[\pi - \cos^{-1} \frac{\sqrt{\alpha}}{2} \right] \quad , \quad 0 < \alpha < 4$$

This implies that for a non-zero α the cancellation band B_c will always be less than the reinforcement band B_r.

3.4.1 Experimental verification

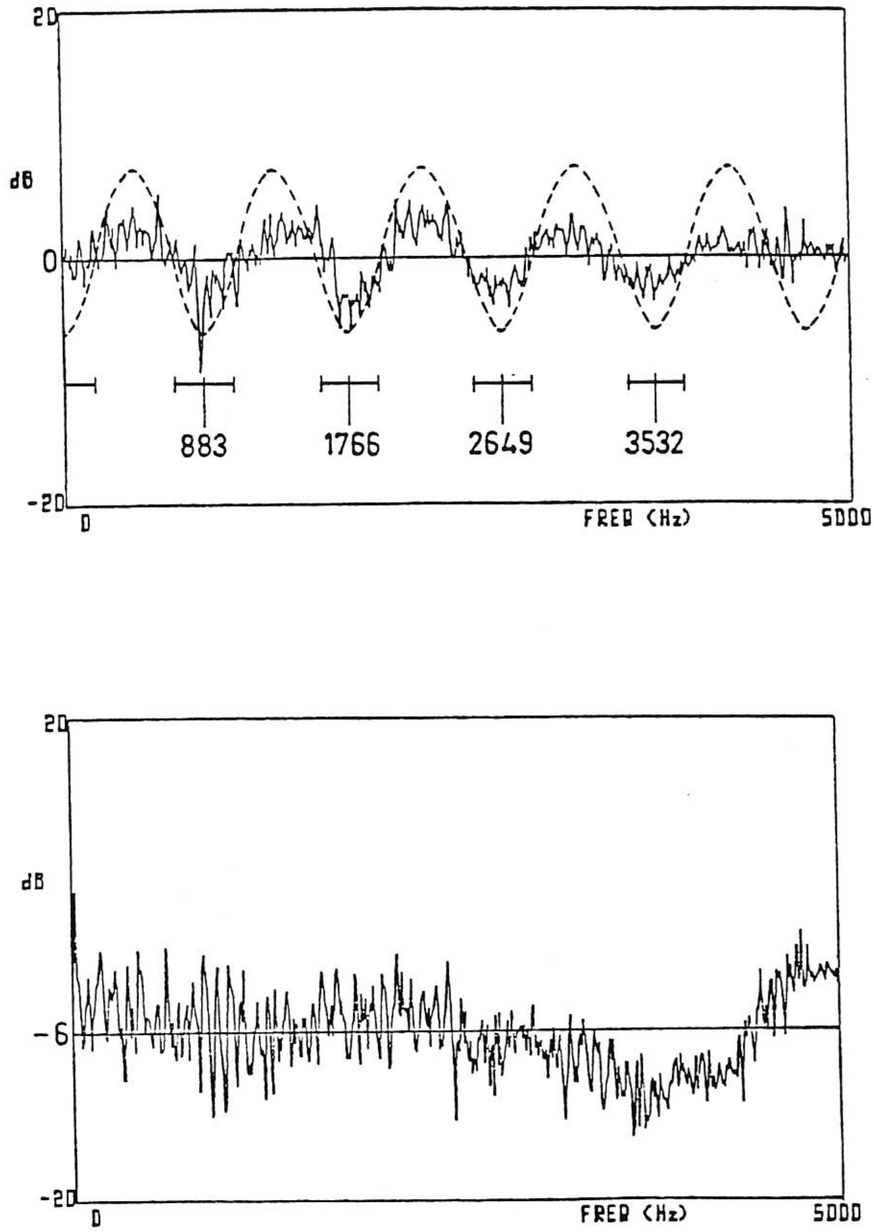
The results of cancellation as function of frequency can be verified experimentally by using two identical loudspeakers as primary and secondary sources and a microphone as an observer located at a specified location with respect to the sources. With the observer placed opposite to the primary source at $r_g = 2$ *metres* and the separation between the sources adjusted to achieve $\Delta r = 0.385$ *metres*, the sources were driven by correlated broadband PRBS signals of $0-5$ KHz and the amplitudes were adjusted so that to achieve a constant power ratio of $\alpha = -6.0$ dB at the observation point over a broad frequency range. The primary wave's spectral density, $G_{ppo}(\omega)$, the secondary wave's spectral density, $G_{sso}(\omega)$, and the spectral density of the combined primary and secondary waves, $G_{cco}(\omega)$, were measured at the observation point, using a Solartron SP1200 signal processor [6], and the ratios $\dfrac{G_{cco}(\omega)}{G_{ppo}(\omega)}$ and $\dfrac{G_{sso}(\omega)}{G_{ppo}(\omega)}$ were formed giving, respectively, the cancellation and power ratio of the component waves. These are shown in Fig. 12 where in Fig. 12a the dashed-line curve shows the expected theoretical pattern of cancellation and the 0 dB line partitions regions of cancellation (above the line) and reinforcement (below the line).

The cancellation as function of frequency thus closely follows the theoretically expected cosinusoidal pattern, specifically at the frequency range for which the power ratio is at a, nearly, constant level. The cosinusoidal pattern of cancellation is slightly deteriorated at frequencies for which the power ratio is deviating from its constant level of -6 dB. This deviation at frequencies below 100 Hz is due to the large attenuation by loudspeakers and at frequencies above 4 KHz is due to mismatch between characteristics of the two loudspeakers.

Substituting for α, Δr and $c = 340$ *metres/sec* into equations (32) and (33) and simplifying yields the frequencies f_l (in Hz) corresponding to the limit of cancellation and f_m at which maximum cancellation occurs as

$$f_l = 883n \pm 185 \quad , \quad f_m = 883n \quad ; \quad n = 0,\, 1,\, 2,\, \cdots$$

These frequencies, marked in Fig. 12a, show a close agreement with the experimental values.



(a)

(b)

Fig. 12: (a) Cancellation, (b) the corrresponding power ratio as a function of frequency.

(a)
d=100 cm

(b)
d=50 cm

(c)
d=10 cm

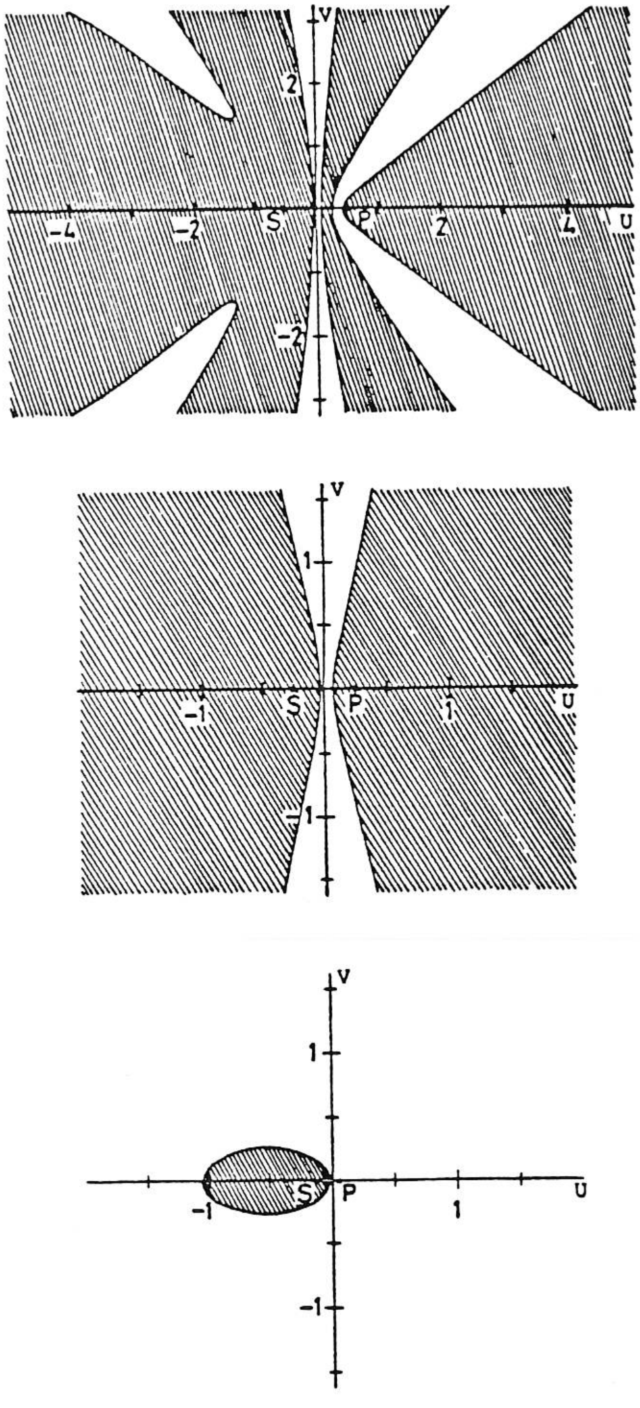


Fig. 13: Zones of cancellation and reinforcement
(f=500 Hz).

(a)
d=100 cm

(b)
d=50 cm

(c)
d=30 cm

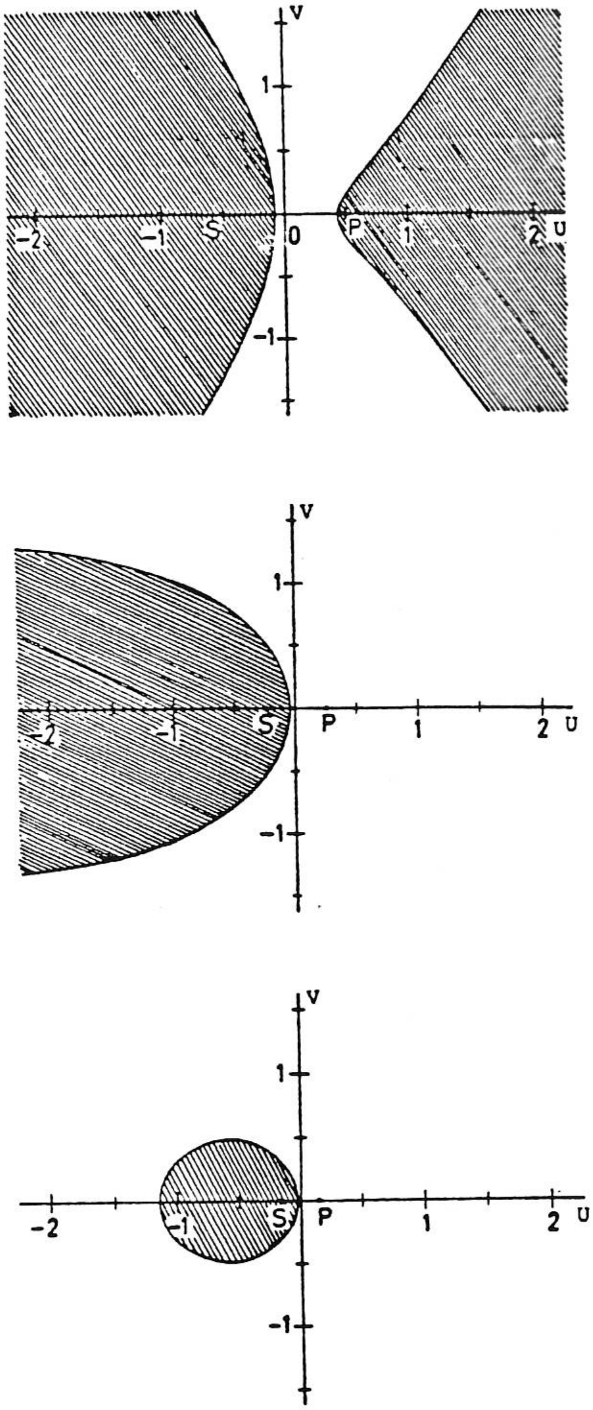


Fig. 14: Zones of cancellation and reinforcement
(f=100 Hz).

areas indicate zones of cancellation and dark areas indicate zones of reinforcement. By revolving the diagrams around the U–axis the corresponding three-dimensional pictures are obtained.

It is evident from a comparison of the set of diagrams in Figs. 13 and 14 that the effect of signal frequency on the extent of zones of cancellation and reinforcement is similar to that of separation between the sources: a decrease in frequency leads to an increase in the range of cancellation.

Figs. 13 and 14 further demonstrate that a feature of the pattern of cancellation and reinforcement resulting by an ANC system in a stationary environment is the distinct geometric symmetry, about a straight line passing through the locations of the sources, in the regions of cancellation in space. This results, from equation (34), due to such a symmetric nature in space of the power ratio α and cross-spectral density factor β.

4 Conclusion

The process of field cancellation by active means in a three-dimensional non-dispersive acoustic medium has been investigated. This involves the derivation of the field cancellation factor, that gives a quantitative measure of cancellation, in terms of amplitude and phase parameters of the primary and secondary waves at an observation point in the medium and the necessary conditions of cancellation on which the design of an ANC system can be based.

The process of cancellation basically involves two sets of conditions: (*i*) source-related and (*ii*) geometry-related. An artificially generated secondary source of noise is required to produce a wave that is coherent with the unwanted noise over the existing frequency range of the primary wave. This, in turn, will require the electronic components, such as sensors, amplifiers, and loudspeakers, to have suitable characteristics as function of frequency so that an undistorted secondary wave is generated.

In propagating through the medium the primary and secondary waves are affected in amplitude and in phase by the properties of the medium. This results in a combined field forming patterns of cancellation and reinforcement. The three-dimensional description of cancellation is determined by a consideration of the functional relation between field cancellation factor and geometry-related parameters, such as separation between the sources and the location of an observation point in the medium. It is clear from this consideration that it is possible to cancel an unwanted noise over large volumes of the medium by proper adjustment of the separation between the sources and, furthermore, even obtain an overall elimination of the noise by a carefully chosen secondary source.

The cancelling of a single tone of noise requires simple electronics and a careful consideration of geometry-related conditions of cancellation. However, due to the frequency-dependent nature of field cancellation factor the cancelling of broadband noise requires a combined consideration of the source-related and geometry-related conditions of cancellation.

5 References

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