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ROBUSTNESS AND SENSITIVITY OF SMITH PREDICTOR CONTROLLERS FOR TIME-DELAY SYSTEMS

by

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1. Introduction

There is now an extensive literature on the subject of the control of time delay systems [1]. The well known Smith predictor [2] and its extensions are often used in this case. Recent work pays particular attention to the robustness of the Smith predictor scheme [3] to copy with the plant/model mismatch and to retain stability in the presence of the changes in plant dynamics. A general theory describing the robustness of the Smith scheme is given by [3] and some problems, particularly the choice of approximate model, are investigated in [4].

In this report, we will first extend the result of [3] to the more general case - nonunity feedback case in both inner and outer loops. Then, by looking at a multivariable process control example, we will discuss the parameter optimal problems and the effects of nonunity feedback. In addition the sensitivity function in multivariable case will be derived and discussed.

2. Robustness of the Extended Smith Schemes

The plant is regarded as a linear operator mapping input linear vector space U^{ℓ} into output linear vector space y^{m} and suppose that it can be expressed into separable form (Fig.1)

$$Y = TZ \qquad \ldots(1)$$

$$Z = GU \qquad \ldots(2)$$

whre the linear operator T maps Y^{m} into itself and represents the delays at the plant output and G maps U^{ℓ} into Y^{m}. Because of the generality of the theory which will be given it can be applied to plants expressed as above [4]. Where T is a pure delay matrix and G is a rational and strictly proper TFM but not necessarily a 'delay free' component (i.e. it can be a component with time delay).

The extended Smith scheme is illustrated in Fig.2 where the linear operator G_A and T_A represent models of the plant components G and T respectively and K is a forward path controller regarded as a mapping of Y^m into U^ℓ. The only difference with the usual Smith scheme is that the feedback components are, instead of unity, $F_1(s)$ in the inner loop and $F_2(s)$ in the outer loop respectively. Here F_1 and F_2 are regarded as mapping of Y^m into itself. As to be mentioned in section 3.2, F_1 and F_2 may lead to some benefits sometimes, e.g. improve stability or performance or increase permissible mismatch.

We now derive the general theory of robustness [3] for the scheme of Fig.2. The signal $\gamma \in Y^m$ is the demand signal. If the initial conditions are zero, the dynamics can be expressed as follows

$$Y = TGU \qquad \ldots(3)$$

$$U = K(\gamma - F_1 G_A U - F_2(Y - T_A G_A U)) \qquad \ldots(4)$$

After a little manipulation, we get

$$U = K^*(\gamma - F_2 TGU) \qquad \ldots(5)$$

where $$K^* = (I + KF_1 G_A - KF_2 T_A G_A)^{-1} K \qquad \ldots(6)$$

is an uniquely defined linear mapping of Y^m into U^ℓ. It is trivially verified from equation (5) that scheme Fig.2 is equivalent to scheme Fig.3.

For convenience of comparison and use, we follow the method of reference [3] to give a thoerem to characterize the stability of the extended Smith scheme. Let U_o^ℓ and Y_o^m be linear vector subspace of U^ℓ and Y^m respectively (regarded as spaces of 'stable' inputs and outputs respectively).

Theorem 1

The scheme of Fig.2 is stable in the BIBO sense if

(i) the plant component G and its model G_A map U_o^{ℓ} into Y_o^m and that their restrictions to U_o^{ℓ} have finite induced norms,

(ii) the delay component T and its model T_A map Y_o^m into itself with restrictions to Y_o^m of finite induced norms,

(iii) the restriction to Y_o^m of the mapping

$\gamma \rightarrow U_A \triangleq (I+KF_1G_A)^{-1}K$ has range in U_o^{ℓ} and finite induced norm,

(iv) $$\lambda_1 \triangleq \|(I+KF_1G_A)^{-1}KF_2\Delta TG_A\| < 1 \quad \ldots(7)$$

(v) $$\lambda_2 \triangleq \frac{1}{1-\lambda_1} \|(I+KF_1G_A)^{-1}KF_2T\Delta G\| < 1 \quad \ldots(8)$$

where $$\Delta G = G-G_A \quad \ldots(9)$$

$$\Delta T = T-T_A \quad \ldots(10)$$

represent the mismatch between plant and its model.

Proof: As G and T are stable and bounded by assumption it is sufficient to prove that $U \in U_o^{\ell}$ whenever $\gamma \in Y_o^m$.

From $$U = K^*(\gamma-F_2TGU)$$

$$= K^*(\gamma-F_2TGU-F_2T_AG_AU+F_2T_AG_AU) \quad \ldots(11)$$

we can get

$$U = (I+K^*F_2T_AG_A)^{-1}K^*(\gamma-F_2(TG-T_AG_A)U) \quad \ldots(12)$$

This is an equation in U^{ℓ} of form $U = W_r(U)$. Suppose that W_r map U_o^{ℓ} into itself whenever the demand $\gamma \in Y_o^m$. Clearly the BIBO stability is ensured if W_r is a contraction[5]. This is the case if

$$\lambda_o \triangleq \|(I+K^*F_2T_AG_A)^{-1}K^*F_2(TG-T_AG_A)\| < 1 \quad \ldots(13)$$

We then prove that the following equality is true

$$(I+K^{*}F_2T_AG_A)^{-1}K^{*} = (I+KF_1G_A)^{-1}K \qquad \ldots(14)$$

Write $U_A = V_1-V_2$

with

$$V_1 = K^{*}\gamma \qquad \ldots(15)$$

$$V_2 = K^{*}F_2T_AG_AU_A \qquad \ldots(16)$$

and express U_A in the form

$$U_A = (I+K^{*}F_2T_AG_A)^{-1}K^{*}\gamma \qquad \ldots(17)$$

By definition (6) we get

$$V_1 = (I+KF_1G_A-KF_2T_AG_A)^{-1}K\gamma \qquad \ldots(18)$$

Rewriting it yields

$$V_1 = K(\gamma-(F_1-F_2T_A)G_AV_1) \qquad \ldots(19)$$

By similar means, we can get

$$V_2 = K\left[F_2T_AG_AU_A-(F_1-F_2T_A)G_AV_2\right] \qquad \ldots(20)$$

Subtracting (19) and (20), we can obtain

$$U_A = V_1-V_2 = K\gamma-KF_1G_AU_A \qquad \ldots(21)$$

or its equivalent

$$U_A = (I+KF_1G_A)^{-1}K\gamma \qquad \ldots(22)$$

and equality (14) is then proved.

Substitute (14) into (13), the result follows by noting

$$\lambda_o \triangleq \|(I+KF_1G_A)^{-1}KF_2(TG-T_AG_A)\|$$
$$= \|(I+KF_1G_A)^{-1}KF_2(T\Delta G+\Delta TG_A)\|$$
$$\leq \|(I+KF_1G_A)^{-1}KF_2T\Delta G\| + \|(I+KF_1G_A)^{-1}KF_2\Delta TG_A\|$$
$$< (1-\lambda_1) + \lambda_1 = 1 \quad ...(23)$$

The condition (i)-(iii) ensures that W_r maps U_o^{ℓ} into itself for all $\gamma \in Y_o^m$. In physical view-point, condition (i) and (ii) is the requirement of open-loop stability of plant TG and its model T_AG_A, conditions (iii) simply require that the feedback scheme of Fig.4 is stable in normal practical sense. Condition (iv) and (v) provide upper bounds on the mismatch ΔG and ΔT that guarantee the BIBO stability of Fig.2.

For simplicity of application, following [3], we suppose that G and G_A are rational and strictly proper TFMs, that K is rational and proper and that both $T = \text{diag}\{e^{-\tau_j s}\}_{1\leq j\leq m}$ and $T_A = \text{diag}\{e^{-\tau_{Aj} s}\}_{1\leq j\leq m}$ are mxm diagonal matrices of pure delay. In this case, the theorem has following simple form[3]:

Theorem 2

If the plant component G and its model G_A are asymptotically stable and the feedback system of Fig.4 is input-output stable then the extended Smith scheme of Fig.2 is BIBO stable if

$$\lambda_1 = \max_{1\leq i\leq m} \sup_{s\in\Omega} \sum_{j=1}^{m} |((I+KF_1G_A)^{-1}KF_2\Delta TG_A)_{ij}| < 1 \quad ...(24)$$

$$\lambda_2 \triangleq \frac{1}{1-\lambda_1} \max_{1\leq i\leq m} \sup_{s\in\Omega} \sum_{j=1}^{m} |((I+KF_1G_A)^{-1}KF_2T\Delta G)_{ij}| < 1$$

...(25)

where s is complex variable and Ω is the Nyquist contour.

Proof: The stability assumptions are equivalent to conditions (i)-(iii) of theorem 1 whilst (24) and (25) are identical to (7) and (8) according to reference [7].

This result is easily used to evaluate λ_1 and λ_2 by numerical calculation.

3. Parameter Optimal Control

In reference [4] we discussed the choice of approximate model G_A and forward path controller K for a kind of process control. But the choice of time delay component T_A has not yet been discussed in detail. Hence we shall investigate this problem from a parameter optimal view-point. The parameters of the controller K and the model G_A can also be analysed from this view-point. The effects of the components F_1 and F_2 in the feedback path will also be investigated in this section.

Throughout this report the performance indexes used are defined as integral square-errors:

$$J_i = \int_0^\infty e_i^2 dt \qquad \ldots(26)$$

where

$$e_i = y_i(t) - x_i(t-\tau_i) \qquad \ldots(27)$$

and τ_i is the time delay in output y_i.

3.1. Temporal Optimality

The term 'temporal optimality' is used to present the optimal choice T_A for a certain G, T, K and G_A. We will indicate that the presence of temporal mismatch ΔT can lead some benefits sometimes. In other words, the matched case is not necessarily the optimal case[6]. The choice of temporal model T_A is hence still a problem which should be handled carefully.

Let's consider the same example used in ref. [4], where the plant can be expressed into separate form TG:

$$G = \begin{pmatrix} \frac{119.3}{1+812.8s} & \frac{-62}{1+904s} \\ \frac{55.3}{1+776s} & -\frac{109.7}{1+715s} \end{pmatrix}, \quad T = \begin{pmatrix} e^{-35x22.8s} & 0 \\ 0 & e^{-35x3s} \end{pmatrix}$$

For simplicity suppose that $F_1 = I$, $F_2 = I$, i.e. its a usual Smith predictor scheme.

We choose the first order model[8] as an approximate model G_A, which is defined as

$$G_A = (A_o s + A_1)^{-1} \qquad \text{...(28)}$$

where

$$A_o^{-1} = \lim_{s\to\infty} sG(s)$$
$$A_1^{-1} = \lim_{s\to 0} G(s) \qquad \text{...(29)}$$

The controller K is chosen as proportional plus integral control[8], which is

$$K = (k + c + \frac{kc}{s})A_o - A_1 \qquad \text{...(30)}$$

where k and c are constant scalar.

Regarding m as a parameter we choose temporal component T_A as

$$T_A = \begin{pmatrix} e^{-mx35x22.8s} & 0 \\ 0 & e^{-mx35x3s} \end{pmatrix} \qquad \text{...(31)}$$

Fig.5 gives the variation of $J-J_o$ with m under the stability condition (24) and (25). Here J_o is the performance index when m = 1, i.e. the matched performance index. It is evident from Fig.5 that the mismatch optimal is lower than matched case. The similar conclusion was indicated in [6] for single input/single output. In other words, the performance can be improved by 'correct' mismatch. So the minimization of the performance index can be, in the authors' opinion, one of the criterions of choosing temporal component.

3.2. The Effects of Feedback Components

We investigate the effects of F_1 and F_2 from the following view-point:

(i) do they improve the stability characteristics?

(ii) do they improve the performance?

(iii) do they increase the permissible mismatch?

The effects of F_1 is first investigated by looking at the same example with previous subsection.

Let

$$T_A = \begin{pmatrix} e^{-30x22.8s} & 0 \\ 0 & e^{-30x3s} \end{pmatrix} \quad \ldots(32)$$

and choose both F_1 and F_2 to be of diagonal form and regard n as a parameter

$$F_1 = \begin{pmatrix} 1+30xnxs & 0 \\ 0 & 1+20xnxs \end{pmatrix} \quad \ldots(33)$$

$$F_2 = I$$

The norms λ_1 and λ_2 can be evaluated by (24) and (25) and are illustrated in Fig.6 as a function of frequency for various fixed value of n. The performance index is shown in Table 1.

n	J
-1	138.66
0	151.45
1	164.53

Table 1

It is evident from this example that when n>0 F_1 can reduce the max value of the norms but increase the performance index. In other words, it can increase the permissible mismatch and hence robustness but deteriorate the performance. Conversely, when n<0, F_1 can improve the performance but reduce the permissible mismatch. So F_1 provides a margin of choice between the robustness and performance. When the main purpose is to increase the robustness, designers should put n>0 (proportional plus differential) in inner loop. And the performance can be improved by putting n<0 in inner loop.

We then investigate the effects of F_2 using the same example but supposing

$$F_2 = \begin{pmatrix} \frac{1}{1+30xnxs} & 0 \\ 0 & \frac{1}{1+20xnxs} \end{pmatrix} \qquad \ldots(34)$$

$$F_1 = I$$

The norms λ_1 and λ_2 are shown in Fig.7 for various fixed values of n and the performance index are shown in Table 2.

n	J
0	151.45
1	154.62
2	153.07

Table 2

We can see from Fig.7 that when n>0, the presence of F_2 can increase the permissible mismatch and hence increase the robustness. The effect of F_2 on performance is very small and can be neglected. The reason of this small effect on performance is that because this example is a small mismatch case, the outer-loop feedback is of no importance in practice. In extreme cases, when it is exactly matched, F_2 has no effect on the performance. However, it is expected that in serious mismatched cases, F_2 might produce more obvious effect on the performance.

As for the effect of F_1 and F_2 on stability characteristics, we only indicate that F_1 is in fact a parallel compensator for the scheme of Fig.4. We know very well that in single-input/single-output case the addition of a zero to a open-loop transfer function has the effects of tending to make the system more stable and to speed up the settling of the response. Hence a F_1 formed as (33) can lead the scheme of Fig.4 to be more easily stabilized and it is this stability that is a necessary condition for stabilizing the scheme of Fig.2.

In short, the presence of F_1 and F_2 may yield some benefits and provide a margin of choice to the designer. Even though the general analysis has not yet been achieved, we can expect that the extended Smith scheme may have some advantage over the usual Smith scheme.

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3.3. The Steady State Error

We here note the steady state error in $F_1 \neq I$, $F_2 \neq I$ case. From Fig.3 the closed loop TFM can be obtained as

$$H_c(s) = T(1+GK^*F_2T)^{-1}GK^* \qquad ...(35)$$

where

$$K^* = (I+KF_1G_A-KF_2T_AG_A)^{-1}K \quad \text{as before (see formula (6))}$$

According to final value theorem, steady state value for unit step demand is

$$Y(\infty) = T(o)(I+G(o)K^*(o)F_2(o)T(o))^{-1}G(o)K^*(o) \qquad ...(36)$$

When both T and T_A are diagonal matrix of pure time delays, the final value is

$$Y(\infty) = (I+G(o)K^*(o)F_2(o))^{-1}G(o)K^*(o) \qquad ...(37)$$

where

$$K^*(o) = (I+K(o)(F_1(o)-F_2(o))G_A(o))^{-1}K(o) \qquad ...(38)$$

We then look at some special case:

(1) When $F_1(o) = F_2(o)$, it is clear that

$$K^*(o) = K(o)$$

and

$$Y(\infty) = (I+G(o)K(o)F_2(o))^{-1}G(o)K(o) \qquad ...(39)$$

In this case if integral action is included in the controller, $K(o) \to \infty$ then

$$Y(\infty) = F_2^{-1}(o) \qquad ...(40)$$

That means steady state error will exist if $F_2(o) \neq I$ even though integral action is included in the controller.

(2) $F_1(o) \neq F_2(o)$ and controller includes integral action, in this case

$$K^*(o) = G_A^{-1}(o)(F_1(o)-F_2(o))^{-1} \qquad \ldots(41)$$

the steady state value of y is

$$Y(\infty) = \left[I+G(o)G_A^{-1}(o)(F_1(o)-F_2(o))^{-1}F_2(o)\right]^{-1}G(o)G_A^{-1}(o)(F_1(o)-F_2(o))^{-1} \qquad \ldots(42)$$

If the approximate model is chosen such that

$$G_A(o) = G(o)$$

then $Y(\infty)$ is of more simple form,

$$Y(\infty) = \left[I+(F_1(o)-F_2(o))^{-1}F_2(o)\right]^{-1}(F_1(o)-F_2(o))^{-1} = F_1^{-1}(o) \qquad \ldots(43)$$

The steady state error will again exist if $F_1(o) \neq I$.

Summarising the analysis above, we can conclude that the steady state error exists in general even though integral action is included in the controller. This is perhaps the expense for obtaining the increase in permissible mismatch or the improvement of the performance. The designer should hence check the steady state error when either inner loop or outer loop feedback is not unity.

4. Sensitivity Function for Multivariable Smith Scheme

The sensitivity problem for time delay systems in single input/single output case has been studied in [6]. We will here investigate this problem in multivariable case. Following [6], the term 'temporal sensitivity' is used to describe the sensitivity of system performance to the time delay parameter and 'parameter sensitivity' for the other parameters. Throughout this report we suppose that the perturbation of the parameter is time invariant. For simplicity assume both F_1 and F_2 in scheme Fig.2 are unity.

4.1. General Sensitivity Function

From Fig.3 we obtain

$$Z = GK^*(\gamma - TZ) \qquad \ldots(44)$$

or

$$Z = (1+MT)^{-1}M\gamma \qquad \ldots(45)$$

where $M = GK^*$, includes all parameters except temporal parameter. The output y can then be expressed as

$$Y = TZ \qquad \ldots(46)$$

$$= T(1+MT)^{-1}M\gamma \qquad \ldots(47)$$

Differentiate both sides of (46) with respect to a general parameter α

$$\dot{Y} = T\dot{Z} + \dot{T}Z \qquad \ldots(48)$$

where the notation '.' means $\frac{\partial}{\partial\alpha}$.

It is clear from (45) that

$$Z + MTZ = M\gamma \qquad \ldots(49)$$

and by differentiating (49) we get

$$\dot{Z} + MT\dot{Z} + (\dot{M}T + M\dot{T})Z = \dot{M}\gamma \qquad \ldots(50)$$

Noting (45), after a little manipulation, we obtain

$$\dot{Z} = (I+MT)^{-1}\dot{M}(I-T(I+MT)^{-1}M)\gamma - (I+MT)^{-1}M\dot{T}(I+MT)^{-1}M\gamma \qquad \ldots(51)$$

The general sensitivity function is

$$\dot{Y} = T(I+MT)^{-1}\dot{M}(I-T(I+MT)^{-1}M)\gamma + (I-T(I+MT)^{-1}M)\dot{T}(I+MT)^{-1}M\gamma \qquad \ldots(52)$$

The parameter sensitivity function $\dot{Y}_\alpha$ can be obtained by taking $\dot{T} = 0$ in (52)

$$\dot{Y}_\alpha = T(I+MT)^{-1}\dot{M}(I-T(I+MT)^{-1}M)\gamma \qquad \ldots(53)$$

The corresponding temporal sensitivity $\dot{Y}_T$ is

$$\dot{Y}_T = (I-T(I+MT)^{-1}M)\dot{T}(I+MT)^{-1}M\gamma \qquad \ldots(54)$$

4.2. Sensitivity Function for Matched Case

In the matched case the outer loop disappears in fact and the Smith scheme is simplified to be Fig.8. The output y is

$$Y = TZ \qquad \ldots(55)$$

where $\quad Z = (I+M)^{-1}M\gamma \qquad \ldots(56)$

and $\quad M = GK$

Differentiate both sides of (55) with respect to a parameter α

$$\dot{Y} = T\dot{Z} + \dot{T}Z \qquad \ldots(57)$$

$\dot{Z}$ can be obtained by the similar means with 4.1

$$\dot{Z} = (I+M)^{-1}\dot{M}(I-(I+M)^{-1}M)\gamma \qquad \ldots(58)$$

It is simpler than (51) because the disappearance of the outer-loop. The sensitivity function in matched case is then

$$\dot{Y} = T(I+M)^{-1}\dot{M}(I-(I+M)^{-1}M)\gamma+\dot{T}(I+M)^{-1}M\gamma \qquad \ldots(59)$$

because

$$(I-(I+M)^{-1}M)\gamma = (I+M)^{-1}\gamma \qquad \ldots(60)$$

substitute (60) into (59) we then get

$$\dot{Y} = T(I+M)^{-1}\dot{M}(I+M)^{-1}\gamma + \dot{T}(I+M)^{-1}M\gamma \qquad \ldots(61)$$

The parameter sensitivity function can be easily obtained from (61)

$$\dot{Y}_\alpha = T(I+M)^{-1}\dot{M}(I+M)^{-1}\gamma \qquad \ldots(62)$$

The corresponding temporal sensitivity is

$$\dot{Y}_T = \dot{T}(I+M)^{-1}M\gamma \qquad \ldots(63)$$

The parameter sensitivity can be further analysed as follows. If only a parameter α in G changes, then

$$\dot{M} = \dot{G}K \qquad \ldots(64)$$

We use the term 'plant sensitivity function' and notation $\dot{Y}_G$ to describe the sensitivity of system performance to this parameter. Which is

$$\dot{Y}_G = T(I+GK)^{-1}\frac{\partial G}{\partial \alpha}K(I+GK)^{-1}\gamma \qquad \ldots(65)$$

When only a parameter α in the controller K changes, so called 'controller sensitivity function' $\dot{Y}_K$ can be defined by similar way:

$$\dot{Y}_K = T(I+GK)^{-1}G\frac{\partial K}{\partial \alpha}(I+GK)^{-1}\gamma \qquad \ldots(66)$$

It describes the sensitivity of system performance to a parameter of controller.

4.3. Sensitivity Function for Mismatched Case

We rewrite the parameter sensitivity function (53) and make further observations to it.

$$\dot{Y}_\alpha = T(I+MT)^{-1}\dot{M}(I-T(I+MT)^{-1}M)\gamma \qquad \ldots(67)$$

where $M = GK^*$

$$= G(I+KG_A-KT_AG_A)^{-1}K \qquad \ldots(68)$$

(1) When only a parameter α in G changes,

$$\dot{M} = \dot{G}(I+KG_A-KT_AG_A)^{-1}K \qquad \ldots(69)$$

then the plant sensitivity function $\dot{Y}_G$ is

$$\dot{Y}_G = T(I+MT)^{-1}\frac{\partial G}{\partial\alpha}(I+KG_A-KT_AG_A)^{-1}K(I-T(I+MT)^{-1}M)\gamma \qquad \ldots(70)$$

(2) When only a parameter α in G_A changes, define 'model sensitivity function' and use notation $\dot{Y}_{G_A}$ to describe the sensitivity of system performance to this parameter.

From (68), we have

$$(I+KG_A-KT_AG_A)G^{-1}M = K \qquad \ldots(71)$$

Differentiate both sides of (71) with respect to α included in G_A, after a little manipulation, obtain

$$\dot{M} = GM(T_A-I)\dot{G}_AG^{-1}M \qquad \ldots(72)$$

The model sensitivity function is obtained by substituting (72) into (67)

$$\dot{Y}_{G_A} = T(I+MT)^{-1}GM(T_A-I)\frac{\partial G_A}{\partial\alpha}G^{-1}M(I-T(I+MT)^{-1}M)\gamma \qquad \ldots(73)$$

We can see from (72) that the nearer the T_A to I, the smaller the model sensitivity. This is easy to be understood - if $T_A = I$, the feedbacks of G_A in inner loop and outer loop will offset each other (see Fig.2 when $F_1 = F_2 = I$) and the model sensitivity will be zero.

(3) Similarly, when only a parameter in K changes, we can find $\dot{M}$ from (71),

$$\dot{M} = G(I+KG_A-KT_AG_A)^{-1} \frac{\partial K}{\partial \alpha} (I-(I-T_A)G_AG^{-1}M) \qquad \ldots(74)$$

The controller sensitivity function $\dot{Y}_K$ can be easily found by substituting (74) into (67) but will be omitted here for brevity.

4.4. The Application of Sensitivity Functions

It is well known that sensitivity plays an important part in system synthesis. The sensitivity functions have been found in both matched and mismatched case without difficulty but we suffer from their complex structure. The structure of the sensitivity function in the multivariable case is much more complex than it is in single-input/single-output case. Because of this the application of the sensitivity function has to be carried forward by numerical method rather than analysis.

If the topic is the investigation of the sensitivity of performance to one parameter α which is included in plant or model (or controller, temporal component), regard the corresponding sensitivity function as a function of s and parameter α: $\dot{Y}(s,\alpha)$. Where s is the complex variable. We then can obtain the induced norm of the sensitivity function for a fixed α. The induced norm is of course different for different α. The best α, from the sensitivity view-point, is such a α which makes norm reach their smallest value under the stability condition. In other words the proper α should tend to minimize the induced norm of sensitivity function.

Like in theorem 2, suppose that G and G_A are rational and strictly proper TFMs, that K is rational and proper and that both T and T_A are diagonal matrices of pure time delay, and suppose that $\frac{\partial G}{\partial \alpha}$, $\frac{\partial G_A}{\partial \alpha}$,

$\frac{\partial K}{\partial \alpha}$, $\frac{\partial T}{\partial \alpha}$ are all proper and rational, then the induced norm is as follows

$$\eta \overset{\Delta}{=} \max_{1 \leq i \leq m} \sup_{s \in \Omega} |(\dot{Y}(s,\alpha))_i| \qquad \ldots(75)$$

where Ω is usual Nyquist contour.

The proper α should tend to minimize the η, i.e. for a series of α, the best α is the α which make η get their smallest value under the stability condition.

We here give an example to illustrate the application of the sensitivity function. The example is to investigate the controller sensitivity in the matched case. Suppose that the plant components T and G are the same as in section 3.1 and that the controller is of form

$$K = (k + c + \frac{kc}{s})A_o - A_1 \qquad \ldots(76)$$

where A_o and A_1 are as in (29).

The parameter which changes is k, i.e. a 'multiple gain'. It is clear that

$$\frac{\partial K}{\partial k} = (1 + \frac{c}{s})A_o \qquad \ldots(77)$$

The controller sensitivity function can be found by substituting (77) into (66), which is

$$\dot{Y}_K = (1 + \frac{c}{s})T(1+GK)^{-1}GA_o(I+GK)^{-1}\gamma \qquad \ldots(78)$$

Choose a unit-impulse function as the input, which Laplace transformed is $\begin{pmatrix}1\\0\end{pmatrix}$. The norm can be calculated according (75) and is illustrated in Fig.9 as a function of s for a series fixed k.

It is evident from this graph that the larger the k the smaller the norm η. So designers should choose k as large as possible under the stability condition for reducing the sensitivity of performance to parameter k.

The sensitivity to other parameters or temporal can be evaluated by the same means.

5. Conclusions

This report has derived the robustness theorem for the extended Smith scheme. Because the nonunity feedback in the inner loop or outer loop may lead to some benefits, so the extension of the robustness theorem is helpful in general.

Some parameter optimality problems are also discussed. The results indicate that the performance may be improved by temporal mismatch[6] or proper chosen feedback component. Minimization of the performance index may be one of the criterions for choosing temporal model T_A.

When the feedback in either inner or outer loop is not unity, the steady state error should be carefully checked even though an integral action is included in the controller.

The sensitivity functions for Smith scheme in the multivariable case have been established in this report. In spite of their complex structure they can be checked numerically.

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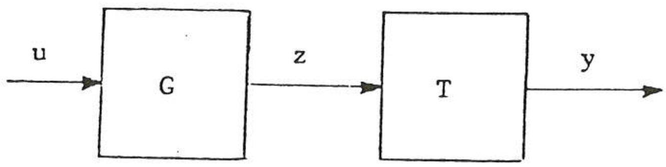

Fig.1. Plant decomposition

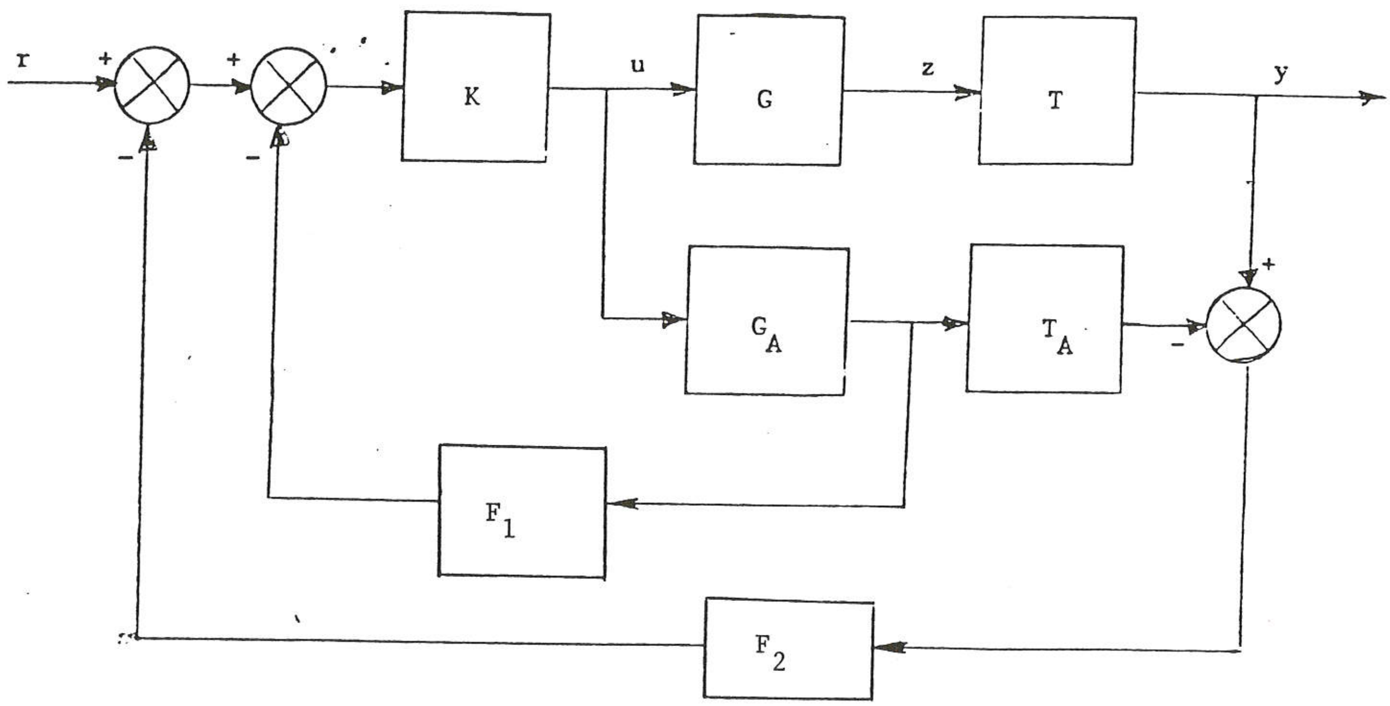

Fig.2. Extended Smith Control Scheme

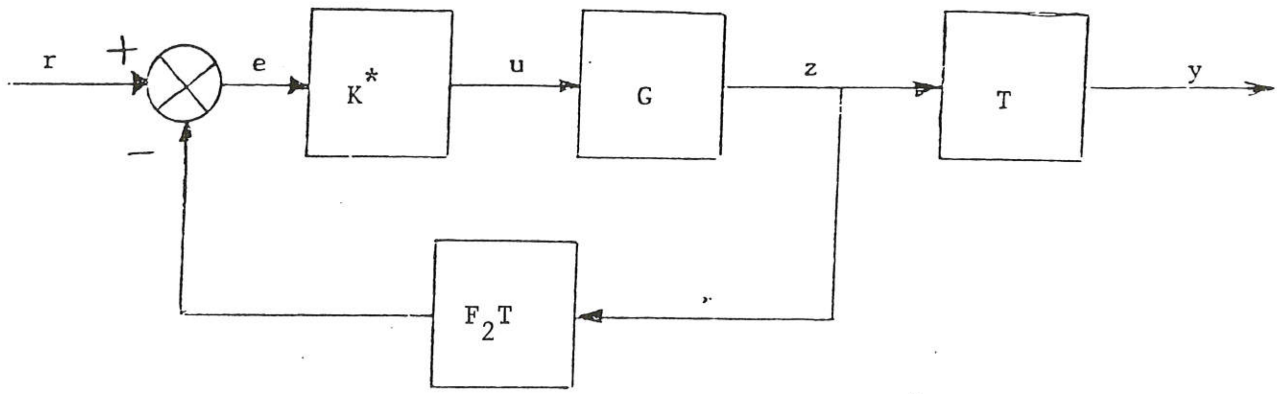

Fig.3. Equivalent Smith Scheme

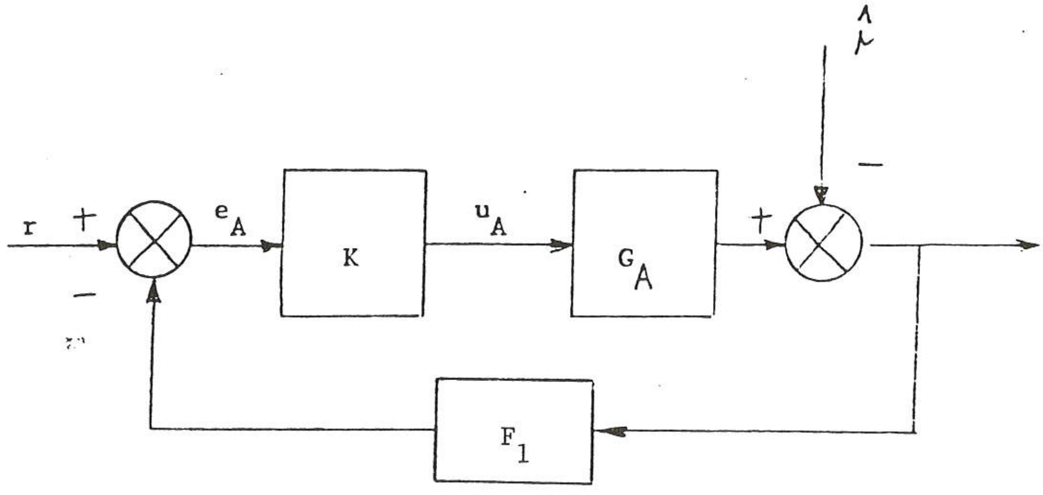

Fig.4. Delay-Free Control Scheme

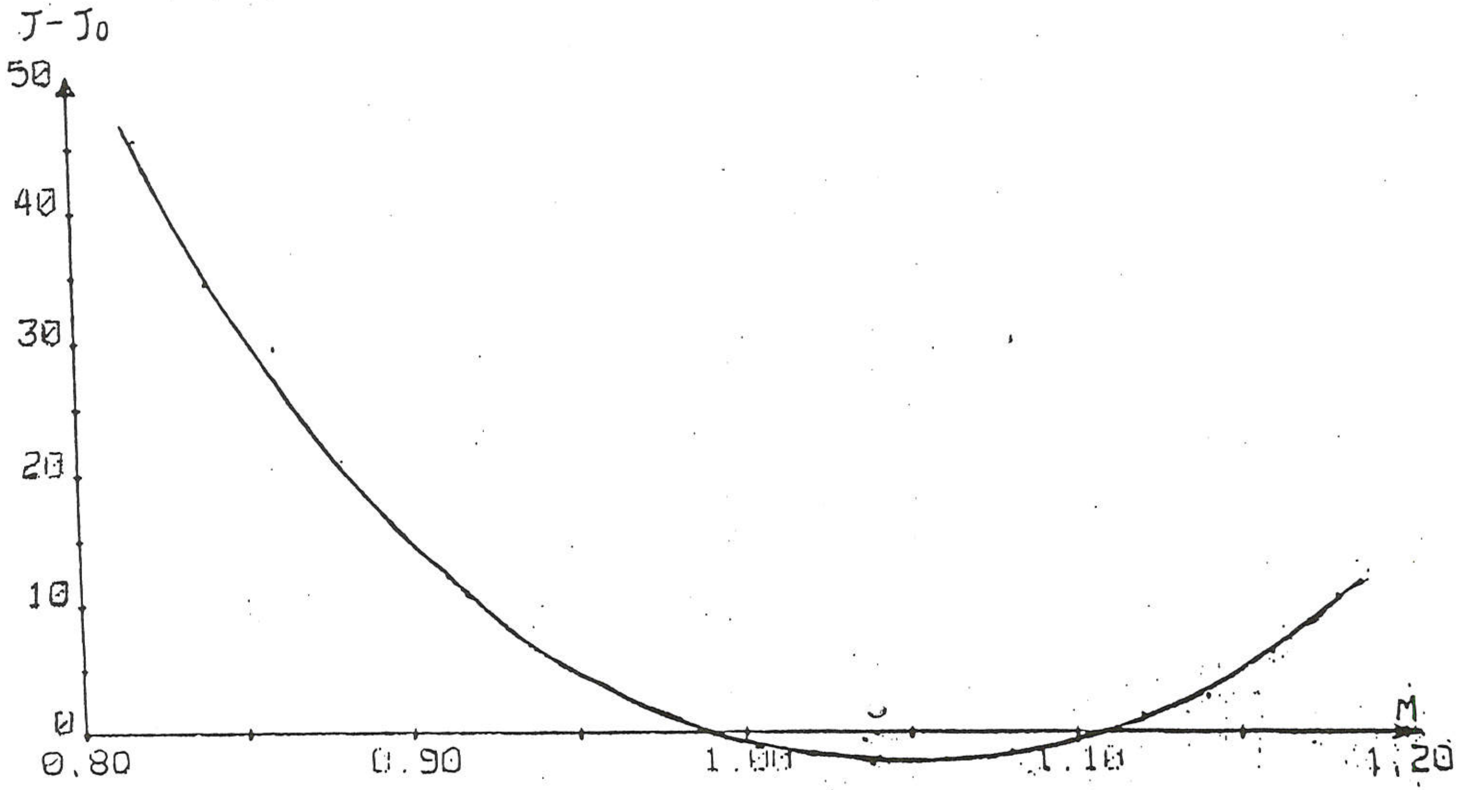

Fig.5. Variation of $(J-J_o)$ with M

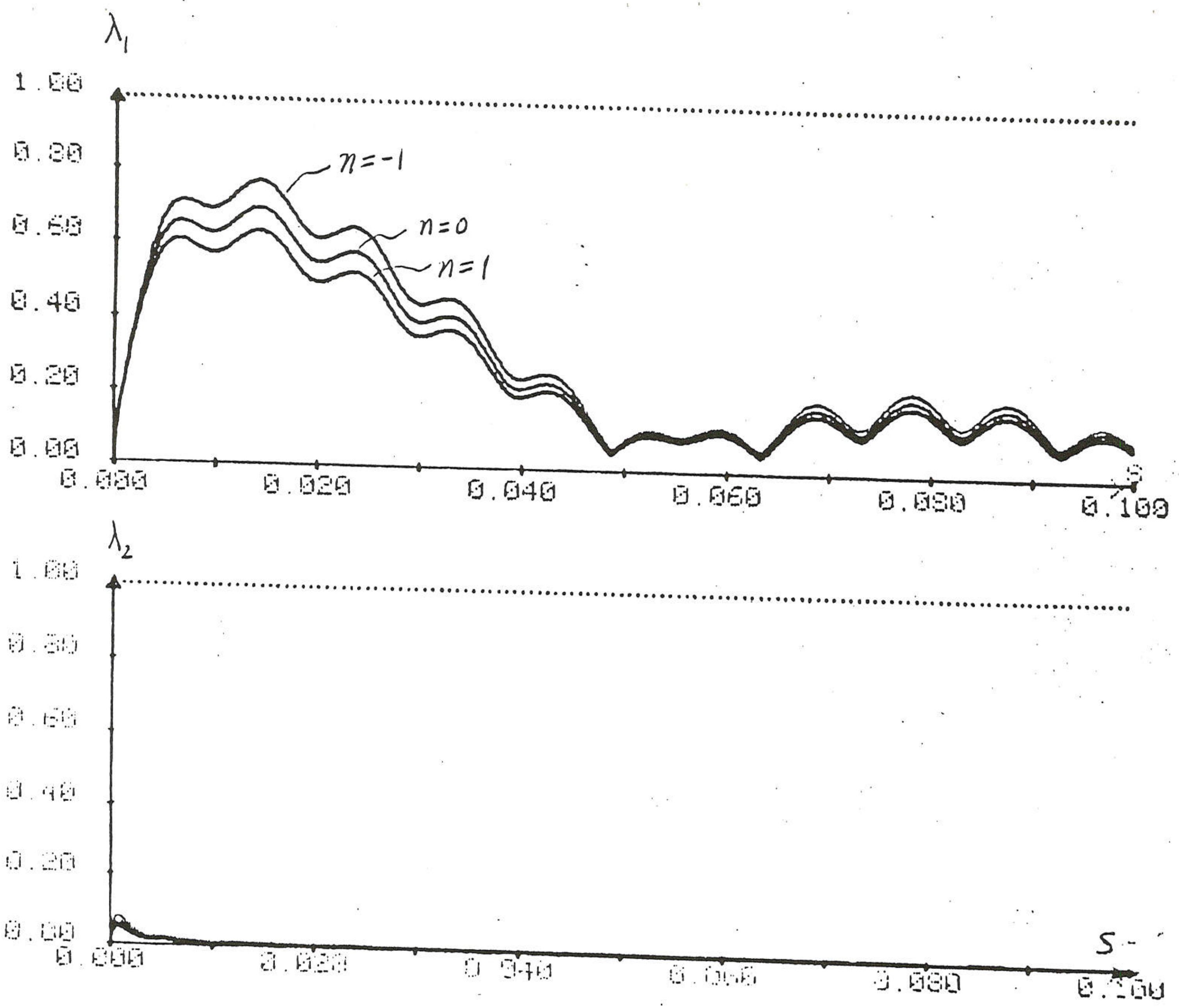

Fig.6. Induced Norm

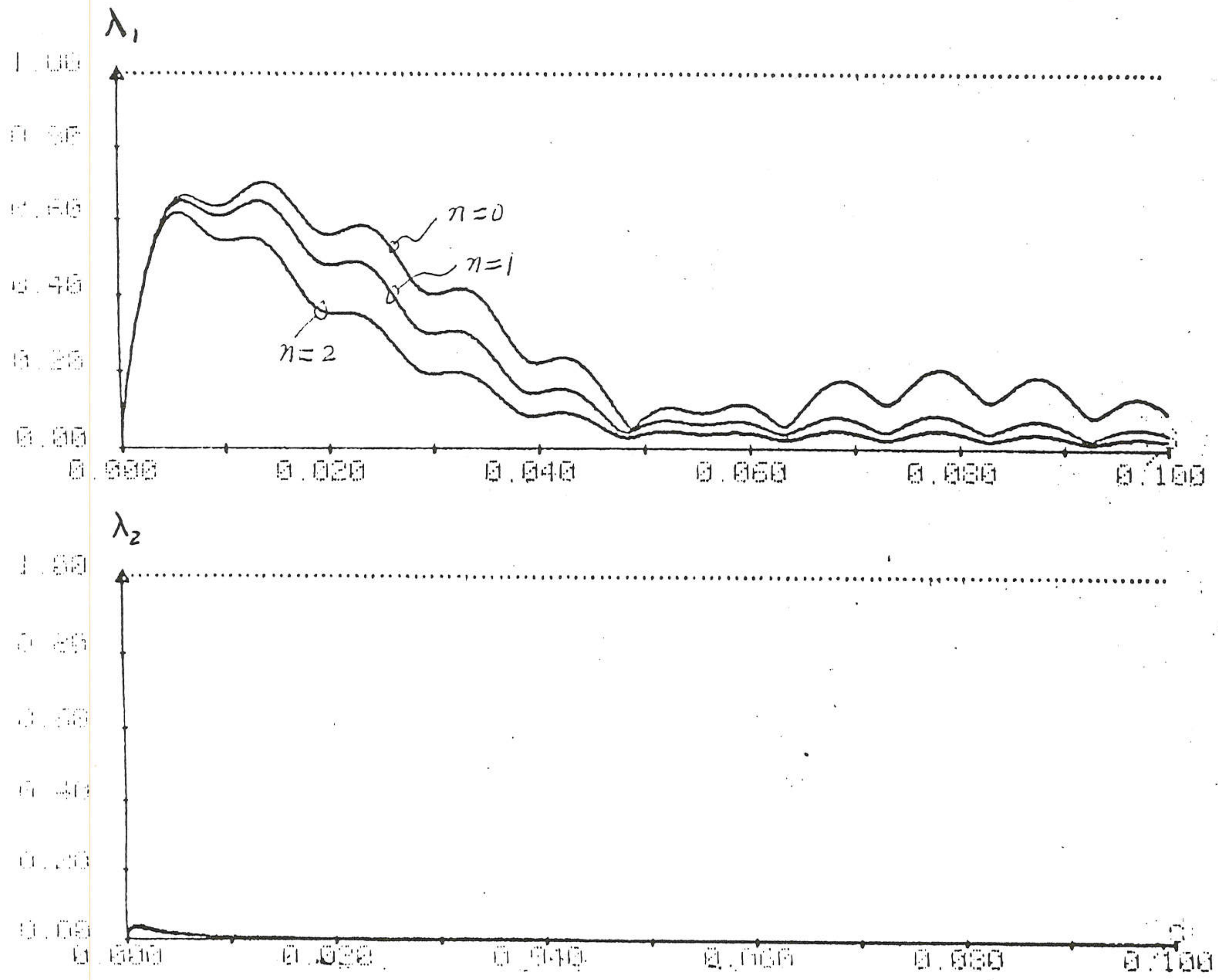

Fig.7. Induced Norm

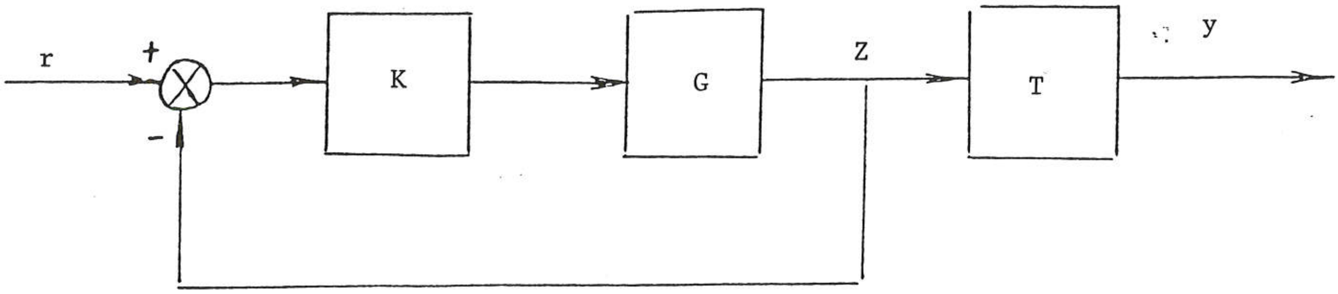

Fig.8. Matched Case

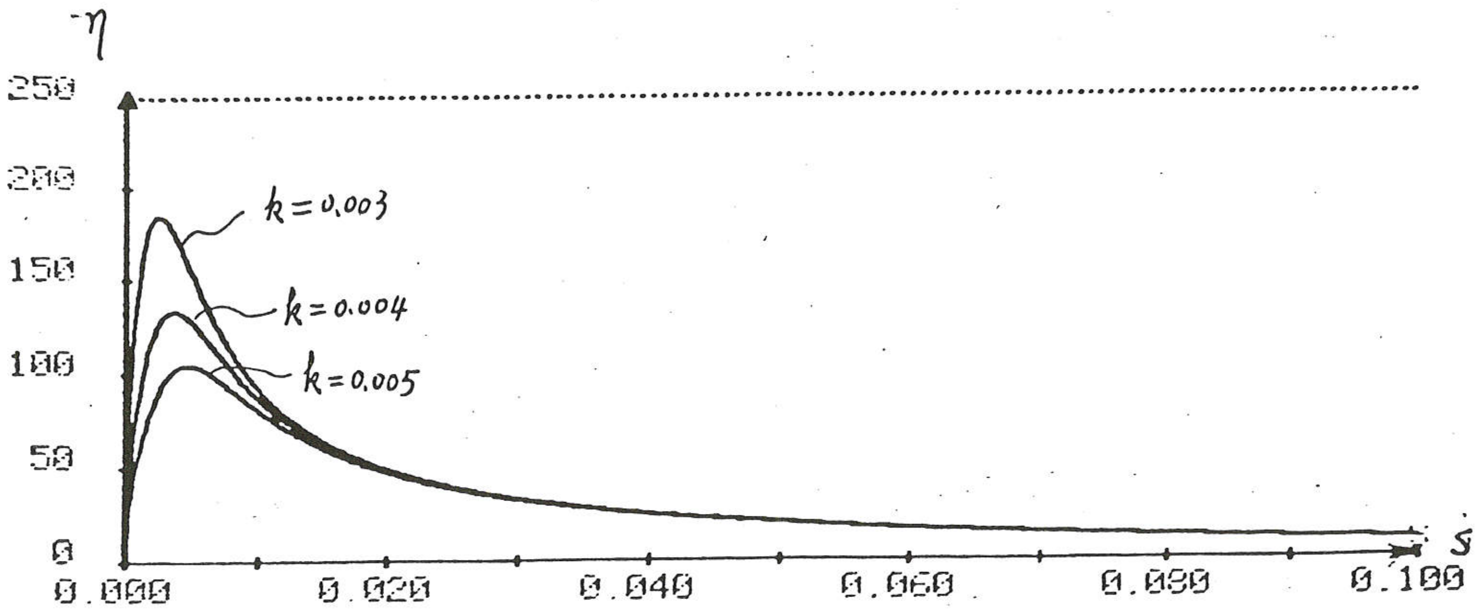

Fig.9. Controller Sensitivity