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Innovation and trade policy coordination: the role of firm heterogeneity*

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Abstract

Recent studies have concluded that R&D grants can induce firms to export and that exporting and innovating can be complementary activities at the firm level. Yet the trade literature has paid little attention to the scope of innovation policy as a stimulus to both trade and innovation. To investigate this question we rely on a general work-horse model of trade and firm heterogeneity with firm investments in R&D activities. The multiplicity of equilibria together with the interplay of innovation and trade policies uncover novel results. In particular, we show that the effects of either policy depend on the degree of protectionism in a country. Therefore, countries can respond differently to the same policy, and similarly to different policies. In such a context, different governments may face different degrees of freedom regarding how to achieve a given target. This finding leads us to discuss the issue of policy coordination.

JEL: F12, F13, F15, F61, O32.

Keywords: innovation, innovation policy, heterogenous firms, technology adoption, trade policy.

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1 Introduction

All industrialized countries use some measures of innovation policy, like R&D grants and tax allowances, to sustain their research activity and economic growth (OECD, 2005). While originally aimed at increasing the productivity of firms, these measures have also contributed to stimulating the exports of firms (Goerg et al., 2008). When trading is the *conditio sine qua non* for innovating, trading and innovating become complementary activities (Lileeva and Trefler, 2010). The other side of this complementarity is policy substitution: if innovation policy favors export initiation by strengthening innovation efforts, it renders trade policies with the same objective superfluous, and vice versa.

The importance of policy substitution and, more generally, of policy coordination is underrated in the current models of trade with heterogenous and monopolistically competitive firms. The reason, however, is not related to the inadequacy of these models for investigating this issue, but rather to their distinct objective. Motivated by trade liberalization in several countries, the models have offered thorough analysis of the micro effects of these policies, but they have hardly explored the concomitant role of innovation policy for stimulating both trade and innovation, which is the focus of this paper.

Once we have clarified the merits of innovation policies relative to trade policies, we also endeavor to analyze the issue of policy coordination that inevitably arises when governments have multiple instruments to achieve similar goals. Policy selection and coordination become relevant only when policy makers face some trade-offs about stimulating one activity (e.g., innovation), while harming the other (e.g., trade). Therefore, a model that features export and innovation only as complementary activities is not adequate for our purpose, but we would ideally need equilibria without such complementarity. To maintain analytical tractability, yet have multiple equilibria, we opt for a model with technology adoption which nests in the class of models that have followed the seminal work of Montagna (2001) and Melitz (2003).

An advantage of our approach is that this model features three equilibria that can encompass the evidence presented in Castellani and Zanfei (2007), in Lileeva and Trefler (2010), and in Bustos (2011). Each equilibrium features different implications for the effects of each policy: only in one of these equilibria are innovation and trade policies substitutes, while in the other equilibria the ability to promote both innovation and export is policy-specific, so that tariff cuts and R&D grants can have different implications. We can rationalize this result by referring to the status of the marginal innovating firm in each equilibrium. As in Melitz

(2003), a pro-trade policy will always cause a reallocation of market share from low to high productivity firms and exit from the market of the least productive domestic enterprises. Consequently, if the marginal innovating firm is an exporter, and its profits are increasing, this policy will also stimulate technology adoption; otherwise, if the marginal innovating firm is a shrinking domestic firm, technology adoption will be discouraged. Because industry productivity growth depends both on the reallocation of market shares and on increased productivity investments by firms, the final outcome of a policy is also prospectively different in each equilibrium.

Since it is the policy space of a country that ultimately determines the final equilibrium selection, the effects of either policy depend on the degree of protectionism of a country. Therefore, countries can have both heterogenous responses to similar policies and similar responses to heterogenous policies. A tariff cut, for instance, increases the share of exporting and innovating firms only in liberal countries, but it harms the share of firms adopting new technologies in protectionist countries. Likewise, an R&D grant promotes innovation and hurts exporting only in the most liberal and most protectionist countries, but favors both activities in the other countries. Therefore, liberal countries undergoing tariff cuts experience similar effects to more protectionist countries introducing R&D grants. Importantly, these results do not stem from the assumption of a specific productivity distribution.

It is indisputable that the availability of an array of policies can confer some degree of freedom on governments as how to pursue a policy objective, but policy makers also face some trade-offs. Indeed, each instrument sustaining one activity can unfold different "side-effects" in terms of harming the other activity. We argue that the order in which policies are implemented can be relevant for mitigating these policy trade-offs.

In this paper we focus on a positive approach. We describe the effects of various policies and analyze the trade-offs that arise with multiple instruments and briefly examine their implications for industry productivity growth. This is a necessary step towards a deep understanding of the consequences of these policies for society. Given the already complex interaction between these policies we leave a normative approach to future research.

Our paper is structured as follows. In section 2 we present our research in the context of the trade literature. We proceed by presenting our model and deriving the equilibrium in a closed and open economy in section 3. Section 4 briefly outlines the closed economy. We then analyze in section 5 the implications of trade and innovation policies in the open economy and synthesize our results by means of one simple graph, our policy space. This graph illustrates our discussion of the issue of policy coordination in section 6. Before concluding, we briefly present the macro-implications of our policies on the industry productivity growth.

2 The background

Given the increasing availability of micro-datasets linked to trade statistics, firm-heterogeneity and its effects have been an important part of recent trade research.[1] The seminal works of Montagna (2001) and Melitz (2003) were extended to include process innovation besides product innovation. Navas and Sala (2007) and Bustos (2011) consider a firm's technology adoption as a form of process innovation, whereas Atkeson and Burstein (2010) focus on a firm's R&D's investments.

In all these models trade liberalization favors process innovation. For instance, Bustos (2011) and Bas and Ledezma (2010) show that trade liberalization has induced Argentinian and Chilean exporters to upgrade their technology. In their review, Greenaway and Kneller (2007) present further evidence from studies in other countries.

All these models, however, are hardly reconcilable with the evidence disclosed in Lileeva and Trefler (2010), as they cannot possibly predict the behavior of some Canadian firms that have both upgraded their technologies and simultaneously started to export with the creation of the US-Canada free trade area. Our work shows that the limitation of these models to encompass these facts rather originates in neglecting a multiplicity of equilibria.

We consider a simple framework in which firms pay a fixed cost to introduce a new technology that reduces the marginal cost in a fixed proportion. Because the reduction in the marginal cost is proportional, firms will experience heterogeneous innovation gains. While this approach departs from more complex innovation technologies (Atkeson and Burstein, 2010; Long et al. 2011), its analytical tractability allows us to introduce the important issue of policy coordination that arises when a broad range of instruments are available to policy makers. The discussion of this matter is based on a comparative static analysis of our steady states. While we can investigate the benefits to policy makers of different policy mixes and different sequences of different policies, we cannot explain the transition dynamic to the steady state that characterizes the discussion in Costantini and Melitz (2007) of alternative tariff scenarios. Finally, our model suggests that extending the type of counterfactual analysis presented in Corcos et al. (2011) to innovation policy scenarios could be fruitful.

[1] See Greenaway and Kneller (2007) for an extensive review of this literature.

To present our model in the next section, we build on Navas and Sala (2007).

3 The model

Preferences
A continuum of households of measure L have preferences described by a standard C.E.S. utility function,

$$U = \left[\int_{\omega \in \Omega} [q(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1,$$

where σ is the elasticity of substitution across varieties, and Ω is the set of available goods.

Technology
The amount of labor required to produce a quantity $q(\omega)$ of variety ω is

$$l(\omega) = f_D + cq(\omega)$$

where f_D is the fixed labor requirement, and $c \in [0, \overline{c}]$ is the firm-specific marginal labor requirement.

Entry - exit
There is a large (unbounded) pool of prospective entrants into the industry, and prior to entry all firms are identical. To enter the industry, a firm must make an initial investment, modeled as a fixed cost of entry $f_E > 0$ measured in labor units, which is thereafter sunk. An entrant then draws a labor-per-unit-output coefficient c from a known and exogenous distribution with cdf $G(c)$ and density function $g(c)$ on the support $[0, \overline{c}]$. Upon observing this draw, a firm, like in Melitz (2003), may decide to exit or to produce. If the firm does not exit, it bears the fixed overhead labor costs, f_D. Additionally, it can improve on its technology: by investing f_I units of labor, it can adopt a more productive technology and produce at a lower cost γc ($\gamma < 1$). Ultimately, it is a choice between a well established "*baseline*" technology - characterized by low implementation costs, normalized to 0, and variable costs of production c - and an *innovative* one - featuring lower variable costs (γc), but higher fixed costs of adoption (f_I).

We assume that technological uncertainty and heterogeneity of the Melitz-type relates to what we have called a "*baseline*" technology, reflecting that firms have to learn about their market and their productivity before they can plan to improve it. Having found out about their idiosyncratic productivity, all firms face the option of adopting an alternative technology, what we have referred to as the "*innovative*" one. While the fixed cost of implementation is the same for each firm, the reduction in variable cost is proportional to the firm's intrinsic marginal cost. Since the Melitz-type entry leads to productivity heterogeneity, the option to adopt is differently attractive to firms with different intrinsic marginal costs. This could be rationalized as some firms being more successful than others in implementing the new technology (i.e. better implementation makes new technologies more productive). [2]

Finally, as in Melitz (2003), every incumbent faces a constant (across productivity levels) probability δ in every period of a bad shock that would force it to exit.

Trade

We shall assume that the economy under study can trade with other $n \geq 1$ symmetric countries. Trade is, however, not free, but involves both fixed and variable costs: the firm has to ship $\tau > 1$ units of a good for each unit to arrive at a destination and has to incur a fixed cost f_X during the period in which starts exporting.

The symmetry of countries ensures that factor price equalization holds, and all countries share the same aggregate variables.

Prices and profits

Given the CES preferences, the demand of each variety ω is

$$q(\omega) = \frac{R}{P}\left[\frac{p(\omega)}{P}\right]^{-\sigma}, \tag{1}$$

where $R \equiv \int\limits_{\omega\in\Omega} p(\omega)q(\omega)d\omega$ is the aggregate total expenditure, and $P \equiv \left[\int\limits_{\omega\in\Omega} [p(\omega)]^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}$ is the price index of the economy.

Facing this demand function, a producer of variety ω with labor output coefficient c charges the price:

$$p(\omega) = \frac{\sigma}{\sigma - 1} wc \equiv p_D(c), \tag{2}$$

where w is the common wage rate, hereafter taken as the numeraire

[2] "Technology implementation processes" are in the data the main source of site-to-site variations in the success of the adopter. See Comin and Bart (2007) and Bikson et al. (1987). Note that in this framework the productivity ratio of two firms with different intrinsic marginal costs will be constant if both firms adopt the new technology.

$(w = 1)$.[3]

If the firm has opted for the innovative technology, it charges the lower price, $p_I(c) = \gamma p_D(c)$. Therefore, the profits that firm type D (producer with a "traditional" technology) and firm type I (firm with innovative technology) make on the domestic market are, respectively,

$$\pi_D(c) = \frac{r_D(c)}{\sigma} - f_D = Bc^{1-\sigma} - f_D \quad \text{and} \tag{3}$$

$$\pi_I(c) = \frac{r_I(c)}{\sigma} - f_D - \delta f_I = B(\gamma c)^{1-\sigma} - f_D - \delta f_I, \tag{4}$$

where $r_s(c)$ is the revenue of firm type $s \in \{D, I\}$ and $B = (1/\sigma)\frac{R}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$ is a constant from the prospective of a single producer.[4]

It is worth noting that $r_I(c)/r_D(c) > 1/\gamma$, and therefore the income of the firm increases more than proportionally following the introduction of process innovations.

The imported products are more expensive than domestically produced goods due to transportation costs. The effective consumer price for a variety shipped from abroad by a non-innovating exporter is $p_X(c) = \tau p_D(c)$, and by a firm adopting the innovative technology it is $p_{XI}(c) = \gamma p_X(c)$. Therefore, the profits of an exporter (firm type X) and an innovator-exporter (firm type XI) earned on the foreign market are, respectively,

$$\pi_X(c) = \tau^{1-\sigma} Bc^{1-\sigma} - \delta f_X \quad \text{and} \tag{5}$$

$$\pi_{XI}(c) = (\gamma\tau)^{1-\sigma} Bc^{1-\sigma} - \delta f_X, \tag{6}$$

where δf_X is the amortized per-period fixed cost of the overhead fixed cost f_X that firms have to pay to export.

As in Melitz (2003), no firms will ever export without also producing for its domestic market, and a firm will either export to all n countries in every period or never export. (5) is therefore the profit from exporting conditional on being a domestic firm and, likewise, (6) is the profit from exporting conditional on being a domestic innovator.

[3] Alternatively, a freely traded homogenous good produced under constant returns to scale could be introduced as the numeraire good to set the wage to unity in all countries.

[4] Given that there is no additional uncertainty or time discounting other than the exogenous probability of exiting, firms are indifferent between paying the one time investment cost f_I or the per-period amortized cost δf_I.

4 Equilibrium in a closed economy

The equilibrium entry cost cutoff c_0 and innovation cost cutoff c_I must satisfy:

$$\pi_D(c_0) = 0 \Longleftrightarrow B\,(c_0)^{1-\sigma} = f_D \tag{7}$$

$$\pi_I(c_I) = \pi_D(c_I) \Longleftrightarrow (\gamma^{1-\sigma} - 1)B\,(c_I)^{1-\sigma} = \delta f_I. \tag{8}$$

To close the model and determine the two equilibrium cost cutoffs c_0 and c_I, as well as B and the number of incumbent firms M, free entry into the market and a stability condition are imposed additionally. Free entry (henceforth FE) ensures that firms equate the per-period expected profit from entry to the equivalent amortized per-period entry cost,

$$\delta f_E = \int_0^{c_I} \pi_I(c)dG(c) + \int_{c_I}^{c_0} \pi_D(c)dG(c).$$

The stationary-equilibrium condition,

$$M_e G(c_0) = \delta M,$$

requires the aggregate variables to remain constant over time, as the mass of successful entrants, $M_e G(c_0)$, exactly replaces the mass, δM, of incumbents who are hit by the bad shock and exit.

Combining (7) with (8), we have the relation between the innovation and the entry cutoff

$$(c_I)^{1-\sigma} = \frac{\delta f_I}{\gamma^{1-\sigma} - 1}\frac{1}{f_D}\,(c_0)^{1-\sigma} = \Psi\,(c_0)^{1-\sigma}\,, \tag{9}$$

where $\frac{\delta f_I}{\gamma^{1-\sigma}-1}$ is the cost-to-benefit ratio of innovation. It follows that a necessary and sufficient condition for having selection into the innovation status is $\Psi > 1$, which is assumed to hold throughout since the empirical evidence suggests that only a subset of more productive firms undertakes process innovations.[5]

Finally, we note that the entry productivity cutoff level is higher in our economy than in Melitz (2003).[6] The possibility to innovate allows the most efficient firms that perform process innovation to "steal" market share from the least efficient firms for which it is harder to survive in

[5] See for instance Parisi et al. (2006) for evidence on Italian firms and Baldwin et al. (2004) for evidence on Canada.

[6] The proof of this result has been left to an online appendix.

the market. Consequently, our economy is more efficient, because some varieties are produced at a lower cost, but less varied because some varieties have disappeared. This trade-off has been well emphasized in the growth literature (see Peretto, 1999) and more recently Gustafsson and Segerstrom, 2010).

5 The open economy

To differentiate the entry cutoff and the innovation cutoff from the ones in the closed economy, we denote them in the open equilibrium by c_0^f and c_I^f, respectively.

Guided by her empirical results, Bustos (2011) focuses only on one possible selection, namely $c_I^f < c_X < c_0^f$, so that the marginal innovating firm is an exporter and responds to tariff cuts with the adoption of a better technology. We label this kind of selection *equilibrium BW* to point to the fact that the growth of the industry productivity has two sources. One, as in Melitz (2003), comes from the reallocation of market shares from low to high productivity firms induced by the selection effect of trade. The second comes from the adoption of better technologies by firms that trade. In its decomposition of the industry productivity growth, Bartelsman et al. (2004) refer to the first source as the *between* variation and to the the second source as the *within* variation. The letters BW (*between-within*) indicate that in this type of equilibrium both sources of variation are present.

An equilibrium where both exporters and non-exporters are performing innovation is also plausible and is consistent with a different type of selection, namely $c_X < c_I^f < c_0^f$.[7] In this equilibrium, the marginal innovating firm is not an exporter, and therefore it does not respond to a fall in transportation costs with the adoption of innovative technologies. We label this selection *equilibrium B*, as only the *between* variation contributes to the growth of the industry aggregate productivity.

As a final case, we scrutinize the limiting case of both these selections where firms engage either in both activities or neither of them ($c_X = c_I^f < c_0^f$). In this equilibrium, trade and innovation become complementary activities (henceforth denoted selection C). Interestingly, our paper shows that the evidence presented in Lileeva and Trefler (2010) is indeed compatible with the Melitz type model in which there is process innovation if one considers the limiting case of two different possible equilibria.[8]

In what follows, we analyze each equilibrium separately and investi-

[7] See Castellani and Zanfei (2007).

[8] We thank an anonymous referee for highlighting this possibility.

gate which are the effects of both innovation and trade policies on the innovation and export activities of firms. We then turn to the issue of policy coordination.

We consider only stationary equilibria in the sense that all aggregate variables are constant over time. We therefore impose on each equilibrium the stationary condition

$$\delta M = M_e G(c_0^f), \tag{10}$$

so that the firms exiting the market are just replaced by the new entrants. Although denoted by the same letter as in the closed economy, the equilibrium values of the aggregate variables Q, R, and B are, in general, different in this equilibrium as compared to the closed economy equilibrium.

5.1 Selection *BW*

We start by determining the equilibrium cost cutoffs for, respectively, market entry, exporting, and innovating. Given that in equilibrium BW we have $c_I^f \leq c_X \leq c_0^f$, the cost cutoffs must satisfy the following conditions:

$$\pi_D(c_0) = 0 \Leftrightarrow B\left(c_0^f\right)^{1-\sigma} = f_D \tag{11}$$

$$\pi_X(c_X) = 0 \Leftrightarrow B c_X^{1-\sigma} = \frac{\delta f_X}{\tau^{1-\sigma}} \tag{12}$$

$$\pi_I(c_I^f) + n\pi_{XI}(c_I^f) = \pi_D(c_I^f) + n\pi_X(c_I^f) \Leftrightarrow B\left(c_I^f\right)^{1-\sigma} = \frac{\delta f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})}. \tag{13}$$

The parameter restriction that sustains this equilibrium and will be important for our policy analysis below is

$$\frac{\delta f_I}{(\gamma^{1-\sigma}-1)}\frac{1}{(1+n\tau^{1-\sigma})} \geq \delta f_X \tau^{\sigma-1} \geq f_D. \tag{14}$$

The FE condition

$$\int_{c_X}^{c_0^f} \pi_D(c)dG(c) + \int_{c_I^f}^{c_X} (\pi_D(c)+n\pi_X(c))dG(c) + \int_0^{c_I^f} (\pi_I(c)+n\pi_{XI}(c))dG(c) = \delta f_E \tag{15}$$

together with the stationary condition (10) close the model to determine B and M.

To compare the share of firms in the trading equilibrium and in the autarky equilibrium, it is instructive to write the innovation cutoff as a function of the entry cut-off,

$$\left(c_I^f\right)^{1-\sigma} = \frac{\delta f_I}{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})}\frac{1}{f_D}\left(c_0^f\right)^{1-\sigma},$$

and note that this relation differs from equation (9) only by the term $1/(1+n\tau^{1-\sigma})$. This term is 1 in the closed economy (set $n=0$ or $\tau \to \infty$) and represents precisely the further revenue differential associated with innovation for each of the foreign markets that become available with trade. Although this term is smaller than unity, we cannot conclude that, for a positive n and a non-prohibitive transportation cost τ, the share of innovating firms in the trading equilibrium is higher than in autarky. As we know from Melitz (2003), the selection effect of trade induces the least productive firms to exit, so that c_0^f is lower than in the closed economy. The overall effect is a priori ambiguous, and we prove in the appendix that $c_I^f \geq c_I$. While this result is common to Bustos (2011), we show that it is more general because it can be derived for a general cumulative distribution function of the firm's productivity without requiring the Pareto assumption.

Crucial to this result is the selection of firms into exporting activities. In the absence of the fixed costs of trading ($f_X = 0$), all firms would export as in Krugman's (1979) model. With CES preferences, the increased revenue from increased sales abroad induced by trade opportunities would be exactly offset by the loss of domestic revenues due to increased import competition from foreign varieties. Given that profits would be unchanged, no firms would exit the market. Without exit, there is no reallocation of market share to exporters. With unchanged output and market share the incentive to adopt a more efficient technology in free trade is also unchanged relative to the autarky equilibrium.

Although not as evident as for τ, f_X and f_I also have an impact on the equilibrium cutoffs in a general equilibrium model. We now turn our attention to the effects that trade policies or innovation policies have on this economy. A pro-trade policy can consist of a reduction in either the transportation cost or the regulatory cost of trade (f_X). A pro-innovation policy is in our model a reduction of f_I. Although we prove all results in the appendix, here we limit ourselves to summarizing the effects of the various policies in table 1, our policy matrix.

INSERT TABLE 1 ABOUT HERE.

The most striking result is that trade policies have very different implications for innovation. Only the reduction of the variable trade costs can affect both the extensive margin of exporting and innovation positively. The reduction of f_X, on the contrary, contracts the extensive margin of innovating while increasing the extensive margin of exporting, and vice versa for a reduction of f_I. Therefore, the reduction of the fixed cost of trade or innovation can just expand the share of firms performing the related activity at the expense of the share of firms performing the other activity.

This is because only transportation costs are affecting the variable profits of all exporting firms. When these costs decline, these enterprises can further lower prices and increase sales abroad (the intensive margin adjustment). Because selling internationally has become cheaper, exporting becomes attractive to some domestic firms (higher c_X), and because exporters now face lower variable costs, some of the non-innovator exporters find it profitable to start innovating (higher c_I^f). The increase in innovation or export either at the intensive or extensive margins raises labor demand and the real wage. The firms most hurt are the domestic ones that cannot compensate the increased costs of production with the expansion of foreign activity. The least productive are therefore forced to exit (lower c_0^f), and their market share is redistributed to all incumbent firms. Overall, like in Melitz (2003), high productivity firms expand, and low productivity firms shrink.

If the sector-wide productivity is defined as the weighted average of all firms' productivity in the industry, with the weights consisting of the firm's market share, there are two sources of productivity growth at the industry level: changes in the firms' productivity or changes in the firms' market share. In Bartelsman et al.'s (2004) terminology, both the redistribution of market share from low to high productivity firms - the *between-firm* effect - and the technology adoption by some exporters - the *within-firm* effect - are contributing to the industry productivity growth in this equilibrium.

5.2 Selection B

In equilibrium B ($c_X \leq c_I^f \leq c_0^f$) the marginal exporting firm is an innovator, but the marginal innovator is not an exporter. Consequently, some domestic firms are innovating. The innovating firms are both the I-type and the XI-type, but only the latter are present in the international market. Therefore, $\pi_D(c_0^f) = 0, \pi_I(c_I^f) = \pi_D(c_I^f)$ and $\pi_{XI}(c_X) = 0$ define our cutoff conditions, and the necessary and sufficient condition for this equilibrium to hold is

$$\delta f_X \tau^{\sigma-1} \geq \frac{\delta f_I}{(\gamma^{1-\sigma} - 1)} \gamma^{1-\sigma} \geq \gamma^{1-\sigma} f_D. \tag{16}$$

From our policy matrix, we deduce that none of the pro-trade policies can stimulate innovation in the sense of leading to a higher share of innovating firms in the economy. Since the marginal innovator is a domestic firm, a reduction of τ or f_X has no direct effect on its profit, but it makes the profits of exporting firms bigger. Higher profits induce these firms to expand their activity abroad (either at the intensive or extensive margin), which causes labor demand and real wages to increase. The rise of production costs affects the non-exporters, including the marginal innovating firm, more severely. Those firms that cannot break even will be forced out of the market. In the new steady state more domestic firms will stick to the baseline technology (lower c_I^f), and less firms will enter (lower c_0^f). Because market share is redistributed from low to high productivity firms and some firms give up technology adoption, it is clear that industry productivity can grow only because of the *between-firm* component.

Note, however, that in our model technology adoption occurs only at the extensive margin because of the exogeneity of γ. With an endogenous γ and a convex cost of innovation, some of the XIfirms, those that expand their activity abroad and enlarge their market shares, would find it optimal in this equilibrium to choose a lower γ and, therefore, promote technology adoption at the intensive margin, too (see Vannoorenberghe, 2008).

It is instructive to graph the policy space which sustains these two equilibria. We represent each term of the inequalities (14) and (16) in figure 1 along with an index of variable trade costs ($\tau^{\sigma-1}$). In the graph we omit the term $\gamma^{1-\sigma} f_D$ as it is irrelevant for equilibrium B provided it lies below the curves $\delta f_X \tau^{\sigma-1}$ and $\delta f_I/(\gamma^{1-\sigma}-1)$. Condition (14) holds in the region labeled with BW for transportation costs below $\tau_{BW}^{\sigma-1}$ and if n not too large. Likewise, condition (16) is satisfied in the region labeled with B for transportation costs above $\tau_B^{\sigma-1}$. Therefore, these two equilibria are not contiguous in this policy space, and no equilibrium would be defined for intermediate value of τ. Note that the size of this middle region crucially depend on the magnitudes of the fixed costs of trading and innovating as well as on the number of trading partners.

INSERT FIGURE 1 ABOUT HERE

Intuitively, this situation is only compatible with the existence in equilibrium BW or B of a profitable deviation from the optimal chosen strategy when transportation costs are in this intermediate range. And this new strategy also has to be different from the strategies the firm could implement in the other equilibrium. The restriction that equilibrium BW imposes is that some firms should be exporting without innovating. However, as transportation costs increase beyond $\tau_{BW}^{\sigma-1}$, some firms find this situation suboptimal, but they cannot yet justify being a domestic innovator until transportation costs have increased at least to $\tau_{B}^{\sigma-1}$. Likewise, the restriction that equilibrium B imposes is that some firms innovate without exporting. As transportation costs decrease below $\tau_{B}^{\sigma-1}$, they cannot any longer justify being innovators without exporting. This intuition points towards the Lileeva and Trefler (2010) equilibrium where innovation and exporting are complementary activities sustaining each other.

To develop the intuition, consider a domestic firm willing to both export and adopt the innovative technology. Such a strategy is wise if

$$[n\pi_{XI}(c)+\pi_I(c)]-\pi_D(c) = [n\pi_{XI}(c)+\pi_I(c)-n\pi_X(c)+\pi_D(c)]+n\pi_X(c) \geq 0,$$

where the first equation is the profit differential from implementing both options at the same time compared to the profit earned on the domestic market. Note that for a firm with $c \leq c_X$ in equilibrium B, the first equation is necessarily positive as $n\pi_{XI}$ and $\pi_I - \pi_D$ are positive. Likewise, for a firm with $c \leq c_I$ in equilibrium BW this condition holds as the squared bracket and the last addend in the second equation are both positive. This is not surprising as we know that this kind of firm performs both activities in each equilibrium, respectively. Yet this condition could more generally hold in either equilibrium for other firms too, although all terms may not necessarily be positive. Indeed, a domestic firm would want to pursue a complementary strategy if the double option is also more profitable than just either exporting or innovating, so if

$$n\pi_{XI}(c) + \pi_I(c) - \pi_D(c) \geq n\pi_X(c) + \pi_D(c) - \pi_D(c) \tag{17}$$

and

$$n\pi_{XI}(c) + \pi_I(c) - \pi_D(c) \geq \pi_I(c) - \pi_D(c). \tag{18}$$

Equation (17) implies $Bc^{1-\sigma} \geq \frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}}$, which evaluated at the marginal exporting firm with $c = c_X$ in equilibrium BW translates into $\delta f_X \tau^{\sigma-1} \geq \frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}}$. For this parameter range the marginal exporting firm finds in the double option a profitable deviation. So will all other firms with higher productivity too.
Equation (18) implies $Bc^{1-\sigma} \geq \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1}$, or for the marginal innovating firm at $c = c_I^f$ in equilibrium B, $\frac{\delta f_I (\tau\gamma)^{1-\sigma}}{\gamma^{1-\sigma}-1} \geq \delta f_X$. Under these circumstances, the marginal innovating firm finds the double option more profitable, and so will all other firms with higher productivity.

Therefore, only when transportation costs are sufficiently low - below $\tau_{BW}^{\sigma-1}$ - some firms find exporting to be more profitable than undertaking both investments simultaneously. Likewise, only for relatively high transportation costs - above $\tau_{B}^{\sigma-1}$ - some firms innovate exclusively for the domestic market without seeking participation in foreign markets. In the intermediate range of transportation costs, when converging to this limiting zone from BW, no firms would ever export without innovating and would ever innovate without exporting like in B. Likewise, when converging to this limiting zone from B, one does not fall straight into BW, as no firm would ever export without innovating.

We consider next the equilibrium where this double strategy or complementary strategy between innovating and exporting is optimal.

5.3 Export and Innovate: complementary activities (selection *C*)

The innovator and the exporter types coincide ($c_X = c_I^f$) because exporting and innovating are complementary activities.[9] Rearranging (14) and (16), the parameter space that is complementary to both equilibria BW and B can conveniently be expressed as

$$\frac{\delta f_I}{(\gamma^{1-\sigma}-1)}\frac{1}{(1+n\tau^{1-\sigma})} \leq \delta f_X \tau^{\sigma-1} \leq \frac{\delta f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma}.$$

We prove in the appendix that only under this parameter restriction can the strategy of innovating and exporting simultaneously be optimal. We are mainly concerned with the representation of this condition in figure 1. Not surprisingly, being the limiting case of both BW and B, this condition is defined for transportation costs between the two other equilibria. We label this region between $\tau_{BW}^{\sigma-1}$ and $\tau_{B}^{\sigma-1}$ with the letter C.

The boundary of this region clearly still depends on the relative sizes of f_X and f_I. But looking at the policy matrix, table 1, we note that because the activity of exporting and innovating are complementary, every pro-trade policy in this equilibrium is also a pro-innovation policy, and vice versa every pro-innovation policy is a pro-trade policy, too. In other words, innovation and trade policies become substitutes, and this feature sets this equilibrium apart from the other two.

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[9] We thank an anonymous referee for pointing this limiting case out to us as the Lileeva and Trefler (2010) equilibrium.

6 The Policy Space

Figure 1 proves to be particularly useful for a discussion of the effects of various policies and, especially, the issue of policy coordination.

Each country's combination of trade policy (a given τ, and a given f_X) and innovation policy (a given f_I), together with the other parameters of the model, jointly select the equilibrium (BW, B, or C) in which each country will fall. In this sense, we refer to this figure as the country's policy space.

Since the fixed costs of trade and innovation determine firms' selection into the different activities, they also delineate the boundary of each equilibrium in this space and trace out the relative size of each region. But it is the country's level of trade costs that determines the region into which each country falls and the effects of trade and innovation policies.

Furthermore, each type of policy instrument has potentially different effects. Changes to the regulatory costs of trade (f_X) and innovation (f_I) affect both the boundaries of the three regions in figure 1 and the innovation and exporting cost cut-offs as summarized in table 1. But a change in transportation costs can affect only the cost cutoffs.

The clear implication is that a given policy can impact the economy of each country differently because its effects depend on the level of current protection. For example, a reduction of tariff lines could foster both trade and innovation in countries with low initial tariffs (region BW), but depress innovation in other countries with higher tariff levels (region B). This is because the extensive margin of innovation (c_I^f) reacts positively in the first case and negatively in the second case (see table 1).[10] Likewise, an innovation policy can be pro-trade, in the sense of enlarging the share of exporting firms, only for intermediate levels of trade protection (region C). In high or low trade cost situations, respectively region BW and B, an innovation policy deters some firms from selling internationally (lower c_X).

Moreover, for a country located in region BW at the boundary with region C, a reduction of f_I has the effect of shrinking region BW in favor of region C as $\tau_{BW}^{\sigma-1}$ shifts to the left. This change, if sufficiently large, may push the country into region C given its tariff level. This policy would then cease to depress the exporting activity (lower c_X) as it would in region BW, but would become pro-trade (higher c_X) in the new steady state.

It is apparent from table 1 that in equilibrium C policy makers have

[10] Depressing innovation in this context means that the share of innovating firms in the economy shrinks. Likewise, fostering trade in this context means that the share of exporting firms expands.

a higher degree of freedom as both innovation and trade policies sustain each other in increasing the share of firms that export and innovate in the economy. In the other two equilibria, policy makers face some trade-offs about sustaining innovation or export. The only exception is the reduction of transport costs in equilibrium BW. That is the only instrument capable of increasing both the share of exporting firms and the share of innovating firms in the economy. All other types of instruments increase the share of firms performing one activity, but reduce the share of firms performing the other activity. In this sense, the "policy space" in equilibrium C becomes "larger" than in the other two equilibria. The other side of the coin is that "policy coordination" becomes more valuable in equilibrium BW and a necessity in equilibrium B.

To show the need for policy coordination, consider in figure 1 a country in region B at the border with region C. As discussed above, this country always faces the policy dilemma of choosing between sustaining innovation or trade. If it uses a pro-trade policy, it depresses innovation, and if it uses a pro innovation policy, it curtails export. If the policy-target were to incentivize more firms to adopt the more productive technology, the only means to achieve it would be through a reduction of f_I. This policy also has a second effect: for a given tariff level, it pushes the country further into region B as the boundary of region B expands at the expense of region C. Recurring repeatedly this type of policy in these circumstances therefore has a lock-in effect, as the border of region C is pushed further to the left. Each attempt of increasing the share of innovating firms would result in side effects in terms of a smaller share of exporting firms. Unfortunately, counterbalancing this effect with the subsequent use of trade policy would not yield the desired effect either, but would only nullify the effects obtained with the innovation policy.[11] The graph suggests that a different strategy centered on a different policy mix could prove to be more beneficial in this circumstance. Reducing either the tariff or the regulatory costs of trade would tend to push this country toward region C. The reduction of trade costs τ would make the country slide toward that equilibrium, while the reduction of f_X shrinks region B in favor of region C. And in this equilibrium these policies would also be pro-innovation. Furthermore, innovation policies would not entail the side cost of depressing exporting activities.

The lock-in effect described above is not a prerogative of equilibrium B, and under different circumstances it can occur in equilibrium BW, too. Consider a country in figure 1 within region BW toward its left

[11] Note that we cannot here discuss the effects of using innovation and trade policies simultaneously. Table 1 indeed summarizes only the partial derivatives, not the cross-derivatives.

boundary. Such a country has exhausted its possibility to resort to tariff cuts to sustain both trade and innovation, as it is already in a free trade situation. Without the tariff instrument at its disposal, it is therefore trapped in a policy dilemma about sustaining innovation (lower f_I) or exporting (lower f_X). And using the two policies sequentially is only detrimental as they tend to cancel each other out.

If instead this country were at the other extreme of the same equilibrium, bordering on region C, it would not face any policy trade-offs. Tariff cuts could indeed be used to increase both the proportion of firms that adopt new technologies and the proportion of firms that export. But if this country were to pursue this policy repeatedly, it would slide toward the low end of this region and progressively exhaust its degrees of freedom. Starting with a different policy mix can be more beneficial, especially when the intended purpose of the policy is to raise productivity investments in the economy. By lowering f_I the country may find itself in region C as this region expands and region BW contracts. And in the steady state of this equilibrium pro-innovation policies are also pro-trade policies, and vice versa.

In conclusion, the graph clearly shows two results. First, the boundaries and the size of each region potentially differ from country to country as they respond to the regulatory costs of trade and innovation. For this fact alone, countries with similar levels of transport costs may find themselves in different equilibria. Second, depending on the current level of trade costs, the effects of these policy mixes are potentially different. Interestingly, putting both these results together, it is possible to reach two apparently antithetic conclusions: the same tariff cut may impact countries with similar levels of tariff protection differently and countries with different levels of tariff protection similarly. Indeed, think in our figure 1 of two countries having different boundaries delineating each equilibrium and the same level of transportation costs. It is then possible that they fall into two different regions and face heterogenous responses to the same policy. Likewise, it is possible that two countries with different transportation costs and different region boundaries fall in the same equilibrium and have identical responses to the same policy.

Moreover, when policy makers can resort to multiple instruments, the choice they make in the first place affects the effectiveness of future policy options and their degree of freedom. In the examples above, some type of policies may lock countries into a policy dilemma that could be avoided with other measures (which we have referred to above as "policy mixes"). In this sense, the model justifies the recent OECD emphasis on the desirability of coordination among trade and innovation policies.[12] But it also stresses that the level of coordination needed is heterogenous and depends upon the level of trade costs. The graph and the discussion above suggest that in low trade-cost situations innovation policies, eventually followed by trade policies, are beneficial to avoid “policy traps”. In high trade-cost situations, the opposite order, namely trade policies first and then innovation policies, is a more appropriate policy mix to avoid policy dilemmas. Finally, with an intermediate level of trade costs, coordination is not stringent.

A further element of consideration for policy makers is the ease with which a policy can be adopted. Institutional arrangements may restrict the range of possibilities available to policy makers, making some policy mixes less attractive or viable than theoretically conceivable. For instance, it can be argued that for most European countries changes to innovation policies are institutionally easier than changes to trade policies. Provided they comply with European laws, innovation policies are largely a national matter, whereas changes in trade policies fall under the European competence and, possibly, the WTO. Return briefly to the example of our country in region B in figure 1 at the border with region C. We have argued that pursuing trade policies in the first place was desirable, but it may be not feasible or more difficult than pursuing innovation policies because of the institutional context.

These results are based on a comparative static analysis between three different steady states and three different policies considered one at the time. It is undeniable that analyzing multiple policy scenarios and/or characterizing the transitional dynamic between different states should become a prerogative for future research on the comparative analysis of different policy mixes across countries.

7 Trade and the moments of the productivity distribution

The redistribution of market share from exiting firms to incumbent firms (the “between” component) contributes to increase the average productivity in the industry. Indeed, to a higher c_0^f corresponds a higher truncation point of the lower tail of the productivity distribution $G(\cdot)$ and therefore a higher mean. But whether the average productivity will increase also depends on the “within” component. Only when a higher share of firms adopts the innovative technology can the first moment admittedly increase. This is a different prediction from the Melitz’s (2003) model where the “within” effect is absent and trade unambiguously increases the average productivity. Likewise, the prediction of our model for the second moment of the distribution differs from Melitz (2003). While in the latter a higher productivity truncation point associated

[12] See Onodera (2008) for a recent OECD discussion.

with trade necessarily translates into a lower variance of the productivity distribution, in our model this effect is counterbalanced in some equilibria by the *within-firm* effect. Firms introducing the new technologies indeed contribute to widening the variance of the distribution.

This means that the different policies we have analyzed will also have different effects on the moments of the productivity distribution, and these effects will depend on the current "mix" of policies adopted.

8 Conclusion

To study the effects of innovation and trade policies on the share of exporting and innovating firms, we include the option of technology adoption in a workhorse trade model with firm heterogeneity. Leaving aside any discussion of whether the notion of productivity-enhancing investment is the same thing as innovation, we explore the richness of this model, and show that it has multiple equilibria. This multiplicity is clearly an asset. Not only can it unify within one paradigm several empirical findings, but it has novel implications for the effects of both trade and innovation policies.

In particular, we show that the same policy does not need to produce equal effects in all countries. Because the effects of a policy depend on the current level of protection, countries can respond differently to the same policy and similarly to different policies. Therefore, a tariff cut is beneficial to innovation in liberal countries, but harmful to innovation in protectionist countries. Likewise, both tariff cuts and R&D grants can promote innovation in countries with different tariff levels.

When policy makers have an array of instruments, there are multiple ways to pursue a given policy objective, but each way entails different effects and therefore different side effects. For instance, a liberal country could promote innovation by means of tariff cuts or R&D grants, but the two instruments have opposite implications for the share of exporting firms. In this sense, policy makers face policy trade-offs, and we have argued that policy coordination becomes essential in mitigating them.

This paper could be extended in several directions. One could consider what the effects of these policies are when they are costly to implement. In this simple exercise we have followed a standard in the literature, in which we consider that reducing trade barriers or innovation barriers is costless. However, we must recognize that in some cases, to implement these policies, society may need to dedicate a non-negligible amount of resources. Another interesting extension could consider what the welfare implications of different policy mixes in different equilibria are. We consider these issues relevant for a future research agenda.

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Appendix

In this section we focus only on the (more relevant) open economy scenario.

.1 Appendix A - Definitions

For deriving the aggregate properties of the model, the cost distributions should be defined for each type of equilibria, since innovators and exporters price differently than domestic firms and the sorting process changes with each type of equilibria.

Equilibrium BW

Cost distributions and productivity indexes

Let $\mu_D(c) = \left\{ \begin{matrix} \frac{g(c)}{G(c_0^f)-G(c_X)}, & c_X < c \leq c_0^f \\ 0 & otherwise \end{matrix} \right\}$,

$\mu_{XN}(c) = \left\{ \begin{matrix} \frac{g(c)}{G(c_X)-G(c_I^f)}, & c_I^f < c \leq c_X \\ 0 & otherwise \end{matrix} \right\}$,

$\mu_I(c) = \left\{ \begin{matrix} \frac{g(c)}{G(c_I^f)}, & 0 \leq c \leq c_I^f \\ 0 & otherwise \end{matrix} \right\}$

denote the cost distributions in each subgroup prior to innovation. Let $\left(\widetilde{c_D}^f\right)^{1-\sigma} = \int\limits_{c_X}^{c_0^f} c^{1-\sigma}\mu_D(c)dc$, $\left(\widetilde{c_I}^f\right)^{1-\sigma} = \int\limits_0^{c_I^f} c^{1-\sigma}\mu_I(c)dc$ and $\widetilde{c_{XN}}^{1-\sigma} = \int\limits_{c_I^f}^{c_X} c^{1-\sigma}\mu_{XN}(c)dc$ be the respective average productivities for domestic firms, innovators, and exporting non-innovators prior to innovation.

Aggregate variables

Analogous to the closed economy version we obtain $P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M^f \left(\widetilde{c}^f\right)^{1-\sigma}$, where $\left(\widetilde{c}^f\right)^{1-\sigma} = \frac{1}{M^f}\left[M_D\left(\widetilde{c_D}^f\right)^{1-\sigma} + (1+n\tau^{1-\sigma})\left(M_{XN}\widetilde{c_{XN}}^{1-\sigma} + M_I\gamma^{1-\sigma}\left(\widetilde{c_I}^f\right)^{1-\sigma}\right)\right]$ is again the weighted average productivity index of the economy. As in the closed economy it can be shown that

$$R = M^f r(\widetilde{c}^f) = M^f \bar{r}^f \tag{19}$$

$$\bar{\pi}^f = \frac{\bar{r}^f}{\sigma} - f_D - \frac{G(c_I^f)}{G(c_0^f)}\delta f_I - \frac{G(c_X)}{G(c_0^f)}n\delta f_X. \tag{20}$$

The latter equations together with

$$\bar{\pi}^f = \frac{\delta f_e}{G(c_0^f)} \tag{21}$$

and

$$c_X = \left(\frac{f_D}{\delta f_X}\right)^{\frac{1}{\sigma-1}} \frac{c_0^f}{\tau} = f(f_D, f_X, \delta, c_0^f, \tau)$$

and

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})f_D}{\delta f_I}\right)^{\frac{1}{\sigma-1}} c_0^f = g(f_D, f_I, \delta, c_0^f, \tau, n, \gamma) \quad (22)$$

determine the unique equilibrium. For the latter proofs it is useful to express the productivity cutoff as

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)(1+n\tau^{1-\sigma})f_X}{f_I}\right)^{\frac{1}{\sigma-1}} \tau c_X = h(f_X, f_I, c_X, \tau, n, \gamma)$$

Equilibrium B

In equilibrium B all exporters are innovators, but not all innovators are exporters. There is a subset of domestic firms which also innovate. To derive the aggregate properties of the model, let us define the following cost functions:

$$\mu_D(c) = \begin{Bmatrix} \frac{g(c)}{G(c_0^f)-G(c_I^f)}, & c_I^f < c \leq c_0^f \\ 0 & otherwise \end{Bmatrix},$$

$$\mu_{NI}(c) = \begin{Bmatrix} \frac{g(c)}{G(c_I^f)-G(c_X)}, & c_X < c \leq c_I^f \\ 0 & otherwise \end{Bmatrix},$$

$$\mu_X(c) = \begin{Bmatrix} \frac{g(c)}{G(c_X)}, & 0 \leq c \leq c_X \\ 0 & otherwise \end{Bmatrix}.$$

Then we have

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M^f \left(\widetilde{c}^f\right)^{1-\sigma},$$

where $\left(\widetilde{c}^f\right)^{1-\sigma} = \frac{1}{M^f}\left[\gamma^{1-\sigma} M_{NI}\, \widetilde{c_{NI}}^{1-\sigma} + M_D \left(\widetilde{c_D}^f\right)^{1-\sigma} + (1+n\tau^{1-\sigma}) M_X \gamma^{1-\sigma} \widetilde{c_X}^{1-\sigma})\right]$,

where $\left(\widetilde{c_D}^f\right)^{1-\sigma} = \int\limits_{c_I^f}^{c_0^f} c^{1-\sigma}\mu_D(c)dc$ and $\widetilde{c_{NI}}^{1-\sigma} = \int\limits_{c_X}^{c_I^f} c^{1-\sigma}\mu_{NI}(c)dc$. As in the equilibrium BW equations (19), (20), (21) can be derived. These together with

$$c_X = \left(\frac{f_D}{\delta f_X}\right)^{\frac{1}{\sigma-1}} \frac{c_0^f}{\tau\gamma} = f^*(f_D, f_X, \delta, c_0^f, \tau, \gamma)$$

and

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)f_D}{\delta f_I}\right)^{\frac{1}{\sigma-1}} c_0^f = g^*(f_D, f_I, \delta, c_0^f, \tau, \gamma)$$

determine the unique equilibrium. For latter proofs it is also useful to express the productivity cutoff as

$$c_I^f = \left(\frac{(\gamma^{1-\sigma}-1)f_X}{f_I}\right)^{\frac{1}{\sigma-1}} (\tau\gamma)\, c_X = h^*(f_X, f_I, c_X, \tau, \gamma).$$

Equilibrium C

In equilibrium Call exporters are innovators, and all innovators are exporters. The conditional productivity distributions of exporters and innovators prior to innovation are the same. The expression for the aggregate price index now becomes:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M^f \left(\widetilde{c}^f\right)^{1-\sigma},$$

where$\left(\widetilde{c}^f\right)^{1-\sigma} = \frac{1}{M^f}\left[M_D\left(\widetilde{c_D}^f\right)^{1-\sigma} + (1+n\tau^{1-\sigma})\left(M_I\gamma^{1-\sigma}\left(\widetilde{c_I}^f\right)^{1-\sigma}\right)\right]$, and the relationships between the export and innovation productivity cutoffs are given by

$$c_X = \left(\frac{\delta(f_I + nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)f_D}\right)^{\frac{1}{1-\sigma}} c_0^f = f^{**} = g^{**}(f_X, f_I, c_0^f, \tau, n, \gamma, f_D, \delta)$$

and since $c_X = c_I \Rightarrow h^{**} = 1$. As in equilibrium BW (19), (20), (21) are easily derived. For the next propositions we are able to express the ZP condition in all three equilibria as

$$f_D j_0(c_0^f) + n\delta f_X j_X(c_X) + \delta f_I j_I(c_I^f) = \delta f_E, \tag{23}$$

where

$$j_i(c_i) = \left(\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} - 1\right) G(c_i),\ c_i = c_0^f, c_X, c_I^f \text{ and } i = 0, X, I$$

and $\left(\tilde{c}_0^f\right)^{1-\sigma} = \int_0^{c_0^f} c^{1-\sigma}\mu(c)dc$ and $\mu(c) = \left\{ \begin{array}{cc} \frac{g(c)}{G(c_0^f)}, & 0 \leq c \leq c_0^f \\ 0 & otherwise \end{array} \right\}.$

This productivity distribution corresponds to the incumbent productivity distribution prior to innovation. The $j_i(c_i)$ functions are continuous. In the online appendix we show that

$j_i'(c_i) = \frac{(\sigma-1)}{c_i}\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} G(c_i) > 0,$

and therefore these functions are monotonically increasing in their respective arguments. We also show that the elasticities are given by:

$$\frac{j_i'(c_i)}{j_i(c_i)}c_i = \frac{(\sigma-1)\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma}}{\left(\left(\frac{\tilde{c}_i}{c_i}\right)^{1-\sigma} - 1\right)}. \tag{24}$$

These results will be useful in the following section.

.2 Appendix B - Parameter restriction for equilibrium C

In equilibrium C, firms consider the joint option of exporting and innovating. According to what is derived in section 3.3, no firm will innovate without being an exporter, and no firm will export without being an innovator. The firm evaluates whether to export and innovate is better than not doing both and remain local. The partition in this equilibrium is between exporting and innovating firms, and local firms. Let us call c_I *w.l.o.g.* the marginal cost associated with the firm which is indifferent between both options.

Then this marginal firm will satisfy the following condition:

$$(1+n\tau^{1-\sigma})\gamma^{1-\sigma}B\left(c_I^f\right)^{1-\sigma} - \delta(f_I+nf_X) - f_D - \left(B\left(c_I^f\right)^{1-\sigma} - f_D\right) = 0$$

$$((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)B\left(c_I^f\right)^{1-\sigma} = \delta(f_I+nf_X).$$

The marginal firm being indifferent between staying in the market or not satisfies

$$Bc_0^{1-\sigma} = f_D.$$

Then a necessary condition for this equilibrium to exist is that

$$\left(\frac{c_I^f}{c_0}\right)^{1-\sigma} > 1 \Rightarrow \frac{\delta(f_I+nf_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)f_D} > 1.$$

The previous equation guarantees that not all firms innovate and export. However, for equilibrium C to exist, the marginal firm must be indifferent between innovating and exporting or being local. This implies that the income of the marginal firm in equilibrium C must be larger than the income of this firm when it innovates, provided that the firm will be an exporter, or when it exports, provided that the firm will be an innovator. In other terms:

$$B\left(c_I^f\right)^{1-\sigma} \geq \frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}} \tag{25}$$

$$B\left(c_I^f\right)^{1-\sigma} \geq \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1}. \tag{26}$$

The marginal firm in equilibrium C has the following income:

$$B\left(c_I^f\right)^{1-\sigma} = \frac{\delta(f_I + n f_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}. \tag{27}$$

Substituting (27) in (25), we have

$$\frac{\delta(f_I + n f_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)} \geq \frac{\delta f_I}{\gamma^{1-\sigma}-1}\frac{1}{1+n\tau^{1-\sigma}}.$$

Rearranging terms in the latter equation, we have

$$\delta f_I \gamma^{1-\sigma}\left(1+n\tau^{1-\sigma}\right) - \delta f_I\left(1+n\tau^{1-\sigma}\right) + n\delta f_X\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right) \geq \delta f_I \gamma^{1-\sigma}\left(1+n\tau^{1-\sigma}\right) - \delta f_I.$$

Rearranging terms, we have

$$-\delta f_I n\tau^{1-\sigma} + n\delta f_X\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right) \geq 0,$$

and then this implies

$$\delta f_X \tau^{\sigma-1} \geq \frac{\delta f_I}{\left(\gamma^{1-\sigma}-1\right)\left(1+n\tau^{1-\sigma}\right)}.$$

This is one of the restrictions needed to be satisfied if equilibrium C holds.

Now substituting (27) in (26) we have

$$\frac{\delta(f_I + n f_X)}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)} \geq \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1}.$$

Rearranging terms, we have

$$\delta f_I + n\delta f_X \geq \delta f_X \tau^{\sigma-1} + n\delta f_X - \delta f_X \gamma^{\sigma-1}\tau^{\sigma-1},$$

and this implies

$$\delta f_I \geq \delta f_X \tau^{\sigma-1} - \delta f_X \gamma^{\sigma-1} \tau^{\sigma-1},$$

which implies

$$\delta f_I \geq \delta f_X \tau^{\sigma-1}(1-\gamma^{\sigma-1}).$$

Multiplying both sides by $(\tau\gamma)^{1-\sigma}$ and rearranging terms, we have

$$\frac{\delta f_I}{(\gamma^{1-\sigma}-1)}\gamma^{1-\sigma} \geq \tau^{\sigma-1}\delta f_X$$

which is the other requirement for equilibrium C to hold.

.3 Appendix C - Proof of Propositions.

Proposition 1 *(Trade Liberalization: Equilibrium BW): Trade Liberalization yields the following results:*

1. *Tariff policy:* $\frac{\partial c_0^f}{\partial \tau} > 0,\ \frac{\partial c_X}{\partial \tau} < 0,\ \frac{\partial c_I^f}{\partial \tau} < 0$
2. *Export regulation costs* (f_X) : $\frac{\partial c_0^f}{\partial f_X} > 0,\ \frac{\partial c_X}{\partial f_X} < 0,\ \frac{\partial c_I^f}{\partial f_X} > 0$

Proof. 1. A tariff reduction:

Totally differentiate (23) with respect to τ :

$$f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial \tau} + n\delta f_X j_X'(c_X)\frac{\partial c_X}{\partial \tau} + \delta f_I j_I'(c_I^f)\frac{\partial c_I^f}{\partial \tau} = 0, \tag{28}$$

where, as described above, $c_X = f(c_0^f, \tau, f_D, f_X, \delta)$, $c_I^f = g(c_0^f, \tau, f_D, f_I, n, \delta, \gamma)$, $c_I^f = h(c_X, f_I, \tau, f_X, n, \gamma)$. Applying the chain rule and applying the following results: $\frac{\partial f}{\partial c_0^f} = \frac{c_X}{c_0^f}$, $\frac{\partial f}{\partial \tau} = -\frac{c_X}{\tau}$, $\frac{\partial g}{\partial c_0^f} = \frac{c_I^f}{c_0^f}$, $\frac{\partial g}{\partial \tau} = -\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)$, we obtain

$$\frac{\partial c_0^f}{\partial \tau} = \frac{n\delta f_X j_X'(c_X)\frac{c_X}{\tau} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)}{\left(f_D j_0'(c_0^f) + n\delta f_X j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

Taking into consideration that

$$\frac{\partial c_I^f}{\partial \tau} = \frac{\partial h}{\partial c_X}\frac{\partial c_X}{\partial \tau} + \frac{\partial h}{\partial \tau}$$

in (28) and applying the following results: $\frac{\partial h}{\partial c_X} = \frac{c_I^f}{c_X}$ $\frac{\partial h}{\partial \tau} = \frac{\partial h}{\partial \phi}\frac{\partial \phi}{\partial \tau} = \frac{c_I^f}{\tau}\frac{1}{1+n\tau^{1-\sigma}}$ where $\phi = \tau^{1-\sigma}$, then:

$$\frac{\partial c_X}{\partial \tau} = \frac{-\left(f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial \tau} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{\tau}\frac{1}{1+n\tau^{1-\sigma}}\right)}{\left(n\delta f_X j_X'(c_X) + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_X}\right)} < 0.$$

To get the sign of the derivative of $\frac{\partial c_I^f}{\partial \tau}$ we apply the chain rule

$$\frac{\partial c_I^f}{\partial \tau} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau} + \frac{\partial g}{\partial \tau}.$$

Then substituting the expressions for $\frac{\partial g}{\partial c_0^f}$, $\frac{\partial g}{\partial \tau}$ we get

$$\frac{\partial c_I^f}{\partial \tau} = \frac{c_I^f}{c_0^f}\frac{\partial c_0^f}{\partial \tau} - \frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Then showing $\frac{\partial c_I^f}{\partial \tau} < 0$ implies

$$\frac{\partial c_0^f}{\partial \tau} < \frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Substituting the expression for $\frac{\partial c_0^f}{\partial \tau}$ in the previous equation, we obtain

$$\frac{n\delta f_X j_X'(c_X)\frac{c_X}{\tau} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right)}{\left(f_D j_0'(c_0^f) + n\delta f_X j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} < \frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right).$$

Manipulating the expression we arrive at the following condition

$$f_D j_0'(c_0^f)\frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}}{1+n\tau^{1-\sigma}}\right) - n\delta f_X j_X'(c_X)\frac{c_X}{\tau}\left(\frac{1}{1+n\tau^{1-\sigma}}\right) > 0.$$

Simplifying, we get

$$f_D j_0'(c_0^f)c_0^f\tau^{1-\sigma} > \delta f_X j_X'(c_X)c_X$$

rearranging

$$\left(\frac{\tau^{1-\sigma} f_D}{\delta f_X}\right) j_0'(c_0^f)c_0^f > j_X'(c_X)c_X,$$

where the first element is

$$\left(\frac{c_0^f}{c_X}\right)^{1-\sigma} j_0'(c_0^f)c_0^f > j_X'(c_X)c_X.$$

Substituting the expressions $j_D'(c_0^f)$, $j_X'(c_X)$, we have that

$$\left(\tilde{c}_0^f\right)^{1-\sigma} G\left(c_0^f\right) > \left(\tilde{c}_X\right)^{1-\sigma} G\left(c_X\right),$$

and substituting the expressions $\left(\tilde{c}_0^f\right)^{1-\sigma}$, $(\tilde{c}_X)^{1-\sigma}$, we get

$$\int_0^{c_0^f} c^{1-\sigma} g(c) dc > \int_0^{c_X} c^{1-\sigma} g(c) dc$$

since $c_X < c_0^f$.

2. A decrease in f_X :

Totally differentiating (23) with respect to f_X, we find that

$$f_D j_0'(c_0^f) \frac{\partial c_0^f}{\partial f_X} + n\delta j_X(c_X) + n\delta f_X j_X'(c_X) \frac{\partial c_X}{\partial f_X} + \delta f_I j_I'(c_I^f) \frac{\partial c_I^f}{\partial f_X} = 0. \quad (29)$$

Again applying the chain rule and the following results: $\frac{\partial f}{\partial f_X} = \frac{-1}{\sigma-1}\frac{c_X}{f_X}$, $\frac{\partial h}{\partial f_X} = \frac{1}{\sigma-1}\frac{c_I^f}{f_X}$, $\frac{\partial g}{\partial f_X} = 0$, we have that

$$\frac{\partial c_0^f}{\partial f_X} = \frac{n\delta\left(j_X'(c_X)\frac{c_X}{\sigma-1} - j_X(c_X)\right)}{\left(f_D j_0'(c_0^f) + n\delta f_X j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

The latter is positive. To see this, notice that the numerator is positive iff

$$\frac{j_X'(c_X)}{j_X(c_X)} c_X > \sigma - 1. \quad (30)$$

Using (24) in (30):

$$\left(\frac{\tilde{c}_X}{c_X}\right)^{1-\sigma} > \left(\left(\frac{\tilde{c}_X}{c_X}\right)^{1-\sigma} - 1\right),$$

which is always satisfied.

To get the sign of $\frac{\partial c_X}{\partial f_X}$, we use (29) and the fact that

$$\frac{\partial c_I^f}{\partial f_X} = \frac{\partial h}{\partial c_X}\frac{\partial c_X}{\partial f_X} + \frac{\partial h}{\partial f_X}$$

to obtain

$$\frac{\partial c_X}{\partial f_X} = \frac{-\left(f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial f_X} + \delta f_I j_I'(c_I^f)\frac{1}{\sigma-1}\frac{c_I^f}{f_X} + n\delta j_X(c_X)\right)}{\left(n\delta f_X j_X'(c_X) + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_X}\right)} < 0.$$

Note that from the chain rule we can derive

$$\frac{\partial c_I^f}{\partial f_X} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_X} + \frac{\partial g}{\partial f_X} > 0.$$

■

Proposition 2 *Trade Liberalization (Equilibrium B) : Trade Liberalization experiments in this equilibrium yields the following results:*

1. *Tariff policy:* $\frac{\partial c_0^f}{\partial \tau} > 0,\ \frac{\partial c_X}{\partial \tau} < 0,\ \frac{\partial c_I^f}{\partial \tau} > 0$
2. *Export regulation costs* (f_X) : $\frac{\partial c_0^f}{\partial f_X} > 0,\ \frac{\partial c_X}{\partial f_X} < 0,\ \frac{\partial c_I^f}{\partial f_X} > 0$

Proof. The Proof follows the same scheme as the previous proof. However, we know that now the equations determining the cutoffs have changed and then we have that

$c_X = f^*(c_0^f, \tau, f_X, \delta),\ c_I^f = g^*(c_0^f, \tau, f_I, \delta, \gamma, f_D),\ c_I^f = h^*(c_X, \tau, f_X, f_I, \gamma).$

1. Tariff policy:

Applying the chain rule and using (28) and the following results: $\frac{\partial f^*}{\partial \tau} = \frac{-c_X}{\tau}$ $\frac{\partial f^*}{\partial c_0^f} = \frac{c_X}{c_0^f}$ $\frac{\partial g^*}{\partial \tau} = 0$ $\frac{\partial g^*}{\partial c_0^f} = \frac{c_I^f}{c_0^f}$, we obtain

$$\frac{\partial c_0^f}{\partial \tau} = \frac{n\delta f_X j_X'(c_X)\frac{c_X}{\tau}}{\left(f_D j_0'(c_0^f) + n\delta f_X j_X'(c_X)\frac{c_X}{c_0^f} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)} > 0.$$

Applying the chain rule in (28) and using $\frac{\partial f^*}{\partial \tau} = \frac{-c_X}{\tau}$ $\frac{\partial f^*}{\partial c_0^f} = \frac{c_X}{c_0^f}$, we obtain

$$\frac{\partial c_X}{\partial \tau} = \frac{-\left(f_D j_0'(c_0^f)\frac{c_0^f}{\tau} + \delta f_I j_I'(c_I^f)\frac{c_I^f}{\tau}\right)}{\left(f_D j_0'(c_0^f)\frac{c_0^f}{c_X} + n\delta f_X j_X'(c_X) + \delta f_I j_I'(c_I^f)\frac{c_I^f}{c_X}\right)} < 0$$

and for the innovation cutoff we just apply the chain rule:

$$\frac{\partial c_I^f}{\partial \tau} = \frac{\partial g^*}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau} + \frac{\partial g^*}{\partial \tau} > 0,$$

but we know that: $\frac{\partial g^*}{\partial \tau} = 0$ and $\frac{\partial g^*}{\partial c_0^f} > 0,\ \frac{\partial c_0^f}{\partial \tau} > 0$. So $\frac{\partial c_I^f}{\partial \tau} > 0$.

2. Export regulation costs:

Notice that $\frac{\partial f^*}{\partial f_X} = \frac{-1}{\sigma - 1}\frac{c_X}{f_X}$, $\frac{\partial h^*}{\partial f_X} = \frac{1}{\sigma-1}\frac{c_I^f}{f_X}$, $\frac{\partial g^*}{\partial f_X} = 0$, and $\frac{\partial f^*}{\partial c_0^f} = \frac{c_X}{c_0^f}$, $\frac{\partial g^*}{\partial c_0^f} = \frac{c_I^f}{c_0^f}$, $\frac{\partial h^*}{\partial c_0^f} = \frac{c_I^f}{c_X}$. Since $j_i'(c_i) > 0 \ \ \forall i = 0, X, I$ and $c_i = c_0^f, c_X, c_I^f$. The partial derivatives mentioned above are the same in both equilibria BW and B; therefore the same proof applies. ■

Proposition 3 *(Trade Liberalization. Equilibrium C): Trade Liberalization experiments in this equilibrium yield the following results:*

1. *Tariff policy:* $\frac{\partial c_0^f}{\partial \tau} > 0$, $\frac{\partial c_X}{\partial \tau} < 0$, $\frac{\partial c_I^f}{\partial \tau} < 0$
2. *Export regulation costs* (f_X) : $\frac{\partial c_0^f}{\partial f_X} > 0$, $\frac{\partial c_X}{\partial f_X} < 0$, $\frac{\partial c_I^f}{\partial f_X} < 0$

Proof. The proof is analogous to the other type of equilibria. It is important to remember however that in this equilibrium: $c_I^f = c_X \Rightarrow \tilde{c}_I^f = \tilde{c}_X \Rightarrow j_X(c_X) = j_I(c_I^f)$. Also $c_X = f^{**}(c_0^f, \tau, f_X, f_D, n, \delta, \gamma, f_I)$, $c_I^f = g^{**}(c_0^f, \tau, f_X, f_D, n, \delta, \gamma, f_I)$, $c_I^f = h^{**}(c_X)$, where $f^{**}(c_0^f, \tau, f_X, f_D, n, \delta, \gamma, f_I) = g^{**}(c_0^f, \tau, f_X, f_D, n, \delta, \gamma, f_I)$. Then $c_I^f = h^{**}(c_X) = c_X$.

1. Tariff policy:

Totally differentiating (23) we get

$$f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial \tau} + n\delta f_X j_X'(c_X)\frac{\partial c_X}{\partial \tau} + \delta f_I j_I'(c_I^f)\frac{\partial c_I^f}{\partial \tau} = 0.$$

Applying the chain rule and the following results, $\frac{\partial f^{**}}{\partial \tau} = \frac{\partial g^{**}}{\partial \tau} = \frac{-c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\right)$ $\frac{\partial f^{**}}{\partial c_0^f} = \frac{\partial g^{**}}{\partial c_0^f} = \frac{c_X}{c_0^f}$, we arrive at the following result:

$$\frac{\partial c_0^f}{\partial \tau} = \frac{\delta(f_I + n f_X) j_X'(c_X)\frac{c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\right)}{\left(f_D j_0'(c_0^f) + \delta(f_I + n f_X) j_X'(c_X)\frac{c_X}{c_0^f}\right)} > 0.$$

Remember that in this equilibrium $\frac{\partial c_I^f}{\partial \tau} = \frac{\partial c_X}{\partial \tau}$. To obtain the sign of this derivative, we first apply the chain rule to get

$$\frac{\partial c_X}{\partial \tau} = \frac{\partial f^{**}}{\partial c_0^f}\frac{\partial c_0^f}{\partial \tau} + \frac{\partial f^{**}}{\partial \tau}.$$

Substituting $\frac{\partial c_0^f}{\partial \tau}$ into the previous expression, we get

$$\frac{\partial c_X}{\partial \tau} = \frac{\delta(n f_X + f_I) j_X'(c_X)\frac{c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\right)}{\left(f_D j_0'(c_0^f)\frac{c_0^f}{c_X} + \delta(f_I + n f_X) j_X'(c_X)\right)} - \frac{c_X}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\right).$$

Rearranging terms it can be shown that this expression is negative whenever the following holds:

$$f_D j_0'(c_0^f)\frac{c_0^f}{\tau}\left(\frac{n\tau^{1-\sigma}\gamma^{1-\sigma}}{((1+n\tau^{1-\sigma})\gamma^{1-\sigma}-1)}\right) > 0.$$

2. Export regulation costs:

Note that
$$\frac{\partial f^{**}}{\partial f_X} = \frac{-c_X}{\sigma - 1}\left(\frac{n}{f_I + nf_X}\right).$$
Then totally differentiating and applying the latter results yields the following expression:

$$\frac{\partial c_0^f}{\partial f_X} = \frac{\delta(f_I + nf_X) j_X'(c_X)\frac{c_X}{\sigma-1}\left(\frac{n}{(f_I+nf_X)}\right) - n\delta j_X(c_X)}{\left(f_D j_0'(c_0^f) + \delta(f_I + nf_X) j_X'(c_X)\frac{c_X}{c_0^f}\right)} > 0.$$

Note that the sign of this derivative is determined by the sign of the numerator. Simplifying we obtain

$$\frac{j_X'(c_X)c_X}{j_X(c_X)} > \sigma - 1,$$

which clearly holds as we have shown in previous sections.

To get the sign for $\frac{\partial c_X}{\partial f_X} = \frac{\partial c_I^f}{\partial f_X}$, we totally differentiate expression (23) and we apply $\frac{\partial h}{\partial c_X} = 0$. Then we obtain

$$\frac{\partial c_X}{\partial f_X} = \frac{-\left(f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial f_X} + n\delta j_X(c_X)\right)}{\delta(f_I + nf_X) j_X'(c_X)} < 0$$

since $\frac{\partial c_0^f}{\partial f_X} > 0$. ■

Proposition 4 *(Innovation policies). A reduction in the costs of innovation reduces the export cutoff(c_X) and the domestic cutoff (c_X). It increases the innovation cutoff $\left(c_I^f\right)$.*

Proof. Equilibrium BW

We show that $\frac{\partial c_0^f}{\partial f_I} > 0$, $\frac{\partial c_X}{\partial f_I} > 0$, $\frac{\partial c_I^f}{\partial f_I} < 0$.

Differentiating (23) with respect to f_I we get

$$f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial f_I} + n\delta f_X j_X'(c_X)\frac{\partial c_X}{\partial f_I} + \delta j_I(c_I^f) + \delta f_I j_I'(c_I^f)\frac{\partial c_I^f}{\partial f_I} = 0.$$

Totally differentiating the previous conditions, we have

$$\frac{\partial c_X}{\partial f_I} = \frac{\partial f}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial f}{\partial f_I}$$

$$\frac{\partial c_I^f}{\partial f_I} = \frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial g}{\partial f_I}$$

and also applying the following results: $\frac{\partial f}{\partial f_I}=0$, $\frac{\partial g}{\partial f_I}=\frac{1}{1-\sigma}\frac{c_I^f}{f_I}$

$$\frac{\partial c_I^f}{\partial f_I}=\frac{\delta\left(\frac{j_I'(c_I^f)c_I^f}{\sigma-1}-j_I(c_I^f)\right)}{\left(f_D j_0'(c_0^f)+n\delta f_X j_X'(c_X)\frac{c_X}{c_0^f}+\delta f_I j_I'(c_I^f)\frac{c_I^f}{c_0^f}\right)}. \tag{31}$$

This condition is positive provided that

$$\frac{j_I'(c_I^f)c_I^f}{\sigma-1}-j_I(c_I^f)>0.$$

Rearranging terms and using (24), for the latter expression to be positive the following must hold:

$$\left(\frac{\tilde{c}_I^f}{c_I^f}\right)^{1-\sigma}>\left(\left(\frac{\tilde{c}_I^f}{c_I^f}\right)^{1-\sigma}-1\right),$$

which obviously holds. Moreover, since

$$\frac{\partial c_X}{\partial f_I}=\frac{\partial f}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I}+\frac{\partial f}{\partial f_I}$$

we then have $\frac{\partial c_X}{\partial f_I}>0$, since $\frac{\partial f}{\partial f_I}=0$. To get the sign for $\frac{\partial c_I^f}{\partial f_I}$, we use the following expression:

$$\frac{\partial c_I^f}{\partial f_I}=\frac{\partial g}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I}+\frac{\partial g}{\partial f_I}.$$

We want to show that $\frac{\partial c_I^f}{\partial f_I}<0$. Substituting (31) and rearranging terms, we arrive at the following expression:

$$\frac{\delta\left(\frac{j_I'(c_I^f)c_I^f}{\sigma-1}-j_I(c_I^f)\right)}{\left(f_D j_0'(c_0^f)\frac{c_0^f}{c_I^f}+n\delta f_X j_X'(c_X)\frac{c_X}{c_I^f}+\delta f_I j_I'(c_I^f)\right)}-\frac{1}{\sigma-1}\frac{c_I^f}{f_I}<0.$$

Rearranging terms we arrive at the following expression:

$$f_D j_0'(c_0^f)\frac{c_0^f}{(\sigma-1)f_I}+n\delta f_X j_X'(c_X)\frac{c_X}{(\sigma-1)f_I}+\delta j_I(c_I^f)>0,$$

which clearly holds.

Equilibrium B

Differentiating f^*, g^*, and h^* with respect to f_I, we get $\frac{\partial f^*}{\partial f_I} = 0$, $\frac{\partial g^*}{\partial f_I} = \frac{1}{1-\sigma}\frac{c_I^f}{f_I}$. Together with this we have $\frac{\partial f^*}{\partial c_0^f} = \frac{\partial f}{\partial c_0^f}, \frac{\partial g^*}{\partial c_0^f} = \frac{\partial g}{\partial c_0^f}$. Then the results are analogous to the ones in equilibrium BW. The proof is also analogous.

Equilibrium C

Differentiating f^{**}, g^{**}, and h^{**} with respect to f_I, we get $\frac{\partial f^{**}}{\partial f_I} = \frac{\partial g^{**}}{\partial f_I} = \frac{-c_X}{\sigma-1}\left(\frac{1}{f_I+nf_X}\right)$. Remember that in Equilibrium C $j_X(c_X) = j_I(c_I^f)$. Differentiating (23) with respect to f_I, we get

$$f_D j_0'(c_0^f)\frac{\partial c_0^f}{\partial f_I} + \delta j_X(c_X) + \delta\left(f_I + nf_X\right) j_X'(c_X)\frac{\partial c_X}{\partial f_I} = 0.$$

Applying the chain rule, we have

$$\frac{\partial c_0^f}{\partial f_I} = \frac{\delta\left(\frac{j_X'(c_X)c_X}{(\sigma-1)} - j_X(c_X)\right)}{f_D j_0'(c_0^f) + \delta\left(f_I + nf_X\right) j_X'(c_X)\frac{c_X}{c_0^f}} > 0$$

since, as previously shown, $\frac{j_X'(c_X)c_X}{(\sigma-1)} - j_X(c_X) > 0$. To get $\frac{\partial c_X}{\partial f_I} = \frac{\partial c_I^f}{\partial f_I}$, we apply the chain rule to get

$$\frac{\partial c_X}{\partial f_I} = \frac{\partial f^{**}}{\partial c_0^f}\frac{\partial c_0^f}{\partial f_I} + \frac{\partial f^{**}}{\partial f_I}.$$

Substituting the values for $\frac{\partial c_0^f}{\partial f_I}$ and $\frac{\partial f^{**}}{\partial f_I}$, we have

$$\frac{\partial c_X}{\partial f_I} = \frac{\delta\left(\frac{j_X'(c_X)c_X}{(\sigma-1)} - j_X(c_X)\right)}{f_D j_0'(c_0^f)\frac{c_0^f}{c_X} + \delta\left(f_I + nf_X\right) j_X'(c_X)} - \frac{c_X}{\sigma-1}\left(\frac{1}{f_I + nf_X}\right).$$

Rearranging terms we have

$$\frac{\partial c_X}{\partial f_I} = \frac{-\left(j_X(c_X) + f_D j_0'(c_0^f)\frac{c_0^f}{(\sigma-1)(f_I+nf_X)}\right)}{f_D j_0'(c_0^f)\frac{c_0^f}{c_X} + \delta\left(f_I + nf_X\right) j_X'(c_X)} < 0.$$

■

Table 1: The Policy Matrix

BW	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} < 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} > 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} > 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$
B	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} > 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} > 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} > 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$
C	c_0^f	c_X	c_I^f
τ	$\frac{\partial c_0^f}{\partial \tau} > 0$	$\frac{\partial c_X}{\partial \tau} < 0$	$\frac{\partial c_I^f}{\partial \tau} < 0$
f_X	$\frac{\partial c_0^f}{\partial f_X} > 0$	$\frac{\partial c_X}{\partial f_X} < 0$	$\frac{\partial c_I^f}{\partial f_X} < 0$
f_I	$\frac{\partial c_0^f}{\partial f_I} > 0$	$\frac{\partial c_X}{\partial f_I} < 0$	$\frac{\partial c_I^f}{\partial f_I} < 0$

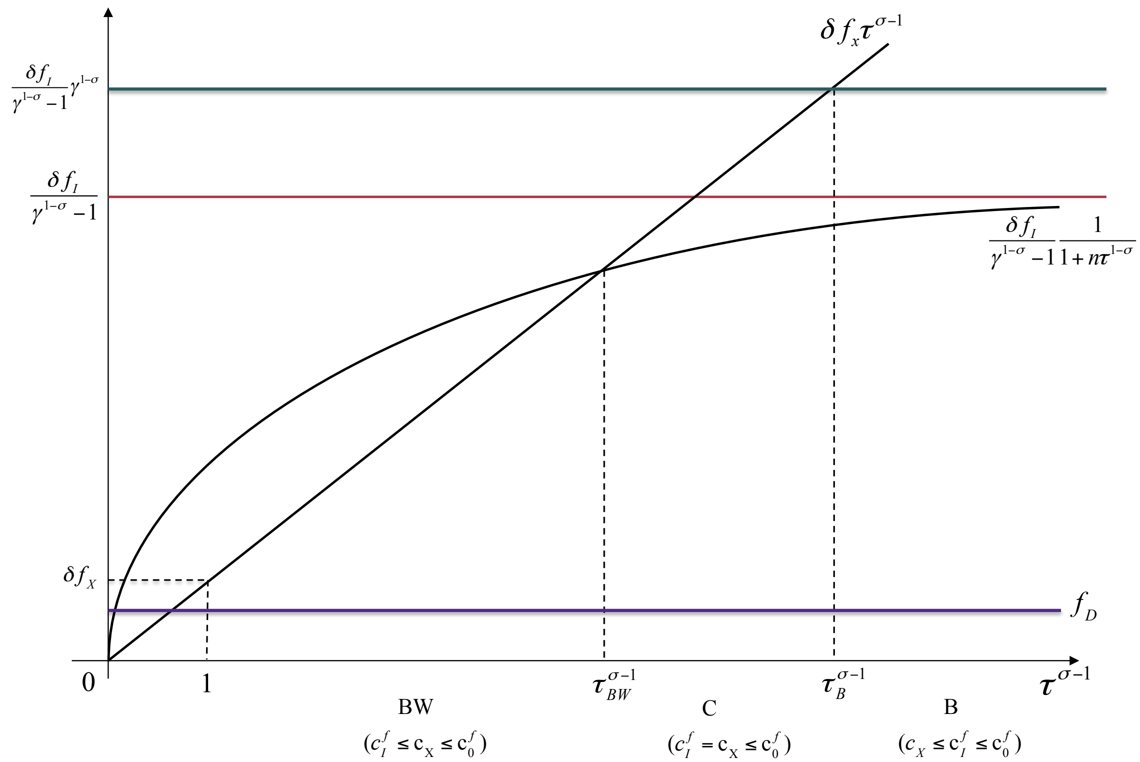

Figure 1: The Policy Space