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IMPROVED STABILITY AND PERFORMANCE BOUNDS USING APPROXIMATE MODELS

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1. INTRODUCTION

In this paper we study the problem introduced by the authors in ref [1] of undertaking multivariable feedback control design for a m-output/ℓ-input stable linear system G based upon the use of a simplified stable approximate or reduced-order model G_A. More precisely, if a forward path controller K is designed to produce acceptable stability and performance characteristics from the model G_A in the presence of linear measurement dynamics F in the configuration of Fig.1(a), we consider how information on the step response characteristics of the real plant G can be used to predict the stability and performance characteristics of the closed-loop system shown in Fig.1(b).

The main results of this paper show that the procedures described in [1] can be improved in both their frequency domain and time-domain form by simulation-based data processing using filters and the technical trick of using exponentially weighted L_∞ spaces in the underlying fixed point problem. The results provide a substantial generalization of those described in ref [1].

2. BACKGROUND MATERIAL AND ASSUMPTIONS

In all that follows all elements G, G_A, K and F are assumed to be described by convolution operators in extended product L_∞ spaces. All impulse responses are assumed to be exponentially bounded and the step response matrices Y(t) and $Y_A(t)$ of G and G_A respectively are assumed to be known. The modelling error will be characterized by the m×ℓ matrix

$$E(t) = [E_1(t),\ldots,E_\ell(t)] \triangleq Y(t) - Y_A(t) \quad , \quad t \geq 0 \tag{2.1}$$

If f is a scalar continuous function defined on $[0,\infty]$ and of bounded variation on any finite interval $[0,t]$, then $N_t(f)$ will denote [1] the norm of f on $[0,t]$ ie

$$N_t(f) \triangleq |f(0+)| + \sum_{j=1}^{k} |f(t_j) - f(t_{j-1})| + |f(t) - f(t_k)| \tag{2.2}$$

where $0 = t_0 < t_1 < t_2 < \ldots$ are the local minima and maxima of f on $[0,\infty]$ and k is the largest integer satisfying $t_k \leq t$. For $t = +\infty$ we define

$$N_\infty(f) = \sup_{t\geq 0} N_t(f) = \lim_{t\to\infty} N_t(f) \tag{2.3}$$

whenever the limit exists.

The development in the paper depends critically on the use of a vector form of the contraction mapping principle for fixed points and, in particular, on the use of vector norms on product Banach spaces and associated induced vector operator norms. The reader is referred to ref [1] for details which are omitted here for brevity.

3. FREQUENCY DOMAIN ANALYSIS

To motivate the general discussion we begin by considering the single-input/single-output case $m = \ell = 1$. If D represents the normal Nyquist contour in the complex plane generated by the imaginary axis $s = i\omega$, $-R \le \omega \le R$, and the semi-circle $|s| = R$, $\mathrm{Re}\, s \ge 0$ with R 'infinitely' large, then the basic sufficient condition ensuring the success of the approximation theorem can be stated as follows [1] in terms of the transfer functions of G, G_A, K and F:

Proposition 3.1: If K stabilizes G_A in the configuration of Fig.1(a), then it will stabilize the plant G in the configuration of Fig.1(b) if

$$\sup_{s \in D} \left| \frac{K(s)F(s)}{1+K(s)F(s)G_A(s)} \right| \Delta(s) < 1 \tag{3.1}$$

where $\Delta(s)$ is any available real-valued function on D with the property that

$$|G(s) - G_A(s)| \le \Delta(s) \quad , \quad \forall\, s \in D \tag{3.2}$$

(Remark: stability is here, and in the following, interpreted as input/output stability. Asymptotic stability requires the addition of (generically satisfied) controllability and observability assumptions on GKF).

The result is a phase-independent result in the sense that it ignores the phase of the error $G-G_A$. Clearly, the best result is obtained by choosing

$$\Delta(s) = |G(s) - G_A(s)| \tag{3.3}$$

as it describes the gain characteristics of the error exactly. The 'best' frequency independent general choice of Δ is [1],

$$\Delta(s) = N_\infty(E) \tag{3.4}$$

obtained from the general result [1]:

Proposition 3.2: If L is a bounded convolution operator from $L_\infty(0,\infty)$ into itself with transfer function L(s) then, for all $s \in D$

$$|L(s)| \le N_\infty(Y_L) \tag{3.5}$$

where $Y_L(t)$ is the unit step response of L.

It can be expected that, between these two extremes, there is an infinity of frequency dependent upper bounds on $|G(s)-G_A(s)|$ that can be used to refine the results of [1] and yet do not need detailed knowledge of G(s). The following result describes a class of bounds that can be obtained by filtering operations on the error data E(t):

<u>Lemma 3.1</u>: Let F_α be a filter with the properties that

(a) $E_\alpha \triangleq F_\alpha E \in L_\infty(0,\infty)$, and

(b) $F_\alpha^{-1}(s)$ is bounded and analytic in the open right-half plane.

Then, for all $s \in D$,

$$|G(s) - G_A(s)| \leq \Delta_\alpha(s) \triangleq |F_\alpha^{-1}(s)| N_\infty(E_\alpha) \tag{3.6}$$

<u>Proof</u>: Write $G-G_A = F_\alpha^{-1}(F_\alpha(G-G_A))$ and apply proposition 3.2 to $F_\alpha(G-G_A)$.

In the case of $F_\alpha = I$, the result reduces to the bound (3.4) used in previous studies. More generally, however, F_α produces a frequency-conscious bound capable of producing more refined results approaching that of (3.3). For example, choose $F_\alpha = (G-G_A)^{-1}$ and note that $E_\alpha(t) \equiv 1$ and hence that (3.6) reduces to (3.3). In practice this choice of F_α is not available but other, more simple choices, can intuitively be used to produce easily computed intermediate estimates.

Estimates obtained from a number of filters can also be used:

<u>Theorem 3.1</u>: Suppose that K stabilizes G_A and that $\{F_\alpha\}_{\alpha\in A}$ is a collection of filters satisfying the conditions of lemma 3.1, then K stabilizes G if (3.1) holds with

$$\Delta(s) = \Delta_A(s) \triangleq \inf_{\alpha\in A} \Delta_\alpha(s) \tag{3.7}$$

The result follows trivially from proposition 3.1 with

$$\Delta(s) \triangleq \inf_{\alpha\in A} \Delta_\alpha(s) \tag{3.8}$$

and is omitted for brevity. It is clear that suitable choice of $\{F_\alpha\}$ will enable a considerable refinement of the results of [1]. The result has a simple graphical interpretation described below:

<u>Corollary 3.1</u>: The conditions of theorem (3.1) are satisfied if the following conditions are satisfied

(i) $$\lim_{\substack{|s|\to\infty \\ \mathrm{Re}\, s\geq 0}} \left| \frac{K(s)F(s)}{1+K(s)F(s)G_A(s)} \right| \Delta_A(s) < 1 \tag{3.9}$$

(ii) the inverse Nyquist plot of $G_A KF$ with $s = i\omega$, $\omega \geq 0$, and superimposed 'confidence' circles of radius

$$r_A(\omega) \triangleq |G_A^{-1}(i\omega)|\, \Delta_A(i\omega) \tag{3.10}$$

at each point generates a 'confidence band' that does not touch or contain the (-1,0) point of the complex plane.

Turning now to the multivariable case, it can be expected that the general structure and conclusions of the single-input/single-output case will be retained. The main result is stated as follows:

Theorem 3.2: Suppose that K stabilizes G_A and that $\{F_\alpha^{(i,j)}\}_{\alpha\in A(i,j)}$, $1\le i\le m$, $1\le j\le \ell$, are filters satisfying the conditions of lemma 3.1 in the sense that

(a) $E_\alpha^{(i,j)} \triangleq F_\alpha^{(i,j)} E_{ij} \in L_\infty(0,\infty)$, and

(b) $(F_\alpha^{(i,j)}(s))^{-1}$ is bounded and analytic in the open right-half-plane

for all $\alpha\in A(i,j)$, $1\le i\le m$, $1\le j\le \ell$. Then K stabilizes the real plant G if

$$\sup_{s\in D} r\left(\left\| (I_\ell+K(s)F(s)G_A(s))^{-1}K(s)F(s) \right\|_P \Delta_A(s)\right) < 1 \tag{3.11}$$

where $\Delta_A(s)$ is the $m\times\ell$ matrix with elements

$$(\Delta_A(s))_{ij} \triangleq \inf_{\alpha\in A(i,j)} (F_\alpha^{(i,j)}(s))^{-1} N_\infty(E_\alpha^{(i,j)}) \tag{3.12}$$

Proof: Following the development of [1], a sufficient condition for stability is that

$$\sup_{s\in D} r((I_\ell+K(s)F(s)G_A(s))^{-1}K(s)F(s)(G(s)-G_A(s))) < 1 \tag{3.13}$$

and the result follows trivially from the inequality $\|G(s)-G_A(s)\|_P \le \Delta_A(s)$.

The result has an interpretation identical to the results of [1] and, in the case of the choice of K, F and G_A diagonal, it can be realized in the form of an INA-type design process [1]-[3].

4. TIME DOMAIN ANALYSIS

The simulation philosophy implicit in the discussion of section 3 can also be used to bound the possible performance deterioration due to the approximation in a similar manner to that described in ref [1].

4.1. Input Assessment

The basic stability result underlaying the performance assessment can be stated as follows [1]:

Proposition 4.1: If K stabilizes G_A, then it will also stabilize G if

$$r(N_\infty^P(W_A)) < 1 \tag{4.1}$$

where $W_A(t) = [W_A^{(1)}(t),\ldots,W_A^{(\ell)}(t)]$ and, $1\le j\le\ell$, $W_A^{(j)}(t)$ is the response from zero initial conditions of the system $(I+KFG_A)^{-1}KF$ to the error data $E^{(j)}(t)$.

The response of the input is described by [1]

$$u = W(u) \triangleq u_A - (I+KFG_A)^{-1}KF(G-G_A)u \tag{4.2}$$

which is here regarded as a fixed point equation in $X_\alpha^{\ell}(t_\alpha)$ with

$$X_\alpha(t_\alpha) \triangleq \{f : e^{\alpha t} f \in L_\infty(0,t_\alpha)\} \tag{4.3}$$

where t_α is to be determined from a contraction condition and the norm of f is taken to be the norm of $e^{\alpha t}f$ in $L_\infty(0,t_\alpha)$.

Proposition 4.2: Let L be a convolution operator from L $(0,\infty)$ into itself with impulse response satisfying $|h_L(t)| \leq h_o e^{-\lambda t}$, $t \geq 0$, for some $\lambda > 0$. If $t_\alpha < +\infty$, the induced operator norm of L restricted to $X_\alpha(t_\alpha)$ satisfies $\|L\| \leq N_{t_\alpha}(Y_L;\alpha)$ for all α where

$$N_{t_\alpha}(Y_L;\alpha) \triangleq |Y_L(0+)| + \sum_{k=1}^{N} e^{\beta_k(\alpha)} (N_{t_k}(Y_L) - N_{t_{k-1}}(Y_L)) \quad , \tag{4.4}$$

Y_L is the unit step response of L, $\beta_k(\alpha) = \alpha t_k$ if $\alpha \geq 0$ or αt_{k-1} if $\alpha < 0$ and $0 = t_o < t_1 < t_2 < \ldots < t_N = t_\alpha$ is any partition of $[0,t_\alpha]$ with the property that $0 < h_1 \leq t_k - t_{k-1} \leq h_2 < +\infty$, $k \geq 1$. Moreover, if $t_\alpha = +\infty$, L maps $X_\alpha(t_\alpha)$ into itself for $\alpha < \lambda$ with $\|L\| \leq N_\infty(Y_L;\alpha) < +\infty$.

Proof: The induced norm of L in $X_\alpha(t_\alpha)$ is

$$\|L\| = |Y_L(0+)| + \int_0^{t_\alpha} e^{\alpha t} |h_L(t)| dt$$

$$\leq |Y_L(0+)| + \sum_{k=1}^{N} e^{\beta_k(\alpha)} \int_{t_{k-1}}^{t_k} |h_L(t)| dt \tag{4.5}$$

which is just (4.4). It is clearly finite if t_α is finite. If $t_\alpha = +\infty$ then $N = +\infty$. If $\alpha \leq 0$ then $e^{\beta_k(\alpha)} \leq 1$ so that $N_\infty(Y_L;\alpha) \leq N_\infty(Y_L) < +\infty$. If $0 < \alpha < \lambda$ then

$$e^{\beta_k(\alpha)} \int_{t_{k-1}}^{t_k} |h_L(t)| dt \leq h_o(t_k - t_{k-1}) e^{\alpha t_k - \lambda t_{k-1}}$$

$$\leq h_o(t_k - t_{k-1}) e^{(\alpha-\lambda)t_k + \lambda(t_k - t_{k-1})}$$

$$\leq h_o h_2 e^{(\alpha-\lambda)kh_1} e^{\lambda h_2} \tag{4.6}$$

and the (infinite) series in (4.4) converges. This completes the proof.

It is easily verified that $N_{t_\alpha}(Y_L;\alpha)$ is continuous and monotonically increasing in both α and t_α with $N_o(Y_L;\alpha) = 0$ and $N_t(Y_L;0) = N_t(Y_L)$. Also

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Proposition 4.3: If condition (4.1) holds, then

(i) for each choice of α, there exists $t_\alpha^* > 0$ such that, for $t_\alpha \leq t_\alpha^*$,

$$r(N_{t_\alpha}^P(W_A;\alpha)) < 1 \tag{4.7}$$

where $N_{t_\alpha}^P(W_A;\alpha)$ is the $\ell x \ell$ matrix with $(i,j)^{th}$ entry $N_{t_\alpha}((W_A)_{ij};\alpha)$,

(ii) there exists $\alpha^* \geq 0$ such that we can choose $t_\alpha^* = +\infty$ for $\alpha \leq \alpha^*$, and

(iii) if $L = (I+KFG_A)^{-1}KF(G-G_A)$ satisfies the conditions of proposition 4.2 then we can choose $\alpha^* > 0$.

Proof: Follows simply from the previous discussion and is omitted for brevity.

We now state the main result of this section.

Theorem 4.1: Suppose that the conditions of proposition 4.1 hold and define

$$\eta(t) = -\left(\int_0^t W_A(t-t')H_o(t')dt'\right)\beta \tag{4.8}$$

where $H_o(t)$ is the impulse response matrix of $(I+KFG_A)^{-1}K$ and $\beta \in R^\ell$. If $u_A(t)$ (resp $u(t)$) is the input response of the approximating (resp real) feedback scheme of Fig.1(a) (resp Fig.1(b)) to the step set point signal $r(t) \equiv \beta$, $t \geq 0$, then, for any choice of α, $1 \leq j \leq \ell$,

$$|u_j(t) - u_j^{(1)}(t)| \leq \varepsilon_j^\alpha(t) \quad , \quad 0 \leq t \leq t_\alpha \tag{4.9}$$

where

(a) $$u^{(1)}(t) \triangleq u_A(t) + \eta(t) \quad , \quad t \geq 0 \tag{4.10}$$

(b) $$\varepsilon^\alpha(t) \triangleq e^{-\alpha t}(I_\ell - N_t^P(W_A;\alpha))^{-1} N_t^P(W_A;\alpha) \sup_{0 \leq t' \leq t} e^{\alpha t'} \|\eta(t')\|_P \tag{4.11}$$

(the supremum being interpreted with respect to the partial ordering in R^ℓ [1]), and

(c) t_α is any time choice such that (4.7) holds true.

Proof: Condition (c) ensures that the unique solution to (4.2) can be obtained by successive approximation [1] as W is a P-contraction in $X_\alpha^\ell(t)$ for any finite $t \leq t_\alpha$ with

$$\|W(x) - W(y)\|_P \leq N_t^P(W_A;\alpha) \|x-y\|_P \tag{4.12}$$

for all $x,y \in X_\alpha^\ell(t)$. Let the initial guess be $u^{(o)} = u_A$ to yield the first iterate $u^{(1)}$ and the norm estimate in $X_\alpha^\ell(t)$

$$\|u-u^{(1)}\|_P \le (I-N_t^P(W_A;\alpha))^{-1}N_t^P(W_A;\alpha)\|\eta\|_P \tag{4.13}$$

Equation (4.9) then follows from the definition of the vector norm.

In the case of $\alpha = 0$, the result reduces to previously published work [1]. The exponential weighting adds a refinement that can in principle be used to tighten the bounds. In practice, it is envisaged that ε_j^α will be computed for a variety of choices of α in a selection set A and extending (4.9) to the form

$$|u_j(t) - u_j^{(1)}(t)| \le \inf_{\substack{\alpha\in A \\ t\le t_\alpha}} \varepsilon_j^\alpha(t) \tag{4.14}$$

by noting that both u and $u^{(1)}$ are independent of α. The possibilities inherent here can be illustrated by recalling that previous studies suffered from the problem that the bound $\varepsilon^0(t)$ is monotonically increasing and hence that the uncertainty at infinity $\varepsilon^0(\infty)>0$. In contrast, the bound (4.9) may have the property that $\varepsilon^\alpha(t) \to 0$ $(t\to+\infty)$ provided that we choose $\alpha>0$ such that (i) $t_\alpha = +\infty$ (ii) condition (4.7) holds and (iii) $e^{\alpha t}\eta(t)$ is uniformly bounded. The technical background to the existence of such choices is omitted for brevity but we note that it is necessary that $\eta(t)\to 0$ $(t\to+\infty)$ which can only be ensured if $E(t)\to 0$ $(t\to+\infty)$ ie the plant and model must have identical steady state characteristics!

4.2. Output Assessment: The Single-input/single-output Case

In the case of $m = \ell = 1$, output performance deterioration can be assessed in a similar manner to input as described in section 4.1. The output response is described by [1]

$$y = W_0(y) \triangleq y_A + (I+KFG_A)^{-1}K(G-G_A)(r-Fy) \tag{4.15}$$

which is regarded as a fixed-point equation in $X_\alpha(t_\alpha)$. Input-output stability is guaranteed if Proposition 4.1 holds which, in the case of $m = \ell = 1$, reduces to

$$N_\infty(W_A) < 1 \tag{4.16}$$

where W_A is the response of $(I+KFG_A)^{-1}KF$ to the error data E. A similar technique to that used in the proof of theorem 4.1 then yields the result:

Theorem 4.2: Let (4.16) hold and define $\eta(t)$ to be the response of the linear system $(I+KFG_A)^{-1}K(I+KFG_A)^{-1}$ to the error data $E(t)$. If $y_A(t)$ (resp $y(t)$) is the output response of the approximating (resp real) feedback scheme of Fig.1(a) (resp Fig.1(b)) to a unit step demand signal r, then, for any choice of α,

$$|y(t) - y^{(1)}(t)| \le \varepsilon^\alpha(t) \quad , \quad 0 \le t \le t_\alpha \tag{4.17}$$

where $y^{(1)}(t) \triangleq y_A(t)+\eta(t)$, $t\ge 0$,

$$\varepsilon^{\alpha}(t) \triangleq e^{-\alpha t}(1 - N_t(W_A;\alpha))^{-1} N_t(W_A;\alpha) \sup_{0 \le t' \le t} e^{\alpha t'}|\eta(t')| \tag{4.18}$$

and t_α is any time choice such that $N_{t_\alpha}(W_A;\alpha) < 1$.

<u>Proof</u>: The result follows in a similar manner to Theorem 4.1 noting that W_o is a contraction on $X_\alpha(t)$ for any <u>finite</u> $t \le t_\alpha$ with contraction constant $N_t(W_A;\alpha)$ and using successive approximation with initial iterate $y^{(o)} = y_A$ and consequent second iterate $y^{(1)} = y_A + \eta$. The details are omitted for brevity.

As in the discussion following theorem 4.1 the choice of several α in an index set A can be used to refine the result to the form

$$|y(t) - y^{(1)}(t)| \le \inf_{\substack{\alpha \in A \\ t \le t_\alpha}} \varepsilon^{\alpha}(t) \tag{4.19}$$

and by choosing $\alpha > 0$, it is possible to ensure that $\varepsilon^{\alpha}(t) \to 0$ $(t \to \infty)$ by careful choice of α providing that the modelling error $E(t) \to 0$ $(t \to \infty)$.

Similar results hold for the multivariable case but are omitted for brevity.

5. REFERENCES

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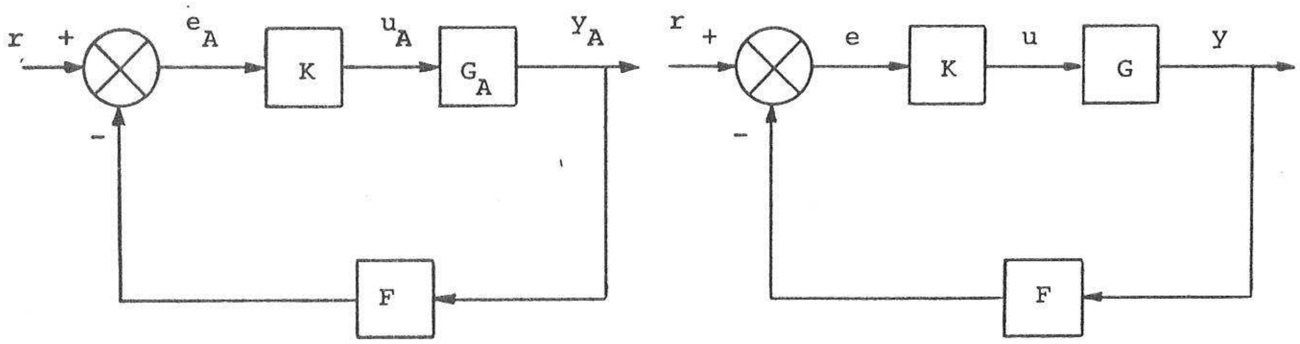


Fig.1. (a) Approximating and (b) implemented feedback schemes

6. APPENDIX

Illustrative Examples

Example 1

Suppose that a single-input/single-output system has an (unknown) transfer function

$$G(s) = \frac{1}{(1+s)^3} \qquad (6.1)$$

with step response Y illustrated in Fig.2(a). The second-order model of the form

$$G_A(s) = \frac{1}{2s^2+3s+1} \qquad (6.2)$$

was fitted, with step response Y_A again illustrated in Fig.2(a) and the error $E(t) = Y - Y_A$ shown in Fig.2(b). The total variation, N_∞ (E) was found to be 0.25 and hence

$$|G - G_A| = \left|\frac{-s^2}{2s^4+7s^3+9s^2+5s+1}\right| \leq 0.25, \quad \forall s \in D \qquad (6.3)$$

To use (3.6), we need to find a suitable filter. For example, choosing

$$F_\alpha = \frac{(1+2s)^2}{s(1+s\alpha)} \qquad (6.4)$$

we obtain the following data

α	$N_\infty(E_\alpha)$	$\Delta_\alpha(s)$
0.1	1.173	$\left\|\frac{s(1+0.1s)}{(1+2s)^2}\right\| \times 1.173$
1.0	0.774	$\left\|\frac{s(1+s)}{(1+2s)^2}\right\| \times 0.774$
2.0	0.541	$\left\|\frac{s}{1+2s}\right\| \times 0.541$

$\Delta_\alpha(s)$ is shown in Fig.3 for the cases α= 0.1, 1.0 and 2.0, together with $|G(s)-G_A(s)|$. Using (3.7), a better upper bound on $|G-G_A|$ is obtained and this is illustrated in Fig.4. Choosing the P+I controller of the form $K(s)=2.0+0.7s^{-1}$ with F(s)=1, the inverse Nyquist plot of $G_A KF=G_A K$ with superimposed confidence circles is given in Fig.5 for the cases $\Delta(s) = N_\infty(E)$ and $\Delta(s) = \inf_{\alpha \in [0.1,\, 1,\, 2]} \Delta_\alpha(s)$

Note that the radii of the confidence circles are smaller in the Fig.5(b).

Using the time-domain method for the above example, the response $W_A(t)$ was computed to be as in Fig.6 and graphical analysis of this response leads to the conclusion that $N_\infty(W_A) = 0.61<1$, hence verifying the stability predictions for the real system.

Using (4.4) we obtain following data

α	$N_{30}(W_A;\alpha)$
0.10	0.84
0.08	0.79
0.06	0.74
0.04	0,69
0.02	0.65
0	0.61
-0.05	0.55
-0.1	0.48
-0.3	0.31

Figure 7(a) shows the error bound $\varepsilon^{\alpha}(t)$ for various fixed values of α, and the error bound $\varepsilon^{o}(t)$ and improved $\varepsilon(t) = \inf_{\substack{\alpha \in A \\ t \leq 30}} \varepsilon^{\alpha}(t)$ are shown in Fig.7(b). The bounds $y^{(1)} \pm \varepsilon^{o}(t)$, together with the responses y and y_A are illustrated in Fig.8 and the improved bounds $y^{(1)} \pm \varepsilon(t)$, together with the responses y and y_A are illustrated in Fig.9.

Example 2

Consider a plant with transfer function

$$G(s) = \frac{1}{1 + 4s} \qquad (6.5)$$

The first-order model of the form

$$G_A(s) = \frac{1}{1 + 5s} \qquad (6.6)$$

was fitted. The total variation, $N_\infty(E)$ was found to be 0.164. Using the frequency-domain technique with $\Delta(s) = N_\infty(E)$ and the P + I controller

$$K(s) = 5 + 5s^{-1} \qquad (6.7)$$

with $F(s) = 1$, the inverse Nyquist plot of $G_A K$ with superimposed confidence circles shown in Fig.10 indicates that the (-1,0) point does lie in the confidence band, hence the theory cannot predict stability or instability of the real plant.

Now using the filtering method with

$$F_\alpha = \frac{(1+4.5s)^2}{s(1 + \alpha s)} \qquad (6.8)$$

$\Delta_\alpha(s)$ is shown in Fig.11 for the cases α=0.1 and 1,0 together with $|G-G_A|$, using $\Delta(s) = \inf_{\alpha \in [0.1,1]} \Delta_\alpha(s)$, the inverse Nyquist plot of $G_A K$ with superimposed confidence circles shown in Fig.12 indicates that the (-1,0) point does not lie in or on the confidence band and stability of real plant is hence guaranteed as the controllability and observability condition is satisfied.

The time-domain analysis lead to the error bound $\varepsilon^{\alpha}(t)$, shown in Fig.13(a). the error bounds $\varepsilon^{o}(t)$ and $\varepsilon(t) = \inf_{\alpha} \varepsilon^{\alpha}(t)$ are shown in Fig.13(b). The bounds $y^{(1)} \pm \varepsilon^{o}(t)$ and $y^{(1)} \pm \varepsilon(t)$, together with the responses y and y_A are illustrated in Fig.14.

Example 3

Consider a system with transfer function

$$G(s) = \frac{4}{(s^2+2s+4)\ (s+1)} \qquad (6.9)$$

with step response Y(t) illustrated in Fig.15(a). The first-order model of the form

$$G_A(s) = \frac{1}{1+1.6s} \qquad (6.10)$$

was fitted, with step response $Y_A(t)$ again illustrated in Fig.15(a) and the error $E(t)=Y - Y_A$ shown in Fig.15(b). The total variation, $N_{\infty}(E)$ was found to be 0.72.

Using the filtering method with

$$F_{\alpha} = \frac{(1 + 2.5s)^2}{s(1+ \alpha s)} \qquad (6.11)$$

we obtain $\Delta_{\alpha}(s)$ for various fixed values of α, shown in Fig.16. Choosing the p + I controller of the form $K(s) = 0.7 + 0.6s^{-1}$ with $F(s)=1$, the inverse Nyquist plot of $G_A KF$ with superimposed confidence circles is shown in Fig.17 for the cases $\Delta(s)= N_{\infty}(E)$ and $\Delta(s) = \inf \Delta_{\alpha}(s)$.

The time-domain analysis lead to the error bound $\varepsilon^{\alpha}(t)$, shown in Fig.18(a). The error bounds $\varepsilon^{o}(t)$ and $\varepsilon(t) = \inf_{\alpha} \varepsilon^{\alpha}(t)$, are shown in Fig.18(b). The bounds $y^{(1)} \pm \varepsilon^{o}(t)$ and $y^{(1)} \pm \varepsilon(t)$, together with the responses y and y_A are illustrated in Fig.19.

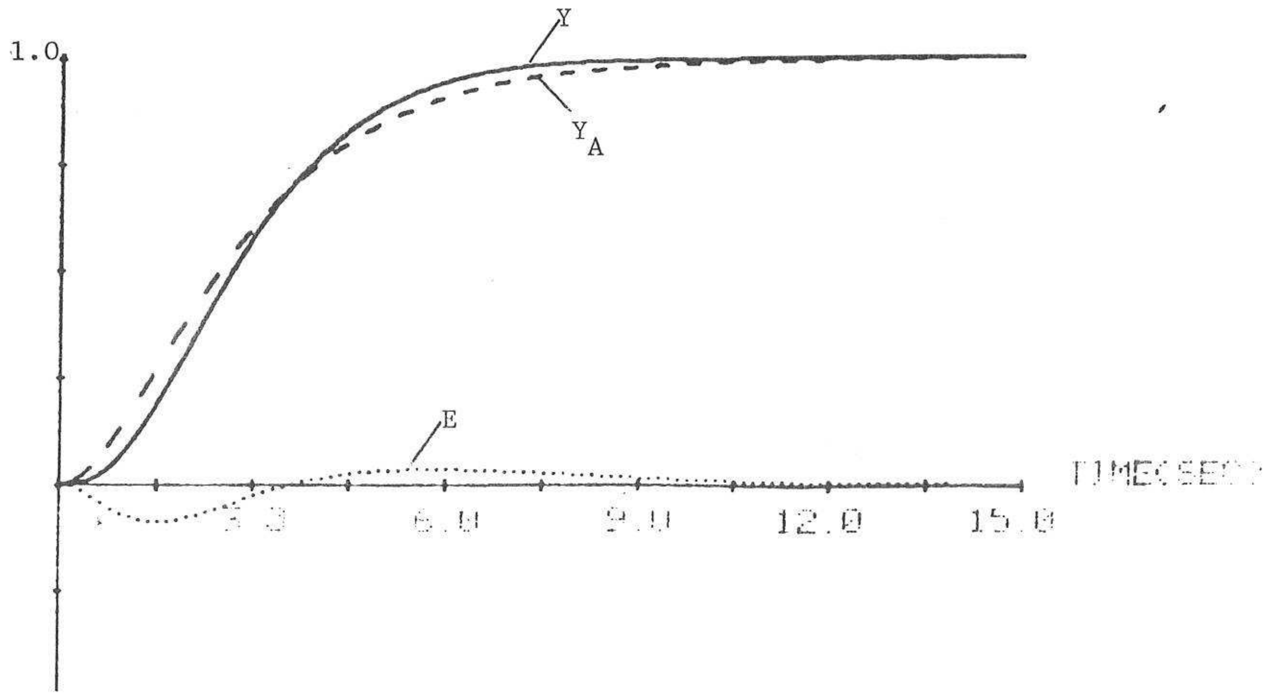


Fig.2(a).

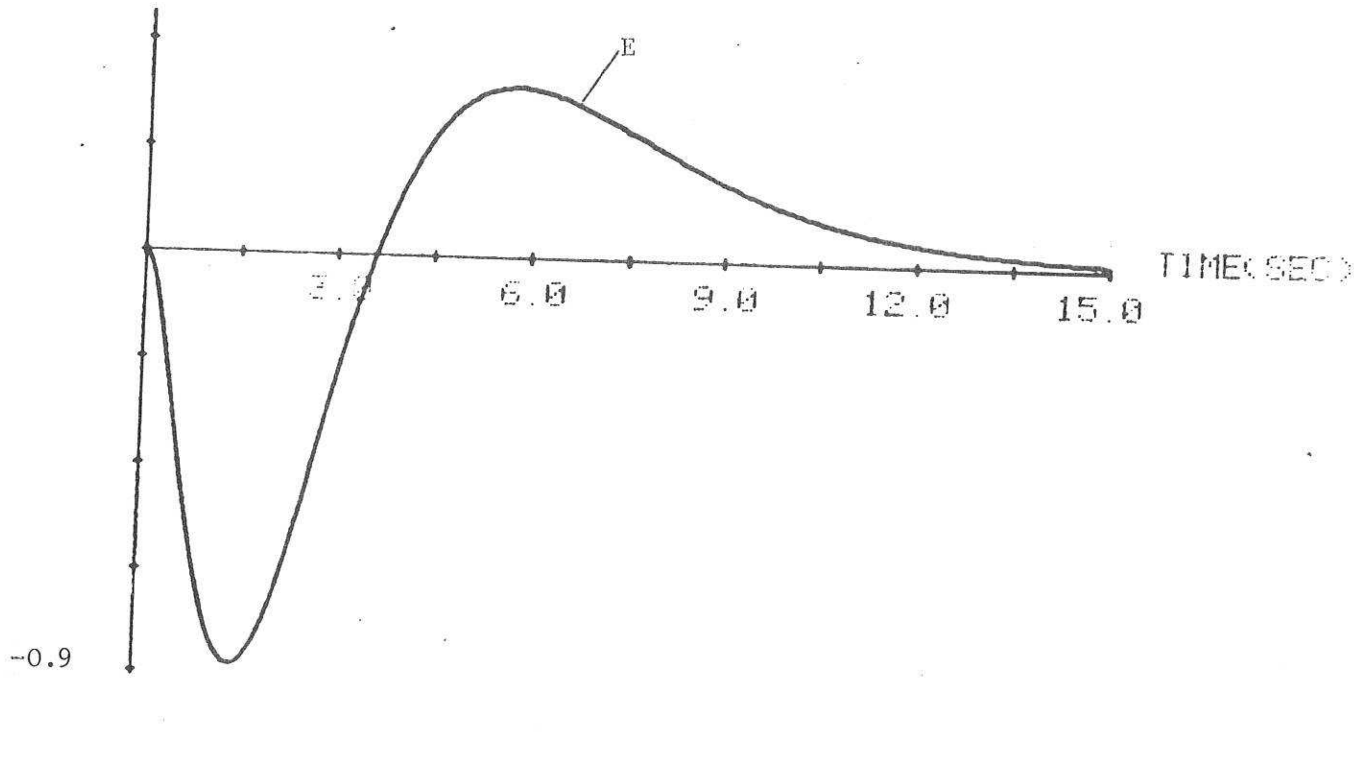


Fig.2(b)

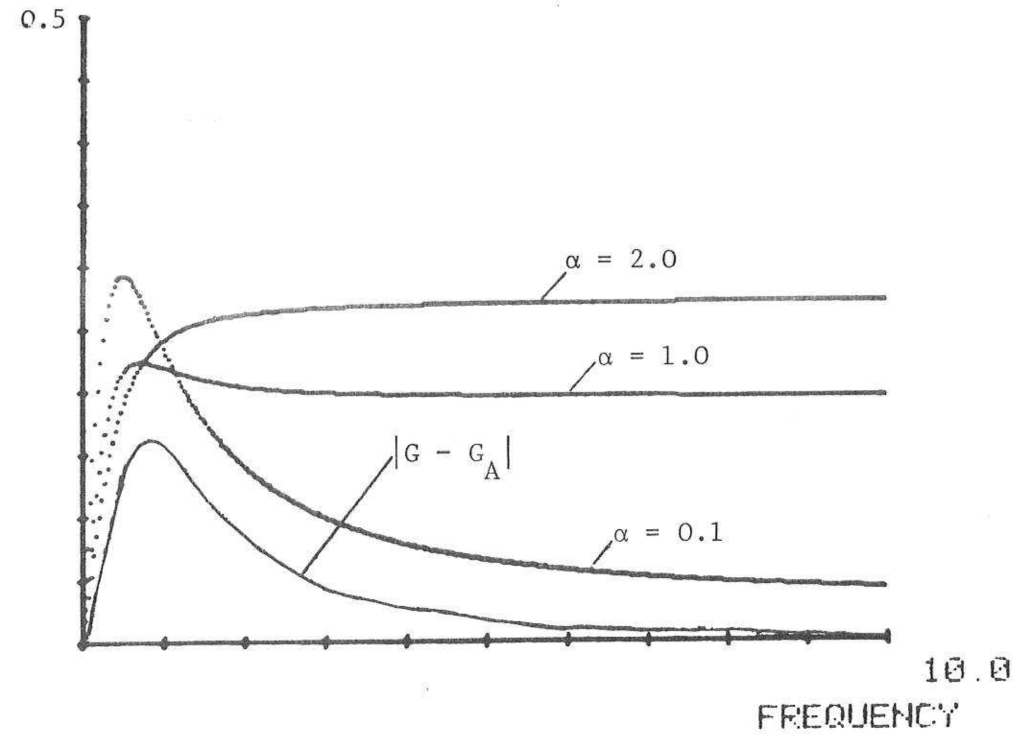


Fig.3.

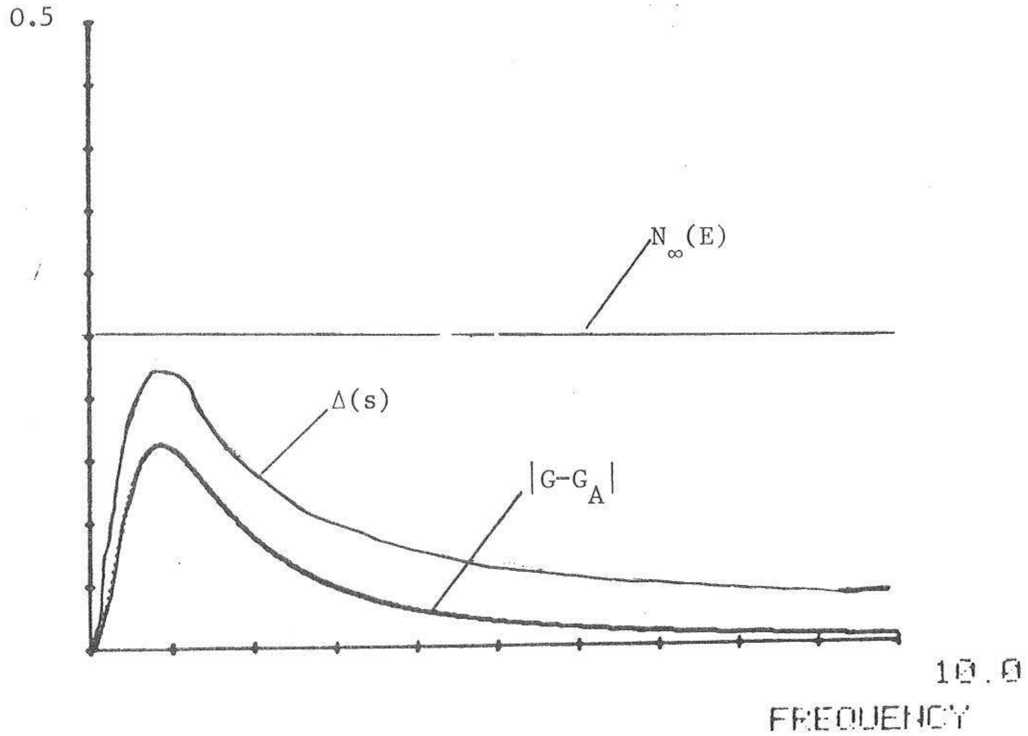


Fig.4.

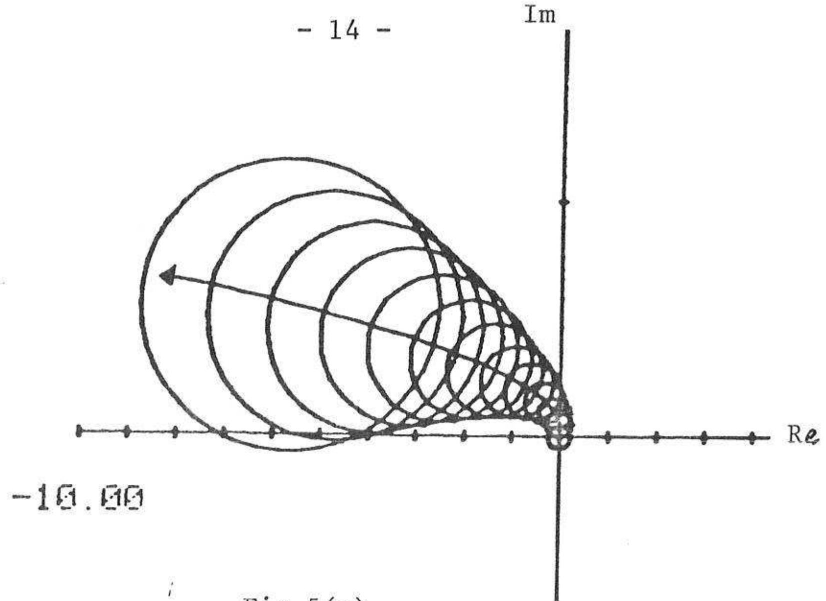


Fig.5(a)

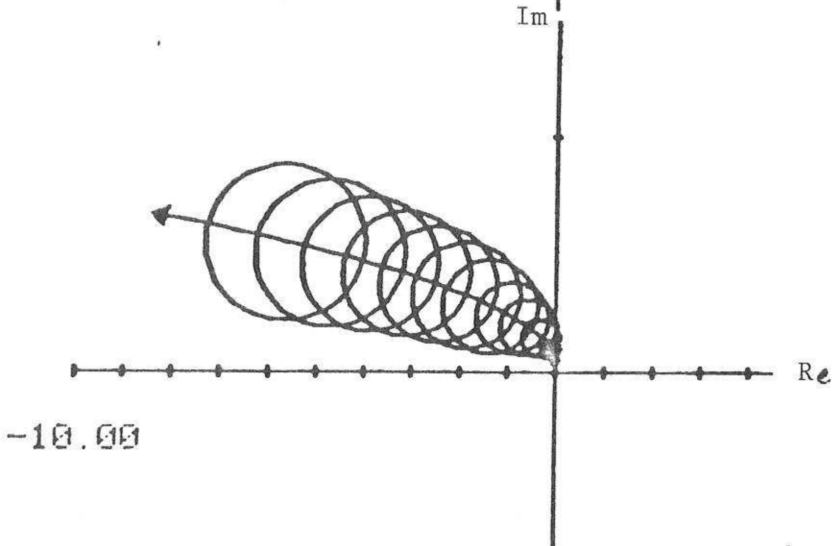


Fig.5(b)

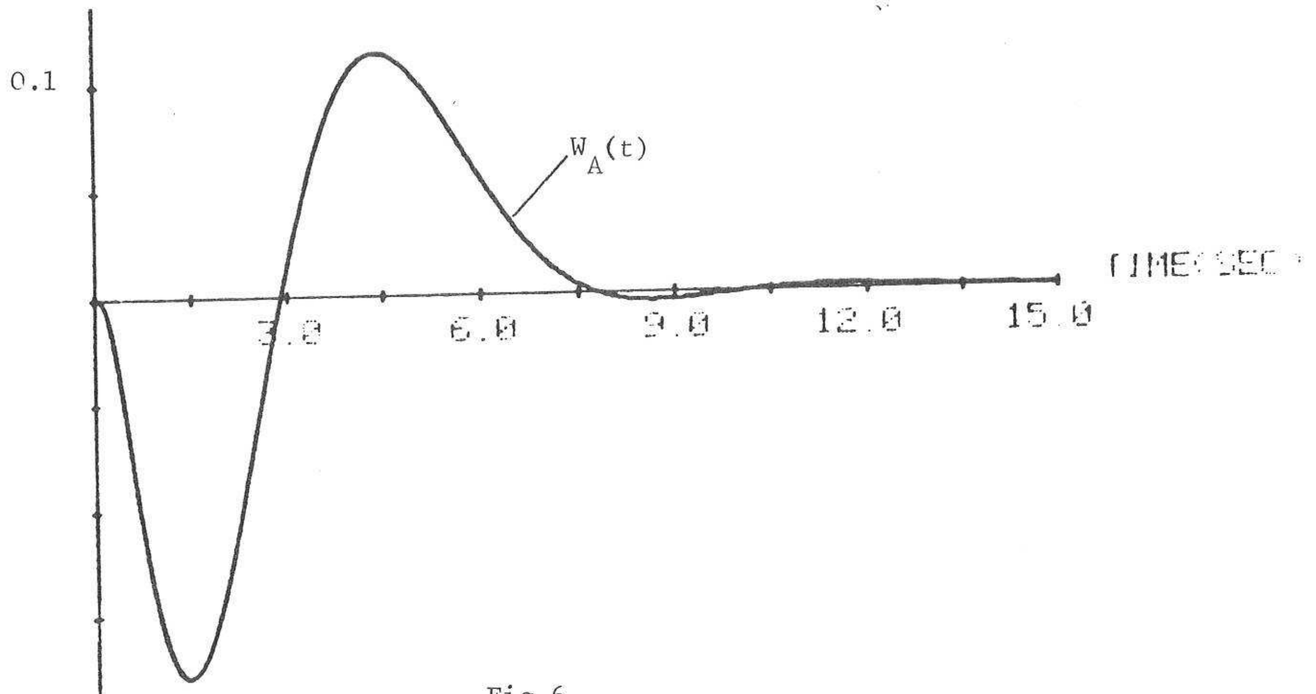


Fig.6.

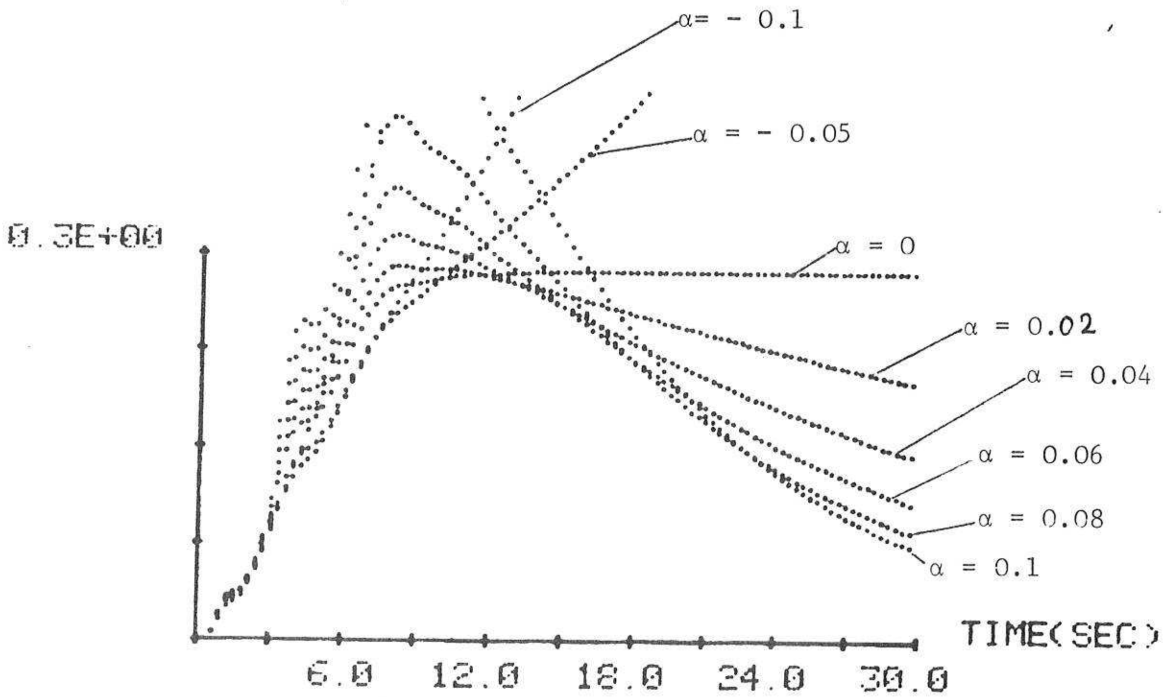

α= - 0.1
α = - 0.05
0.3E+00
α = 0
α = 0.02
α = 0.04
α = 0.06
α = 0.08
α = 0.1
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

Fig.7(a).

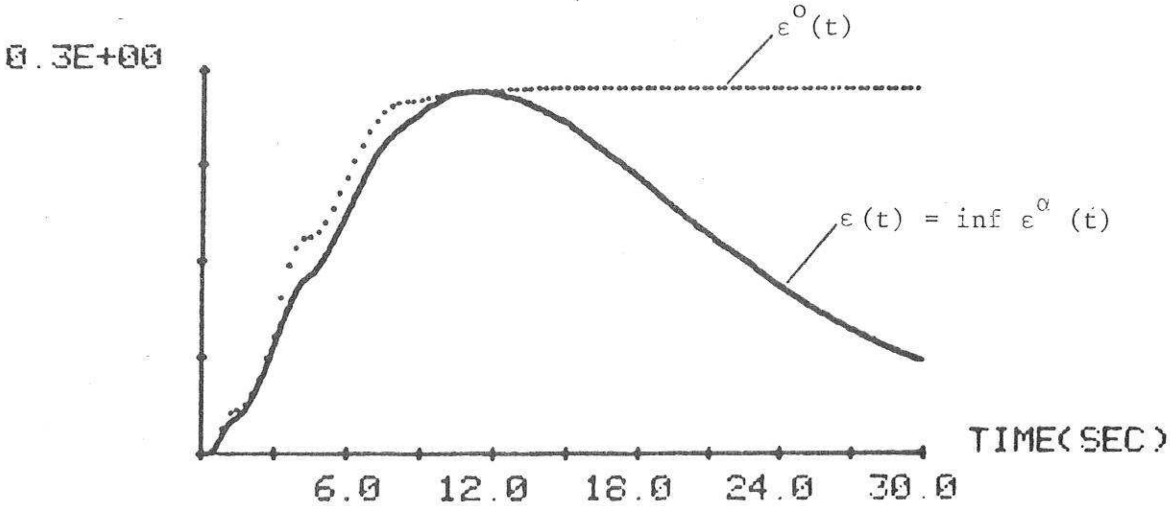

$\varepsilon^{o}(t)$
0.3E+00
$\varepsilon(t) = \inf \varepsilon^{\alpha}(t)$
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

Fig. 7 (b).

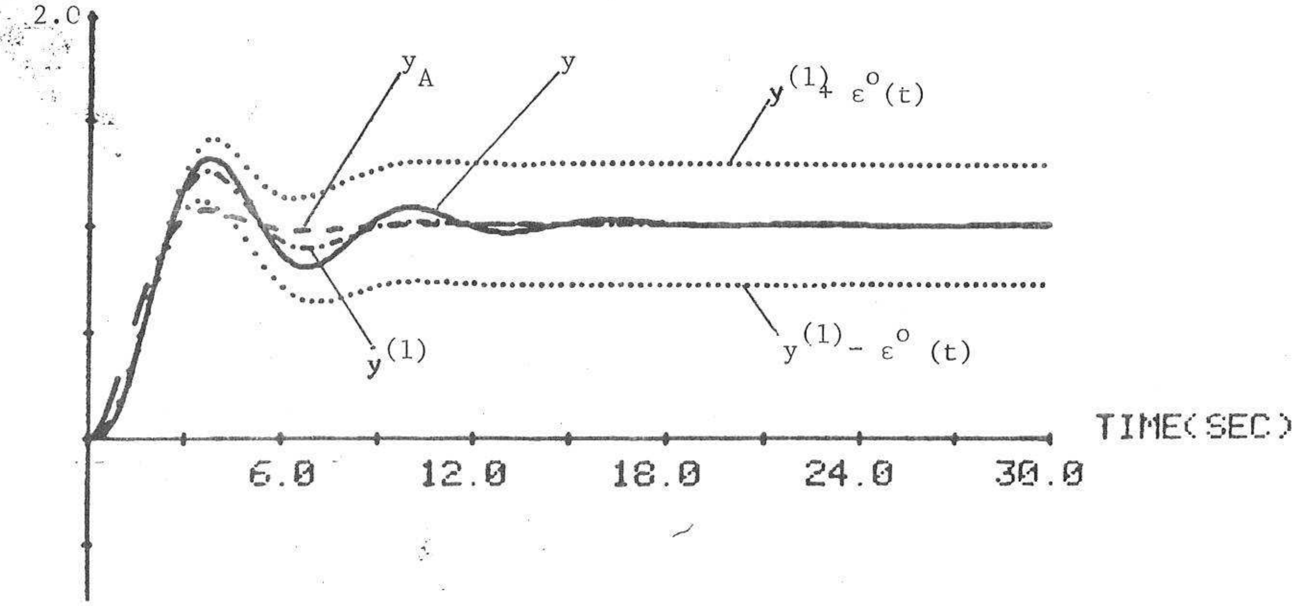

2.0
y_A
y
y(1)+ εo(t)
y(1)
y(1)- εo (t)
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

Fig.8.

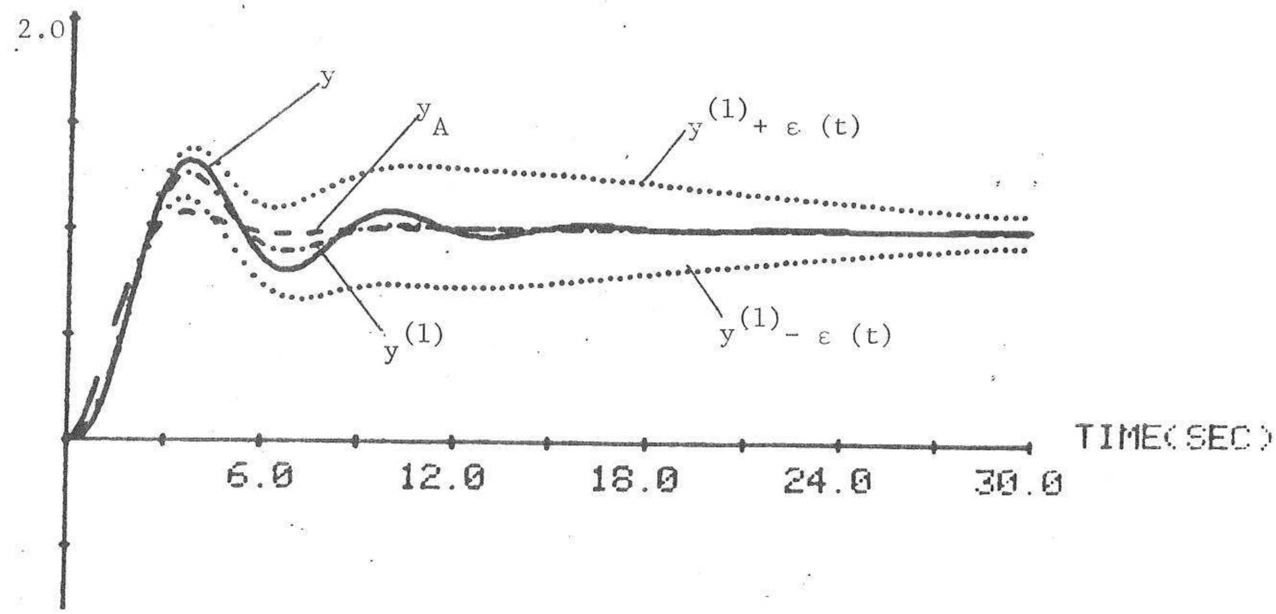

2.0
y
y_A
y(1)+ ε (t)
y(1)
y(1)- ε (t)
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

Fig.9.

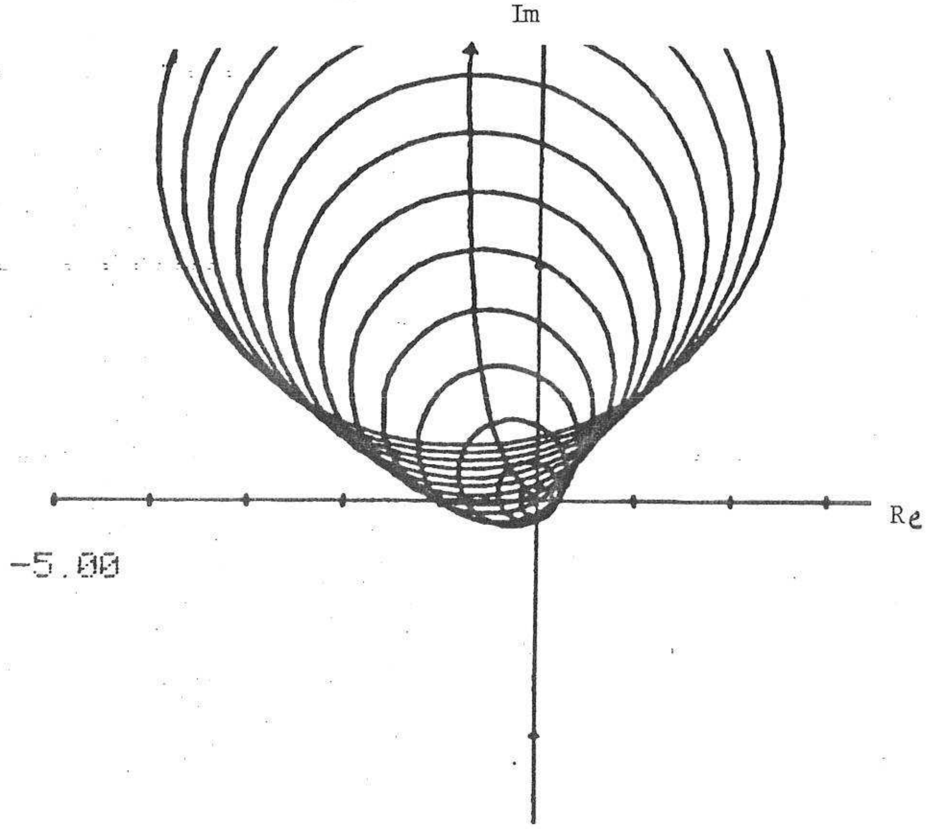


Fig.10

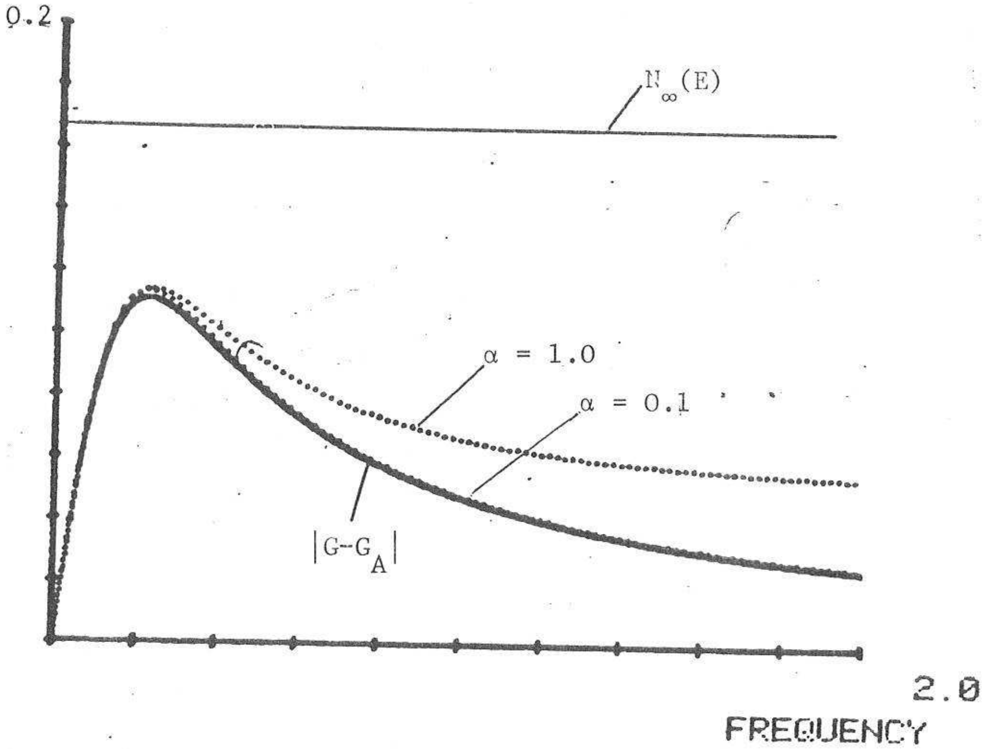


Fig.11.

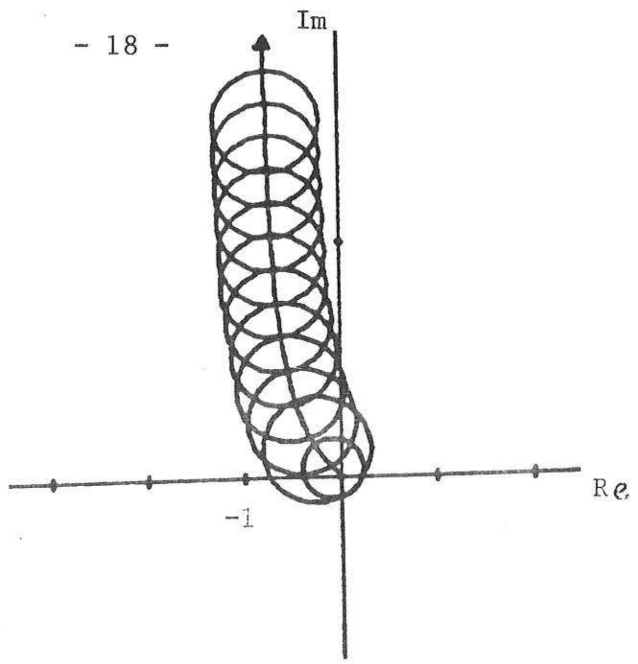


Fig.12.

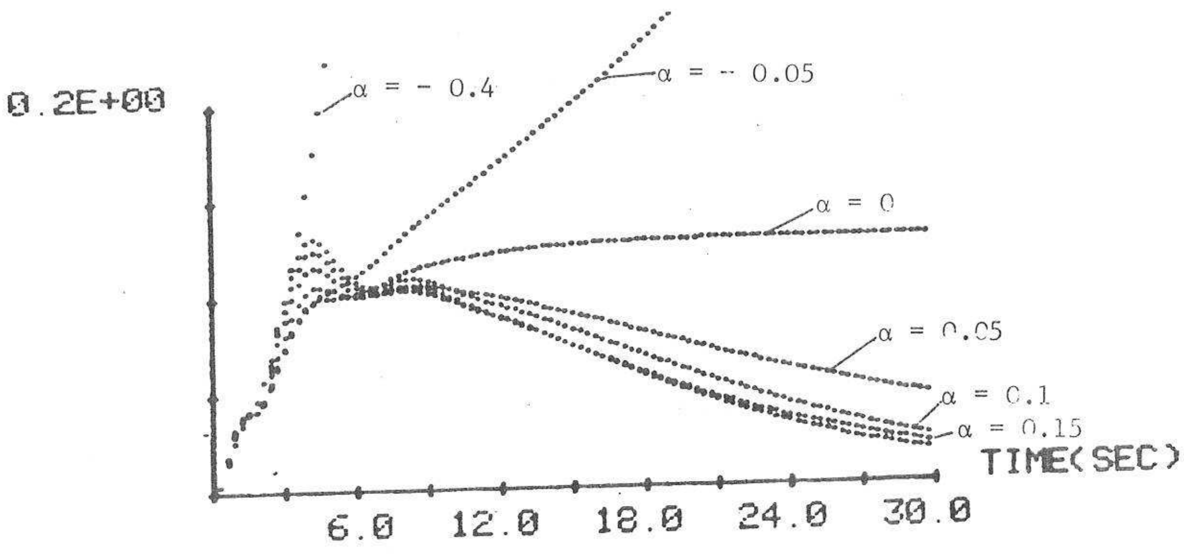


Fig.13(a)

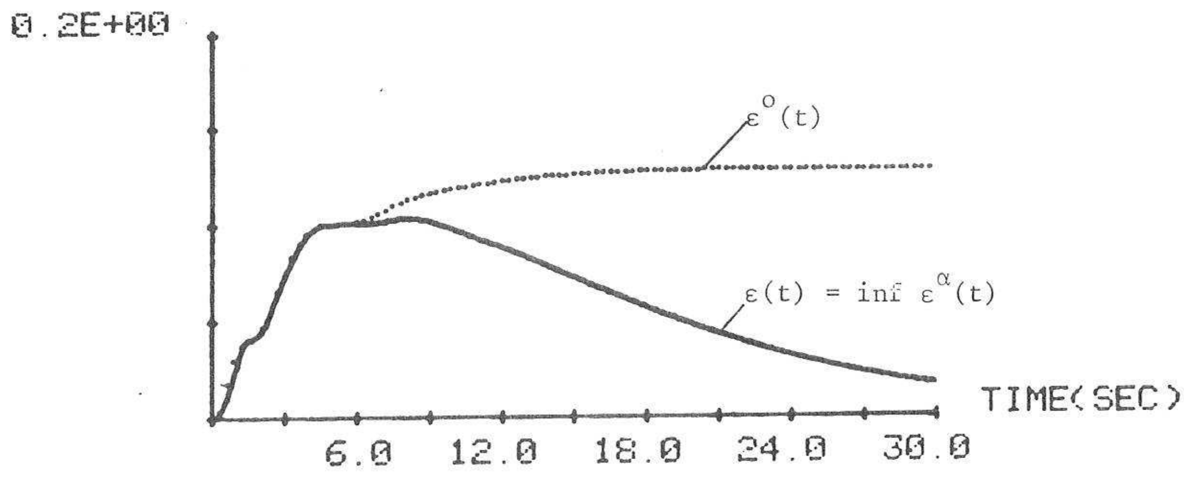
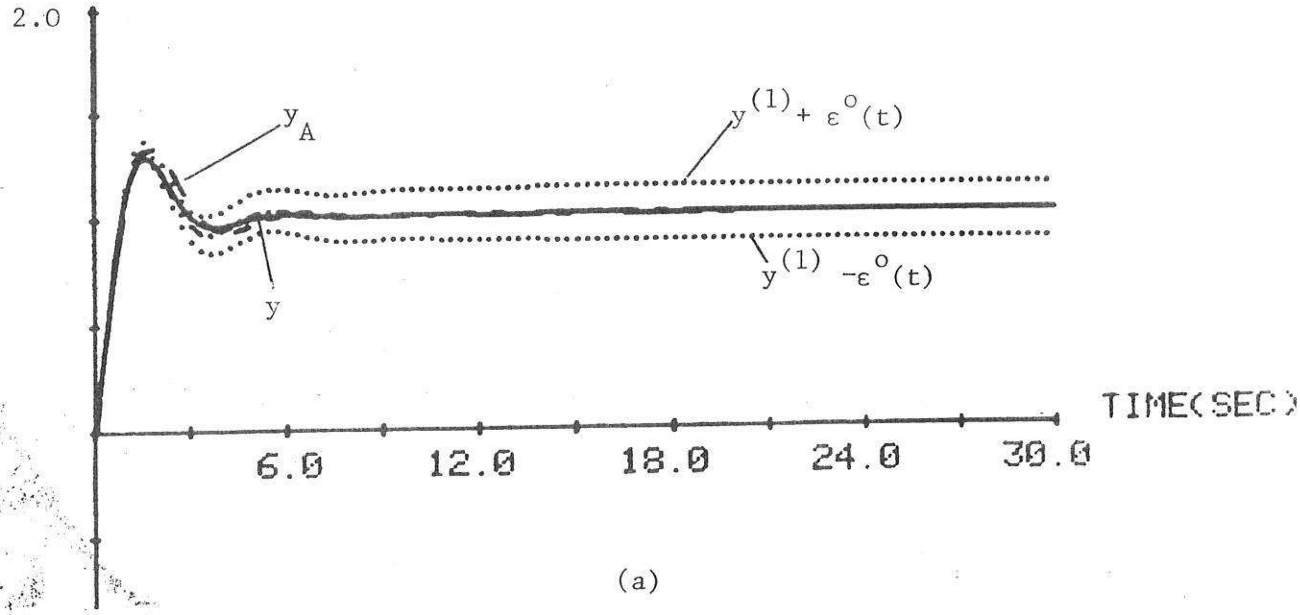
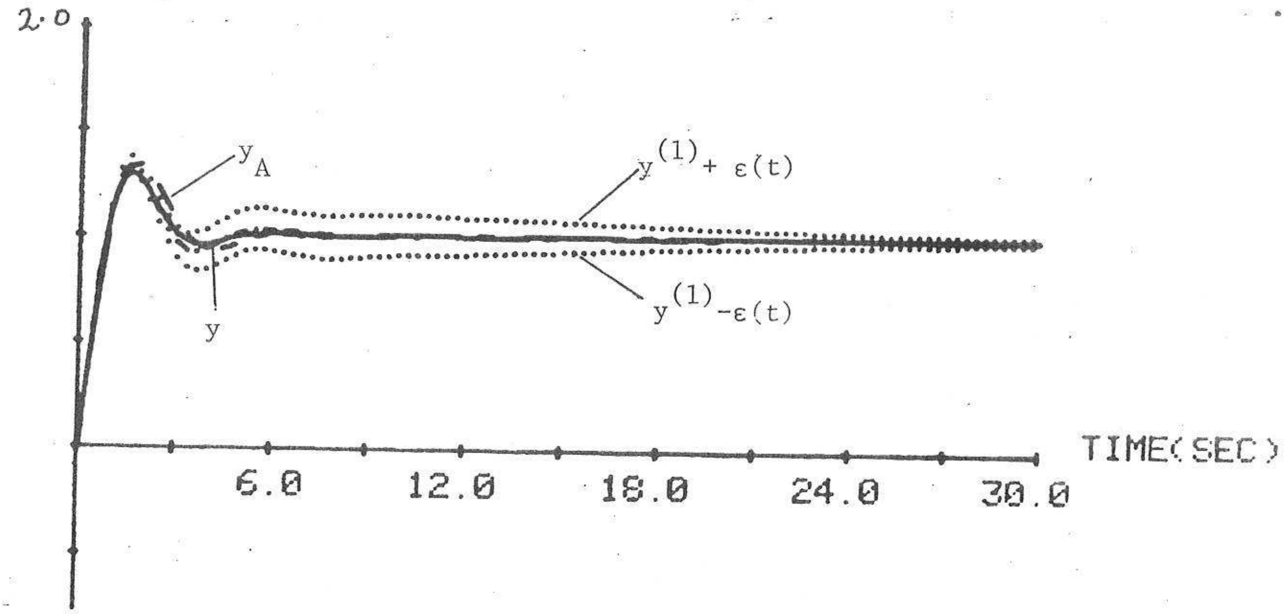


Fig.13(b)


2.0
y_A
$y^{(1)} + \varepsilon^o(t)$
$y^{(1)} - \varepsilon^o(t)$
y
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

(a)


2.0
y_A
$y^{(1)} + \varepsilon(t)$
$y^{(1)} - \varepsilon(t)$
y
TIME(SEC)
6.0
12.0
18.0
24.0
30.0

(b)

Fig.14.

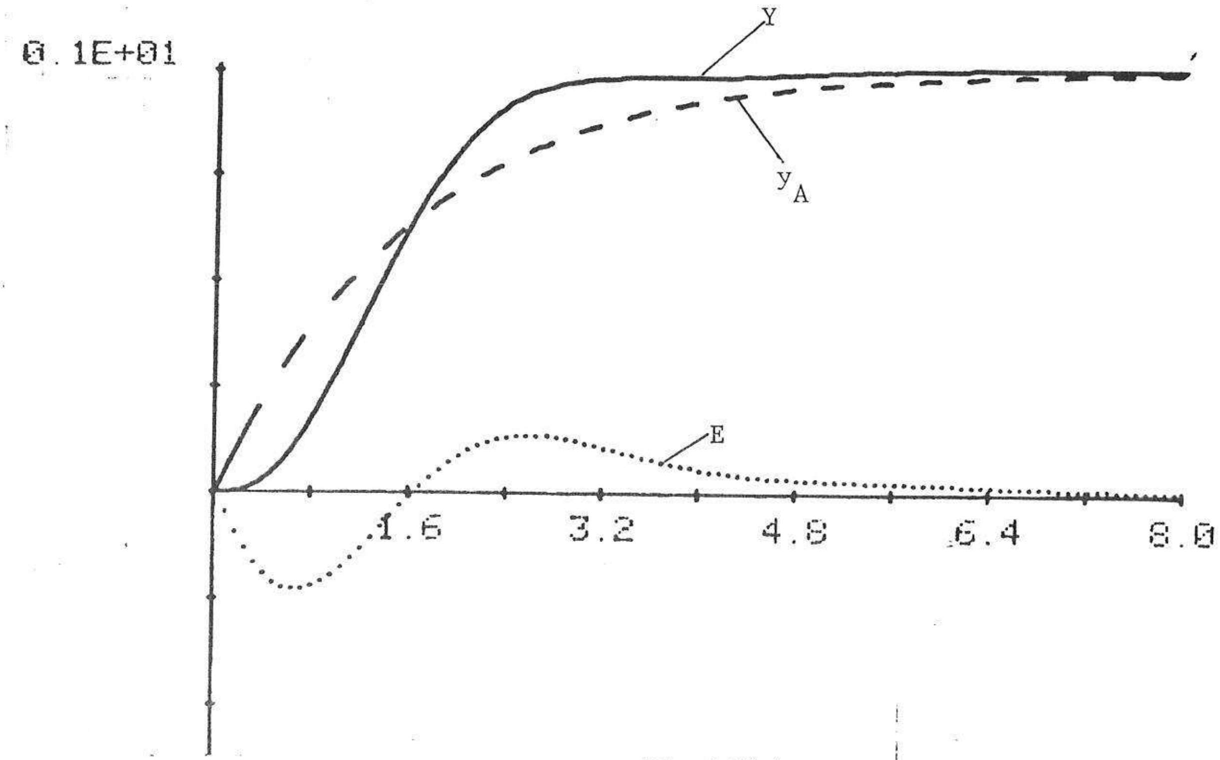


Fig.15(a)

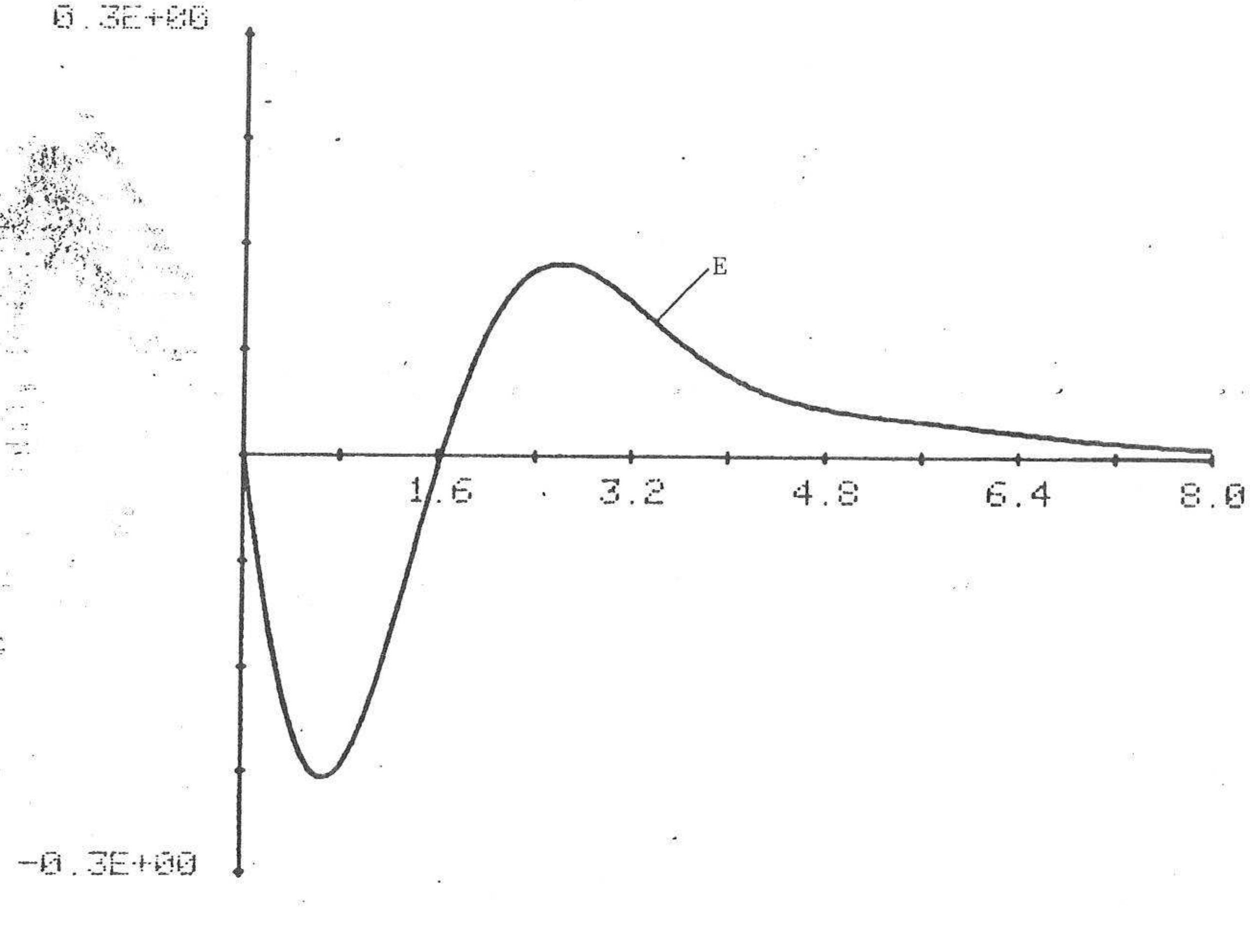


Fig.15(b)

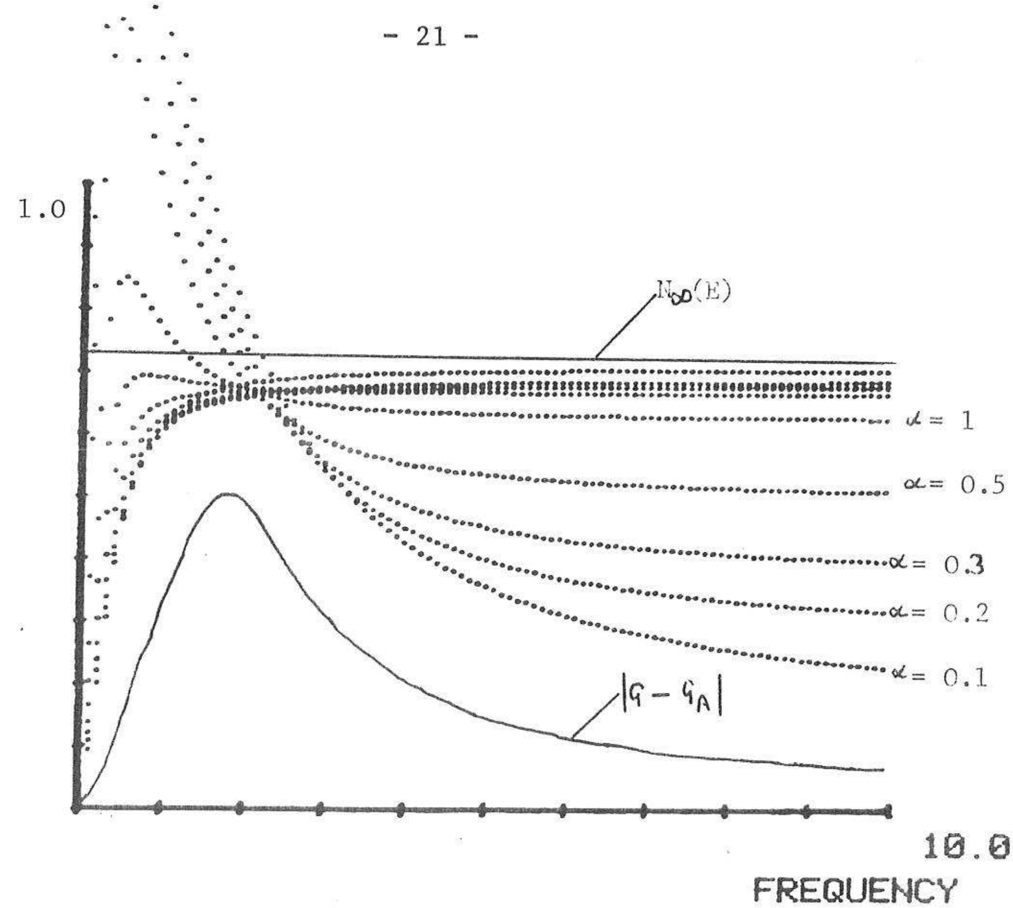


Fig.16

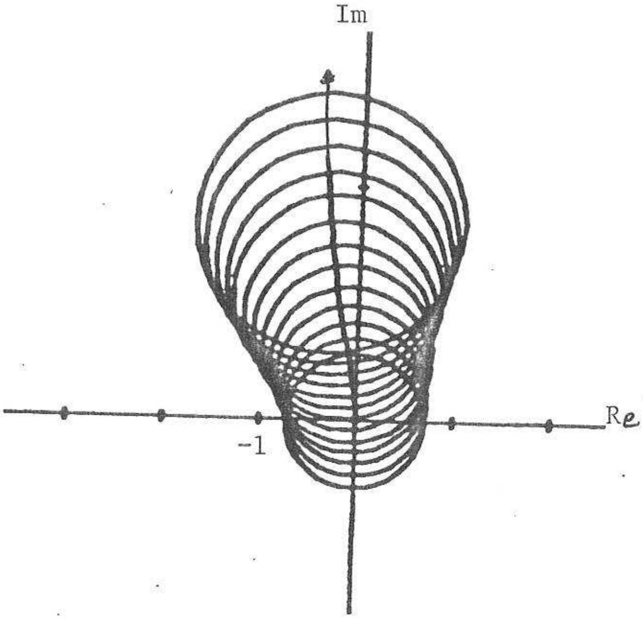


Fig.17(a)

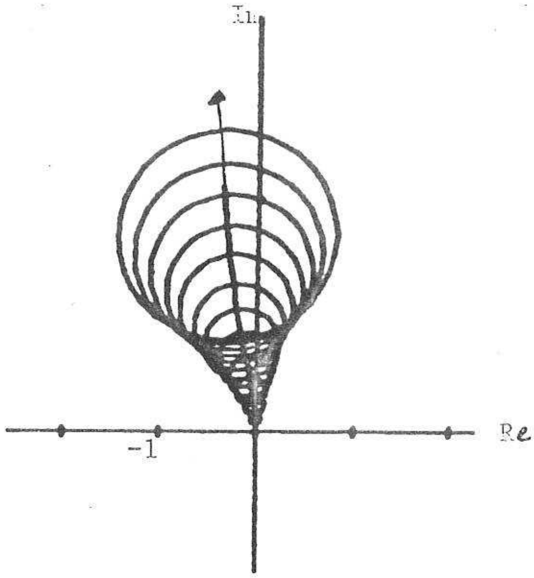


Fig.17(b)

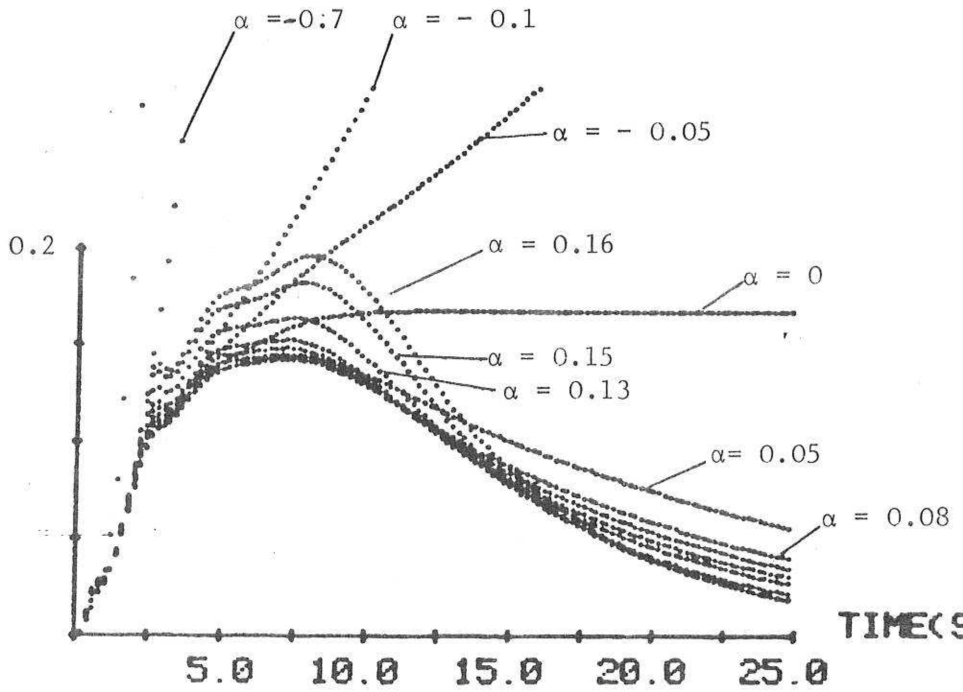


Fig.18(a)

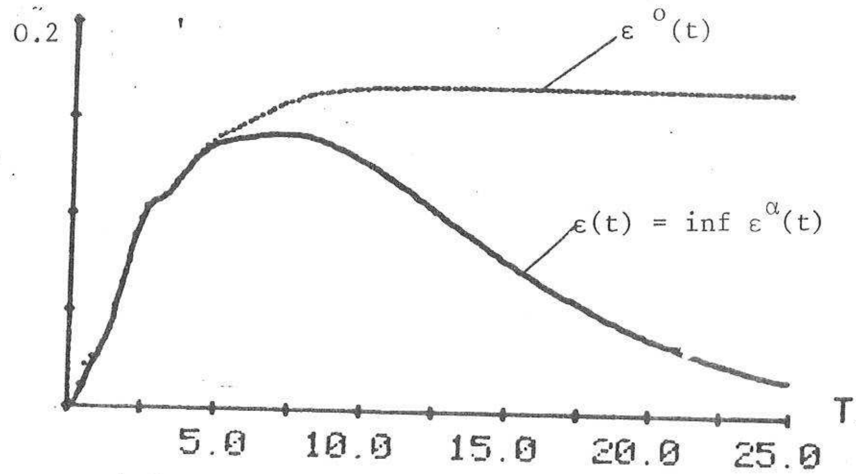
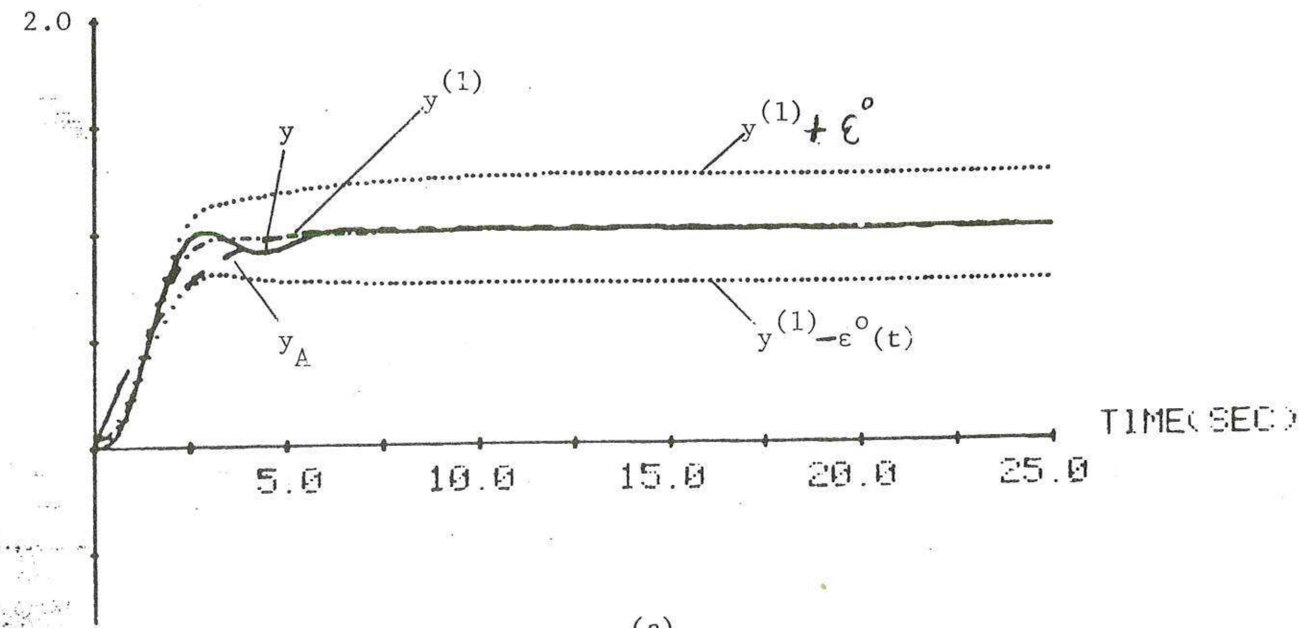
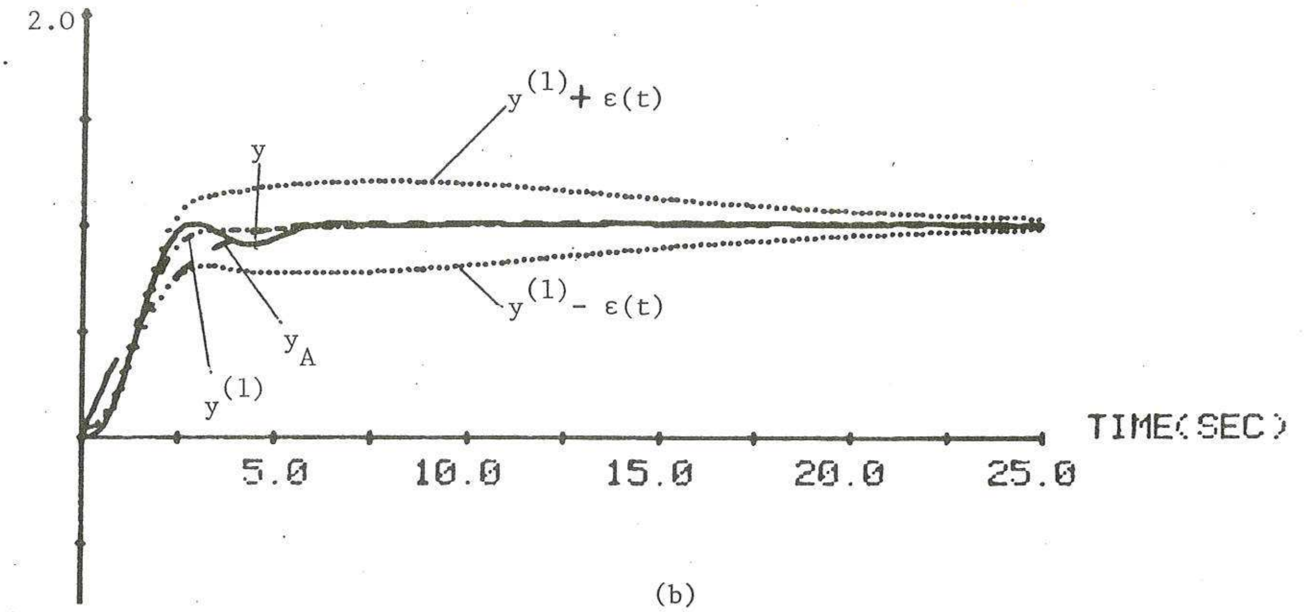


Fig.18(b)


2.0
y
y^(1)
y^(1) + ε°
y_A
y^(1) − ε°(t)
TIME(SEC)
5.0
10.0
15.0
20.0
25.0

(a)


2.0
y^(1) + ε(t)
y
y^(1) − ε(t)
y_A
y^(1)
TIME(SEC)
5.0
10.0
15.0
20.0
25.0

(b)

Fig.19.