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Published paper

Nasir, A.N.K, Tokhi, M.O., Ghani, N.M.A. and Raja Ismail, R.M.T. (2012) *Novel adaptive spiral dynamics algorithms for global optimization*. In: Proceeding of the 11th IEEE International Conference on Cybernetic Intelligent Systems 2012. 11th IEEE International Conference on Cybernetic Intelligent Systems, 22-24th August 2012, Guildford. IEEE , pp. 334-341.

Novel Adaptive Spiral Dynamics Algorithms for Global Optimization

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Abstract—This paper presents adaptive versions of spiral dynamics algorithm (SDA) referred to as adaptive SDA (ASDA). SDA is known as fast computing algorithm due to its simplicity in the structure and it has stable convergence response when approaching the optimum point in the search space. However, the performance of SDA is still poor due to incorporation of single radius value during the whole search process. In ASDA, the spiral radius is made dynamic by employing novel mathematical equations and incorporating non-mathematical fuzzy logic strategy establishing the relationship between fitness value and spiral radius. This results in better performance in terms of convergence speed, accuracy, and total computing time while retaining the simple structure of SDA. Several uni-modal and multi-modal benchmark functions are employed to test the algorithm in finding the global optimum point. The results show that ASDA outperforms SDA in all test functions considered.

Index Terms—Adaptive spiral dynamics; optimization algorithm; nature inspired; fuzzy logic.

I. INTRODUCTION

Metaheuristic optimization algorithms have gained a lot of interest among researchers worldwide. These algorithms are inspired by biological phenomena or natural phenomena. Some of the newly introduced algorithms include biogeography-based optimization [1], firefly optimization algorithm [2], galaxy-based search algorithm [3], and spiral dynamics inspired optimization [4]. All these algorithms have gained attention due to their simplicity to program, fast computing time, easy to implement, and possibility to apply to various applications. There are a lot of possibilities to improve the algorithms from various aspects. Many attempts have been made to improve performances of the algorithms such as hybridizing two or more algorithms and mostly developing adaptive approaches or incorporating powerful mathematical functions into the algorithms.

Adaptive approach is a common strategy used in metaheuristic to enhance capability of optimization algorithms. It may increase convergence speed, accuracy and reduce total computational time by varying step size of search point through simple mathematical function or through an intelligent approach. The advantage of adaptive approach is that the simplicity of the original algorithm is preserved thus resulting in better performance without requiring extra computational

cost. Various adaptive approaches of metaheuristic optimization algorithms have been proposed by researchers with the aim to increase system performance. The adaptive approach by incorporating mathematical function into bacterial foraging algorithm (BFA) with improved performance has been reported in [5], [6], where the performance of the algorithm has been analysed based on incorporating mathematical equation into the BFA. On the other hand, the adaptation scheme of varying step size of BFA through intelligent approach has been reported in [7], [8]. Intelligent approaches such as fuzzy logic have shown not only to improve the algorithm performance, but their simplicity in determining fuzzy rules based on intelligent human logic thinking to vary step size of a point in search space is offering more flexibility and very promising results. However, since the introduction of SDA, the literature has been lacking further development and its application to real world problems.

This paper presents four new approaches of adaptable step size of SDA. The first approach employs a non-mathematical fuzzy logic scheme to establish relationship between fitness value of a particular point in the search space and spiral radius of SDA while the rest of the proposed approaches utilize novel mathematical equations based on linear, quadratic and exponential functions to establish similar relationships. The rest of the paper is organized as follows. Section II provides a brief literature review of the original spiral dynamics inspired optimization. The proposed adaptive spiral dynamics algorithm (ASDA) and its details are described in section III. Validation of the proposed adaptive algorithms in comparison to SDA with uni-modal and multi-modal test functions is presented in section IV. Section V provides concluding remarks.

II. SPIRAL DYNAMICS ALGORITHM

The original version of SDA is briefly described in this section. The SDA is a metaheuristic algorithm adopted from spiral phenomena in nature [4]. The essential feature of SDA is the dynamic step size in its spiral path trajectory. The step size is larger at the beginning of the search process and becomes smaller when approaching the optimum point, which is always located at the centre of the spiral form. The length of the dynamic step size from generation to generation is determined by spiral radius r . The angular displacement θ on the other

The input is absolute fitness value of a particular search point and the output is the spiral radius value, which can be defined within $[0, 1]$ to ensure the search process converges towards best fitness location. The overall FASDA mechanism can be represented in a block diagram as shown in Fig. 2. In this work, three Gaussian membership functions are used to fuzzify the crisp value of the fitness and defuzzify fuzzy sets to a single value representing the spiral radius. This is to ensure the optimization algorithm does not have very long computation time, and in turn to increase speed of convergence. Moreover, Gaussian membership function is smooth and concise, which can represent uncertainty in measurement more effectively.

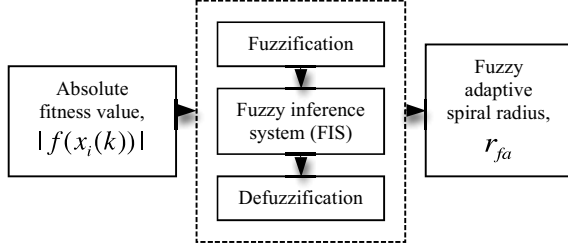


Fig. 2. Structure of fuzzy adaptive spiral dynamics algorithm.

The Mamdani-type with centre of area defuzzification method is used due to its intuitiveness, widespread acceptance and suitability in dealing with human reasoning [8], [9]. Another important feature of fuzzy logic scheme is linguistic rule that comprises IF-THEN statement to establish relationship between antecedent and consequence. The general form of fuzzy logic linguistic rule for FASDA is defined as:

$$\text{IF } |f(x_i(k))| \text{ is } A \text{ THEN } r_{fa} \text{ is } B \quad (3)$$

where A and B are linguistic values defined by fuzzy sets in the range of absolute fitness function value, $|f(x_i(k))|$ and adaptive spiral radius, r_{fa} respectively. The ' $|f(x_i(k))|$ is A' is known as antecedent while ' r_{fa} is B' is known as consequent. Thus, Equation 3 can be defined such that if $|f(x_i(k))|$ is small then r_{fa} is big or approaching maximum spiral radius and if $|f(x_i(k))|$ is big then r_{fa} is small or approaching minimum spiral radius.

B. Linear adaptive spiral dynamics algorithm

In linear adaptive SDA (LASDA), a novel linear mathematical equation is used to establish relationship between spiral radius and absolute fitness value of a particular point in the search space. The mathematical equation is formulated as:

$$r_{la} = [r_l - r_u] / [1 + [c_1 / |f(x_i(k))|] + r_u] \quad (4)$$

where r_{la} is linear adaptive spiral radius, c_1 is positive constant value and $|f(x_i(k))|$ is absolute fitness value of a particular point. r_u and r_l are maximum radius and minimum radius of spiral path trajectory for a particular point respectively. r_u and r_l must be chosen within $[0, 1]$ to ensure a point in the search space converges towards current best location, which is always located at the centre of spiral trajectory. On the other hand,

positive constant value of c_1 is rate of change of fitness value and spiral radius. Small value of c_1 tends to select maximum radius, r_u while big value of c_1 tends to select minimum radius, r_l . Employing Equation 4 into SDA will vary the spiral radius between r_u and r_l , which directly changes step size of a search point more dynamically and effectively.

C. Quadratic adaptive spiral dynamics algorithm

In the proposed quadratic adaptive SDA (QASDA), Equation 4 is modified by representing the fitness function $f(x_i(k))$ in a quadratic form. Through quadratic formulation of fitness value, the spiral radius and the step size of a search point are changed dynamically in a quadratic manner. Compared to linear approach, quadratic formulation helps the search point to further accelerate towards best location. The mathematical equation of QASDA is formulated as:

$$r_{qa} = [r_l - r_u] / [1 + [c_1 / [c_2 [|f(x_i(k))|^2 + |f(x_i(k))|]] + r_u]] \quad (5)$$

where r_{qa} is quadratic adaptive spiral radius, c_2 is tuneable constant value which is heuristically determined and $[|f(x_i(k))|^2 + |f(x_i(k))|]$ is quadratic term of fitness function. Other parameters of Equation 5 are similar to those of Equation 4. Quadratic term of fitness function in Equation 5 produces a steeper slope of spiral radius versus fitness value compared to linear term in Equation 4. This indicates that the search point has higher acceleration towards the best location.

D. Exponential adaptive spiral dynamics algorithm

In exponential adaptive SDA (EASDA), Equation 4 is modified by representing the fitness function $f(x_i(k))$ in an exponential form. Through exponential formulation of fitness value, the spiral radius and the step size of a search point are changed dynamically in exponential manner. Compared to linear and quadratic approaches, exponential formulation helps the search point to further accelerate towards best location. The mathematical formulation of EASDA is given as:

$$r_{ea} = [r_l - r_u] / [1 + [c_1 / [\exp[c_2 [|f(x_i(k))|]]]] + r_u] \quad (6)$$

where r_{ea} is exponential adaptive spiral radius, $\exp[c_2 [|f(x_i(k))|]]$ is exponential term of fitness function and c_2 is tuneable constant value which is heuristically determined. Other parameters of Equation 6 are similar to those of Equation 4. Exponential term of fitness function in Equation 6 produces a sharper slope of spiral radius versus fitness value compared to linear term in Equation 4 and quadratic term in Equation 5. This indicates that the search point has higher acceleration towards global best location.

E. The adaptive spiral dynamics algorithm

Most of the steps in ASDA are similar to the steps in SDA. The major difference is that constant spiral radius r in SDA is replaced by adaptive spiral radius r_{fa} , r_{la} , r_{qa} and r_{ea} . The parameters and descriptions used in an n-dimensional adaptive spiral dynamics optimization algorithm are presented in Table I and the step-by-step algorithm is shown in Fig. 3. Notice that

the changes are in step 2 of Fig. 3, where each point within the search space at every iteration has different spiral radius depending on its fitness value in the current location. It also may help the algorithm to avoid getting trapped into local optima. Moreover, the simple structure of SDA is preserved. Dynamic spiral radiuses that directly contribute to more dynamic step sizes result in faster convergence and reduced computing time for the whole algorithm.

Step 0: Preparation

Select the number of search points $m \geq 2$, parameters $0 \leq \theta < 2\pi, 0 < r < 1$ of $S_n(r, \theta)$, and maximum iteration number, k_{\max} . Set $k = 0$.

Step 1: Initialization

Set initial points $x_i(0) \in R^n, i = 1, 2, \dots, m$ in the feasible region at random and centre x^* as $x^* = x_{i_g}(0)$, $i_g = \arg \min_i f(x_i(0)), i = 1, 2, \dots, m$.

Step 2: Updating x_i

$$x_i(k+1) = S_n(r, \theta)x_i(k) - (S_n(r, \theta) - I_n)x^*$$

$$i = 1, 2, \dots, m.$$

where spiral radius, r can be replaced by r_{fa}, r_{la}, r_{qa} or r_{ea} as shown in Equations 2, 4, 5 and 6 respectively.

Step 3: Updating x^*

$$x^* = x_{i_g}(k+1),$$

$$i_g = \arg \min_i f(x_i(k+1)), i = 1, 2, \dots, m.$$

Step 4: Checking termination criterion

If $k = k_{\max}$ then terminate. Otherwise set $k = k + 1$, and return to step 2.

Fig. 3. An n-dimensional adaptive spiral dynamics algorithm.

IV. VALIDATION TEST AND RESULTS

In this section, the proposed adaptive algorithms are validated through simulation tests on two 3-dimensional uni-modal and two 2-dimensional multi-modal benchmark functions. Comparison with the original version of SDA tested on the four-benchmark functions is also given in terms of iteration number and CPU computation time to show the improved performance achieved with ASDA. As a means of comparison, parameters for a test function such as $\theta = \pi/4$, $m = 30$ and $k_{\max} = 200$ were kept the same for all adaptive SDA approaches and standard SDA. The difference is the spiral radius value in which $r = 0.96$ was used for SDA while $r = [r_l, r_u]$ was used for all ASDAs as generated from Equations 2-6. The parameters used in the simulation were chosen heuristically for all test functions.

Table II shows adaptive spiral radius ranges used in the proposed ASDA for all the benchmark functions considered. Notice that different adaptive spiral radius ranges were required to get the best results for different benchmark functions.

TABLE II. ADAPTIVE SPIRAL RADIUS FOR THE TEST FUNCTIONS

Cost function name	FASDA	LASDA	QASDA	EASDA
Sphere	[0.7,0.87]	[0.78,0.87]	[0.71,0.86]	[0.7,0.85]
Ackley	[0.7,0.85]	[0.75,0.83]	[0.2,0.82]	[0.2,0.85]
Rastrigin	[0.75,0.94]	[0.81,0.91]	[0.82,0.91]	[0.82,0.93]
Griewank	[0.8,0.9]	[0.8,0.86]	[0.8,0.84]	[0.51,0.86]

A. Uni-modal sphere function

The sphere function is defined as:

$$f(x) = \sum_{i=1}^n x_i^2 \quad (7)$$

The function has global minimum at $x_i = [0, 0, 0]$ with fitness $f(x) = 0$. In this simulation, the sphere function was considered to have dimension $n = 3$ and variable x_i in the range $[-5.12, 5.12]$. The convergence plot for 3 dimensional sphere function thus achieved is shown in Fig 4.

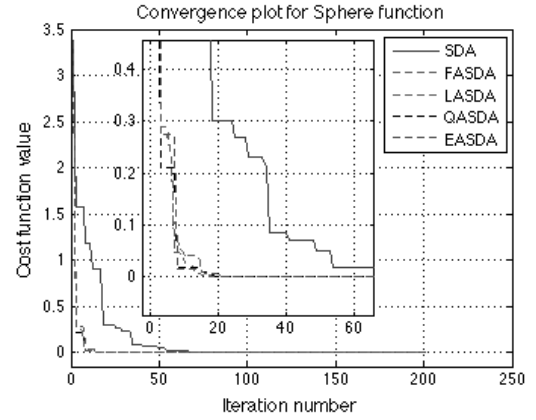


Fig. 4. Convergence plot for 3D sphere function.

B. Uni-modal Ackley function

The uni-modal Ackley function is mathematically defined as:

$$f(x) = -20 \exp(-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^n x_i^2})$$

$$- \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e \quad (8)$$

The function has global minimum at $x_i = [0, 0, 0]$ with fitness $f(x) = 0$. The Ackley function was considered with dimension $n = 3$ and variable x_i in the range $[-32.768, 32.768]$. The resulting convergence plot for 3-dimension Ackley function is shown in Fig 5.

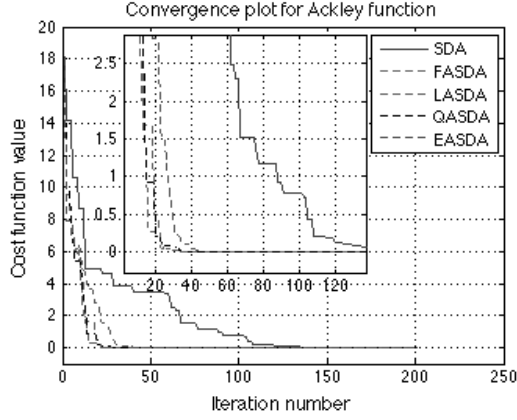


Fig. 5. Convergence plot for 3D Ackley function.

C. Multi-modal Rastrigin function

The Rastrigin function is defined as:

$$f(x) = \sum_{i=1}^n [x_i^2 - 10 \cos(2\pi x_i) + 10] \quad (9)$$

The function has global minimum at $x_i = [0, 0]$ with fitness $f(x) = 0$. The Rastrigin function was considered with dimension $n = 2$ and variable x_i in the range $[-5.12, 5.12]$. The resulting convergence plot for the 2-dimensional Rastrigin function is shown in Fig 6.

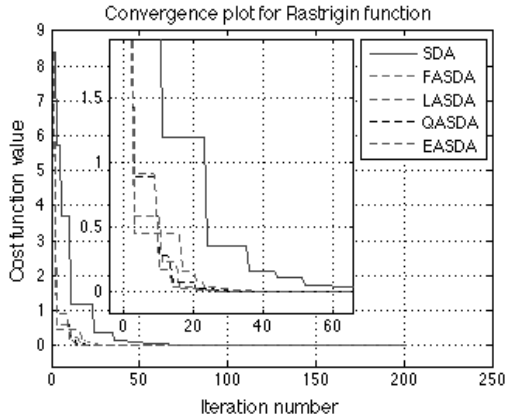


Fig. 6. Convergence plot for 2D Rastrigin function.

D. Multi-modal Griewank function

The Griewank function is defined as:

$$f(x) = \frac{1}{4000} \sum_{i=1}^n x_i^2 - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1 \quad (10)$$

The function has global minimum at $x_i = [0, 0]$ with fitness $f(x) = 0$. The Griewank function was considered with dimension $n = 2$ and variable x_i in the range $[-600, 600]$. The resulting convergence plot for the 2-dimensional Griewank function is shown in Fig 7.

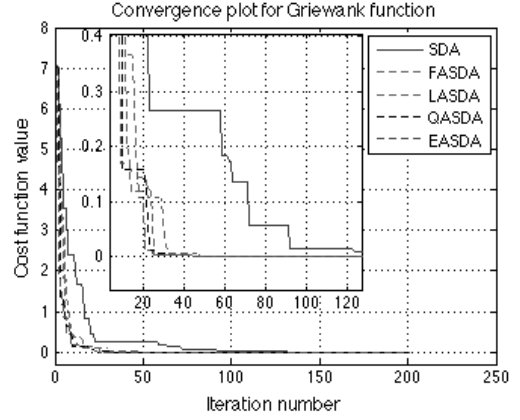


Fig. 7. Convergence plot for 2D Griewank function.

It can be clearly seen in Figures 4-7 that all ASDA approaches outperformed SDA in terms of convergence speed and improved accuracy. Numerical results of SDA, FASDA, LASDA, QASDA and EASDA performance tests with the benchmark functions are shown in Tables III, IV, V, VI and VII respectively in terms of CPU computation time in seconds and iteration number (in bracket) and in terms of accuracy. It is noted that all ASDA approaches have achieved better performance than SDA in terms of speed of convergence based on algorithm iteration number and in terms of accuracy for all test functions. Notice that for the speed of convergence based on CPU computation time, the results show that adaptive fuzzy logic approach consumed more computer CPU time and consequently resulted in lower speed of convergence. This is due to the relatively complex fuzzy structure compared to other proposed adaptive approaches and standard SDA, which used simple mathematical formulation and single constant value respectively to produce spiral radius. Nevertheless, all the adaptive approaches with mathematical formulation such as LASDA, QASDA and EASDA outperformed the standard SDA. On the other hand, for the performance of the optimization algorithms in terms of accuracy, the best fitness value for sphere function was 9×10^{-32} and it was 0 for Ackley function, both achieved by FASDA. Moreover, FASDA, LASDA, QASDA and EASDA resulted in the lowest and the best fitness values for Rastrigin and Griewank functions.

TABLE III. SDA PERFORMANCE ON BENCHMARK FUNCTIONS

Cost Function Name	Performance				
	Best fitness	Converge time (iteration)	X_1	X_2	X_3
Sphere	1×10^{-5}	0.069sec (67)	2×10^{-3}	2×10^{-3}	2×10^{-3}
Ackley	5×10^{-3}	0.170sec(135)	6×10^{-4}	-2×10^{-3}	1×10^{-3}
Rastrigin	4×10^{-7}	0.074sec (96)	-4×10^{-6}	5×10^{-5}	-
Griewank	1×10^{-5}	0.098sec (92)	5×10^{-3}	-7×10^{-4}	-

TABLE IV. FASDA PERFORMANCE ON BENCHMARK FUNCTIONS

Cost Function Name	Performance				
	Best fitness	Converge time (iteration)	X_1	X_2	X_3
Sphere	9×10^{-32}	0.817sec (17)	-2×10^{-16}	-2×10^{-16}	2×10^{-16}
Ackley	0	1.431sec (35)	4×10^{-15}	-3×10^{-17}	-2×10^{-16}
Rastrigin	0	1.670sec (25)	3×10^{-9}	-2×10^{-9}	-
Griewank	0	1.060sec (25)	7×10^{-9}	5×10^{-9}	-

TABLE V. LASDA PERFORMANCE ON BENCHMARK FUNCTIONS

Cost Function Name	Performance				
	Best fitness	Converge time (iteration)	X_1	X_2	X_3
Sphere	7×10^{-28}	0.024sec (16)	3×10^{-15}	3×10^{-14}	7×10^{-15}
Ackley	4×10^{-15}	0.060sec (45)	2×10^{-15}	1×10^{-15}	8×10^{-16}
Rastrigin	0	0.049sec (37)	9×10^{-10}	6×10^{-10}	-
Griewank	0	0.059sec (33)	2×10^{-9}	1×10^{-8}	-

TABLE VI. QASDA PERFORMANCE ON BENCHMARK FUNCTIONS

Cost Function Name	Performance				
	Best fitness	Converge time (iteration)	X_1	X_2	X_3
Sphere	5×10^{-28}	0.023sec (19)	2×10^{-14}	9×10^{-15}	3×10^{-15}
Ackley	0	0.062sec (46)	2×10^{-16}	3×10^{-16}	-2×10^{-16}
Rastrigin	0	0.040sec (30)	2×10^{-10}	8×10^{-10}	-
Griewank	0	0.054sec (27)	8×10^{-9}	4×10^{-10}	-

TABLE VII. EASDA PERFORMANCE ON BENCHMARK FUNCTIONS

Cost Function Name	Performance				
	Best fitness	Converge time (iteration)	X_1	X_2	X_3
Sphere	1×10^{-31}	0.028sec (21)	8×10^{-17}	2×10^{-16}	3×10^{-16}
Ackley	9×10^{-14}	0.087sec (47)	1×10^{-14}	2×10^{-14}	-4×10^{-14}
Rastrigin	0	0.038sec (22)	9×10^{-10}	9×10^{-10}	-
Griewank	0	0.049sec (25)	1×10^{-9}	7×10^{-9}	-

V. CONCLUSION

Four novel adaptive spiral dynamics optimization algorithms have been proposed. Adaptation strategies based on mathematical and non-mathematical fuzzy logic intelligent methods have been presented without adding extra complexity

to the original algorithm structure. Simulation results have shown that the proposed adaptive algorithm outperforms SDA in terms of speed of convergence based on algorithm iteration number and in terms of accuracy. However, in terms of speed of convergence based on CPU computation time, fuzzy adaptive approach needed longer time to execute the algorithm compared to other adaptive approaches and SDA. It has been revealed that further simplification of fuzzy logic approach is required and computation time in seconds need to be taken into account before fuzzy logic approach can be applied to real world problems. The results also show that all the proposed adaptive approaches have high potential for real world applications.

ACKNOWLEDGMENT

Ahmad Nor Kasruddin Bin Nasir is on study leave from Faculty of Electrical & Electronics Engineering, Universiti Malaysia Pahang (UMP) and is sponsored by Ministry of Higher Education (MOHE) Malaysia.

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