Monograph:

Worden, P., Standsby, P.K., Tomlinson, G.R. et al. (1 more author) (1991) The First (1991) International Offshore and Polar Engineering Conference, Edinburgh 11-Aug 1991. "Towards and Improved Wave Force Equation". Research Report. Acse Report 423 . Dept of Automatic Control and System Engineering. University of Sheffield

THE FIRST (1991) INTERNATIONAL OFFSHORE AND POLAR
ENGINEERING CONFERENCE, EDINBURGH 11-16 Aug 1991

Towards and Improved Wave Force Equation

K. Worden, P.K. Stansby and G.R. Tomlinson
Simon Engineering Laboratories
The University
Oxford Road
Manchester
M13 9PL

S.A. Billings
Department of Control Engineering
University of Sheffield
Mappin Street
Sheffield
S1 3JD

March 1991

Research Report No. 423

THE FIRST (1991) INTERNATIONAL OFFSHORE AND POLAR
ENGINEERING CONFERENCE, EDINBURGH 11-16 Aug 1991

Towards an Improved Wave Force Equation

K.Worden, P.K.Stansby and G.R.Tomlinson
Simon Engineering Laboratories
The University
Oxford Road
Manchester M13 9PL

S.A.Billings
Department of Control Engineering
University of Sheffield
Mappin Street
Sheffield S1 3JD

Abstract

An extension to the Morison equation is proposed to improve wave force prediction. A comparison is made between the performance of Morison's equation and the extended model in curve-fitting to measured time data; both regular oscillatory flows in a U-tube and irregular oscillatory flows in a wave flume are considered.

1 Introduction

Since its introduction in 1950 [1], the Morison equation has provided the main means of predicting wave forces on slender cylinders. In the usual notation,

$$\frac{\partial F}{\partial l} = \frac{1}{2}\rho D C_d u|u| + \frac{1}{4}\pi\rho D^2 C_m \dot{u} \qquad (1)$$

where $u(t)$ is the instantaneous flow velocity. The dimensionless drag and inertia coefficients C_d and C_m will of course depend on the characteristics of the flow. In general the main dependence is taken to be on Re, the Reynolds number, and KC, the Keulegan-Carpenter number. Alternatively, they can be considered to be functions of KC and $\beta = Re/KC$. The coefficients C_d and C_m are usually obtained by applying least-squares procedures to measured data.

The equation generally predicts the main trends in measured data quite well; however, some characteristics of the flow are not represented adequately. For example, in a regular oscillatory flow the force variation at the fundamental frequency may be well predicted while that at higher harmonics is not. One result is that peak forces can be underpredicted. Furthermore, it is the high frequency content of the forces which determines to a large extent the fatigue life of a structural element. These are clearly serious limitations. The aim of the present research is to produce an equation which provides a better means of predicting wave forces on cylinders in situations of varying complexity, which will eventually include the dynamic response of the cylinder if possible. The means by which this objective will be pursued will be through the application of system identification techniques.

2 Vortex Shedding and the History Effect

It is well known that the time-history of the wave-force on a cylinder can be complex, even when the flow is regular. Nonlinear effects can be significant and must all be represented by the nonlinear drag term proportional to $u|u|$ if Morison's equation is to predict accurately. The linear inertia component is mainly due the inviscid effect of flow acceleration. Expansion of the drag term as a polynomial gives

$$u|u| = a_1 u + a_3 u^3 + a_5 u^5 \dots \tag{2}$$

which shows that even if the flow velocity is a sinusoid $u(t) = U_m \sin(2\pi t/T)$ the force signal will contain all odd harmonics. One immediately sees that the explanation for the failure of Morison's equation to predict the higher frequency behaviour of the force signal is that the relative size of *all* harmonic components must be fixed by the one coefficient C_d.

The reason why the drag term does not predict high frequency components is because the effect of vortex shedding is not taken into account. During the first half-cycle of flow oscillation starting from rest, simple theory suggests that $u|u|$ is roughly proportional to the rate at which vorticity is shed into the flow. This vorticity is assembled into eddies or vortices which can detach from the cylinder and be convected back past the cylinder during the period of flow reversal, a phenomenon which is sometimes termed 'wake re-encounter'. This will clearly change the harmonic content of the force, producing deviations from the Morison prediction. While the knowledge of the instantaneous flow velocity is sufficient to determine the general trend of the force, its detailed behaviour is strongly dependent on vortex movement which occurs for $KC \geq 5$. This means that at a given time the force can depend strongly on the flow behaviour in the recent past. It is this 'history' effect which is inadequately represented by Morison's equation.

The approach taken here to extend Morison's equation is to include additional terms involving $F(t)$ rather than $u(t)$. Since the first (Morison) approximation is associated with vorticity generation one might expect the history of the motion of shed vorticity to be associated with higher order terms in F. The recent history of the force is thus represented by including derivative terms $\ddot{F}$ and $\dot{F}$. Further nonlinear terms F^2 and $F|F|$ are also included; the first because it allows the model to generate the *all* harmonics including the even ones, the second simply as a means of allowing the model greater freedom in shaping the relative proportions of the harmonics. As a further rationale, one can point out that system identification techniques commonly require the inclusion of output terms in a model as this generally allows the specification of a parsimonious model i.e. one containing a minimal number of terms. The initial choice for the extended Morison equation thus has the following form

$$\alpha_1 \ddot{F} + \alpha_2 \dot{F} + F + \alpha_3 F^2 + \alpha_4 F|F| = \frac{1}{2}\rho D C_d u|u| + \frac{1}{4}\pi\rho D^2 C_m \dot{u} \tag{3}$$

The reasoning which leads to this structure is certainly intuitive. However, it does have a physical basis and the complexity of the problem has effectively ruled out any attempt to extend Morison's equation by direct analytical means.

Previous attempts to extend Morisons equation have concentrated on the addition of input (i.e. velocity and acceleration) terms. Notably, Sarpkaya [2] proposes a two-term extension which allows improved prediction of the wave forces for U-tube data.

It is clear that the main disadvantage of adopting *any* extension to Morison's equation is that it would entail the re-evaluation of all previously determined drag and inertia coefficients together with the determination of the corresponding values of the additional coefficients. This would require a considerable experimental effort. However, it is hoped that the possible improvement in predictive capabilities would be such as to justify such a programme.

3 System Identification and Parameter Estimation.

The first of the two main problems in identifying a mathematical model of an input-output system is that of structure detection, i.e. what is the form of the equation of the underlying process? In this study the problem is bypassed by adopting the form (3) on heuristic grounds. A further systematic approach is presently being carried out using more sophisticated techniques based on NARMAX routines [3] which allow the model structure to be determined as part of the identification process. The NARMAX method produces a representation of the force at a given instant as a function of forces and velocities at previous instants. The techniques have a number of valuable features including the capability of determining the structure of any noise processes present, they can also detect the presence of delays between input and output signals which can arise in experiments due instrumentation.

Having obtained a model structure the next problem is that of parameter estimation, i.e. how to determine C_d, C_m and $\alpha_1 \dots \alpha_4$ in equation (3). This is accomplished by minimising the difference between the model output and the measured output data which correspond to the measured input. Suppose N sampled records of force, velocity and acceleration are available i.e. $\{F_i, u_i, \dot{u}_i : i = 1, \dots, N\}$ where F_i is the i^{th} sampled force value etc. At each sampling instant (by hypothesis) the data satifies the equation

$$\alpha_1 \ddot{F}_i + \alpha_2 \dot{F}_i + F_i + \alpha_3 F_i^2 + \alpha_4 F_i |F_i| = \beta_1 u_i |u_i| + \beta_2 \dot{u}_i + \zeta_i \quad (4)$$

where β_1 and β_2 are introduced as a convenient shorthand for the constants in equation (3). $\alpha_1, \dots, \alpha_4, \beta_1$ and β_2 are *estimates* of the parameters here and ζ_i represents the error or residual in the model at instant i. The least-squares estimates of the parameters are formed by minimising the sum of the squared errors

$$J = \sum_{i=1}^{N} \zeta_i^2 \quad (5)$$

with respect to variations of the parameter estimates. The problem is best expressed in matrices. Assembling all equations (3) for $i = 1, \dots, N$ gives

$$
\begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{Bmatrix} = \begin{bmatrix} \ddot{F}_1 & \dot{F}_1 & F_1^2 & F_1|F_1| & u_1|u_1| & \dot{u}_1 \\ \ddot{F}_2 & \dot{F}_2 & F_2^2 & F_2|F_2| & u_2|u_2| & \dot{u}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \ddot{F}_i & \dot{F}_i & F_i^2 & F_i|F_i| & u_i|u_i| & \dot{u}_i \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \ddot{F}_N & \dot{F}_N & F_N^2 & F_N|F_N| & u_N|u_N| & \dot{u}_N \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \end{Bmatrix} + \begin{Bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_i \\ \vdots \\ \zeta_N \end{Bmatrix} \tag{6}
$$

or

$$\{F\} = [A]\{\beta\} + \{\zeta\} \tag{7}$$

in matrix notation (square brackets denote matrices, curved brackets denote vectors), $[A]$ is called the design matrix, $\{\beta\}$ is the vector of parameters and $\{\zeta\}$ is the vector of residuals. In this notation the sum of squared errors (5) is

$$\{\zeta\}^T\{\zeta\} = (\{F\}^T - \{\beta\}^T[A]^T)(\{F\} - [A]\{\beta\}) \tag{8}$$

Minimising this expression with respect to the parameter estimates yields the well-known normal equations for the least-squares estimates.

$$[A]^T[A]\{\beta\} = [A]^T\{F\} \tag{9}$$

which are trivially solved by

$$\{\beta\} = ([A]^T[A])^{-1}[A]^T\{F\} \tag{10}$$

provided that $[A]^T[A]$ is invertible. Because of random errors in the measurements, different samples of data will contain different noise components, consequently they will lead to slightly different parameter estimates. The parameter estimates therefore constitute a random sample from a population of possible estimates; this population being characterised by a probability distribution. Clearly, it is desirable that the expected value of this distribution should coincide with the true parameters. If such a condition holds, the parameter estimator is said to be ***unbiased***. Now, given that the unbiased estimates are distributed about the true parameters, knowledge of the variance of the parameter distribution would provide valuable information about the possible scatter in the estimates. In fact, this information is readily available; the covariance matrix for the parameters is defined as

$$[C](\hat{\beta}) = E[(\{\hat{\beta}\} - E\{\hat{\beta}\}).(\{\hat{\beta}\} - E\{\hat{\beta}\})^T] \tag{11}$$

where the carets are used to emphasize the fact that quantities are estimates and the expectation E is taken over all possible estimates. The diagonal elements C_{ii} are the variances of the parameter estimates $\hat{\beta}_i$. Under a number of mild assumptions it is possible to show that, given an estimate $\{\hat{\beta}\}$

$$[C] = \sigma_\zeta^2.([A]^T[A])^{-1} \tag{12}$$

where σ_ζ^2 is the variance of the residul sequence ζ_i obtained by using $\{\hat{\beta}\}$ to predict the output. The standard deviation for each parameter is therefore:

$$\sigma_i = \sigma_\zeta \sqrt{([A]^T[A])_{ii}^{-1}} \tag{13}$$

If the parameter distributions are Gaussian, standard theory yields a 95% confidence interval of $\{\hat{\beta}\} \pm 1.96\{\sigma\}$, i.e. there is a 95% probability that the true parameters fall within this interval.

In order to determine whether a term is an important part of the model, a significance factor can be defined as follows. Each model term $\theta(t)$, e.g. $\theta(t) = F(t)$ or $\theta(t) = F(t)|F(t)|$, can be used on its own to generate a time-series which will have variance σ_θ^2. The significance factor s_θ is then defined by

$$s_\theta = 100\frac{\sigma_\theta^2}{\sigma_{\hat{F}}^2} \tag{14}$$

where $\sigma_{\hat{F}}^2$ is the variance of the estimated force, i.e. the sum of all the model terms. Roughly speaking, s_θ is the percentage contributed to the model variance by the term θ.

Having obtained a set of model parameters, it is necessary to check the accuracy of the model. The simplest means of doing this is to plot and compare the measured force F_i with the curve-fit value

$$\hat{F}_i = -\alpha_1\ddot{F}_i - \alpha_2\dot{F}_i - \alpha_3 F_i^2 - \alpha_4 F_i|F_i| + \beta_1 u_i|u_i| + \beta_2\dot{u}_i \tag{15}$$

based on the estimated parameters. One can also use a numerical measure of the closeness of fit; the measure adopted here is the normalised mean-square error or MSE defined by

$$MSE(\hat{F}) = \frac{100}{N\sigma_F^2}\sum_{i=1}^{N}(F_i - \hat{F}_i)^2 \tag{16}$$

This MSE has the following useful property; if the mean of the force signal $\overline{F}$ is used as the model i.e. $\hat{F}_i = \overline{F}$ for all i, the MSE is 100%, i.e.

$$MSE(\hat{F}) = \frac{100}{N\sigma_F^2}\sum_{i=1}^{N}(F_i - \overline{F})^2 = \frac{100}{\sigma_F^2}.\sigma_F^2 = 100 \tag{17}$$

A more stringent test of the model validity is to predict the wave force from equation (4) using measured velocities and accelerations only, via some time-stepping procedure. This can then be compared with the measured force.

The most comprehensive set of model validity tests are those of Billings [4]. Briefly, the validity of the model is contingent on the vanishing of certain correlation functions between the input (in this case velocity), and residual data.

4 Application of the New Model Structure to U-Tube Data.

The U-tube data here were obtained by digitising the force time-history figures from published papers and reports. The data sets examined are from the experimental study by Obasaju et.al. [5]. In the experiment, at various different values of KC, the time-history of the force on a cylinder in a regular planar oscillatory flow was measure. For each KC value considered, two types of force histories were distinguised in the experiment. Elements of the first class of force histories were produced by carefully averaging over cycles which exhibited the same form or mode of vortex shedding. Elements of the second class were obtained by averaging over *all* cycles, irrespective of the mode of vortex shedding. It is the second class which is considered here. For a fixed β value of 417, the KC values for which averaged force cycles were available were 3.31, 6.48, 11.88, 17.5 and 34.68.

For comparison purposes, the Morison equation was fitted to each data set. The resulting coefficient estimates and model MSE's were

KC	C_d	C_m	MSE
3.31	1.13 ± 0.38	2.28 ± 0.11	0.168
6.48	1.75 ± 0.15	2.02 ± 0.08	0.422
11.88	2.51 ± 0.30	0.92 ± 0.31	7.362
17.50	2.08 ± 0.05	1.03 ± 0.07	0.680
34.68	1.69 ± 0.05	1.28 ± 0.13	1.067

Table 1: Results for Morison equation fit to the U-tube data.

The comparisons for the data sets with $KC = 11.88$, 17.5 and 34.68 are shown in Figures 1, 2 and 3 respectively, (the other two force-histories are inertia dominated and essentially harmonic. The Morison equation is clearly inadequate when there is a significant effect from vortex shedding. This is most marked in Figure 1.

In order to assemble the data for fitting the model equation (3), the force data was differentiated twice using a five-point centred difference giving $\dot{F}$ and $\ddot{F}$. The resulting MSE's for the extended model fit are tabulated below.

KC	MSE	$\frac{MSE}{MSE(Morison)}$
3.31	0.092	0.548
6.48	0.120	0.284
11.88	1.538	0.209
17.50	0.289	0.425
34.68	0.510	0.750

Table 2: MSE values for curve-fit of equation (3) to data.

These results show distinct improvements over Morison's equation. In fact consideration of the significance factors indicated that the F^2 term made no contribution and the $\dot{F}$ and $\ddot{F}$ terms only made a limited contribution. This raised the possibility of having an improved model with only one additional term. As a consequence, parameter estimates for the model structure

$$F + \alpha F|F| = \beta_1 u|u| + \beta_2 \dot{u} \tag{18}$$

were obtained. The resulting MSE values are given below.

KC	MSE	$\frac{MSE}{MSE(Morison)}$
3.31	0.162	0.964
6.48	0.186	0.445
11.88	2.210	0.300
17.50	0.529	0.870
34.68	0.746	0.699

Table 3: MSE values for curve-fit of equation (18) to data.

Still showing an improvement over Morisons equation, particularly when vortex shedding is important. Unfortunately, this model structure can not be used to *predict* (as distinct from *curve-fit to*) the wave forces. The reason for this is that prediction requires the solution of the equation (18) for F at each sampling instant. The exact solution has been obtained and was found to demonstrate bifurcation phenomena as the RHS of the equation changed. Specifically, at one sampling instant the equation can have three real roots, at the following it may have only one. This effect is shown clearly in Figure 4 where the force history for $KC = 11.88$ is predicted from the model. This problem effectively rules out equation (18) as a possible extended wave force equation.

In order to recover the improvement, it is therefore necessary to consider the full extended model (3) in the hope that this will prove stable for the prediction of wave forces. The problem here is that the derivative terms did not appear to be significant. However, it was thought that this might be due to the fact that the estimated derivatives $\dot{F}$ and $\ddot{F}$ are quite noisy as they were estimated on data digitised from plots. In order to test this hypothesis, the discrete difference or NARMAX [3] form of equation (3) was adopted. The forward-difference version of (3) was obtained (this corresponds to the centred-difference form of a first order differential equation model).

$$F_i = a_1 F_{i-1} + a_2 F_{i-2} + a_3 F_{i-1}|F_{i-1}| + b_1 u_{i-1} + b_2 u_{i-2} + a_b u_{i-1}|u_{i-1}| \qquad (19)$$

The resulting MSE values on fitting this model structure were:

KC	MSE	$\frac{MSE}{MSE(Morison)}$
3.31	0.014	0.080
6.48	0.001	0.020
11.88	0.044	0.005
17.50	0.011	0.016
34.68	0.101	0.096

Table 4: MSE values for curve-fit of equation (19) to data.

This constitutes a very significant improvement. The comparisons corresponding to Figures 1 to 3 are given in Figures 5 to 7. Equation (19) was then used to step the force data F_i forward in time using the measured velocities only (the values F_1 and F_2 are used to start the iteration). This procedure gave the predicted output. The MSE values for the comparison between the predicted forces and measured forces are given below.

KC	MSE	$\frac{MSE}{MSE(Morison)}$
3.31	0.034	0.202
6.48	0.499	1.180
11.88	20.40	2.771
17.50	0.295	0.434
34.68	0.936	0.877

Table 5: MSE values for predicted forces using equation (19).

Unfortunately, the improvement is not as expected. This is a little surprising given that there are no obvious bifurcation type effects and that the least-squares curve-fits (Figures 5 to 7) are nearly perfect. However, the lack of bifurcation is encouraging. A stability study of the model indicated that this behaviour may be due to a fundamental instability in the model/data structure which may be curable. Research is currently proceeding on this effect.

5 Application to Wave Flume Data.

The data was obtained from the Delta flume of the De Voorst facility of Delft Hydraulics Limited. The particular data considered here comes from the run OA1F1 which used a fixed smooth cylinder. The unidirectional wave profiles were generated so that the the surface elevation spectrum approximated to a JONSWAP spectrum. More details of the experiment can be found in [6] which contains an exhaustive wave-by-wave Morison analysis of the full De Voorst data set. In the experiment, the velocity signal was obtained from electromagnetic flowmeters placed adjacent to the cylinder at the same distance from the wave maker. The forces were recorded from force sleeves placed at three levels (Stations 2,3 and 4) on the cylinder. The data from Station 2 had to be discarded as the sleeve fell within the crest to trough region of the wave. Of those remaining, Station 3 was nearest to the surface while fully immersed and was consequently subjected to the highest nonlinear forces. For this reason data from Station 3 was used in the following analysis.

Only one example of a curve-fit to De Voorst data will be presented here. The entire velocity and transverse force records for the data were plotted and 1000 points of the data around the instant where the maximum transverse force occurred was chosen for analysis. This is because the transverse force can be taken as an indicator of the intensity of vortex shedding. The Morison fit to the chosen data is shown in Figure 8. C_d and C_m were estimated as 0.601 and 1.83 respectively. The MSE value for the fit was 2.47. This good prediction is consistent with KC values being comparitively low. As a result the dominant part of the measured force is the linear inertia component. The extended model (19) was then fitted to the data and the resulting curve-fit is given in Figure 9, the MSE value being 0.775. As in the case of the U-tube data, this represents a substantial improvement. The model-predicted output for the model, shown in Figure 10, gave an MSE of 2.34 which is slightly better than Morison. Unfortunately, the improvement is small compared to that for the curve-fit. This seems to be the same problem as that which beset the U-tube data. Further research is in progress.

6 Conclusion.

In conclusion, the model structures proposed here are the product of some intuitive reasoning based on a knowledge of flow mechanisms. Unfortunately, they cannot yet provide a final form for an extension of Morison's equation as they appear to be plagued by possible stability or bifurcation problems which prevent the accurate prediction of wave force data from velocities. However, the tremendous improvements in the curve-fits to the data indicate that the structures would be of value if the stability problems could be overcome. The new model structures produce comparable improvements for both regular U-tube data and irregular wave-flume data. The techniques applied here have quite general applicability and will be applied to the multidirectional sea data from the Christchurch Bay Tower experiment.

7 Acknowledgements.

Thanks are due to Malcolm Birkenshaw, Professor P.W.Bearman and particularly Martin Davies for their help in acquiring data from the De Voorst experiments. Thanks are also due to Dr. E.Obasaju for making available unpublished data from his experimental work. This project is supported by the Department of Energy through the Fluid Loading Programme.

References

[1] Morison (J.R.), O'Brien (M.P.), Johnson (J.W.) & Schaf (S.A.) 1950 *Petroleum Transactions* 189 pp.149-157. The force exerted by surface waves on piles.

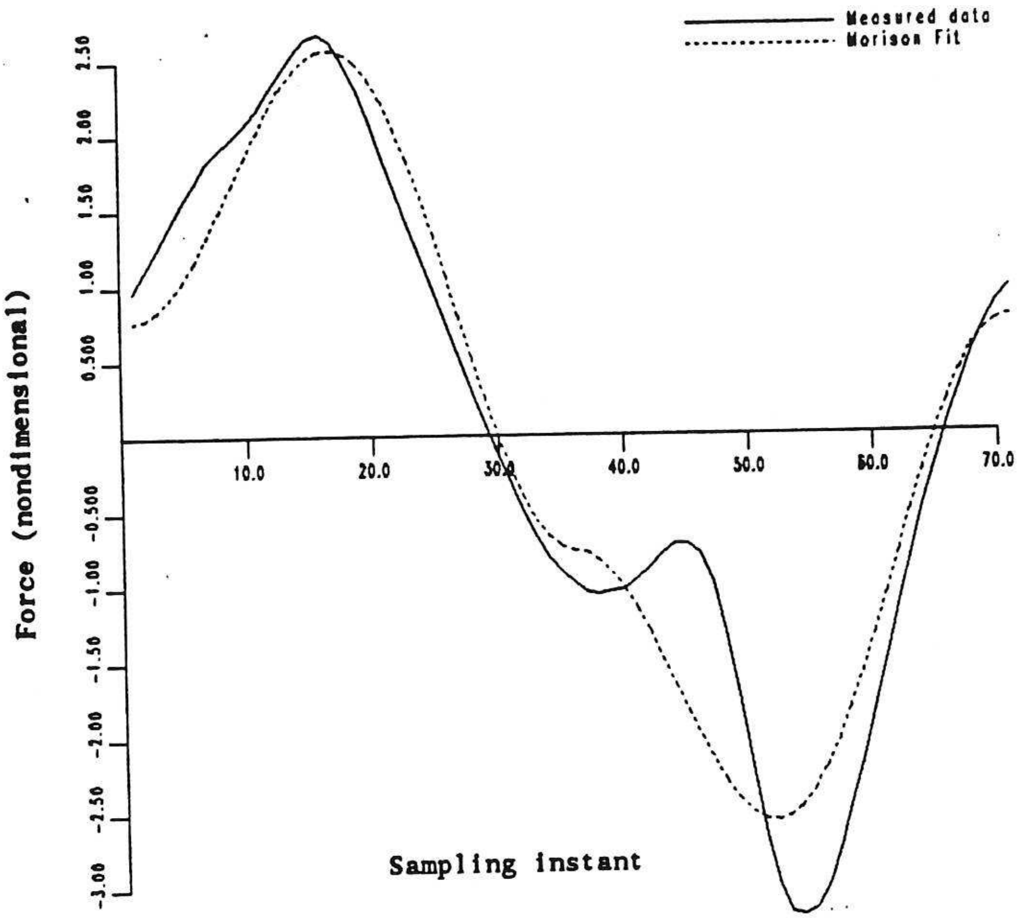
[2] Sarpkaya (T.) 1981 *US Naval Civil Engineering Laboratory Report no. CR 82.008*. Morison's equation and the wave forces on offshore structures.

[3] Chen (S.), Billings (S.A.) & Liu (Y.P.) 1987 *International Journal of Control* **50** pp.1873-1896. Orthogonal least-squares methods and their application to nonlinear system identification.

[4] Billings (S.A.) & Voon (W.S.F.) 1983 *IEE Proceedings* **130** pp.193-199. Structure detection and model validity tests in the identification of nonlinear systems.

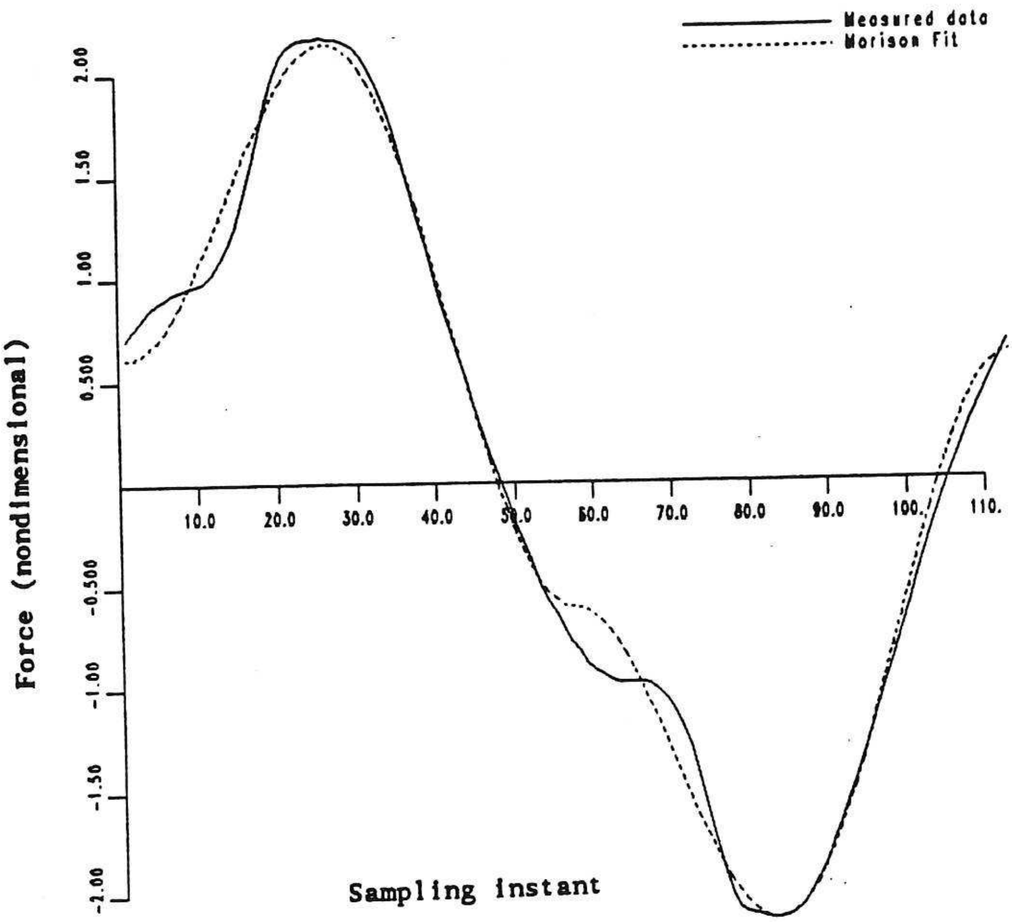
[5] Obasaju (E.D.), Bearman (P.W.) & Graham (J.M.R.) 1988 *Journal of Fluid Mechanics* **196** pp.467-494. A study of forces, circulation and vortex patterns around a circular cylinder in oscillating flow.

[6] Davies (M.J.S.), Graham (J.M.R.), & Bearman (P.W.) 1990 in *Environmental Forces on Offshore Structures and Their Prediction* pp.113-136. Kluwer Academic Press.



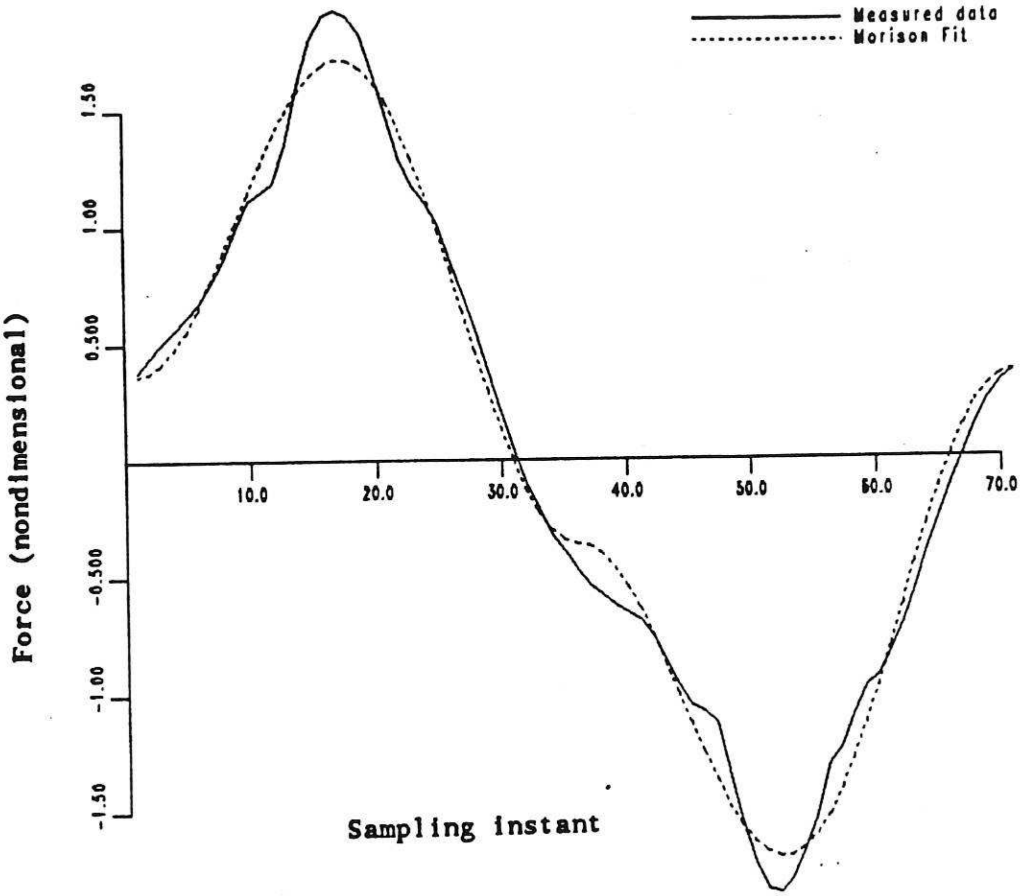
Normalised MSE : 7.36

Figure 1. Comparison between measured U-tube data and Morison equation prediction. KC = 11.88.



Normalised MSE : 0.680

Figure 2. Comparison between measured U-tube data and Morison equation prediction. KC = 17.5.



Normalised MSE : 1.07

Figure 3. Comparison between measured U-tube data and Morison equation prediction. KC = 34.68.

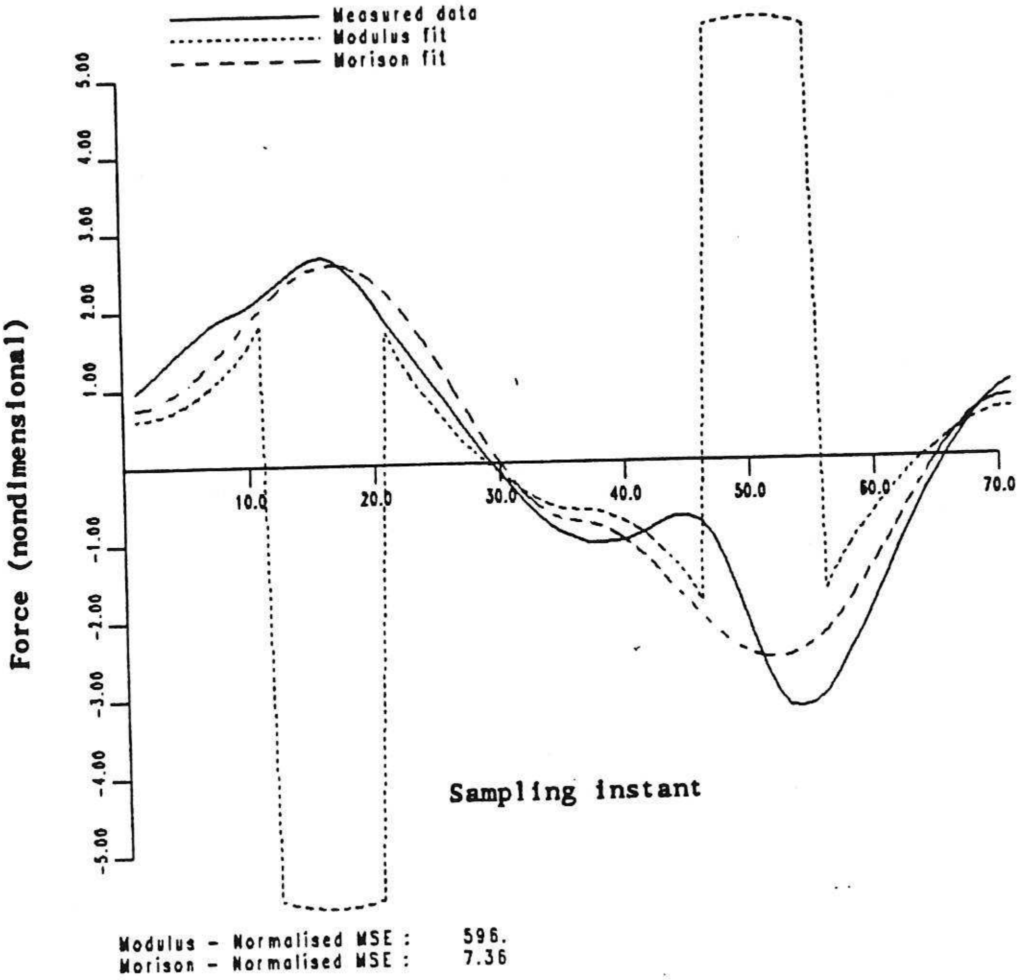
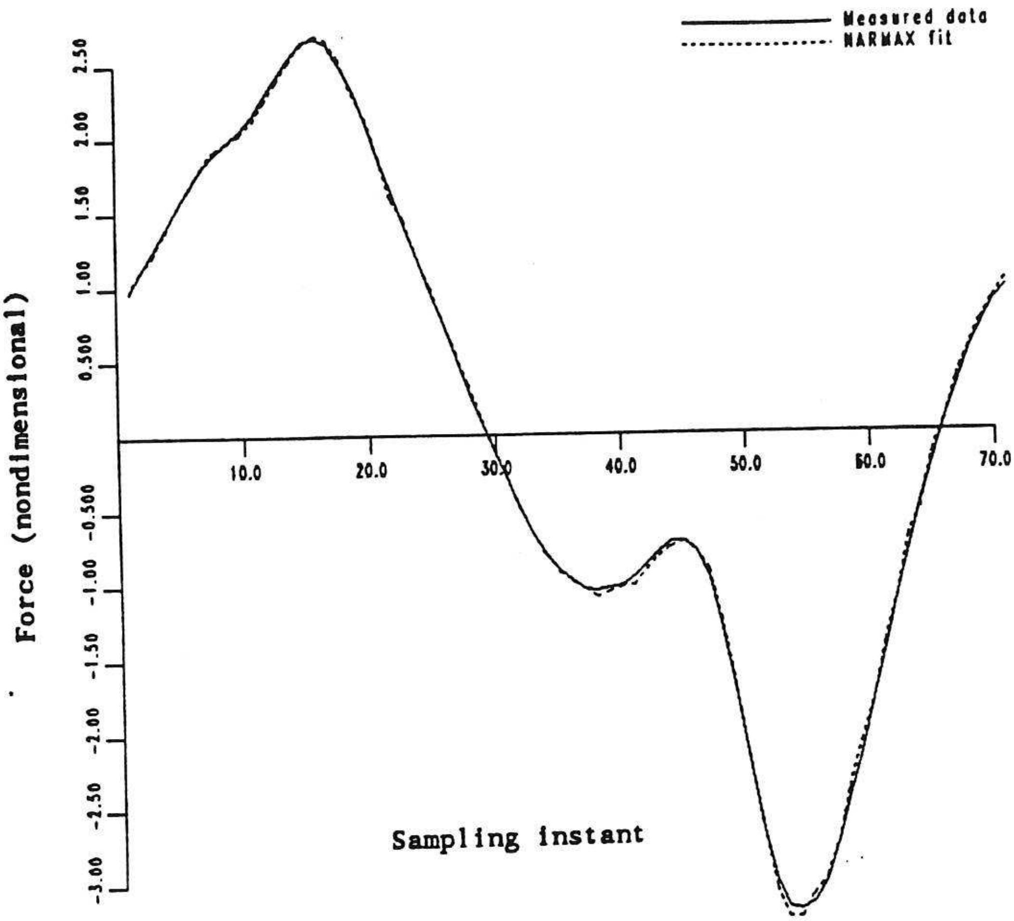
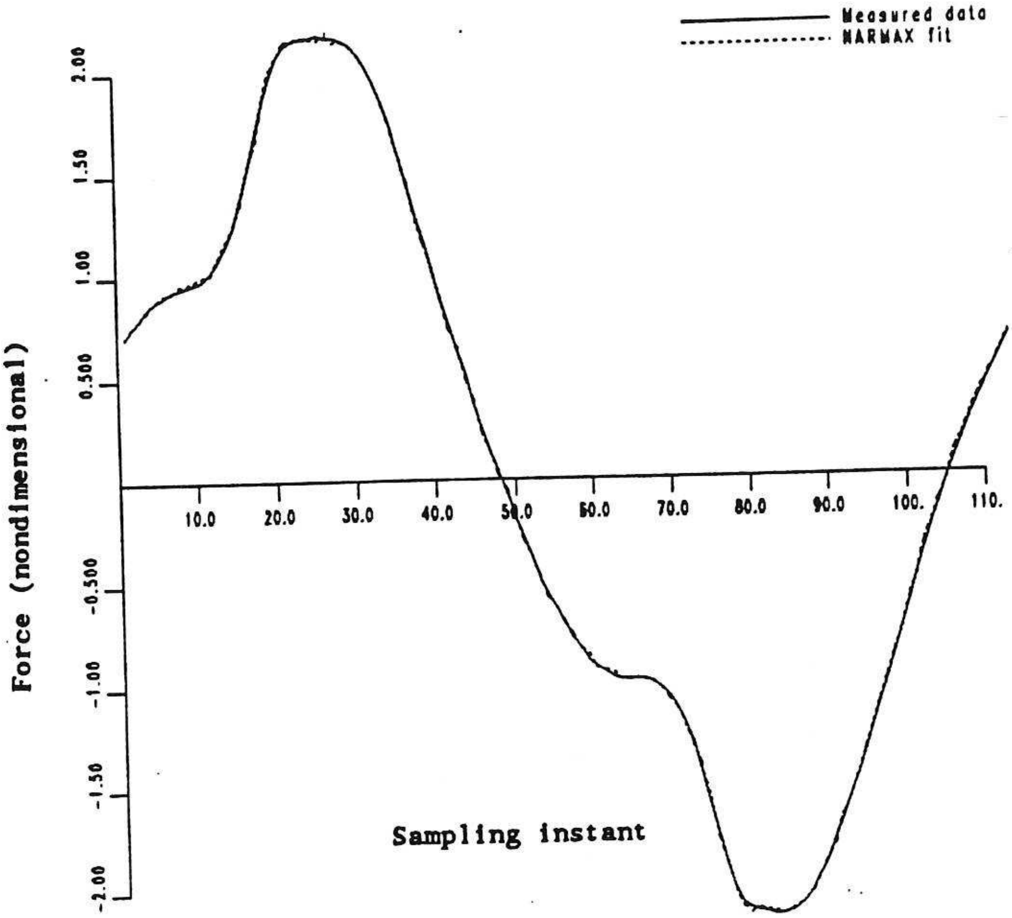


Figure 4. Comparison between measured data and reconstruction using extended model with a F|F| term. KC = 11.88.



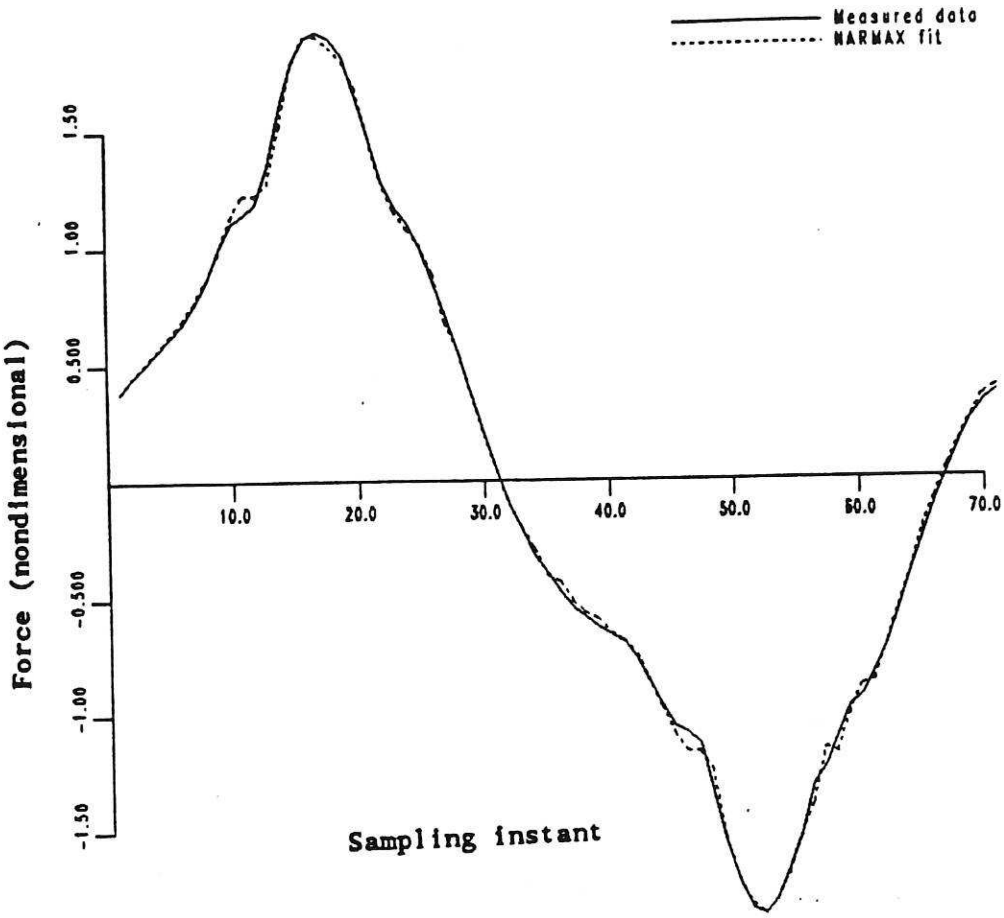
Normalised MSE : 0.044

Figure 5. Comparison between measured data and least-squares curve-fit for the NARMAX extended model. KC = 11.88.



Normalised MSE : 0.011

Figure 6. Comparison between measured data and least-squares curve-fit for the NARMAX extended model. KC = 17.5.



Normalised MSE : 0.101

Figure 7. Comparison between measured data and least-squares curve-fit for the NARMAX extended model. KC = 34.68.

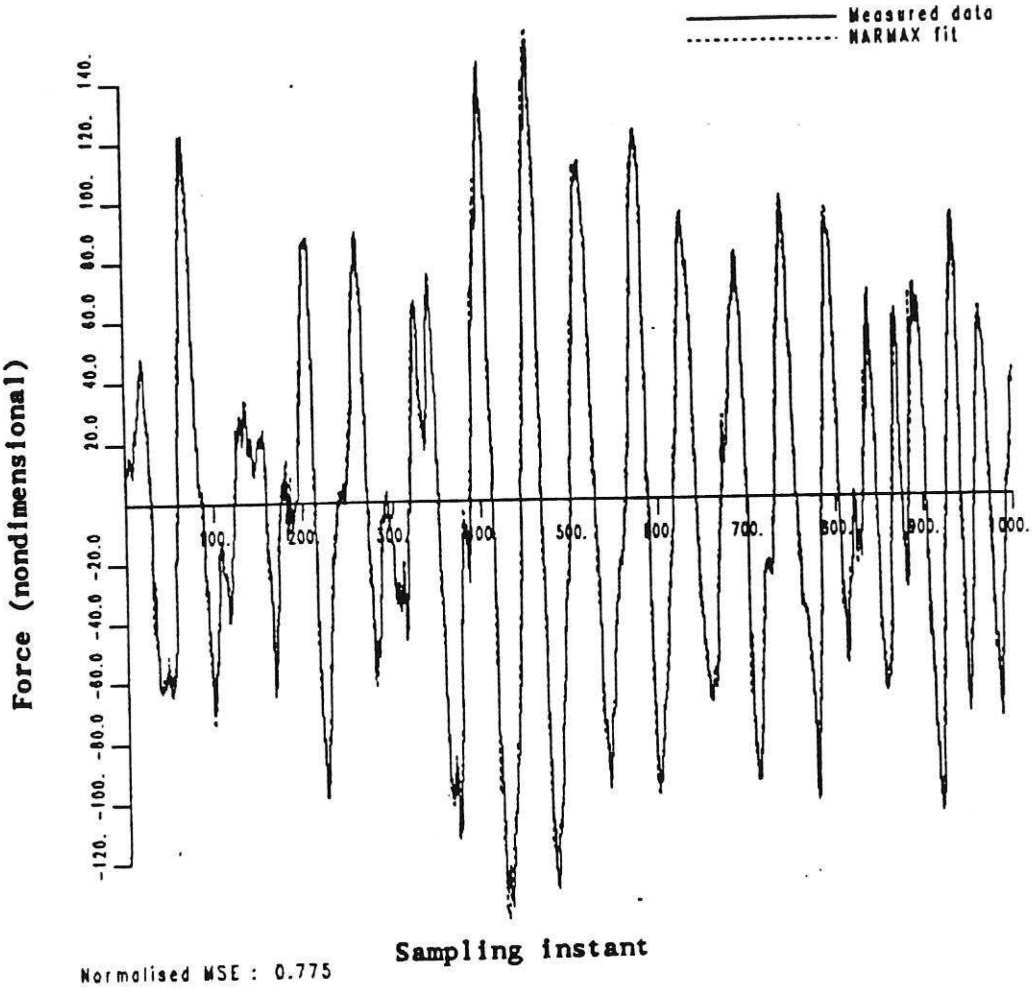


Figure 9. Comparison between the De Voorst data and the least-squares curve-fit using the extended model.

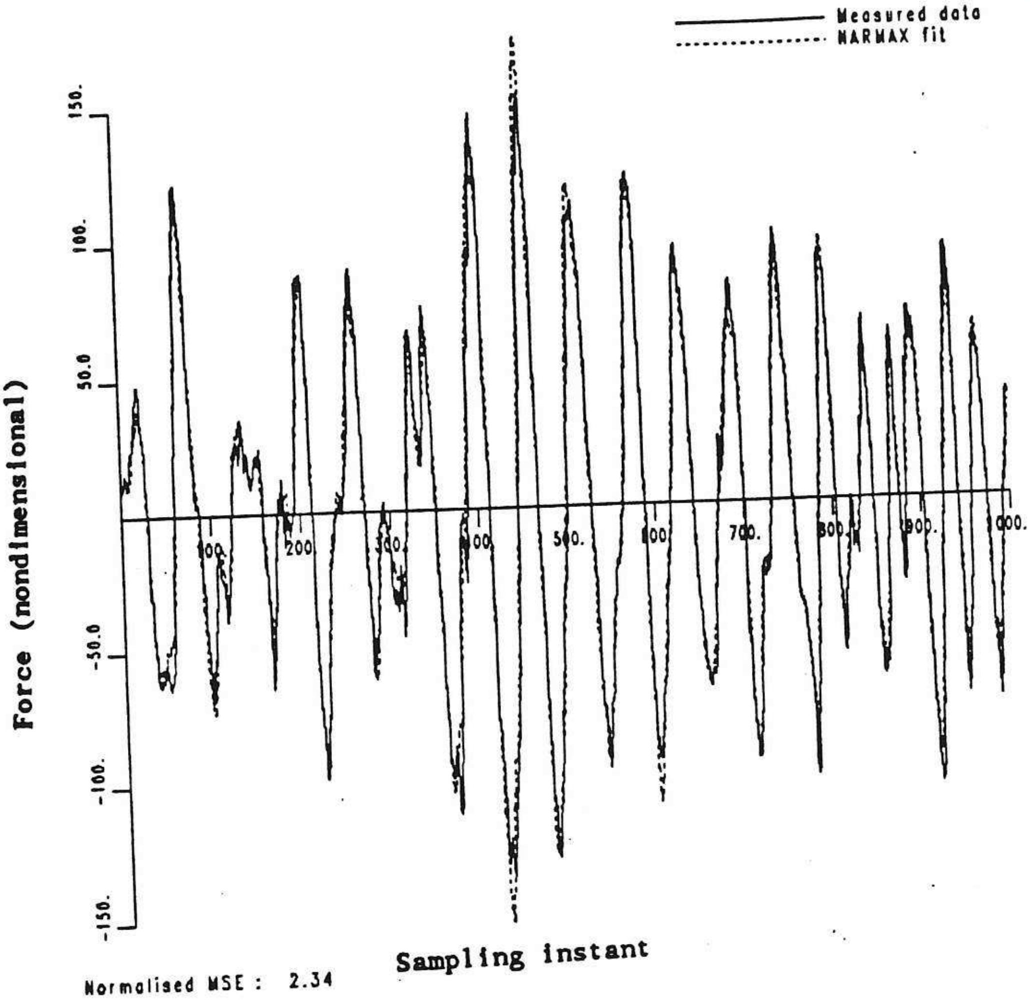


Figure 10. Comparison between the De Voorst data and that predicted by the extended model.