APPROXIMATE MODELS IN MULTIVARIABLE PROCESS CONTROL: AN INVERSE NYQUIST ARRAY AND ROBUST TUNING REGULATOR INTERPRETATION

By

D.H.Owens B.Sc., A.R.C.S., Ph.D., FIMA., C.Eng., MIEE

and

A. Chotai B.Sc., Ph.D.

Research Report No. 261

July 1984

Department of Control Engineering
University of Sheffield
Mappin Street,
Sheffield S1 3JD

This work was supported by SERC, under grant GR/B/23250

Keywords: Multivariable systems; process control; approximation; reduced order models; inverse Nyquist array; robust regulation; tuning regulators

---

Abstract

Using the widely accepted philosophy that simple control design procedures are of great value in producing acceptable controllers for plant with a degree of dynamic and/or structural simplicity, particularly in the situation where closed-loop performance requirements are modest, recent work on the use of approximate models and plant step data in multivariable design is applied to produce simple, robust design algorithms for process plant which, in the presence of suitable precompensation, has 'small' interaction effects and/or simple loop dynamics that can be approximated by delay-lag models. In one form, the results have a structure and generate a procedure identical to the inverse Nyquist array design technique with Gershgorin circles replaced by 'confidence circles' deduced from simple graphical operations on plant step data. Other interpretations of the work lead to estimates of the maximum gain required to retain stability in Davison's robust feedback regulator. An optimization procedure for choice of precompensator using step data is also outlined with strong connections with the method of pseudo-diagonalization.

1. Introduction

In process control, simple design techniques such as that due to Ziegler and Nichols [1] based on simple delay-lag approximate models of plant behaviour play an important role in simplifying the design process. It is natural therefore to conjecture that a similar philosophy could play a useful role in simplifying the process of multivariable design and/or generate preliminary insight into a design problem before more detailed analysis or on-line tuning. In fact, the use of reduced-order models for representation of process dynamics is now an accepted form of design aid and a rigorous theory for the use of such models has been described by the authors elsewhere [2]. In order to use this framework to generate simple design methods for process plant however, it is necessary to recognize that the enriched structure of multivariable (MIMO) systems (as compared with single-input-single-output (SISO) systems) will require the use of different forms of approximate models to suit different dynamic and structural properties of the plant under consideration.

It is the purpose of this paper to investigate the possibility of and to provide detailed descriptions of suitable approximate models and simple design procedures for process plant that, in the presence of suitably chosen precompensation, has one or more of the following properties:

(a) Interaction effects that, on physical grounds, are anticipated to have only second order effects on closed-loop stability and performance.

(b) Simple step response dynamics that can be roughly approximated by first-order 'delay-lag' dynamics.

Throughout the analysis, we take the viewpoint strongly proposed by Davison [3] and Åstrom [4] and accept the simplest and most straightforward use of available plant data to produce the simplest possible design procedure on the understanding that many process control applications have modest closed-loop performance requirements and an 'inherent robustness' that makes possible the achievement of a successful design despite the uncertainties introduced by gross neglect of detailed plant dynamics.

The plant data assumed again fits into the framework processed by Davison [3] and Åstrom [4] (together with the authors [2]). More precisely, it is assumed that the designer has access to reliable open-loop plant step-data as described in section 2. With this assumption, section 3 uses diagonal approximate models of the (precompensated) plant to extend the inverse Nyquist array (INA) design procedure [5], [6] to cope with modelling errors and the use of step data by replacing Gershgorin circles by 'confidence circles' deduced from plant step data and producing results on the system integrity properties. Section 4 then considers the use of delay-lag models of (precompensated) plant dynamics in the framework of Davisons robust tuning regulator [3] to produce simple estimates of the maximum stabilizing gain as a prelude to 'on-line' control system tuning. Section 5 suggests computational tools for the construction of suitable constant precompensators. Illustrative examples are given in section 6.

2. Data Requirements and Data Processing.

Throughout the remainder of the paper, it is assumed that the plant is stable, has ℓ-inputs and m-outputs with $\ell \geq m$ and is described by a (possibly unknown) transfer function matrix (TFM) G(s). It is also assumed that plant tests or simulations of an available (assumed complex) model of plant dynamics give the designer access to a reliable estimate of the plant open-loop step-response matrix.

$$Y(t) \triangleq \begin{bmatrix} Y_{11}(t) & \cdots\cdots & Y_{1\ell}(t) \\ & & \\ Y_{m1}(t) & \cdots\cdots & Y_{m\ell}(t) \end{bmatrix} \quad , \; t \gtrless o \qquad (1)$$

where $Y_{ij}(t)$ is the response from zero initial conditions of the i^{th} output y_i to a unit step in the j^{th} input $u_j(t)$ with $u_k(t) \equiv o$, $k \neq j$. We let K_p denote an ℓxm constant precomparator (as yet unspecified) and write the step response matrix of $G^p \triangleq GK_p$ as

$$Y_p\,(t) \triangleq Y(t)\,K_p \quad , \; t \geq o \qquad (2)$$

Note that Y_p is square of dimension mxm.

Following the procedure of [2], $G_A(s)$ denotes a stable (and, possibly, highly simplified) approximate model of the dynamics of the precompensated plant $G^p = GK_p$ and K_c denotes an mxm compensator designed to produce the required stability and performance from G_A in the presence of diagonal measurement dynamics $F(s) = \text{diag}\,\{f_j(s)\}_{1 \leq j \leq m}$. The control system to be implemented on the real plant G has forward path TFM

$$K(s) = K_p\,K_c\,(s) \qquad (3)$$

and measurement dynamics F.

A major objective of the design analysis is to ensure that the design procedure takes full advantage of the simple structure of G_A to reduce conceptual and computational complexity and produce insight into a possible control structure whilst assuming that the resultant design will regulate the plant despite the modelling errors $G^p - G_A$ between the real plant and the model used for design purposes.

To ensure this vital design objective, it is, of course, necessary to use some numerical measures of the effect of the modelling errors on stability. Following the philosophy of Davison [3] and Åstrom [4], we will take the simplest and most straightforward approach to data usage as outlined in the introduction. More precisely, the designer is expected to compute the step response matrix $Y_A(t)$, $t \geq o$, of G_A and to store the modelling error data

$$E^p(t) \triangleq Y_p(t) - Y_A(t) \ , \ t \geq o \tag{4}$$

This error can be processed in many ways but, in the remainder of the paper, the error will be represented by its 'matrix total variation' [2] or matrix total variations of filtered versions. More precisely, let $F_{ij}(s)$, $1 \leq i,j \leq m$, denote a chosen set of proper filters and define $E^p_F(t)$ to be the mxm matrix whose (i,j) element is the result of passing E^p_{ij} through the filter F_{ij}. Using the easily proved identity

$$|W(s)| \leq |s|^{-1} \ |w(\infty)| + \int_0^\infty |w(t) - w(\infty)| \, dt \ , \ \text{Re}s \geq o \tag{5}$$

valid for any scalar function $w(t)$, $t \geq o$, with the property that $w(\infty) = \lim_{t \to \infty} w(t)$ exists and possessing a Laplace transform $W(s)$, it follows that, for $1 \leq i,j \leq m$, and $\text{Re}s \geq o$,

$$\begin{aligned} &|F_{ij}(s)\ (G^p_{ij}(s) - (G_A(s))_{ij})| \\ &\leq \ |s|^{-1} \ |e^F_{ij}(\infty)| + \int_0^\infty | e^F_{ij}(t) - e^F_{ij}(\infty) | dt \end{aligned} \tag{6}$$

where $e^F_{ij}(t)$ is the impulse response of $F_{ij}\ (G^p_{ij} - (G_A)_{ij})$ and

$$e^F_{ij}(\infty) = \lim_{t \to \infty} \ e^F_{ij}(t) = \lim_{s \to o} s\, F_{ij}(s)\ (G^p_{ij}(s) - (G_A(s)_{ij}) \tag{7}$$

Defining

$$(Z^p_F(t))_{ij} = \int_o^t (e^F_{ij}(t') - e^F_{ij}(\infty))dt' \equiv (E^p_F(t))_{ij} - e^F_{ij}(\infty)\ t \tag{8}$$

and, using the results of [2], we can bound the gain of the modelling error by

$$|G^p_{ij}(s) - (G_A(s))_{ij}| \leq |F_{ij}^{-1}(s)| \ \{ |s|^{-1} \ |e^F_{ij}(\infty)| + N_\infty((Z^p_F)_{ij})\} \tag{9}$$

where $N_\infty(f)$ denotes [2] the total variation of f as deduced from graphical inspection of its time variation.

The choice of filters F_{ij} is open to the designer subject to the constraint of boundedness of $N_\infty((Z_F^P)_{ij})$ and existence of $e_{ij}^F(\infty)$. Two cases are considered in this paper as representative of the simplest possible procedures:

Case 1: Choosing $F_{ij} \equiv 1$ leads to $(Z_F^P)_{ij} = E_{ij}^P$ and $e_{ij}^F(\infty) = o$ so that, from (6),

$$\left| G_{ij}^P(s) - (G_A(s))_{ij} \right| \leq \Delta_{ij}^{(1)}(s) \triangleq N_\infty(E_{ij}^P) \quad , \ \mathrm{Re}\, s \geq o \tag{10}$$

which is precisely the bound used in [2]. It represents the best possible constant upper bound in the sense that it is generally applicable and equality holds at $s = o$ if E_{ij}^P is monotonic. Other choices of F_{ij} permit some frequency dependence of the bound and hence, in principle, allows tighter characterizations to hold. For example,

Case 2: Choosing $F_{ij}(s) = {}^1/s$ leads to $e_{ij}^F(t) = E_{ij}^P(t)$ and hence

$$(Z_F^P(t))_{ij} = \int_o^t (E_{ij}^P(t') - E_{ij}^P(\infty))\, dt' \tag{11}$$

is just the integrated modelling error. This gives the bound

$$|G_{ij}^P(s) - (G_A(s))_{ij}| \leq \Delta_{ij}^{(2)}(s) \triangleq |E_{ij}^P(\infty)| + |s|\, N_\infty((Z_F^P)_{ij}) \quad \mathrm{Re}\, s \geq o \tag{12}$$

Note that (10) and (11) can be combined to yield the improved bound

$$\left| G_{ij}^P(s) - (G_A(s))_{ij} \right| \leq \min\{\Delta_{ij}^{(1)}(s), \Delta_{ij}^{(2)}(s)\} \quad , \ \mathrm{Re}\, s \geq o \tag{13}$$

The improvement is guaranteed at low frequencies as $|E_{ij}^P(\infty)| \leq N_\infty(E_{ij}^P)$ from the definition of the total variation.

Finally, we note that non-proper filters can be incorporated in the formulation. To illustrate this, let

$$V^P(t) \triangleq \frac{dE^P(t)}{dt} \quad , \ t \geq o \ , \tag{14}$$

then clearly, for Re $s \geq o$,

$$\begin{aligned} |s(G_{ij}^P(s) - (G_A(s))_{ij})| &= \left| V_{ij}^P(o+) + \int_o^\infty e^{-st} \dot{V}_{ij}^P(t)\, dt \right| \\ &\leq |V_{ij}^P(o+)| + \int_o^\infty |\dot{V}_{ij}^P(t)|\, dt \\ &= N_\infty(V_{ij}^P) \end{aligned} \tag{15}$$

leading to the bound

$$|G^p_{ij}(s) - (G_A(s))_{ij}| \leq \Delta^{(3)}_{ij}(s) \triangleq |s|^{-1} N_\infty(V^p_{ij})$$
$$\text{Re } s \geq o \qquad (16)$$

If a model of the plant is available then V^p can be computed reliably but, if Y(and hence E^p) is obtained from plant tests, noise contamination of the signal will make the derivative unreliable. It is expected therefore that the result will only be of practical use when a detailed model of the plant S(A,B,C) is available to be used to obtain V^p as the step response matrix of S(A,B,CA,CB). Comparing $\Delta^{(3)}_{ij}$ with $\Delta^{(1)}_{ij}$ and $\Delta^{(2)}_{ij}$ we note that it provides tighter bounds on the modelling error at <u>high</u> frequencies. It can be combined with these estimates to produce the bound

$$|G^p_{ij}(s) - (G_A(s))_{ij}| \leq \min \{\Delta^{(k)}_{ij}(s) : k=1,2,3\} \quad , \text{Re} s \geq o \qquad (17)$$

Finally, the bounds described above are easily computed but do contain a degree of conservatism when compared with bounds from a detailed plant model. For example, if

$$G^p_{ij}(s) - (G_A(s))_{ij} = \frac{s}{(1+4s)(1+5s)} \qquad (18)$$

the bounds are shown in Fig 1 together with the exact form of the error gain indicating that the characteristics are overestimated by up to 150 %. This situation can easily be improved by the use of more detailed calculation but is not pursued further here as it is inconsistant with our declared philosophy of minimal data usage for a simplest possible design procedure.

3. Diagonal Models for Systems with Small Interaction

With the notation defined in section 2 we now concentrate on the special case of process plant G for which it is possible to choose a constant precompensator K_p such that the interaction effects in the precompensated plant $G^p = GK_p$ (as described by the off-diagonal terms of Y_p) are sufficiently 'small' as to make the designer suspect that the performance objectives can be achieved by neglecting them during the design process. In a similar manner to the work described in [2], thus suspicion can be converted into a systematic design procedure by choosing a diagonal approximate model

$$G_A(s) = \text{diag}\{g_j(s)\}_{1 \leq j \leq m} \qquad (19)$$

of the precompensated plant G^p by inspection of $Y_p(t)$ followed by the construction of models of the diagonal terms of the required complexity. The resultant procedure is similar in structure to that implied by theorem 3 in [2]. The contribution of this section therefore is primarily to demonstrate that those results can be extended to provide a complete generalization of the INA [5], [6] design procedure to cope with approximate plant representations and limited plant data obtained from step response experiments. In particular, the work in [2] is extended to include

(i) more detailed frequency domain bounds deduced from plant step data,

(ii) constant precompensation to improve the accuracy of a diagonal model,

(iii) graphical design aids based on 'row' and 'column' dominance and spectral radius methods using the magnitudes of interaction effects as observed in the time-domain, and

(iv) integrity assessment procedures based on time-domain data in a frequency-domain context.

The resultant technique has all the basic ingredients of the INA method with the exception of performance assessment using Ostrowski bands. This can however be approached using the simulation method described in [2].

Given the diagonal model (19) of plant dynamics, it is natural to consider the use of single-loop compensators in the form of the diagonal compensator.

$$K_c(s) = \text{diag}\,\{k_j(s)\}_{1 \leq j \leq m} \tag{20}$$

where the loop control $k_j(s)$ is designed to produce stability and desirable loop dynamics for the loop model $g_j(s)$ in the presence of loop measurement dynamics $f_j(s)$. As in the INA procedure the interaction effects (and, in this case, errors in modelling the diagonal terms) are ignored at this stage. As in the INA procedure however, the prediction of stability of the implemented closed-loop feedback scheme for the real plant G with control $K=K_pK_c$ and measurement dynamics F must take into account the modelling error. It is therefore assumed that the data $E^p(t)$, $t \geq o$, has been processed in the way described in section 2 (or otherwise) to provide an array of error bounds.

$$|G^p_{ij}(s) - (G_A(s))_{ij}| \leq \Delta_{ij}(s),\ 1 \leq i,j \leq m,\ \text{Re}s \geq o \tag{21}$$

Given this data, the following result provides a systematic INA-type graphical technique for assessing the stability of the implemented scheme. The proof is similar to that of theorem 3 of [2] and is outlined only for brevity.

<u>Theorem 1:</u> Suppose that the control elements $k_j(s)$ $1 \leq j \leq m$, are designed to produce stability and desirable loop dynamics from the loop models $g_j(s)$, $1 \leq j \leq m$, in the presence of measurement dynamics $f_j(s)$, $1 \leq j \leq m$, and that the composite system GK_pK_cF is both controllable and observable. Then the control $K=K_pK_c$ will stabilize the real plant G in the presence of measurement dynamics F if

(i) the inequality

$$\lim_{\substack{\text{Re } s \geq 0 \\ |s| \to \infty}} \sup \left| \frac{k_j(s)\; f_j(s)}{1+k_j(s) f_j(s)\; g_j(s)} \right| d_j(s) < 1 \tag{22}$$

is satisfied for $1 \leq j \leq m$, and

(ii) the 'confidence bands' generated by plotting the inverse Nyquist locus of $g_j(s)\; k_j(s)\; f_j(s)$ for $s=i\omega, \omega \geq 0$, with superimposed 'confidence circles' at each point of radius

$$r_j^p(s) \triangleq |g_j^{-1}(s)|\; d_j(s) \tag{23}$$

does not contain or touch the (-1,0) point of the complex plane.

In the above, at each point s of interest, the functions $d_j(s)$, $1 \leq j \leq m$, can be taken to be equal to any of the following error dependent quantities,

(a) $d_j(s) \triangleq \sum_{i=1}^{m} \Delta_{ji}(s)$ (row-sum) (24)

(b) $d_j(s) \triangleq \sum_{i=1}^{m} \Delta_{ij}(s)$ (column-sum) (25)

or

(c) $d_j(s) \triangleq r\,(\Delta(s))$ (spectral-radius estimate) (26)

where $\Delta(s)$ is the mxm matrix with $(i,j)^{th}$ element $\Delta_{ij}(s)$ and r(M) denotes the spectral radius (maximum modulus of eigenvalues) of a matrix M.

<u>Proof:</u> Using the results of [2] with G_A approximating the precompensated plant G^P, closed-loop stability is guaranteed if the controllability and observability conditions are satisfied and (using the notation of [2])

$$r\,(||(I_m + K_c(s)\; F(s)\; G_A(s))^{-1} K_c(s)\; F(s)||_p \Delta(s)) < 1 \tag{27}$$

for all s on the Nyquist contour. At any point s on the contour, (27) is satisfied if any of the following three conditions holds

$$\max_{1 \leq j \leq m} \left| \frac{k_j(s)\ f_j(s)}{1+g_j(s)\ k_j(s)\ f_j(s)} \right| d_j(s) < 1 \tag{28}$$

where $d_j(s)$ is given by (24), (25) or (26). This requirement leads to the conditions (i) and (ii) of the theorem as required.

We make the following observations

(1) the graphical interpretation of (ii) has been given in [2],

(2) the error data $\Delta_{ij}(s)$ can, in principle, be obtained by any means. It is assumed in this paper that it has been obtained by processing plant step data as described in section 2. It does however permit the designer the flexibility of obtaining the data in other ways if the data is available or if the effort in obtaining the data is feasible or thought to be worthwhile or necessary. In the 'best' case, for example, we could choose $\Delta_{ij}(s) = |G^P_{ij}(s) - (G_A(s))_{ij}|$ when the result boils down essentially to the stability theorem underlying the Direct Nyquist array [5], [6].

(3) The radii of the confidence circles increase as the modelling error increases (in the time-domain). In particular they are small if G_A is a good representation of the step response dynamics of $G^P = GK_p$.

(4) For a given modelling error, the result limits the predicted control gains that can be applied to ensure stability. This problem is shared by the INA method but the effect can be partially offset by choice of K_p or by 'reducing control gains' into the predicted maximum range. It can also be offset by using improved models G_A but, if this option is not taken, it is clear that closed-loop performance is limited by the modelling error. In high performance designs this limitation may not be acceptable, the designer being committed to detailed analysis of plant dynamics.

In process control applications however, where performance requirements are modest, the implicit gain limitation will have little effect as a successful design can still be achieved.

(5) The controllability and observability requirement holds generically but can be reduced to stabilizability and detectability of GK_pK_cF if stable uncontrollable and/or unobservable modes are acceptable. This is described in more detail in ref [2].

Overall, the above analysis indicates that the basic INA methodology carries over with little change to CAD of control systems based directly on processed plant step data and the use of approximate system models. Our final result in this section underlines this fact by showing that integrity of the closed-loop system can also be assessed in a simple manner.

Theorem 2: If the conditions of theorem 1 are satisfied, then the resultant feedback scheme is stable in the presence of simultaneous failures in the sensors $f_j(s)$ and/or actuators $k_j(s)$, $j = j_1, j_2, .., j_q$, provided that there networks are stable.

(Remark: In practice, this reduces to the requirement that there are no integrators in loops $j_1, j_2, ..., j_q$).

Proof: The failure conditions correspond to the situation $f_j(s)k_j(s) \equiv o$, $j = j_1, j_2, ..., j_q$. Condition (28) is hence still satisfied and stability is guaranteed [2].

In the following section, similar results to the above are constructed to provide information on Davidson's tuning regulator [3].

4. Robust Tuning Regulation

Given a stable, linear ℓ-input/m-output, time-invariant process whose model is either unknown or known, the work of Davison [3] and Porter [7] has shown how plant step data can be used as the basis of an on-line tuning procedure for a simple control scheme capable of ensuring plant stability, asymptotic tracking of step set-point changes and rejection of constant disturbances at the output. The basic idea of the method can be expressed (in a mildly generalized form) in terms of the notation of sections 1-3 for the precompensated plant $G^p = GK_p$. More precisely, if $G^p(o)$ in the matrix of precompensated open-loop d.c. gains obtained from plant step responses via the relation $G^p(o) = Y_p(\infty)$, then a unity negative output feedback scheme with controller TFM $K = K_pK_c$ and (similar to the form originally proposed by Rosenbrock [5]),

$$K_c(s) = \varepsilon\, \hat{G}^p(o)\, \frac{1}{s} \tag{29}$$

is capable [3] of achieving the specified objectives for scalar gains ε in a non-empty range

$$o < \varepsilon < \varepsilon^* \tag{30}$$

provided that the required inverse exists,

(Note: In the following, the symbol $\hat{M}$ will frequently be used to denote the inverse of a matrix M).

The possibility of including proportional action has been noted in [3], [7], [8], [9] but, in all cases, the upper gain bound ε^* is unknown, being revealed only at the on-line tuning stage. In this section we consider the problem of obtaining easily computed off-line estimates of lower bounds $\varepsilon_o^* > o$ for ε^*. Given such a lower bound, stability is then known to be guaranteed in the gain range $o < \varepsilon < \varepsilon_o^*$. The availability of such information could be of great practical value in applications (see eg [3], [10]) of tuning regulator theory by providing a range of initial gain settings that guarantee stability and from which on-line tuning can be initiated. A large value of ε_o^* indicates large stability margins whilst a small value suggests that gains may be severely limited in practice.

Exact evaluation of ε^* requires, in principle, root-locus [6] or similar analysis of a detailed plant model. To retain the spirit of Davisons work however, we concentrate here on the use of minimal step response information that is required to produce an estimate i.e. graphical operations on the results of plant step tests in a manner similar to that described in section 2 above.

Following the procedure of sections 2 and 3 suppose that a constant precompensator K_p has been specified and the (precompensated) plant step response $Y_p(t)$, $t \geq o$, computed and stored. To obtain a gain estimate we consider the use of a (conceptual) approximate plant model of the form

$$G_A(s) = \tilde{G}^p(o) \text{ diag } \left\{ \frac{e^{-s\mathcal{T}_j}}{1+sT_j} \right\}_{1 \leq j \leq m} \tag{31}$$

when $T_j \geq o$, $1 \leq j \leq m$, and $\mathcal{T}_j$, $1 \leq j \leq m$ are representative time constants and delays of the (precompensated) plant responses to step inputs in channel j as deduced from inspection of the elements of $Y_p(t)$, $t \geq o$. $\tilde{G}^p(o)$ is a nonsingular estimate or conveniently structured (e.g. diagonal) approximation of the d.c. gain $G^p(o) = Y_p(\infty)$. The consequent modelling error $E^p(t) = Y_p(t) - Y_A(t)$, $t \geq o$, can now be computed. Note that both G^p and G_A have identical steady state characteristics if and only if, $\tilde{G}^p(o) = G^p(o)$ when it is easily seen that

$$\lim_{t \to \infty} E_p(t) = o \tag{32}$$

Throughout the remainder of this section, the basic form of Davisons controller is extended to include proportional action and loop gains of differing magnitude i.e.

$$K_c(s) \triangleq \text{diag}\,\{k_1^{(j)} + s^{-1}\,k_2^{(j)}\}_{1\leq j\leq m}\ \hat{\tilde{G}}^P(o) \tag{33}$$

which reduces to (29) if $k_1^{(j)}=o$, $1\leq j\leq m$, $k_2^{(j)}=\varepsilon$, $1\leq j\leq m$, and $\tilde{G}^P(o) = G^P(o)$.

4.1 ε_o^* and the Modelling Error

The techniques proposed for estimating ε_o^* vary depending upon the chosen means of processing the error data $E_p(t)$. A basic result is stated as follows:

Theorem 3: If the precompensated plant $G^P = GK_p$ is stable with $G^P(o)$ nonsingular and it is possible to choose representative time constants $T_j>o$ and delays $\mathfrak{T}_j\geq o$, $1\leq j\leq m$, such that the modelling error generates the inequality.

$$r_\infty \triangleq r\,(||\hat{\tilde{G}}^P(o)||_p\ N_\infty^P\,(E^P)) < \min\ (1, \min_{1\leq j\leq m} (k_1^{(j)})^{-1}) \tag{34}$$

then the controller $K = K_pK_c$ with K_c given by (33) will stabilize the plant G for any gains $k_1^{(j)}\geq o$ $k_2^{(j)}>o$, $1\leq j\leq m$, satisfying

(a) $k_1^{(j)} + s^{-1}\,k_2^{(j)}T_j$ stabilizes $e^{-s\alpha_j}/(s+1)$, $1\leq j\leq m$, and

(b) $$\gamma_\infty \max_{1\leq j\leq m}\ \sup_{s\in D} \left| \frac{(1+s)\ (k_1^{(j)}s+k_2^{(j)}\ T_j)}{s(s+1) + (k_1^{(j)}s + k_2^{(j)}\ T_j)\ e^{-s\alpha_j}} \right| < 1 \tag{35}$$

where D is the usual Nyquist contour in the complex phase, $\alpha_j = \mathfrak{T}_j/T_j$, $1\leq j\leq m$, $\gamma_\infty < \min\,(1, \min_{1\leq j\leq m} (k_1^{(j)})^{-1})$ is a convenient upper bound on r_∞ and $N_\infty^P\,(E_p)$ denotes the 'matrix' total variation of E^P on $[o, \infty]$ (see [2]) ie the matrix with $(i,j)^{th}$ element $N_\infty\,(E_{ij}^P)$.

Proof: Using the results of [2] stability is guaranteed if

$$\sup_{s\in D} r\,\left(\text{diag}\ \left\{ \left| \frac{(1+sT_j)\ (k_1^{(j)}s + k_2^{(j)})}{s(1+sT_j) + (k_1^{(j)}s+k_2^{(j)})e^{-s\mathfrak{T}_j}} \right| \right\}_{1\leq j\leq m} ||\hat{\tilde{G}}^P(o)||_p N_\infty^P\,(E^P)\right) \tag{36}$$

$$< 1$$

as the nonsingularity of $G^P(o)$ ensures the stabilizability and detectability of GK. Inequality (36) is satisfied if

$$\gamma_\infty \max_{1\leq j\leq m}\ \sup_{s\in D} \left| \frac{(1+sT_j)\ (k_1^{(j)}s+k_2^{(j)})}{s(1+sT_j) + (k_1^{(j)}\ s+k_2^{(j)})e^{-s\mathfrak{T}_j}} \right| < 1 \tag{37}$$

(35) following by replacing s by $s' = T_j s$. The necessity of (34) is seen by noting that (35) must be satisfied at s=o and $|s| \to \infty$ and (9) follows from the requirement [2] that K_c stabilizes G_A.

For practical purposes, the result indicates that a range of gains $k_1^{(j)}$, $k_2^{(j)}$ can be obtained by analysis of (35) provided that the precompensated plant G^p can be modelled by the 'delay-lag' model (31) with error E^p small enough to satisfy (34). This situation corresponds to the case when all plant responses are overdamped with essentially delayed first-order characteristics. In many process control situations (34) is not a severe constraint as, for example, in the case of m = 1 and $k_1 < 1$, it reduces to the requirement that the total variation of the modelling error is strictly less than the plant d.c. gain. In the case of plant with oscillatory or violent reverse-kick characteristics (due to low damping or non-minimum phase elements) or badly conditioned d.c. gain matrix however (34) can be violated as a first order model is then not sufficiently representative of plant dynamics.

There are several ways of approaching the analysis of (35) apart from the obvious numerical search procedures. The following corollary provides a simple graphical technique suitable for CAD.

<u>Corollary 3.1.:</u> The conclusions of theorem 3 hold if (a) and (b) are replaced by the equivalent condition that, for $1 \leq j \leq m$, (a) the point $(-(k_2^{(j)} T_j)^{-1}, o)$ of the complex plane does not lie in or on the band generated by plotting the Nyquist locus of the transfer function

$$g_j(s\ ,\ k_1^{(j)}\ ,\ k_2^{(j)} T_j) \triangleq \frac{(k_1^{(j)} s + k_2^{(j)} T_j)\ e^{-s\alpha_j}}{s(s+1)\ k_2^{(j)} T_j} \tag{38}$$

and superimposing at each frequency a circle of radius

$$r_j(s, k_1^{(j)}, k_2^{(j)} T_j) \triangleq \gamma_\infty |k_1^{(j)} + s\ k_2^{(j)} T_j| \ /(|s| k_2^{(j)}\ T_j) \tag{39}$$

and (b) that the band does not encircle that point.

<u>Proof:</u> Write (35) in the form, $1 \leq j \leq m$,

$$|(k_2^{(j)}\ T_j)^{-1} + g_j(s,\ k_1^{(j)},\ k_2^{(j)} T_j)| > r_j(s,\ k_1^{(j)},\ k_2^{(j)}\ T_j) \tag{40}$$

for all s on the Nyquist contour D and interpret in graphical form as illustrated in Fig.2. for $s=i\omega$, $\omega \gtrless o$. It is automatically satisfied for $|s| \to \infty$ by (34).

The similarity in graphical structure of corollary 3.1. with that of theorem 1 is self-evident with the main difference that the radius of the 'confidence circles' now depends upon the control gains. This dependence can be 'removed', in practice, by choosing the ratio $\eta_j = k_1^{(j)}/k_2^{(j)}T_j$ to correspond to a fixed and specified ratio of reset time to time constant in loop j and regarding $\varepsilon_j \triangleq k_2^{(j)}$ as a design variable. The corollary can then be applied using the identities.

$$g_j(s\ ,\ k_1^{(j)}\ ,\ k_2^{(j)}T_j) \equiv g_j(s\ ,\ \eta_j,\ 1)\ ,\quad 1 \leq j \leq m\ , \tag{41}$$

$$r_j(s\ ,\ k_1^{(j)}, k_2^{(j)}T_j) \equiv r_j\ (s\ ,\ \eta_j,\ 1)\ ,\quad 1 \leq j \leq m, \tag{42}$$

and noting that the plots are now independent of ε_j. In these circumstances an easily computed estimate of the largest (predicted) value of ε_j is obtained as

$$o < \varepsilon_j < \frac{1}{\mu_j T_j} \tag{43}$$

where $(-\mu_j, o)$ is the point where the trailing edge of the uncertainty band cuts the negative real axis as illustrated in Fig.2.

The need for graphical analysis can be removed in the case of integral control only and opens up the possibility of 'back-of-envelope' estimates of stabilizing gains. To the best of the authors knowledge this result, together with that of Åstrom [4], are the only available results of this degree of simplicity.

<u>Corollary 3.2.:</u> The conclusion of theorem 3 remains valid in the delay-free integral control case of $k_1^{(j)} = \tau_j = o$, $1 \leq j \leq m$, if (a) and (b) are replaced by the algebraic inequalities

$$o < k_2^{(j)}T_j < \sqrt{2} - 1\ ,\quad 1 \leq j \leq m \tag{44}$$

<u>Proof:</u> Elementary calculus indicates that the supremum in (35) is achieved for some j and frequencies ω_j satisfying $\omega_j = o$ or, if $\beta_j = k_2^{(j)}T_j$,

$$\omega_j^2 = -1 + \sqrt{\beta_j(\beta_j + 2)} \tag{45}$$

Relation (44) ensures that (45) has no real solution and hence (35) reduces to

$\gamma_\infty < 1$ which is satisfied by assumption. Condition (a) is automatically satisfied as (44) is a connected set containing small gains.

In the case of $k_1^{(j)} = o$, $k_2^{(j)} = \varepsilon$, $1 \leq j \leq m$, the control scheme reduces to Davisons controller and corollary 3.2. produces the estimate

$$\varepsilon_o^* = (\sqrt{2}-1) / \max_{1 \leq j \leq m} T_j \tag{46}$$

which yields an easily computed estimate of ε_o^* but will be more conservative than the estimate obtained from corollary 3.1.

4.2. ε_o^* and the Integrated Modelling Error

Although the techniques of section 4.1. yield values of ε_o^*, the modelling error must be small enough to satisfy (34). If this constraint is not satisfied then more plant data must be included in the design exersize. There are many possibilities such as the use of the techniques of section 3 or ref [2] but here we concentrate on the use of the error bounds (12) based on the integrated model error data (11). It is also assumed that the plant and model have the same steady state step response characteristics in the sense that $E(\infty) = o$. The main result of this section is stated as follows. Note the absence of any constraint on the modelling error!

<u>Theorem 4:</u> With the above conditions, the unity feedback controller $K = K_p K_c$ with integral precompensator

$$K_c(s) = \text{diag}\{\varepsilon_j\}_{1 \leq j \leq m} \hat{G}^P(o) s^{-1} \tag{47}$$

will stabilize the plant G for any choice of gains ε_j, $1 \leq j \leq m$, such that

(a) $\varepsilon_j T_j s^{-1}$ stabilizes $e^{-s\alpha_j}/(s+1)$, $1 \leq j \leq m$, and

$$\text{(b)} \quad \gamma_\infty^o \max_{1 \leq j \leq m} \varepsilon_j \sup_{\substack{s=i\omega \\ \omega \geq o}} \left| \frac{s(s+1)}{s(s+1) + \varepsilon_j T_j e^{-s\alpha_j}} \right| < 1 \tag{48}$$

where γ_∞^o is any convenient upper bound for

$$r_\infty^o \triangleq r\,(\|\hat{G}^P(o)\|_p \, N_\infty^P(Z_F^P)) \tag{49}$$

Proof: Follows in a similar manner to theorem 3 using $\tilde{G}(o) = G(o)$ and the upper bound $|s|\ N_\infty((Z_F^p)_{ij})$ for $|G_{ij}^p(s) - (G_A(s))_{ij}|$ obtained from (12) by noting that $E^p(\infty) = o\ ,\ 1 \leq i,j \leq m$.

There is a graphical interpretation as follows:

Corollary 4.1.: The conclusion of theorem 4 hold if (a) and (b) are replaced by the equivalent conditions that, for each value of j, (a) the point $(-(\varepsilon_j T_j)^{-1}, o)$ of the complex plane does not lie in or on the band generated by plotting the Nyquist locus of the transfer function

$$g_j\ (s,\alpha_j) \triangleq \frac{e^{-s\alpha_j}}{s(s+1)} \tag{50}$$

and superimposing at each frequency a circle of constant radius

$$r_j^o\ (s) \triangleq \gamma_\infty^o / T_j \tag{51}$$

and (b) that the band does not encircle that point.

If the trailing edge of the band cuts the negative real axis at the point $(-\mu_j^o, o)$ then we can choose

$$\varepsilon_j < \varepsilon_j^* \triangleq 1/\mu_j^o\ T_j \tag{52}$$

and, in the Davison case of $\varepsilon_j = \varepsilon$, $1 \leq j \leq m$, we obtain the estimate $\varepsilon_o^* = 1/\max\limits_{1 \leq j \leq m} \mu_j^o\ T_j$.

Proof: Write (48) in the form

$$\left| \frac{e^{-s\alpha_j}}{s(s+1)} + (\varepsilon_j T_j)^{-1} \right| > \frac{\gamma_\infty^o}{T_j}\ \ , s=i\omega\ ,\ \omega > o \tag{53}$$

and interpreting in graphical form as illustrated in a similar manner to Fig.2.

We conclude this section with the observation that

(i) the limitations in gain implicit in the result are revealed by noting that (53) requires that $\varepsilon_j < 1/\gamma_\infty^o$, $1 \leq j \leq m$, by letting $\omega \to \infty$ and hence $\varepsilon_o^* < 1/\gamma_\infty^o$.

(ii) In the case of $T_j = T$, $1 \leq j \leq m$, and $\mathfrak{F}_j = \mathfrak{F}$, $1 \leq j \leq m$, the m plots described by the result are identical and yield a single value of $\mu_j = \mu$, $1 \leq j \leq m$.

5. Choice of Constant Precompensation.

In the techniques described in previous sections, the choice of K_p will play a crucial role in determining the size of the error in modelling (compensated) plant dynamics. In a conceptually similar manner to the INA methodology, it is, in general, necessary to make the error $E^p(t)$ as 'diagonal as possible' for $t \geq o$. This process is then followed by constructing models of the diagonal terms.

The problem of choice of K_p could be approached intuitively by, for example, choosing $K_p = I_m$ (if $m = \ell$) if the plant G has naturally low interaction. Alternatively, if interaction tends to peak in all loops around $t = t_o$ (say), it is natural to try $K_p = Y^{-1}(t_o)$. For example, if $t_o = \infty$, $K_p = Y^{-1}(\infty)$ will remove all steady state interaction from the plant and have the added bonus of ensuring that $E(\infty) = o$ if $g_j(o) = 1$, $1 \leq j \leq m$, in section 3.

More generally, a numerical/algorithmic procedure will be necessary to relieve the load on the designer. The following procedure is a direct parallel to the method of pseudo-diagonalization [5] that has proved to be invaluable in the INA design technique. The following development is geared to the techniques of section 3.

Suppose that the model G to be used is diagonal and, as yet, unspecified. The choice of constant precompensator K_p to minimize the width of the confidence bands as deduced from the column-sum (25) is equivalent to the solution of an optimization problem. As the total variation (10) is an absolute upper bound on the effect of modelling errors, this measure of modelling error will be used as the basis of the optimization procedure by choosing K_p to solve the problem,

$$\min_{K_p} \max_{1 \leq j \leq m} \sum_{\substack{i=1 \\ i \neq j}}^{m} N_\infty (E^p_{ij}) \tag{54}$$

Let T be the length of the data sequence available and let

$$o = t_o < t_1 < \ldots < t_{N-1} < t_N = T \tag{55}$$

be a partition of $[o,T]$. Write K_p in column form

$$K_p = [K_1, K_2, \ldots, K_m] \tag{56}$$

and note that (54) can be approximated by m independent optimization problems $1 \leqslant j \leqslant m$,

$$\min_{K_j} \sum_{\substack{i=1 \\ i \neq j}}^{m} \sum_{k=1}^{N} \left| E^p_{ij}(t_k) - E^p_{ij}(t_{k-1}) \right| \tag{57}$$

as the data E^p_{ij}, $1 \leqslant i \leqslant m$, is independent of K_r, $r \neq j$.

This problem can be regarded as a minimum norm problem and solved algorithmically but is not easily solved analytically. For this reason, we propose that the problem is relaxed to a quadatic form

$$\min_{K_j} \sum_{\substack{i=1 \\ i \neq j}}^{m} \sum_{k=1}^{N} \left[E^p_{ij}(t_k) - E^p_{ij}(t_{k-1}) \right]^2 \tag{58}$$

in a manner similar to the relaxation of the dominance condition to a quadatic condition in pseudo-diagonalization.

Introducing the notation

$$\Delta_k M = M(t_k) - M(t_{k-1}), \quad 1 \leqslant k \leqslant N \tag{59}$$

for any matrix function M(t) on $[o,T]$ and letting $\{e_j\}_{1 \leqslant j \leqslant m}$ denote the natural basis in R^m (ie $e_1=(1,o,\ldots o)^T$, $e_2=(o,1,o,..,o)^T$, etc etc) replaces [58] by

$$\min_{K_j} \sum_{\substack{i=1 \\ i \neq j}}^{m} \sum_{k=1}^{N} \left[e_i^T \Delta_k Y K_j \right]^2 = \min_{K_j} K_j^T Q_j K_j \tag{60}$$

where

$$Q_j = \sum_{\substack{i=1 \\ i \neq j}}^{m} \sum_{k=1}^{N} (\Delta_k Y)^T e_i e_i^T (\Delta_k Y) \tag{61}$$

To avoid the trivial solution $K_j = o$, add the additional constraint that

$$K_j^T R K_j = 1, \quad 1 \leqslant j \leqslant m \tag{62}$$

where $R = R^T > o$ is arbitrary. Elementary analysis then indicates that the optimal solution K_j^* satisfies $QK_j^* = \lambda R K_j^*$ with $\min_{K_j} K_j^T Q_j K_j = \lambda$. That is, K_j^* is an eigenvector of $R^{-1} Q$ corresponding to an eigenvalue of smallest magnitude.

6. Illustrative Examples

6.1 A Boiler-furnace System

Consider the case of unity feedback control of the boiler furnace system described by Rosenbrock [5] with 4X4 TFM

$$G(s) = \begin{bmatrix} \frac{1.0}{1+4s} & \frac{0.7}{1+5s} & \frac{0.3}{1+5s} & \frac{0.2}{1+5s} \\ \frac{0.6}{1+5s} & \frac{1.0}{1+4s} & \frac{0.4}{1+5s} & \frac{0.35}{1+5s} \\ \frac{0.35}{1+5s} & \frac{0.4}{1+5s} & \frac{1.0}{1+4s} & \frac{0.6}{1+5s} \\ \frac{0.2}{1+5s} & \frac{0.3}{1+5s} & \frac{0.7}{1+5s} & \frac{1.0}{1+4s} \end{bmatrix}$$

$$\equiv \frac{1}{1+4s} I_4 + \frac{1}{1+5s} (G(o) - I_4) \qquad (63)$$

Despite the relatively large number of inputs and outputs and the known fact [5] that the process turns out to be relatively easy to control, we include it in the paper to highlight the conceptual and numerical simplicity of the step data techniques described in previous sections and the manner in which they reflect the simple structure of the system.

Considering initially the use of theorem 1 and the simple possibility of loop controllers only, choose the precompensator $K_p = I_4$ and the diagonal compensator $K_c(s) = kI_4$ with identical proportional gain $k>o$ in all loops to reflect the identical diagonal plant dynamics. We choose the model $G_A(s) = I_4/(1+4s)$ and note that it is stabilized for all gains $k>o$. Noting that all interaction effects are monotonic leads to the matrix total variation.

$$N^P_\infty(E^P) = G(o) - I_m \qquad (64)$$

and hence, choosing the error bound (10), we obtain the column sums (25) as

$$d_1(s) \equiv d_4(s) \equiv 1.15 \quad , \quad d_2(s) \equiv d_3(s) \equiv 1.4 \qquad (65)$$

A preliminary estimate of the predicted permissible gain range is now obtained from (22) i.e.

$$k < \min_j \; d_j^{-1}(\infty) \;=\; 0.71 \tag{66}$$

Simulation studies indicate closed-loop stability in this range but sluggish response characteristics and large steady state errors. Consideration is, therefore, given to the use of a precompensator.

(Note: the above result should be compared with that of Rosenbrock [5] where the INA method predicts stability for all k>o. The techniques in this paper are much simpler than the INA but more conservative. This is to be expected however as we have started from a different data base (ie the step response) and need only the simplest representation of the modelling error. The precompensator described below improves the interaction considerably, but in the absence of precompensation, it is necessary to use a more detailed error characterization approaching that used in the INA method).

Following Rosenbrock [5] consider the use of the steady state precompensator

$$K_p = G^{-1}(o) = Y^{-1}(\infty) = \begin{bmatrix} 1.75 & -1.21 & -0.16 & 0.17 \\ -0.98 & 1.87 & -0.23 & -0.32 \\ -0.32 & -0.23 & 1.87 & -0.98 \\ 0.17 & -0.16 & -1.21 & 1.75 \end{bmatrix} \tag{67}$$

leading to the compensated plant

$$G^P(s) = \frac{1}{1+4s} \; G^{-1}(o) + \frac{1}{1+5s} \; (I_4 - G^{-1}(o)) \tag{68}$$

with step response matrix

$$Y_p(t) = (1-e^{-t/4}) \; G^{-1}(o) + (1-e^{-t/5}) \; (I_4 - G^{-1}(o)) \tag{69}$$

Choosing G_A to model the diagonal terms to high accuracy leads to

$$g_1(s) = g_4(s) = \frac{1 + 5.75s}{(1+4s)(1+5s)} \;, \; g_1(s) = g_3(s) = \frac{1+ 5.87s}{(1+4s)(1+5s)} \tag{70}$$

with total variation of the error equal to

$$N_\infty^P(E^P) = \begin{bmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{bmatrix} N_\infty(e^{-t/5} - e^{-t/4}) \tag{71}$$

where graphical analysis indicate that $N_\infty(e^{-t/5} - e^{-t/4}) = 0.164$. This produces column sums

$$d_1(s) \equiv d_4(s) \equiv 0.24 \;, \; d_2(s) \equiv d_3(s) \equiv 0.26 \tag{72}$$

Choosing the compensator $K_c(s) = \text{diag}\{k_i(s)\}_{1 \leq i \leq 4}$ with diagonal terms $k_i(s) = k_1^{(i)} + s^{-1}k_2^{(i)}$, $i = 1, 4$, the stability of the approximating feedback scheme is guaranteed for all positive gains. To check the stability of the implemented scheme using theorem 1, note that (22) requires that

$$\left| k_1^{(i)} \right| < \begin{cases} 4.1 & i = 1\ , \quad i = 4 \\ 3.8 & i = 2\ , \quad i = 3 \end{cases} \tag{73}$$

the approximate model hence indicating that these gains allow an increase in response speed of up to six times the open-loop response speeds. This is more than normally necessary for process control applications.

Choosing the networks

$$k_i(s) = 3 + \frac{1}{2s}\ , \quad i = 1,..,4 \tag{74}$$

to produce time constants of approximately 1.0 and reset times of approximately 6.0 for the approximating feedback system, then (73) is satisfied and condition (23) of theorem 1 is satisfied as shown in Fig.3 (Note: the inverse Nyquist plots with confidence bands are shown only for loops 1 and 2, the plots for loops 3 and 4 are obtained by symmetry and interchanging). Clearly, the (-1,0) point does not lie in or on any confidence band and hence the controller successfully stabilizes the plant. The resulting closed loop performance is shown in Fig.4 indicating excellent loop performance and low interaction effects.

Although the design has been successful without the use of frequency dependent estimates described in section 2, confidence in the design can be increased by calculating the integrated modelling error to yield the total variation

$$N_\infty^P(Z_F^P) = \begin{bmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{bmatrix}$$

Again using column sums and incorporating the bound (12) into the analysis yields the inverse Nyquist plots with confidence bands shown in Fig.5. Note the decrease in the width of the confidence bands at low frequency by comparison with Fig.3. Derivative data can also be used by evaluating the total variation

$$N_\infty^P(V^P) = \begin{bmatrix} 0 & 1.21 & 0.16 & 0.17 \\ 0.98 & 0 & 0.23 & 0.32 \\ 0.32 & 0.23 & 0 & 0.98 \\ 0.17 & 0.16 & 1.21 & 0 \end{bmatrix} \times 0.114 \tag{76}$$

and incorporating (16) into the modelling error bound. The confidence bands are illustrated in Fig.6 and indicate a substantial reduction in the radii of the confidence circles at high frequencies.

Turning now to the analysis of a unity feedback integrating tuning regulator control (29) for the boiler-furnace, we will use a unit precompensator $K_p = I_4$ and choose $\tilde{G}^p(o) = G^p(o) = G(o)$. Analysis of the dynamics suggests the use of identical model time constants $T_j = 4{,}0$, $1 \leq j \leq 4$, (to reflect the similar time constants in all loops) and zero delays $\vartheta_j = o$, $1 \leq j \leq 4$. The modelling error is hence

$$E^P(t) = (G(o) - I_4)\ (e^{-t/4} - e^{-t/5}) \tag{77}$$

leading to the total variation.

$$N_\infty^P\ (E^P) = (G(o) - I_4)\ 0.164 \tag{78}$$

Defining $||M||_m = \max_i \sum_j |M_{ij}|$ for any mxm matrix M, we can choose the bound $\gamma_\infty = ||G^{-1}(o)||_m\ ||N_\infty^P(E)||_m$ for r_∞ to yield $\gamma_\infty = 3.4 X (1.35 X 0.164)$ $= 0.753 < 1$ and hence theorem 3 can be applied for integrating controllers (i.e. $k_1^{(j)} = o$, j=1, 4) to estimate ε_o^*. Using corollary 3.2 in the form of (46) yields

$$\varepsilon_o^* = 0.1 \tag{79}$$

In contrast, corollary 3.1 and (43) yields the improved estimate

$$\varepsilon_o^* \simeq 1\ /\ (1.2\ X\ 4.0) = 0.21 \tag{80}$$

by plotting the Nyquist plot of 1/s(s+1) with circles of radius γ_∞/ω as given in Fig.2 to obtain $\mu_j \simeq 1.2$, $1 \leq j \leq 4$. Finally, noting that the assumptions of theorem 4 are satisfied it is easily verified that the integrated error has matrix total variation.

$$N_\infty^P\ (Z_F^P) = G(o) - I_4 \tag{81}$$

and we can take $\gamma_\infty^o = ||G^{-1}(o)||_m\ ||G(o) - I_4||_m = 4.59$ and

$r_j^o(s) = \gamma_\infty^o / T_j = 1.15$, $1 \leq j \leq 4$. Plotting the Nyquist diagram of 1/s(s+1) with circles of radius $r_j^o(s)$ then yields $\mu_j^o \simeq 1.55$, $1 \leq j \leq 4$, as shown in Fig.7 and hence

$$\varepsilon_o^* \simeq 1/(1.55 X 4.0) = 0.16 \tag{82}$$

by corollary 4.1.

Each technique produces a different estimate, all results being improved somewhat if better choices of K_p (eg $K_p = G^{-1}(o)$) or γ_∞ and γ_∞^o are used (eg. $\gamma_\infty = r_\infty = 0.702$). This is unnecessary in this case as the closed-loop responses to a unit step demand in y_1 indicate in the case of $\varepsilon = 0.1$ (Fig.8). Note the stable response characteristics, zero steady state error and small interaction. The response speed is slow but this can be improved by including proportional action.

6.2. A Level Control Study

Consider now the problem of level control considered by Tomizuka [11]. A two-input-two-output three vessel system is described by the model.

$$\dot{x}(t) = \begin{bmatrix} -0.375 & 0.125 & 0 \\ 0.25 & -1.0 & 0.25 \\ 0 & 0.5 & -1.5 \end{bmatrix} x(t) + \begin{bmatrix} 0.25 & 0 \\ 0 & 0.5 \\ 0 & 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) \qquad (83)$$

where x_j denotes the level in vessel j and u_1, u_2 are input flow variables. Unity output feedback is assumed throughout.

The step response matrix of the process after precompensation by

$$K_p = \begin{bmatrix} 5.0 & 0 \\ 0 & 1.111 \end{bmatrix} \qquad (84)$$

(to normalize the d.c. gains of the diagonal elements to approximately unity) are given in Fig.9. The diagonal approximate model G_A was fitted by visual inspection to be defined by

$$g_1(s) = \frac{e^{-s0.3}}{1+s\,3.8} \quad , \quad g_2(s) = \frac{1.04}{1+s} \qquad (85)$$

with step responses indicated on Fig.9 also. A simple calculation then gave

$$N_\infty^P(E^P) = \begin{bmatrix} 0.219 & 0.888 \\ 0.332 & 0.085 \end{bmatrix} \qquad (86)$$

Choosing a diagonal compensator $K_c(s) = \text{diag}\{k_j(s)\}_{j=1,2}$ with

$k_j(s) = k_1^{(j)} + s^{-1} k_2^{(j)}$, j=1,2, and application of condition (22) of theorem 1 gives the preliminary gain estimate

$$\max_j k_1^{(j)} < 1.43 \qquad (87)$$

where (26) has been used to represent the modelling error with $\Delta(s) = N_\infty^p(E^p)$. Choosing $k_j(s) = 1.2 + s^{-1}0.3$, j=1, 2, the inverse Nyquist plots with confidence bands again deduced from (26) are shown in Fig.10 verify that condition (ii) of theorem 1 is also satisfied and the implemented scheme will be stable. This is verified in Fig.11. Confidence in the predictions can be obtained by reducing the width of the confidence bands using filtered error data (section 2) but is not necessary here and is hence omitted for brevity.

Turning now to the design of an integrating tuning regulator of the form $K_c(s) = \text{diag}\{\varepsilon_j\} \hat{\tilde{G}}(o)s^{-1}$ with K_p given by (84) and

$$\tilde{G}^p(o) = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.04 \end{bmatrix} \qquad (88)$$

chosen to represent the steady state behaviour of the diagonal dynamics and to produce a simple loop controller.

Choosing T_1=4.1, T_2=1.0, $\mathfrak{F}_1 = \mathfrak{F}_2 = 0$, the total variation of the error is given by

$$N_\infty^p(E^p) = \begin{bmatrix} 0.288 & 0.888 \\ 0.332 & 0.085 \end{bmatrix} \qquad (89)$$

and, using theorem 3 with $\gamma_\infty = r_\infty = 0.727 < 1$, we see that we can apply corollary 3.2. to ensure stability in the range (equation (46)).

$$o < \varepsilon_1 < \frac{\sqrt{2}-1}{T_1} = 0.101 \ , \quad o < \varepsilon_2 < \frac{\sqrt{2}-1}{T_2} = 0.414 \qquad (90)$$

Choosing $\varepsilon_1 = 0.1$ and ε_2=0.2, the closed-loop unit step responses are shown in Fig.12 and indicate the excellent tracking capability of the controller. Response speeds can be increased and interaction reduced by incorporating proportional control action but this is not considered here for brevity. Alternatively, the proven existence of stabilizing loop controllers with estimated gain range can be used to initiate on-line tuning operations on the integral gains ε_1 and ε_2.

7. Conclusions

The paper has illustrated that the flexibility introduced into the design process by offering the designer the capability of representing the plant by an approximate model can be used to produce simple design procedures based on plant step data for process plant which, after suitable precompensation, has one or more of the properties,

(a) 'small' interaction effects as observed by inspection of transient step data.

(b) simple response characteristics that suggest that the process can be modelled in its dominant characteristics by first order 'delay-lag' models.

The illustrative examples of simple, but representative systems, indicates that the procedures can work well in practice.

The material of section 2 has demonstrated that filtering and graphical total variation calculations on step response data can be used in a systematic way to produce frequency domain error bounds that can be incorporated (section 3) into a CAD framework for control design that is essentially a general form of INA technique that uses plant step data as its sytem description. A feature of the methodology is hence that a detailed model of the process is not required for the design study. Section 4 has demonstrated that Davison's tuning regulator design method can also be set in the framework in the same way that suitable choice of approximate model yields easily computed estimates of upper gain bounds for feedback regulators. In its simplest form (corollary 3.2) the gains are computed by simple 'back-of-envelope' calculations requiring minimal computational backup.

Finally, we note that, when compared with detailed designs based on a detailed plant model, the predictions obtained from the techniques are, of course, conservative. This is a feature of many design techniques that seek to simplify the design process by neglecting information on modelling errors or interaction effects. Well known examples of this procedure are the INA method (that neglects phase information on off-diagonal terms), the work of Åstrom [4] (that uses only monotonicity and gross time-constant representation), the work of Davison [3] (which uses only known stability and steady state data) and that of Lünze [12] (who uses upper bounds on i/o relationships for modelling errors). In each case, conservatism is present (in general) but this need not prevent the attainment of a satisfactory control design as illustrated by examples included above and in the stated references.

The designer is, in essence, taking advantage of a trade-off between the computational and conceptual simplicity of an approximiate model as a vehicle for design and the conservatism in the treatment of the modelling errors. The trade-off will be successful provided that the performance objectives are compatible with the approximations involved. The theoretical CAD methods described provide, in this context, a numerical indication of the success of the approach.

8. Acknowledgement.

This work was supported by SERC under grant GR/B/23250

9. References.

[1] Ziegler J.G. and N.B.Nichols: 'Optimum settings for automatic controllers', Trans. ASME, 64, 759-768.

[2] Owens D.H. and A.Chotai: 'Robust controller design for linear dynamic systems using approximate models', Proc IEE, 130(D), 1983, 45-46.

[3] Davison E.J.: 'Multivariable tuning regulators: the feedforward and robust control of a general servomechanism problem' IEEE Trans.Aut.Control. AC-21,1976, 35-47.

[4] Åstrom K.J.: 'A robust sampled regulator for stable systems with monotone step responses', Automatica, 1980, 16, 313-315.

[5] Rosenbrock H.H.: 'Computer-aided-design of control systems', Academic Press, London, 1974.

[6] Owens D.H.: 'Feedback and multivariable systems', Peter Peregrinus, Stevenage, 1978

[7] Porter B.: 'Design of tunable set point tracking controllers for linear multivariable plants', Int.J.Control, 35, 1982, 1107-1115.

[8] Pentinnen J and H.N.Koivo: 'Multivariable tuning regulators for unknown systems' Automatica, 16, 1980, 393-398.

[9] Owens D.H. and A.Chotai: 'High performance controllers for unknown multivariable systems' Automatica, 18, 1982, 583-587.

[10] G.W.M.Cappus, S.L.Shah and R.K.Wood: 'Robust multivariable control of a binary distillation column', Proc IEE, 130(D), 1983, 201-208.

[11] Tomizuka M.: 'A simple digital control scheme for a class of multi-input, multi-output industrial processes' Jrnl.Dynamic Systems, Measurement and Control, 101, 1979, 339-344.

[12] Lünze.J.: 'The design of robust feedback regulators for partly unknown systems by optimal control procedures', Int.J.Control, 1982, 36, 611-630.

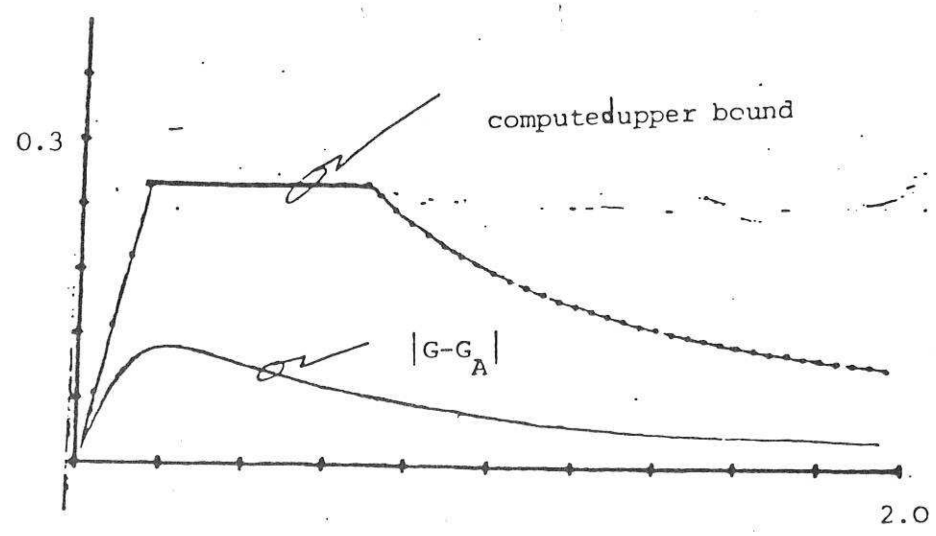


Fig.1. The gain error and its computed upper bound.

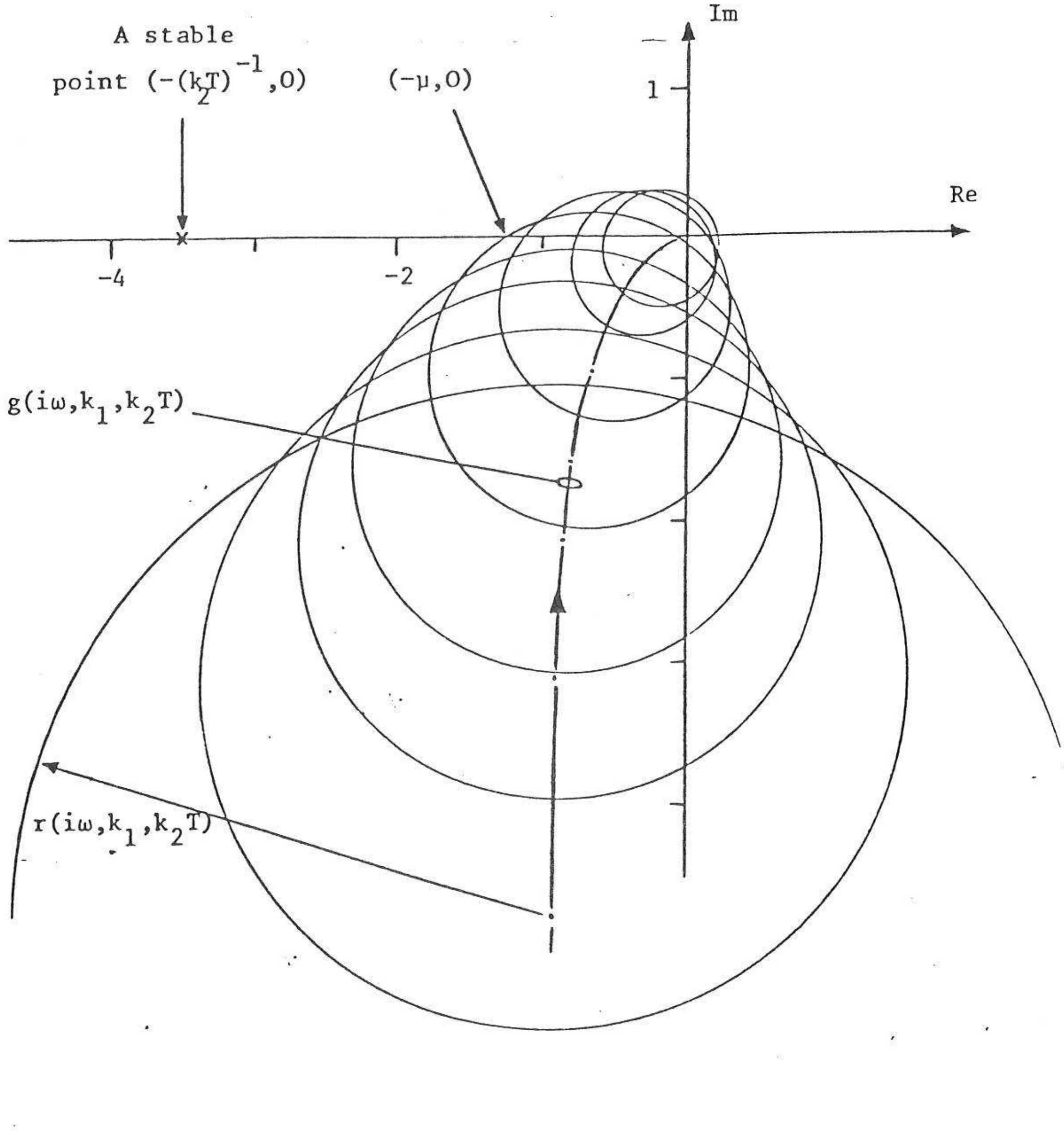


Fig. 2 Graphical Stability Criterion and

Evaluation of μ

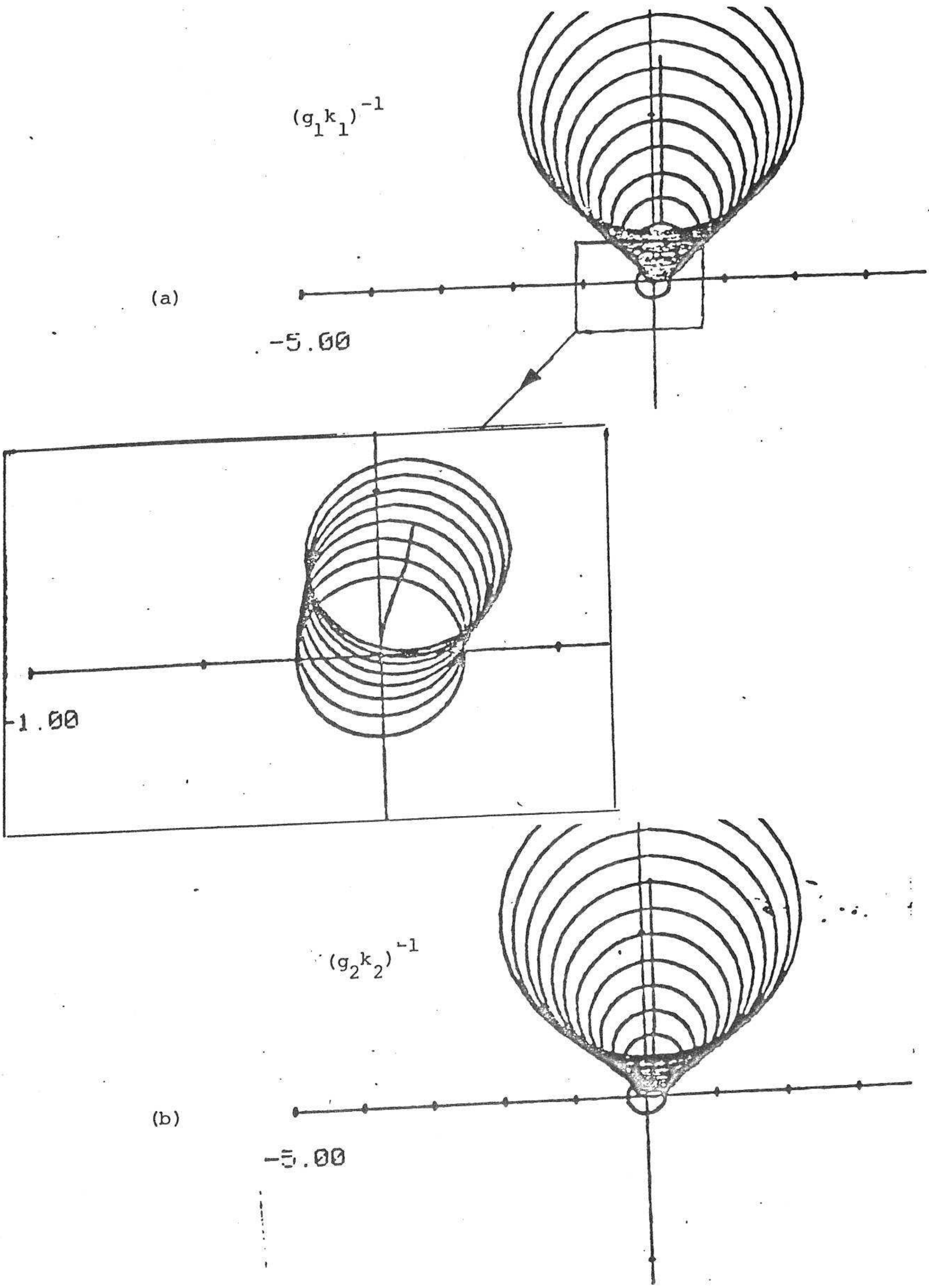


Fig. 3 Inverse Nyquist plots with confidence bands

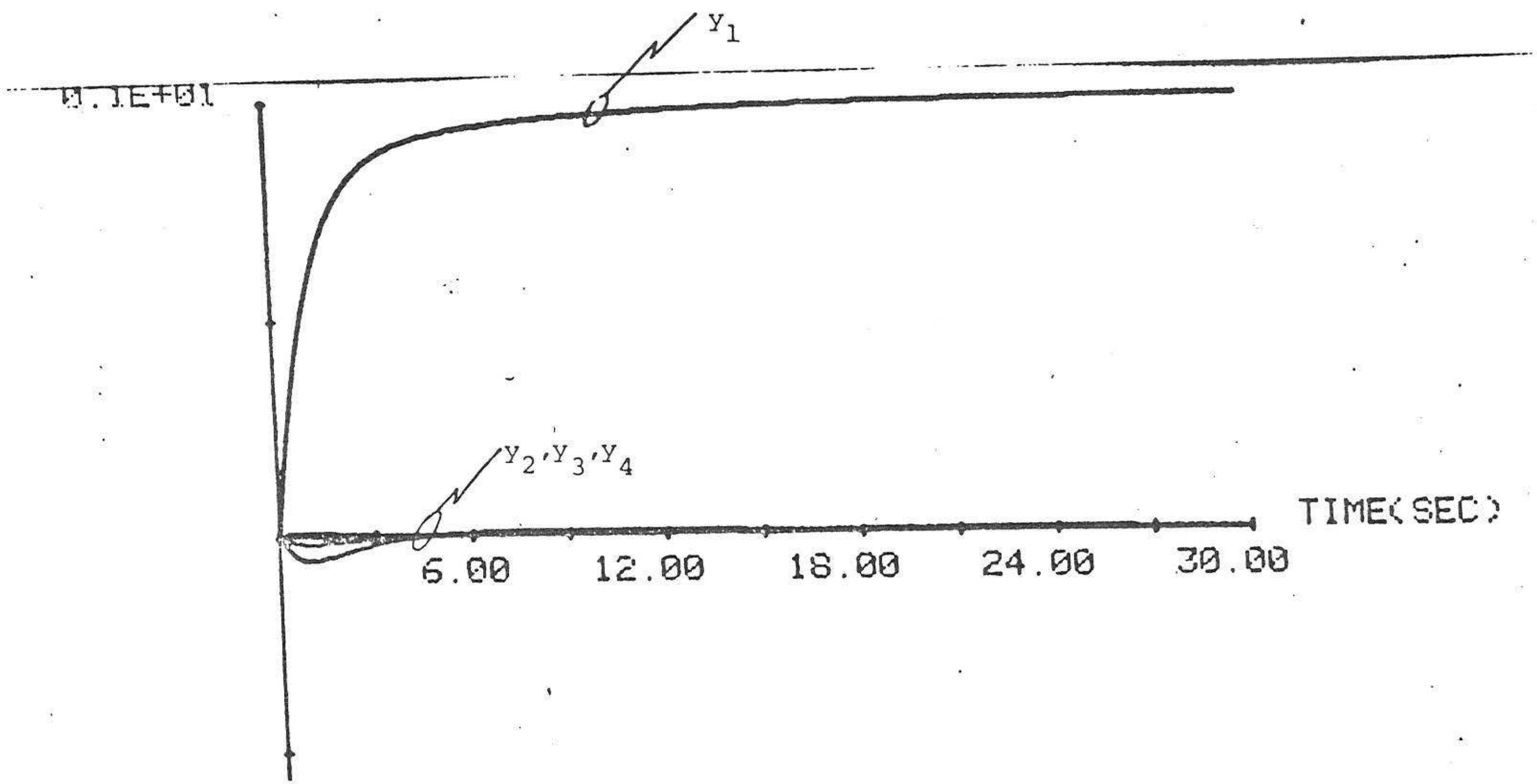


Fig. 4. Closed-loop responses to a unit step demand in y_1

(a)

(b)

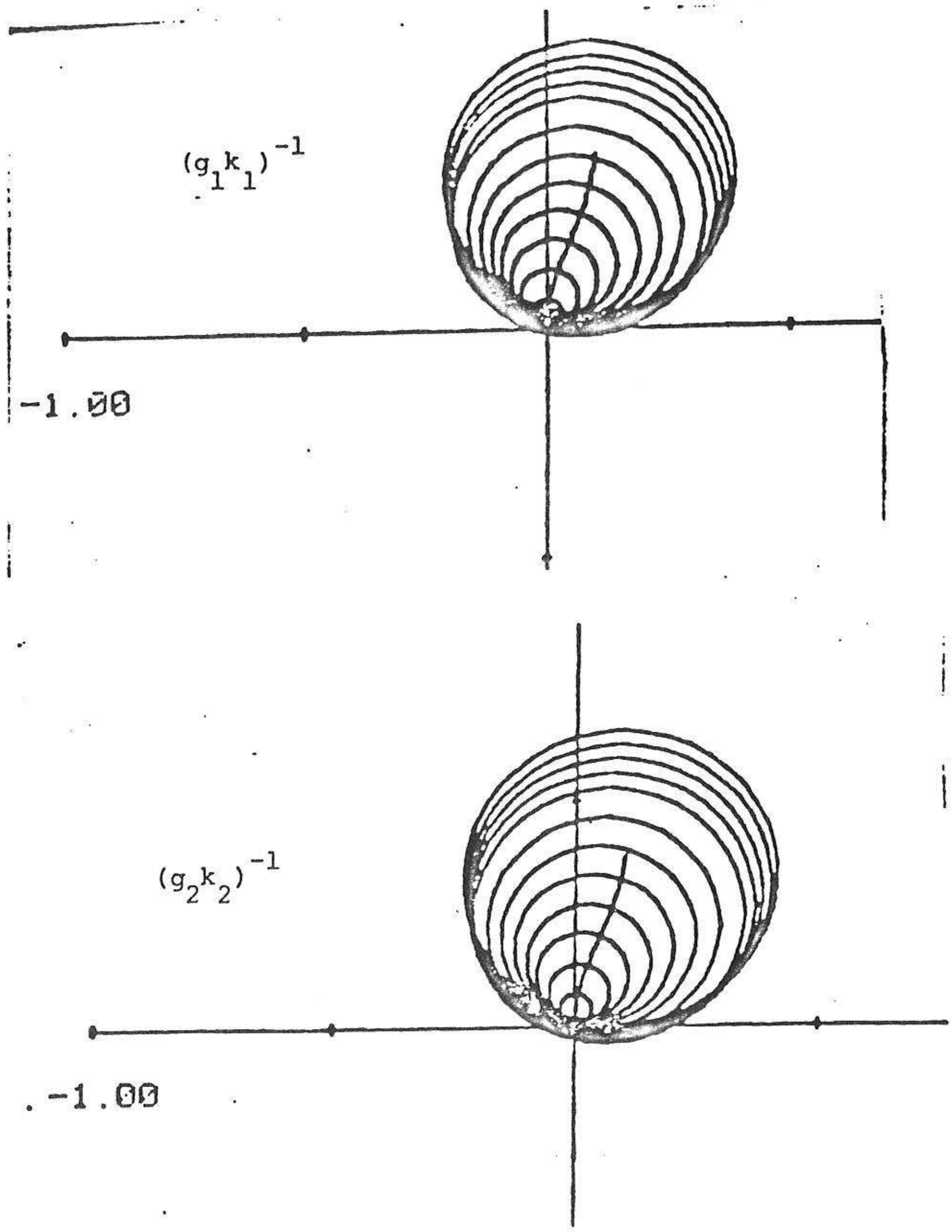
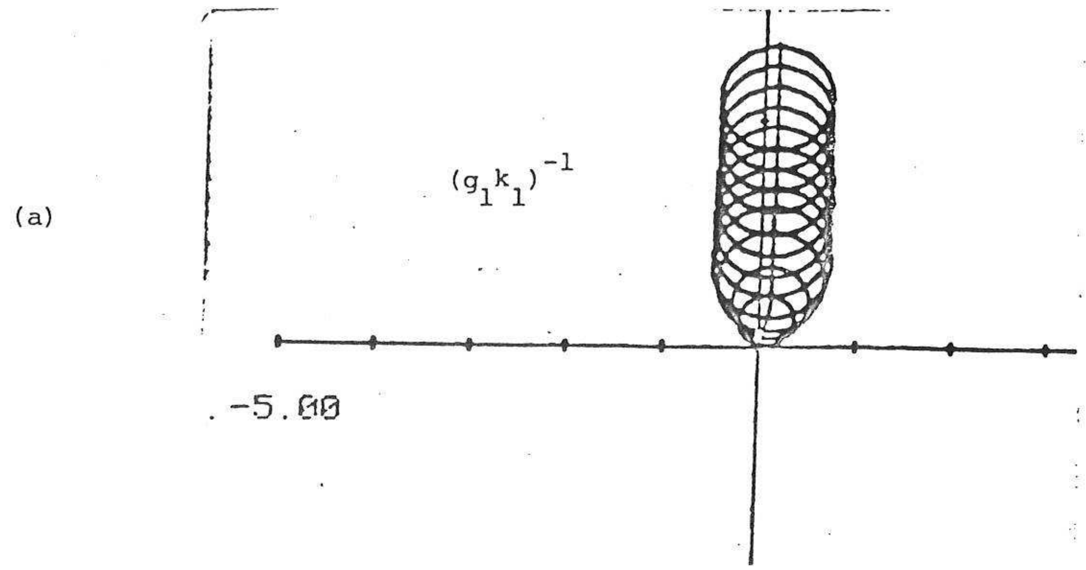


Fig. 5. Low frequency confidence bands with integrated error data



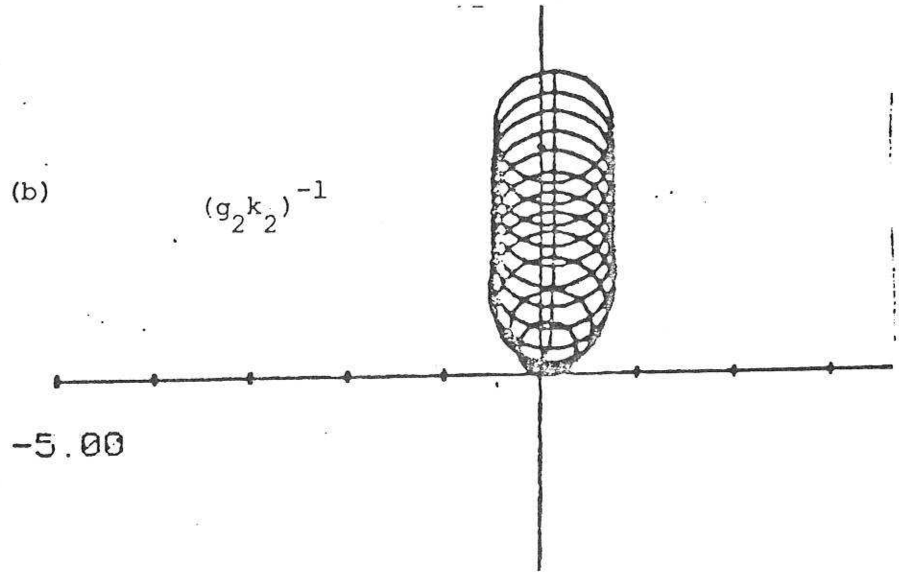


Fig. 6. Confidence bands using derivative data

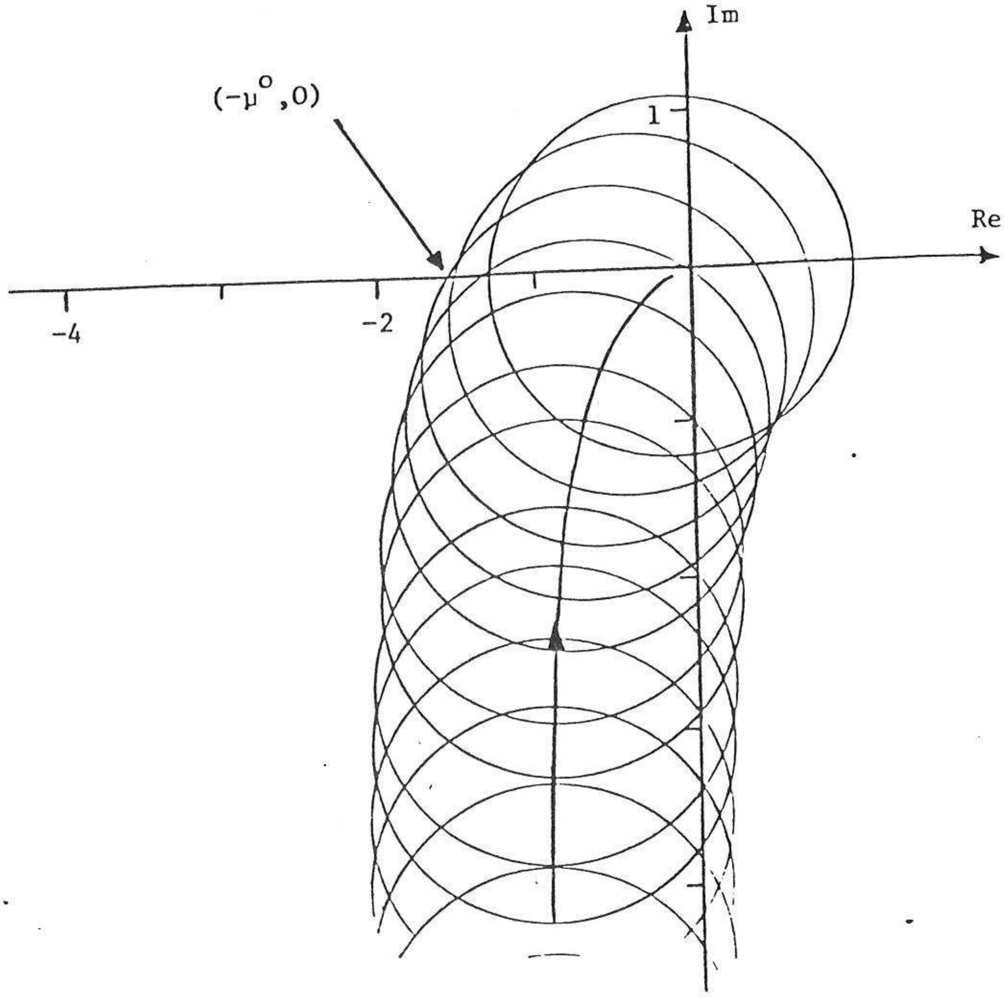


Fig. 7. Evaluation of μ^o

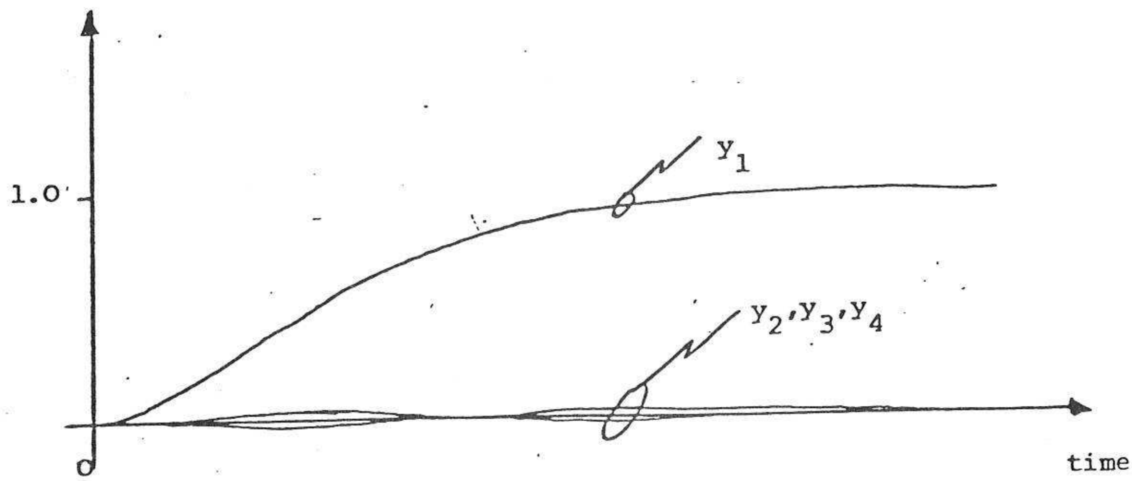
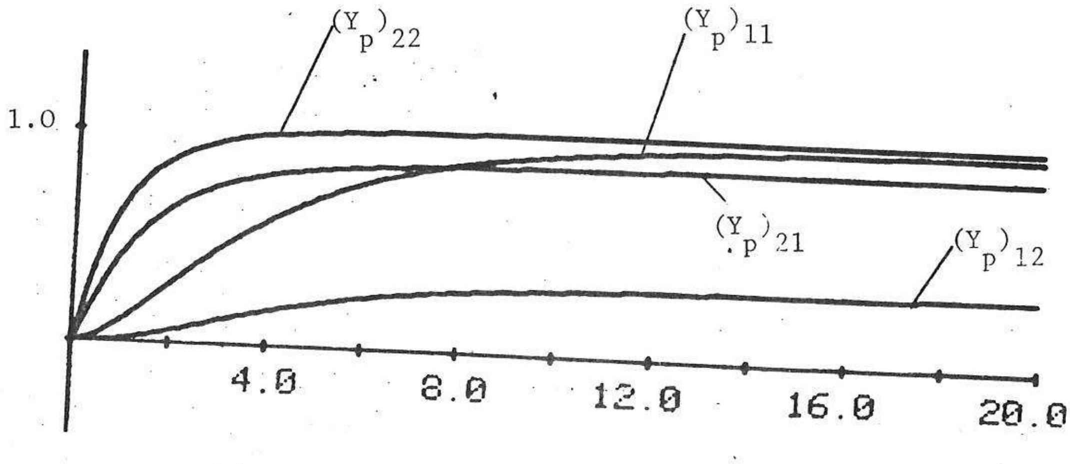
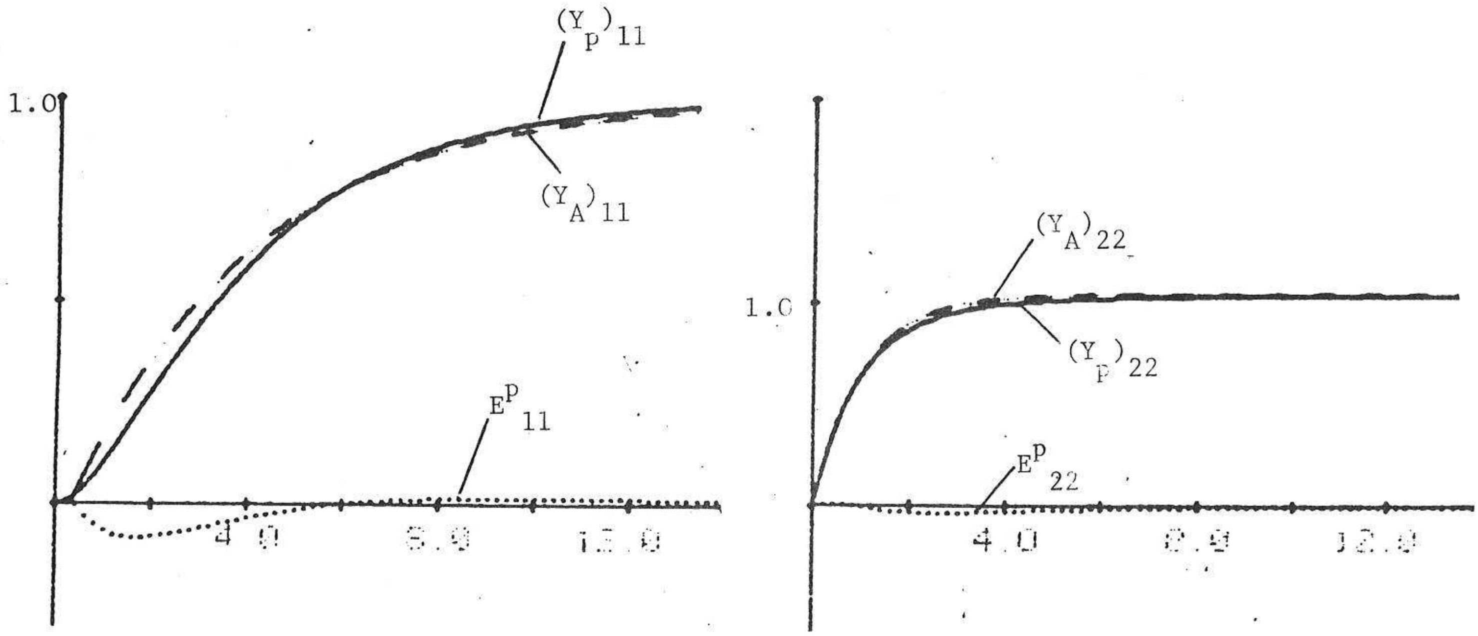


Fig. 8 Closed-loop response to a unit step demand in y_1



(a)



(b)

Fig.9. (a) Open-loop step responses for level control system after precompensation by K_p.

(b) Diagonal elements of Y_p and Y_A

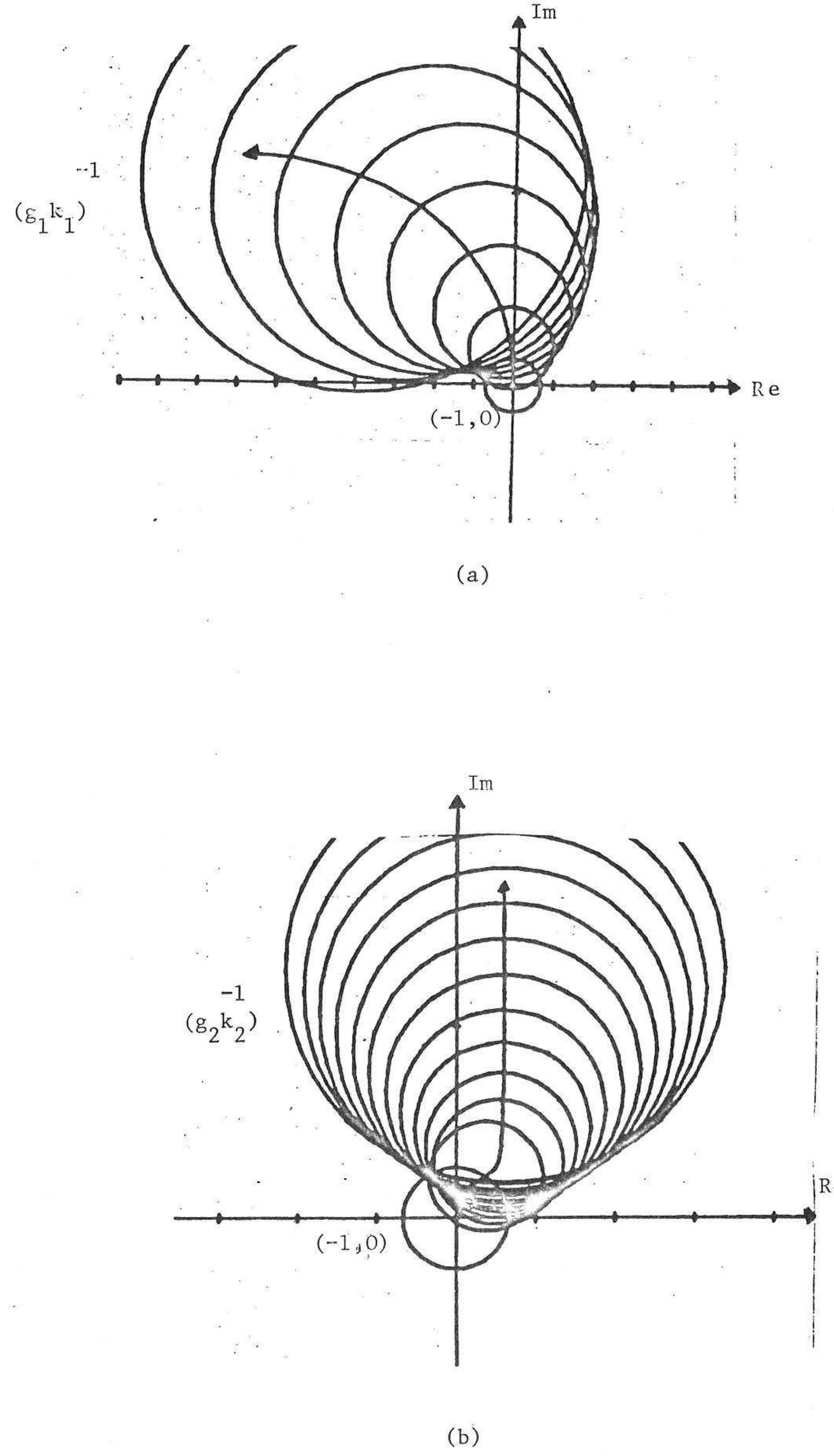


Fig.10. Inverse Nyquist plots with confidence bands.

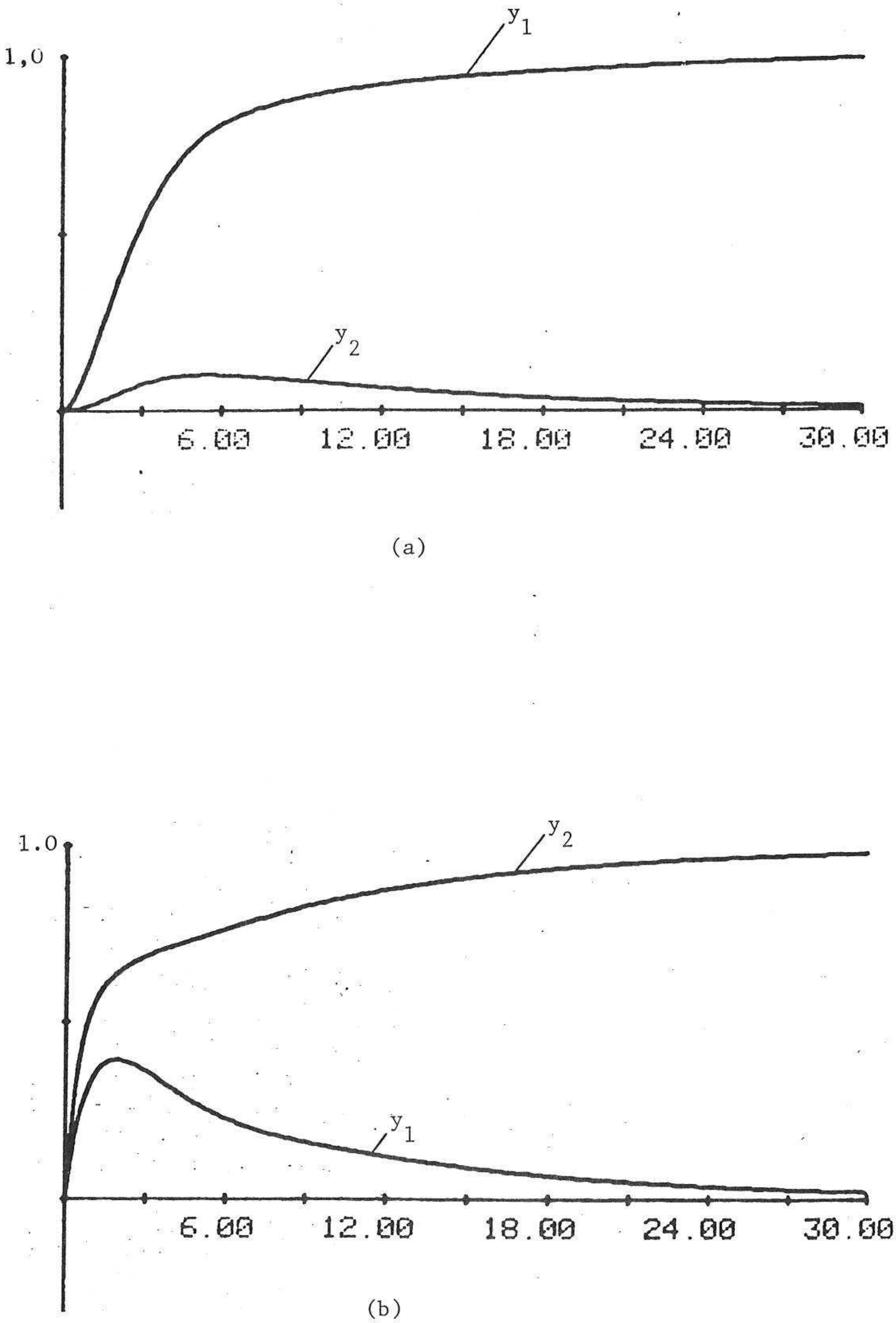


Fig.11. Closed-loop responses to a unit step demand in output (a) one and (b) two.