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Jalel, N.A. and Nicholson, N. (1990) Fault Detection Using Optimal Control Techniques. Research Report. Acse Report 419 . Dept of Automatic Control and System Engineering. University of Sheffield

Fault Detection Using Optimal Control Techniques

by

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Research Report No. 419
November 1990

1 Abstract

The possibility of using optimal control techniques for fault diagnosis is investigated in this paper. To achieve this, two techniques will be used, in the first the performance index will be used to identify the abnormal behaviour in any subsystem of the Loss Of Fluid Test (LOFT) reactor, a small scale pressurised water reactor, and in the other the faulty controller of the LOFT reactor will be identified using the Pontryagin maximum principle .

2 Introduction

The basic optimal control problem is to determine the optimal control u_t for $t_o \leq t \leq t_f$ which will transfer the system

$$\dot{X}(t) = f(x, u, t)$$

from an initial state at t_o to a final state at time t_f such that the performance index

$$PI = \int_{t_o}^{t_f} L(x, u, t)\, dt$$

is a minimum. The applications of optimal control on-line usually imply models operating in parallel with the plant. The models track plant operation while optimizing the performance criteria. Typically, in classical control, the control output is computed as a function of the difference between a measured variable and its setpoint. In optimal control, on the basis of desired performance levels and process characteristics, a functional relationship between plant conditions and corrective actions is developed. Compared to classical control, the optimal control approach promises better performance and enhanced plant safety.

Optimal control techniques have been used in military weapon systems, airspace vehicles, and industrial plants. In the nuclear field, the optimal control technique has been used both in nuclear design and in nuclear plant operation problems [1,2].

To achieve adequate cooling of the pressurized water reactor and proper functioning of the turbine, the water level on the secondary side of the steam generator must remain as close as possible to a preset value. A good steam generator water level controller reduces the occurrence of high-level and low-level trip and improves the overall power plant availability. An optimal control technique has been used to solve this problem and to design a water level controller [3]. They have also been used for the synthesis of a flow-pressure control system in the water supply line of steam generators by controlling the water flow rate and the pressure drop across the control valve by adjusting the valve stem position and pump speed [4]. For the boiling water reactor BWR, an optimal control technique has been used for designing a suitable controller for the recirculation pump and throttle valve [5].

3 Optimal Control Technique and Fault Diagnosis

An optimal control technique has also been used for fault diagnosis and accident analysis of nuclear power plant. Manzetis, used Pontryagin's optimal control principle to detect and identify malfunctions in nuclear power plants. The malfunction control variables are defined and used to initiate the diagnostic procedure, which can be used to identify the malfunctioning variable. In his work, Manzetis used a simple model of a pressurized water reactor to simplify the diagnostic problem [6,7].

A south Korean group developed an expert system for nuclear power plant accident diagnosis. The system is able to diagnose nine accidents using the Prolog language to represent the knowledge base and a backtracking inference strategy. The knowledge base is divided into two parts, the rule-base and the data-base. The rule-base includes rules for accident classification, accident diagnosis and treatment. The data-base contains transient analysis codes, plant design data, and plant operational experience data. The most interesting part of this system is the diagnostic strategy. If there is any upset in the plant, a suspected accident group is selected by the use of accident classification rules. The initial plant variables and data are then sent to the transient analysis codes of each accident in the group and the performance-index for each accident are calculated using the initial plant variables and measured plant variables. The accident and its cause can be diagnosed and identified using a minimum performance-index [8].

In the present work [9], a new fault diagnosis algorithm for the LOFT reactor is investigated, based on dividing the LOFT reactor into six subsystems and calculating the performance index for each part separately. In the second part of this work, the Pontryagin maximum principle will be explored in detail to identify a faulty controller in the LOFT reactor model. In calculating the Euler Lagrange equations, Manzetis has used the analytical solution for calculating the required partial derivatives using a simple model of a pressurized water reactor. However, for the LOFT reactor model which is a highly nonlinear system of 27th order the analytical solution is a very tedious tusk, so in this study a modified approach based on approximating each partial derivative by a first order gradient will be used to calculate the Euler Lagrange equations.

4 Identifying the Abnormal Situation in the LOFT Reactor

4.1 Performance Index

The LOFT reactor is divided into six separate subsystems and the performance index for each individual section is calculated independently. By dividing the reactor into separate parts, it is possible to identify the subsystem with the abnormal behaviour from the value of the performance index.

The performance indices for these subsystems are defined as:

1. Reactor core performance index

$$\begin{aligned} PI1 &= \int_0^{t_f} W_1 (P - P_d)^2 + W_2 (T_f - T_{fd})^2 \\ &+ W_3 (T_c - T_{cd})^2 \, dt \end{aligned} \tag{1}$$

2. Primary loop performance index

$$\begin{aligned} PI2 &= \int_0^{t_f} W_4 (W_{loop} - W_{loopd})^2 + W_5 (T_{in} - T_{ind})^2 \\ &+ W_6 (T_{sgo} - T_{sgod})^2 + W_7 (T_{out} - T_{outd})^2 \\ &+ W_8 (T_{sgi} - T_{sgid})^2 \, dt \end{aligned} \tag{2}$$

3. Pressuriser performance index

$$PI3 = \int_0^{t_f} W_9 (P_p - P_{pd})^2 + W_{10} (P_{level} - P_{leveld})^2 \, dt \tag{3}$$

4. Steam generator and secondary steam flow performance index

$$\begin{aligned} PI4 &= \int_0^{t_f} W_{11} (T_{tsg} - T_{tsgd})^2 + W_{12} (P_{sg} - P_{sgd})^2 \\ &+ W_{13} (L_{sg} - L_{sgd})^2 + W_{14} (W_{stm} - W_{stmd})^2 \, dt \end{aligned} \tag{4}$$

5. Air cooled condenser and condensate receiver performance index

$$\begin{aligned} PI5 &= \int_0^{t_f} W_{15} (T_{tube} - T_{tubed})^2 + W_{16} (P_{cr} - P_{crd})^2 \\ &+ W_{17} (P_f - P_{fd})^2 \, dt \end{aligned} \tag{5}$$

6. Feedwater flow performance index

$$PI6 = \int_0^{t_f} W_{18}\left(T_{fw} - T_{fwd}\right)^2 + W_{19}\left(W_{fw} - W_{fwd}\right)^2 dt \qquad (6)$$

where t_f is the final time and W_1 to W_{19} are weighting constants. The values of the weighting constants and the desired values of the state variables are defined in Appendix A. The state variables are calculated from the transient response of the LOFT reactor model. A computer program for simulating the model and calculating the performance index is coded in Fortran and operates on a Sun workstation.

4.2 Malfunction Diagnosis and Results

The performance indices are calculated for the six parts during normal operation of the reactor power plant model. The performance values settle around a small value, due to the fact that the system follows the desired behaviour during normal operation. The fault diagnosis algorithm is based on calculating the performance index for these subsystems during the fault situation and comparing their values with those for normal operation. The subsystem exhibiting a large deviation in performance index is the part with the abnormal behaviour.

To test the diagnostic ability of this algorithm a malfunction in the feedwater system is assumed to occur in the reactor. The accident is initiated by reducing the feedwater flow to 0 (lbm/sec) at an operational time equal to 5 seconds. Table 1 shows the values of the performance index for the six subsystems during the normal operation case. Due to the drop in the feedwater flow, the value of the performance index for the feedwater system changes to a value of 1 at $t = 6$ sec, as shown in table 2. From table 2 and by comparing the values of the performance indices for the individual subsystems it is possible to see that at $t = 6$ sec there is a large change in the value of the performance index indicating an abnormal behaviour in the feedwater system. Figures 1 and 2 illustrate the values of the six performance indices with respect to time during normal and accident situations respectively. From figure 2 it is possible to see the change in the value of the performance index for the feedwater system at $t = 6$ sec revealing an abnormal behaviour in the feedwater system.

t (sec)	PI1	PI2	PI3	PI4	PI5	PI6
0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	$0.368e^{-5}$	$0.407e^{-9}$	$0.241e^{-5}$	$0.218e^{-11}$	0.0	$0.415e^{-13}$
2.0	$0.6619e^{-5}$	$0.312e^{-8}$	$0.193e^{-4}$	$0.136e^{-9}$	0.0	$0.1638e^{-10}$
3.0	$0.896e^{-5}$	$0.893e^{-8}$	$0.654e^{-4}$	$0.122e^{-8}$	$0.121e^{-14}$	$0.156e^{-9}$
4.0	$0.108e^{-4}$	$0.177e^{-7}$	$0.154e^{-3}$	$0.509e^{-8}$	$0.295e^{-12}$	$0.6513e^{-9}$
5.0	$0.124e^{-4}$	$0.3e^{-7}$	$0.302e^{-3}$	$0.152e^{-7}$	$0.22e^{-11}$	$0.198e^{-8}$
6.0	$0.135e^{-4}$	$0.473e^{-7}$	$0.521e^{-3}$	$0.35e^{-7}$	$0.7593e^{-11}$	$0.455e^{-8}$
7.0	$0.142e^{-4}$	$0.707e^{-7}$	$0.827e^{-3}$	$0.715e^{-7}$	$0.197e^{-10}$	$0.913e^{-8}$
8.0	$0.147e^{-4}$	$0.1e^{-6}$	$0.123e^{-2}$	$0.13e^{-6}$	$0.418e^{-10}$	$0.137e^{-7}$
9.0	$0.149e^{-4}$	$0.133e^{-6}$	$0.175e^{-2}$	$0.215e^{-6}$	$0.762e^{-10}$	$0.157e^{-7}$

Table 1: Values of the performance index during normal operation

t (sec)	PI1	PI2	PI3	PI4	PI5	PI6
0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	$0.368e^{-5}$	$0.407e^{-9}$	$0.241e^{-5}$	$0.218e^{-11}$	0.0	$0.415e^{-13}$
2.0	$0.6619e^{-5}$	$0.312e^{-8}$	$0.193e^{-4}$	$0.136e^{-9}$	0.0	$0.1638e^{-10}$
3.0	$0.896e^{-5}$	$0.893e^{-8}$	$0.654e^{-4}$	$0.122e^{-8}$	$0.121e^{-14}$	$0.156e^{-9}$
4.0	$0.108e^{-4}$	$0.177e^{-7}$	$0.154e^{-3}$	$0.509e^{-8}$	$0.295e^{-12}$	$0.6513e^{-9}$
5.0	$0.124e^{-4}$	$0.3e^{-7}$	$0.302e^{-3}$	$0.152e^{-7}$	$0.22e^{-11}$	$0.198e^{-8}$
6.0	$0.136e^{-4}$	$0.526e^{-7}$	$0.525e^{-3}$	$0.11e^{-3}$	$0.413e^{-7}$	$0.1e^{+1}$
7.0	$0.153e^{-4}$	$0.305e^{-6}$	$0.867e^{-3}$	$0.876e^{-3}$	$0.46e^{-6}$	$0.2e^{+1}$
8.0	$0.221e^{-4}$	$0.138e^{-5}$	$0.138e^{-2}$	$0.292e^{-2}$	$0.194e^{-5}$	$0.3e^{+1}$
9.0	$0.469e^{-4}$	$0.393e^{-5}$	$0.216e^{-2}$	$0.687e^{-2}$	$0.547e^{-5}$	$0.4e^{+1}$

Table 2: Values of the performance index during the drop in the feedwater flow

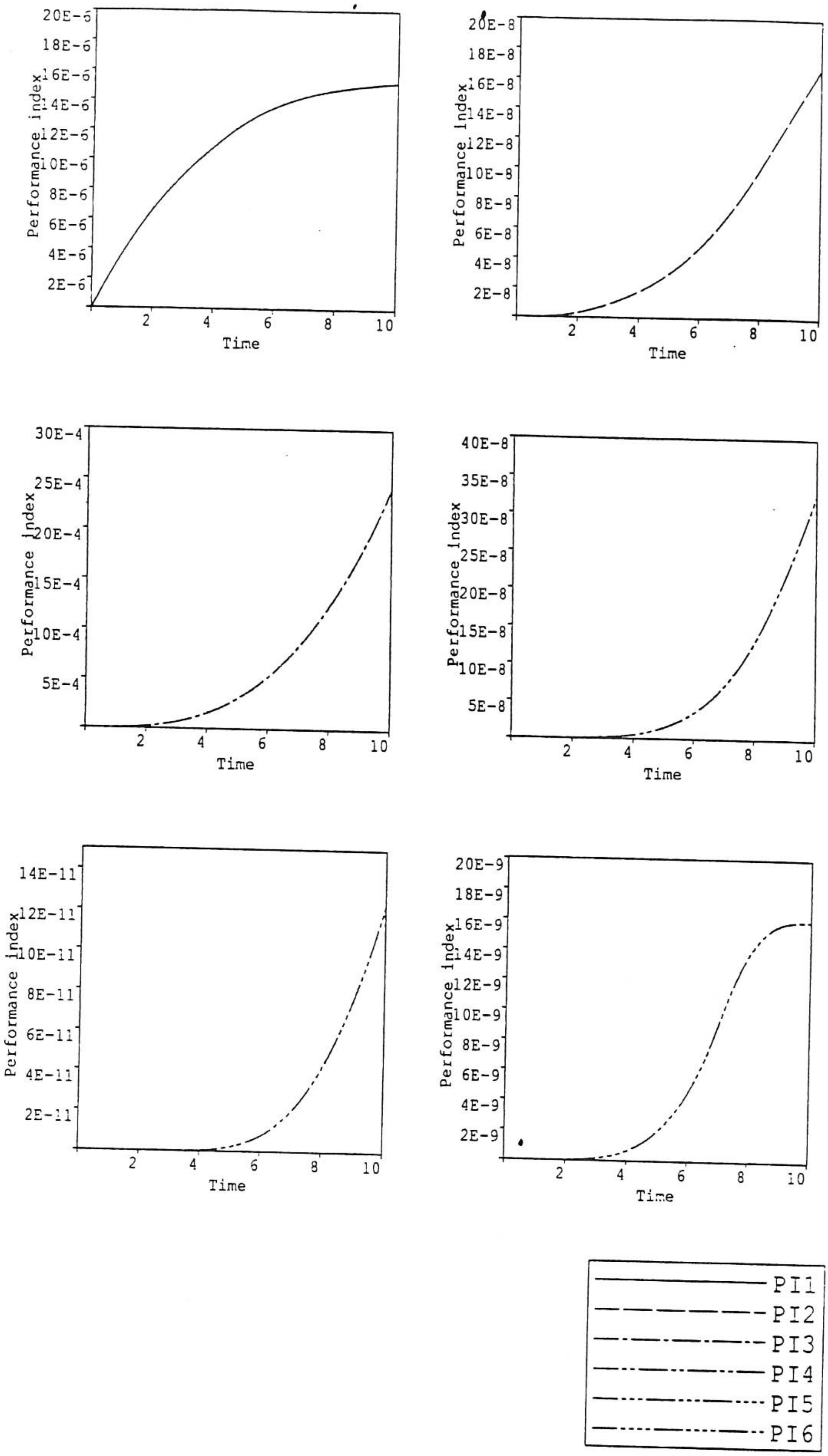

Figure 1: Performance indices during normal operation

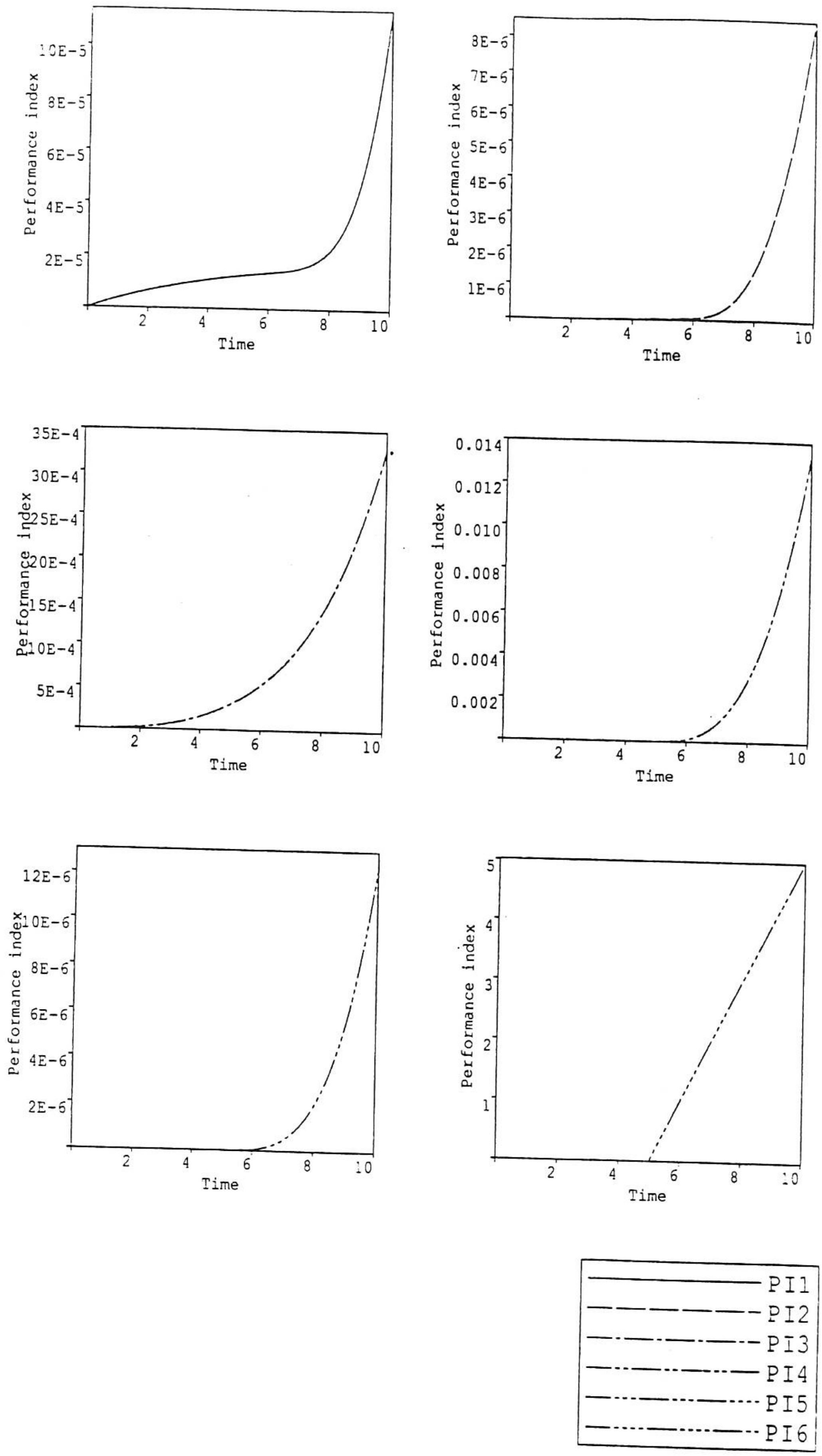

Figure 2: Performance indices during loss in feedwater flow

5 Pontryagin Maximum Principle and Fault Diagnoses

5.1 Pontryagin Theory

The Pontryagin optimization theory is well established and there are many references which give the full details of the mathematical derivation of the theory. The problem can be described briefly as finding the controller u which minimizes the function

$$J = \int_{t_o}^{t_f} L(x, u, t)dt$$

subject to the constraint system equations

$$\dot{X} = f(x, u, t)$$

The solution of this problem involves introducing a Lagrange multiplier λ_o and defining a new cost function as

$$J_{nc} = \int_{t_o}^{t_f} L(x, u, t) + \lambda_o^T \left[f(x, u, t) - \dot{X}\right] dt$$

A new state function called the Hamiltonian function is defined as

$$H(x, u, \lambda_o, t) = L(x, u, t) + \lambda_o^T f(x, u, t)$$

thus

$$J_{nc} = \int_{t_o}^{t_f} H - \lambda_o^T \dot{X}$$

The integral

$$F = H - \lambda_o \dot{X}$$

is a function of the variables x and u, so the Euler Lagrange equations are defined

$$\frac{\partial F}{\partial x} - \frac{d}{dt}\frac{\partial F}{\partial \dot{X}} = 0$$

$$\frac{\partial F}{\partial u} - \frac{d}{dt}\frac{\partial F}{\partial \dot{U}} = 0$$

which can be rewritten as

$$\dot{\lambda_o} = -\frac{\partial H(x, u, \lambda_o, t)}{\partial x}$$

and

$$\frac{\partial H(x, u, \lambda_o, t)}{\partial u} = 0$$

so,

$$\dot{\lambda_o} = -\frac{\partial L(x, u, t)}{\partial x} - \lambda_o^T \frac{\partial f(x, u, t)}{\partial x}$$

$$\frac{\partial L(x, u, t)}{\partial u} + \lambda_o^T \frac{\partial f(x, u, t)}{\partial u} = 0$$

5.2 Euler Lagrange Equations of the LOFT Reactor

The Euler Lagrange equations consist of two sets, the equations of Lagrange multipliers defined to be the variation of the new cost function with respect to the state variables and the set of switching functions achieved by the variation of the new cost function with respect to the control variables. The J integral is defined as

$$\begin{aligned} J &= \int_0^{t_f} WW_1 (P - P_d)^2 + WW_2 (P_p - P_{pd})^2 \\ &+ WW_3 (P_{sg} - P_{sgd})^2 + WW_4 (P_{level} - P_{leveld})^2 \\ &+ WW_5 (P_{cr} - P_{crd})^2 + WW_6 (L_{sg} - L_{sgd})^2 \\ &+ WW_7 (V_{loop} - V_{loopd})^2 \, dt \end{aligned} \tag{7}$$

where t_f is the final time and WW_1 to WW_7 are the weighting constants. The values of the weighting constants are given in Appendix A. The state variables used in the performance index are calculated from the transient response of the LOFT reactor model and were selected because they played important roles in describing the behaviour of the reactor during reactor transients and accidents.

5.2.1 The Set of Lagrange Multipliers

Since the LOFT reactor consists of 27 state equations, there are 27 final value Euler Lagrange equations for the Lagrange multipliers. In general, the LOFT model and the set of the Lagrange multipliers can be written in a functional form as

$$\dot{X} = f(x, u, t) \tag{8}$$

$$\dot{\lambda}_o = -\frac{\partial L(x, u, t)}{\partial x} - \lambda_o^T \frac{\partial f(x, u, t)}{\partial x} \tag{9}$$

where

$$\begin{aligned} L &= WW_1 (P - P_d)^2 + WW_2 (P_p - P_{pd})^2 \\ &+ WW_3 (P_{sg} - P_{sgd})^2 + WW_4 (P_{level} - P_{leveld})^2 \\ &+ WW_5 (P_{cr} - P_{crd})^2 + WW_6 (L_{sg} - L_{sgd})^2 \\ &+ WW_7 (V_{loop} - V_{loopd})^2 \, dt \end{aligned}$$

The difficulty in obtaining the set of Euler Lagrange equations for the Lagrange multipliers lies in finding the values of $\frac{\partial f(x,u,t)}{\partial x}$. One obvious method is to find the analytical solution for the required partial derivatives, but for the highly non linear model this would be quite a difficult tusk. The approach used here was to approximate each partial derivative by a first order gradient.

$$\frac{\partial f(x, u, t)}{\partial x} = \frac{f(x, u, t)\,|_{x=x} - f(x, u, t)\,|_{x=x-\Delta x}}{\Delta x}$$

This technique will lead to a set of 27 Lagrange multiplier equations. These are solved by stepping backward in time using the Euler numerical integration techniques and by assuming the values of the Lagrange multipliers are zero at the end point $t = t_f$.

5.2.2 The Set of Switching Functions

The variation of the new cost function with respect to the control variables will result in a set of switching function equations which can be represented in a functional form as

$$SW = \frac{\partial H(x, u, \lambda_o, t)}{\partial u}$$

$$SW = \frac{\partial L(x, u, t)}{\partial u} + \frac{\partial f(x, u, t)}{\partial u}$$

For the LOFT reactor, eight controllers are modelled which are defined to be the core reactivity, volumetric primary loop flow, pressuriser spray flow, pressuriser heating, desired main steam valve position, injection boration rate and feedwater valve demand position.

As in the case of the Lagrange multipliers, the partial derivative $\frac{\partial f}{\partial u}$ is approximated by a first order gradient

$$\frac{\partial f(x, u, t)}{\partial u} = \frac{f(x, u, t)\mid_{u=u} - f(x, u, t)\mid_{u=u-\Delta u}}{\Delta u}$$

Using this technique, a set of eight switching function equations can be derived.

5.3 Malfunction Diagnostic Algorithm

The optimal control technique is used to identify the failure or fault in any of the modelled controls of the LOFT reactor. The control malfunction detection process is shown in figure 3. This starts by supplying the measured state variables during the transient to the control model; this will initiate the iteration process in which various transient signatures as a function of assumed increment to each malfunction control variable is calculated, then the deviation of each of these transient signatures is related to the selected measured parameters within the performance index. The minimum performance index defines the malfunction control variable which produced the measured state variables as a function of time during the transient.

The calculation for a given fault condition transient starts by setting the unfailed initial set of malfunction control variables. These are needed to start the iteration, and the system equations are solved numerically using the Euler method by stepping forward in time. The Euler Lagrange equations are solved using the

Euler method by stepping backward in time using known final conditions and the current calculated values of the system variables. The switching functions which are algebraic functions of both the system and Euler Lagrange variables are then calculated.

For the next iteration, the assumed malfunction control variables are incremented using the signs of the switching functions. The change of the malfunction control variables are made in the direction opposite in sign to the switching functions as shown in figure 4. Then the system equations and the Euler Lagrange equations are solved forward and backward in time once again. This iteration process is repeated for each new control variable until the malfunctioning control variable can be identified, which occurs when the performance index is at its minimum value.

The most important stage in this algorithm is in the successive increment change in the control variables. The incremental change is chosen proportional to the negative of the switching function such that the change is in a direction which reduces the value of the performance index.

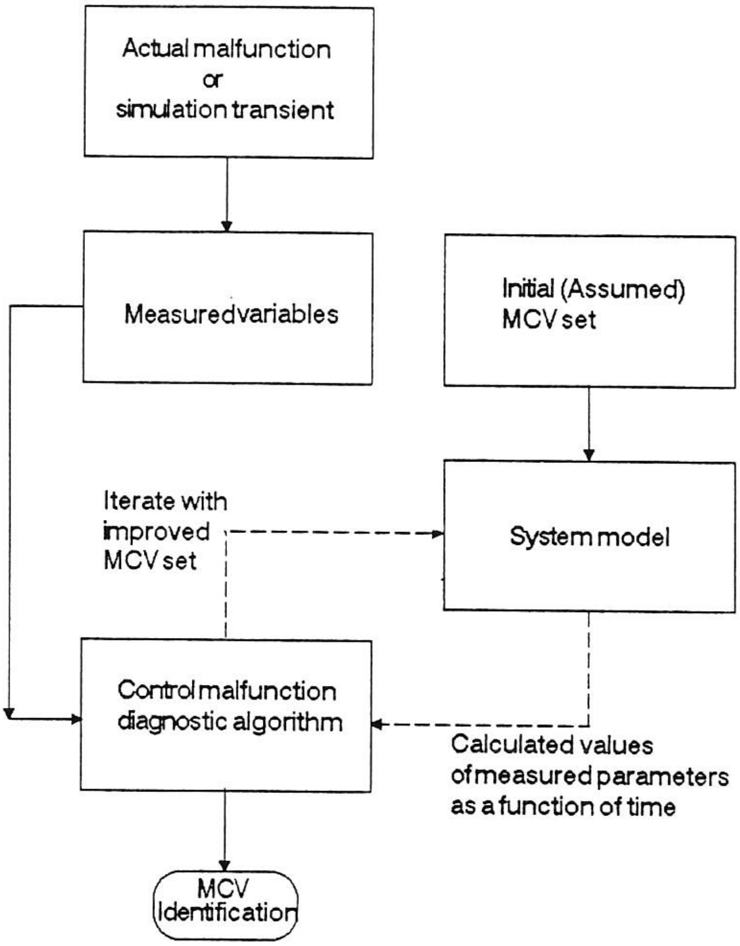

Figure 3: Malfunction detection process

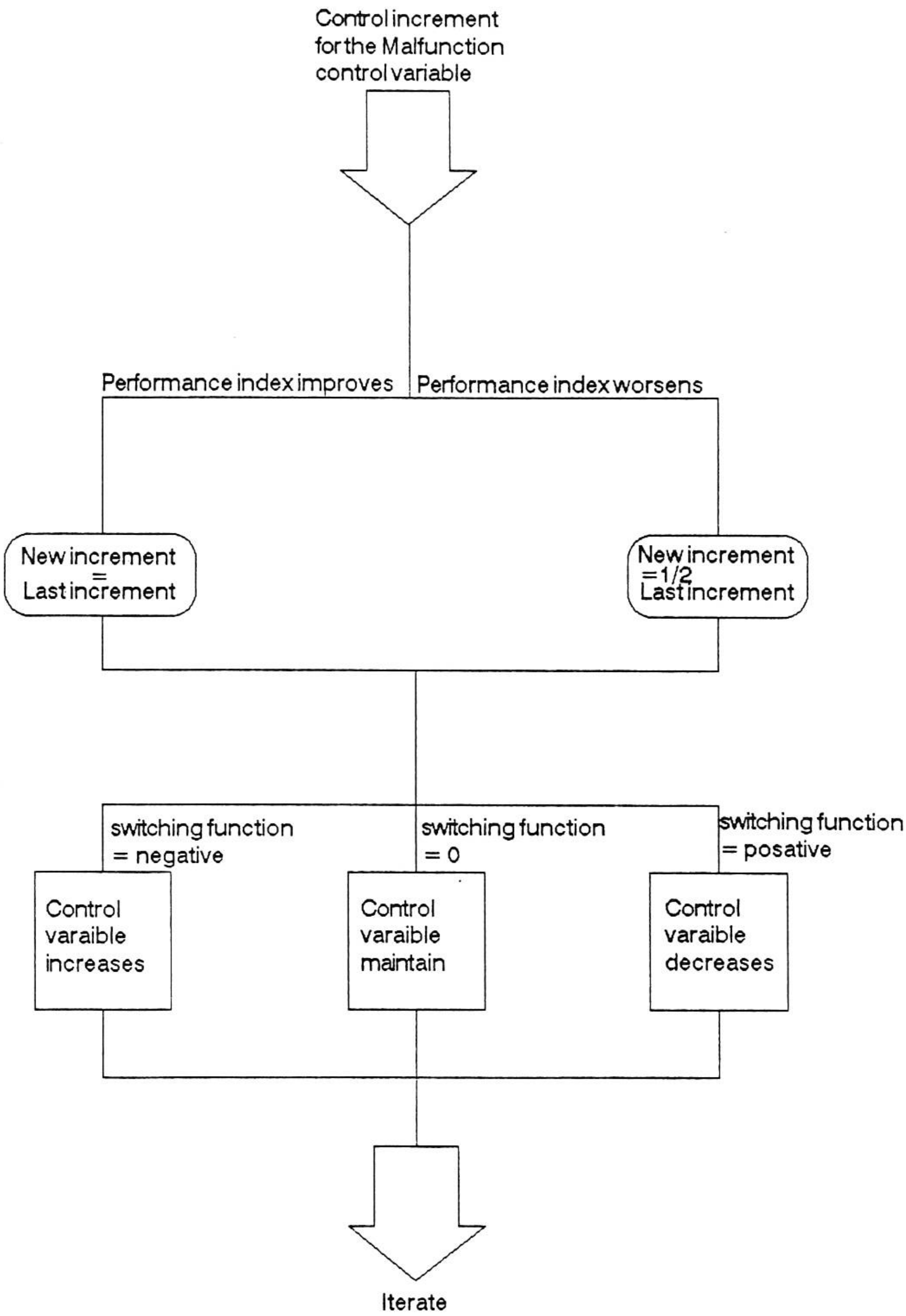

Figure 4: Control adjustment algorithm

5.4 Results and Analysis

To examine the ability of this algorithm to identify the faulty controller, a reactivity malfunction is assumed to occur in the reactor, which could happen as a result of a control rod malfunction or increase or decrease in the primary coolant boron concentration. The reactivity failure is initiated by a drop in the control rod causing an insertion of control rod reactivity ($) into the system model by an amount equal to - 0.5.

Table 3 shows the value of the performance index during the iteration process for the eight controllers. By comparing the last iteration value of the performance index for the each controller we can see that the minimum overall value was calculated for the first controller which is the reactivity control, and this means that the optimal control technique is able to identify the faulty controller. Figure 5 shows the lowest achieved value of the performance index for all eight controllers. This indicates that the lowest achieved value of the performance index belongs to the reactivity component and demonstrates that the reactivity component can be distinguished from the other control components.

The response of the reactor power as a result of the reactivity perturbation is shown in figure 6. In the second iteration process, the reactor model is assumed to be in the normal operation situation, which means that the control rod reactivity is equal to zero. Figure 7 shows the response of the reactor power during the normal operation case. The iteration process is continued by changing the value of the reactivity according to the sign of the switching function, and the response of the power at iteration number 10 is shown in figure 8. At the last iteration, when the failure controller is identified, the modelled reactivity changes in response to instructions from the control algorithm to move the control rod reactivity in a direction which will reduce the performance index, and due to the reactivity change the response of the power is as shown in figure 9. Figure 10 illustrates the response of the power at several iterations as the control algorithm continues to adjust to minimize the performance index.

Iteration Number	Control 1	Control 2	Control 3	Control 4	Control 5	Control 6	Control 7	Control 8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
3	0.14258	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
4	0.05464	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
5	0.03817	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
6	0.04489	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
7	0.036	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
8	0.0245	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
9	0.02455	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
9	0.02455	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
10	0.02304	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
11	0.02459	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
12	0.02336	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
13	0.02309	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
14	0.02303	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
15	0.02309	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
16	0.02306	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
17	0.02304	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
18	0.02304	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
19	0.02304	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006
20	0.02304	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006	0.29006

Table 3: The iteration of the performance index during the reactivity malfunction

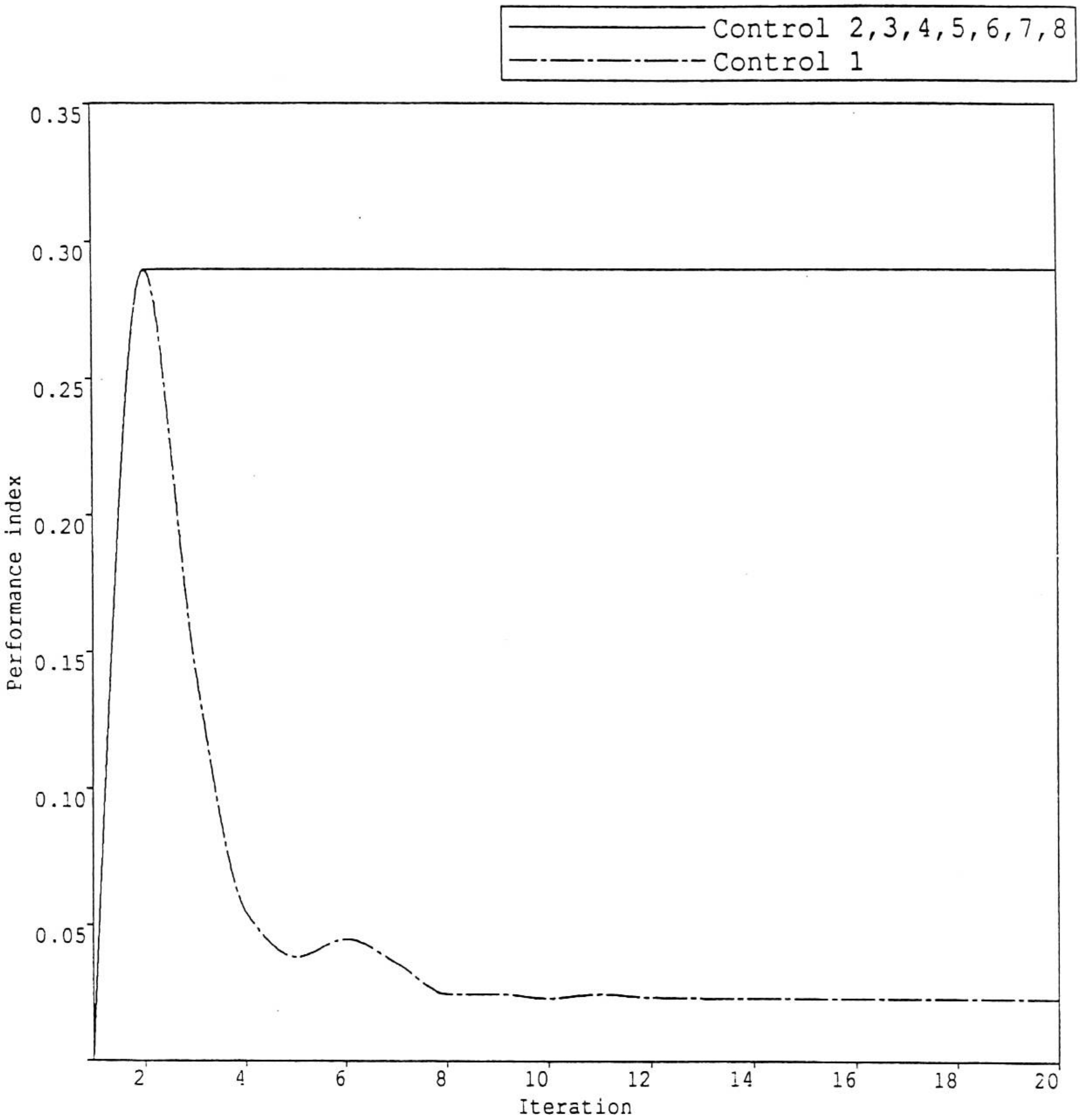

Figure 5: Values of the performance index for the eight controllers during reactivity malfunction

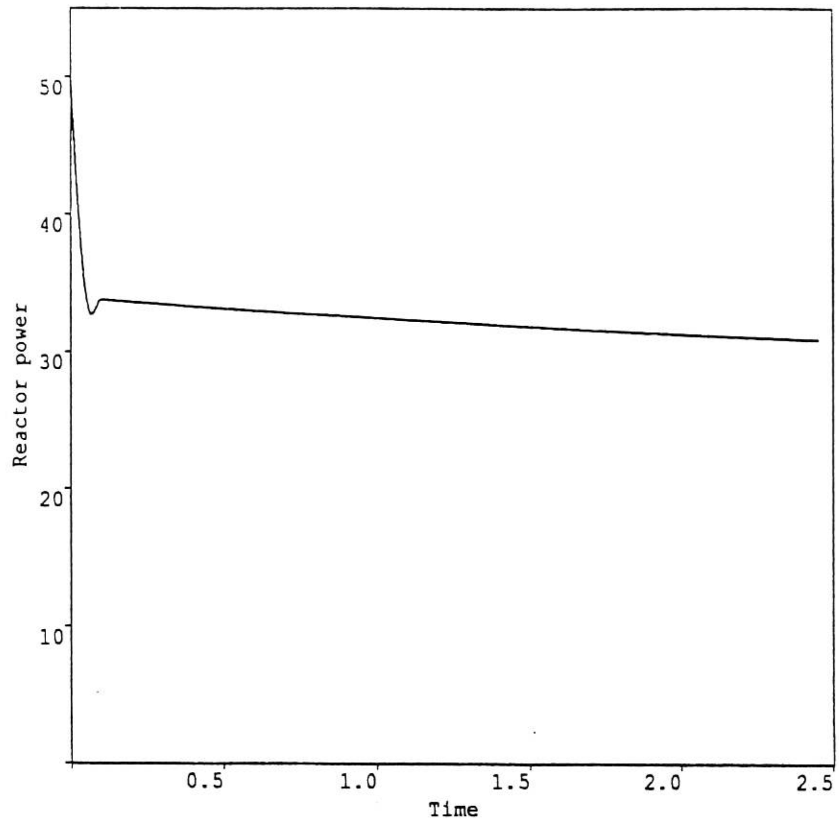

Figure 6: Response of the reactor power due to reactivity malfunction

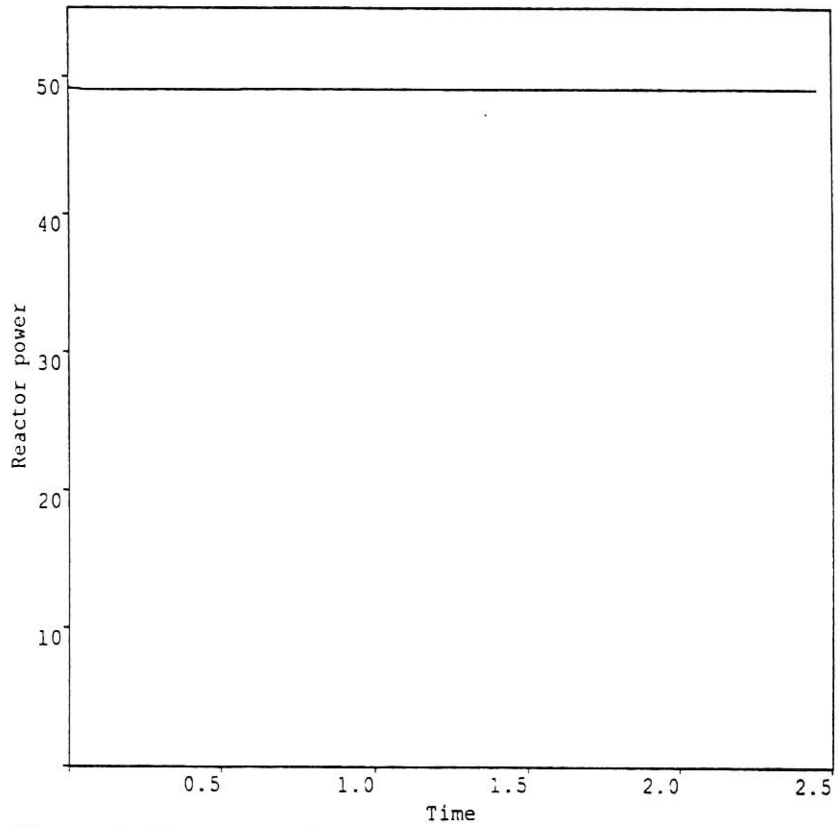

Figure 7: Response of the reactor power during normal operation

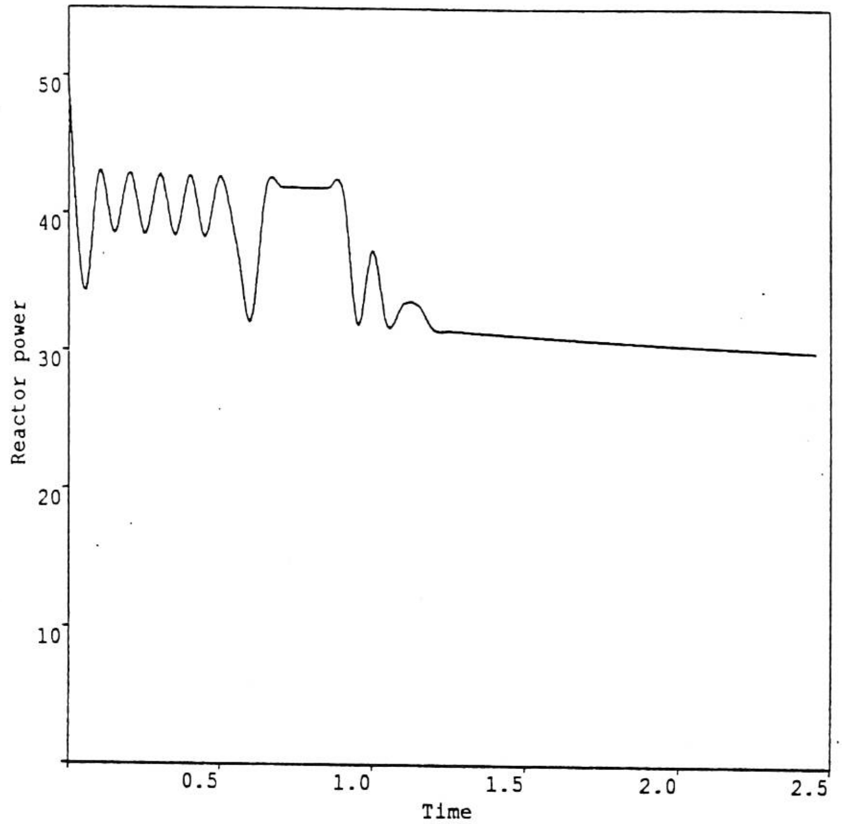

Figure 8: Reactor power at iteration No. 10 during reactivity malfunction

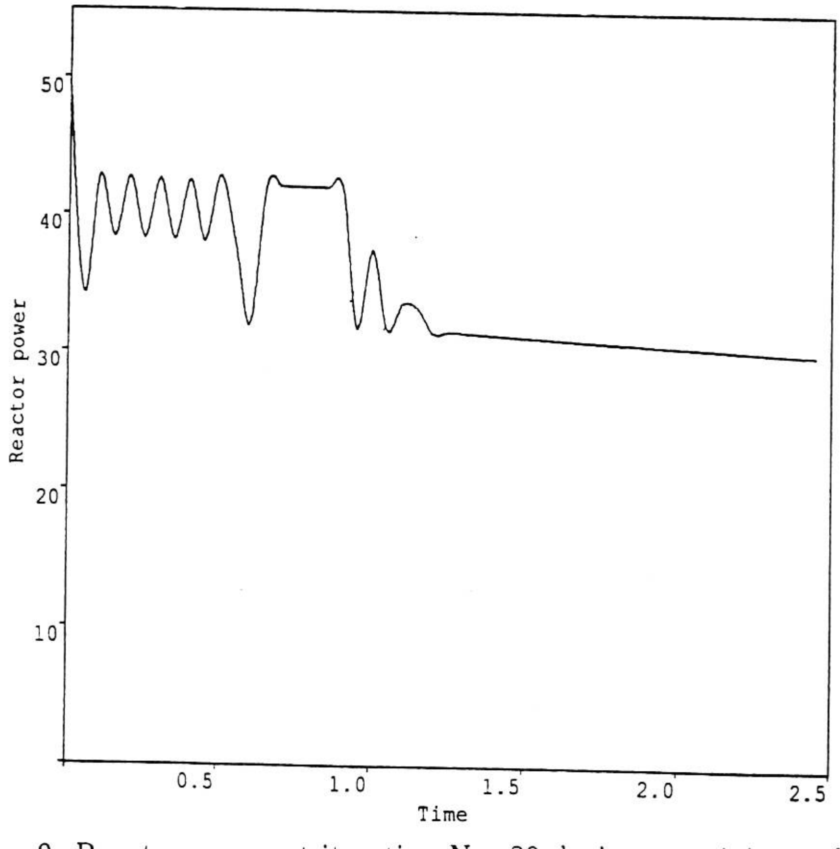

Figure 9: Reactor power at iteration No. 20 during reactivity malfunction

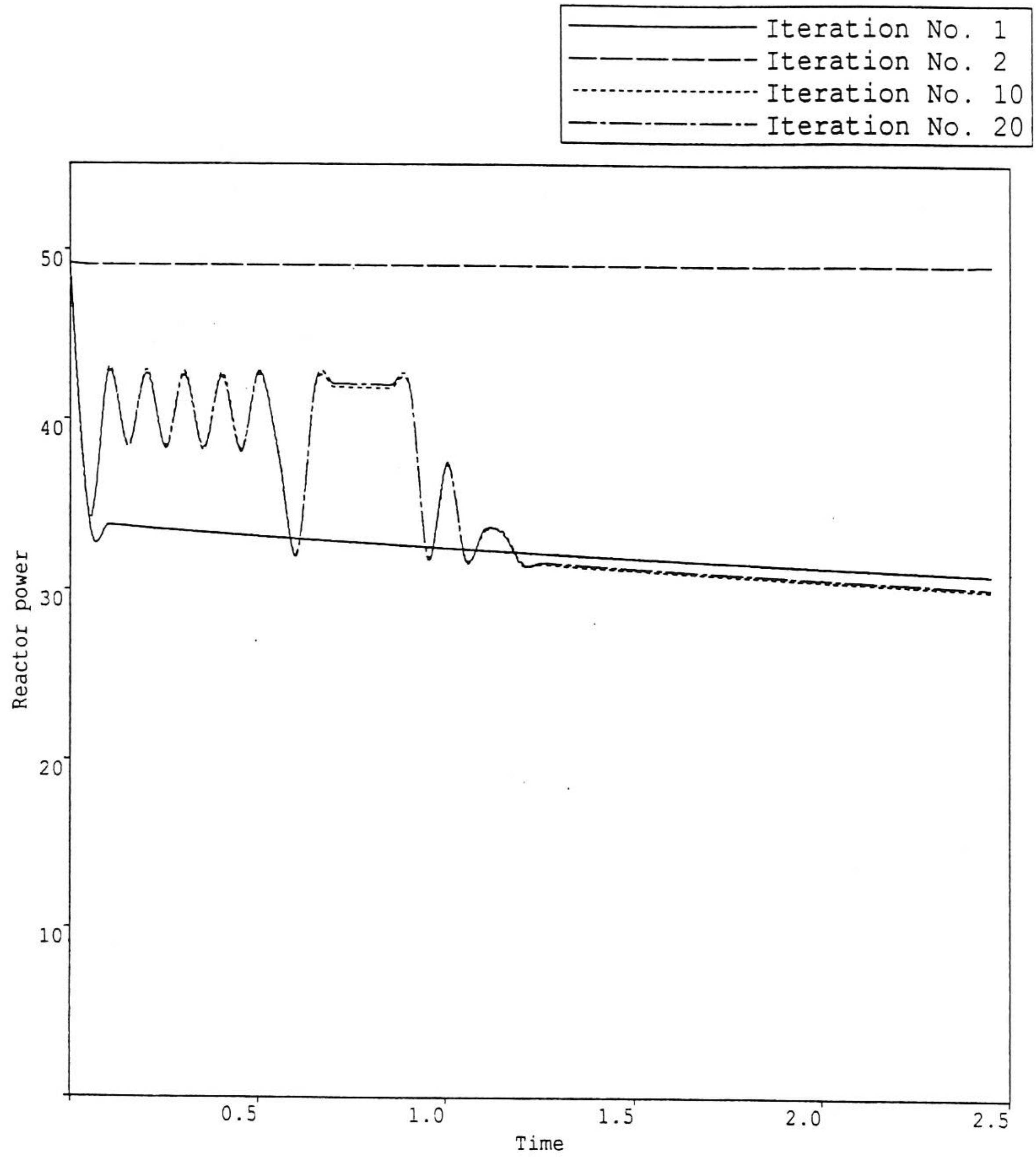

Figure 10: Response of the reactor power at different iteration during reactivity malfunction

6 Conclusion

An optimal control approach has been used to assist the nuclear reactor operator to diagnose any possible faults and eliminate human error. The LOFT model is divided into six subsystems and a fault diagnosis algorithm based on the idea of a performance index technique has been used to assist the operator in identifying any abnormal behaviour in any of these subsystems. The algorithm has been found to achieve a very good result and is able to identify the subsystem with the abnormal behaviour. A new algorithm has been developed and used to help the nuclear reactor operator to identify and diagnose any fault that might occur in one of the eight controllers in the LOFT model. The algorithm is tested by assuming a hypothetical accident in the reactor, and the algorithm results are very encouraging and clearly domenstrate that the faulty controller can be easily identified.

The optimal control technique is useful in that it could help the operator to identify the fault in any subsystem of the reactor using the performance indices approach. It is also able to diagnose the faulty controller using the Pontryagin maximum principle. The main limitation with this approach is the use of the lineraization procedure which is a very difficult task, especially for a highly non linear system such as a nuclear reactor, and the diagnostic procedure is consequently very time consuming. With the performance index approach the diagnosis is limited only to the subsystem level and it can not move deeper inside each subsystem. However, the feasibility of deep diagnosis could be investigated in future work, and the algorithm could be modified to include additional controllers.

The procedure of fault diagnosis using the Pontryagin maximum principle for identifying faulty controller using conventional computers is time consuming, and a parallel processing approach using for example the transputer could reduce the computation time significantly.

A Desired Values and Weighting Constants

Reactor power P_d	49.2 MW
Average fuel temperature T_{fd}	1443.5 $^\circ F$
Fuel cladding temperature T_{cd}	616.7 $^\circ F$
Primary loop flow W_{loop}	3.65555 $\times 10^6$ $\frac{lbm}{hr}$
Inlet coolant temperature T_{ind}	535.09 $^\circ F$
Temperature of steam generator outlet T_{sgod}	535.09 $^\circ F$
Temperature of coolant at core outlet T_{sgod}	570.7 $^\circ F$
Temperature of steam generator inlet T_{sgid}	570.7 $^\circ F$
Pressuriser pressure P_p	2280 psia
Pressuriser water level P_{leveld}	46.9 in
Temperature of steam generator tubes T_{tsgd}	532.19 $^\circ F$
Steam generator pressure P_{sgd}	749.67 psia
Steam generator water level L_{sgd}	125.96 in
Steam flow W_{stmd}	57.64 $\frac{lbm}{sec}$
Temperature of condenser tubes T_{tubed}	381.92 $^\circ F$
Condensate pressure P_{crd}	318.71 psia
Condensate fan blade pitch angle P_{fd}	19.15 degree
Feedwater temperature T_{fwd}	414.6 $^\circ F$
Feedwater flow W_{fwd}	57.64 $\frac{lbm}{sec}$
Volumetric flow of primary coolant W_{fwd}	22.62 $\frac{ft^3}{sec}$

Weight Values

$W_1 = \frac{1}{(49.2)^2}$ $W_2 = \frac{1}{(1443.5)^2}$ $W_3 = \frac{1}{(616.7)^2}$ $W_4 = \frac{1}{(3.65555 \times 10^6)^2}$

$W_5 = \frac{1}{(535.09)^2}$ $W_6 = \frac{1}{(535.09)^2}$ $W_7 = \frac{1}{(570.7)^2}$ $W_8 = \frac{1}{(570.0)^2}$

$W_9 = \frac{1}{(2280.0)^2}$ $W_{10} = \frac{1}{(46.9)^2}$ $W_{11} = \frac{1}{(532.19)^2}$ $W_{12} = \frac{1}{(749.67)^2}$

$W_{13} = \frac{1}{(125.96)^2}$ $W_{14} = \frac{1}{(57.64)^2}$ $W_{15} = \frac{1}{(381.92)^2}$ $W_{16} = \frac{1}{(318.717)^2}$

$W_{17} = \frac{1}{(19.15)^2}$ $W_{18} = \frac{1}{(414.6)^2}$ $W_{19} = \frac{1}{(57.64)^2}$ $WW_1 = W_1$

$WW_2 = W_9$ $WW_3 = W_{12}$ $WW_4 = W_{10}$ $WW_5 = W_{16}$

$WW_6 = W_{13}$ $WW_7 = \frac{1}{(22.62)^2}$

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