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Robust DOA Estimation for a MIMO Array Using Two Calibrated Transmit Sensors

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Abstract

Traditional direction of arrival (DOA) estimation for a MIMO array assumes perfect knowledge of the array manifold. Its performance will degrade severely in the presence of array model errors. In this paper we propose a simple scheme for robust DOA estimation based on a MIMO array configuration, where two well-calibrated transmit sensors are used as the transmit array and no knowledge about the receive array manifold is assumed. Its performance is verified by simulation results.

1 Introduction

The problem of DOA estimation has been widely studied and many DOA estimation methods have been proposed in the past. Among them, the subspace-based methods such as MUSIC [1] and ESPRIT [2] are the most representative ones [3]. However, the subspace-based methods are sensitive to uncertainties in the array manifold (i.e., the collection of array steering vectors for all possible DOA angles) [4, 5, 6]. When there are array model errors, such as sensor position error, gain and phase errors, their performance will degrade significantly. On the other hand, it is time-consuming and expensive to calibrate the system in the case of large arrays or when we need to move the array system frequently from place to place [5]. In addition, it is observed that in practice, even after initial calibration, sensor gain and phase errors still exist due to the change of some environment parameters [7].

In this work, we address the problem of robust DOA estimation using a multi-input multi-output (MIMO) array configuration in the presence of array model errors [8, 9, 10]. In particular, we here consider a MIMO array with only two fully calibrated transmit sensors. Since the two transmit sensors transmit orthogonal waveforms, we can extract the received array data associated with each transmit sensor. Given that the two transmit sensors are well calibrated, a rotational invariance property between the two sets of data can still be maintained without the knowledge of the array manifold of the receive array; then the ESPRIT algorithm can be used to find the DOAs of the targets. The advantage of the proposed scheme is that only two calibrated sensors are needed for accurate DOA estimation and no specific requirement is imposed on the receive array. To our best knowledge, none of the existing DOA estimation methods (see [11, 12, 13, 14, 15, 16, 17, 18] and their references) for MIMO arrays works in this scenario.

This paper is organized as follows. In Section 2, the signal model is provided, with the proposed method given in Section 3. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

2 Signal Model

Consider a MIMO system with a transmit array of $M = 2$ sensors and a receive array of N sensors. Both the transmit and the receive arrays are assumed to be closely located in space so that any target located in the far-field can be seen at the same direction by both arrays. The two transmit sensors form a 2-sensor (linear) array and its steering vector $\mathbf{a}_t(\theta)$ is given by

$$\mathbf{a}_t(\theta) = [1, e^{-j2\pi d \sin(\theta)/\lambda}]^T \quad (1)$$

where θ is the angle of the pointing direction, d is the adjacent sensor spacing and λ denotes the signal wavelength.

For simplicity of notation and without loss of generality, we assume that the steering vector of the receive array is also a function of θ, given by

$$\begin{aligned}\mathbf{a}_r(\theta) = [&\alpha_1 e^{j\phi_1}, \alpha_2 e^{j\phi_2} e^{-j2\pi D_2 \sin(\theta)/\lambda}, \\ &\cdots, \alpha_N e^{j\phi_N} e^{-j2\pi D_N \sin(\theta)/\lambda}]^T \quad (2)\end{aligned}$$

where α_i and ϕ_i denote the gain and phase errors, respectively, and D_i represents the location of the ith receive sensor. Here we have effectively assumed a linear array model for the receiver side. However, we will see later that the layout of the receive array has no effect on the result since no information about the receive array manifold is needed and any receive array geometry can be employed.

Assume that the transmit array are fully calibrated and arranged with half-wavelength spacing between adjacent sensors and K non-coherent targets are present. The output of the matched filters at the receiver is given by [12, 19]

$$\begin{aligned}\mathbf{x}[n] &= [\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \mathbf{a}_t(\theta_2) \otimes \mathbf{a}_r(\theta_2), \cdots, \\ &\quad \mathbf{a}_t(\theta_K) \otimes \mathbf{a}_r(\theta_K)]\mathbf{b}[n] + \mathbf{n}[n] \\ &= \mathbf{A}\mathbf{b}[n] + \mathbf{n}[n] \quad (3)\end{aligned}$$

where θ_k is the DOA of the kth target, $\otimes$ stands for the Kronecker product operator, $\mathbf{b}[n] = [b_1[n], b_2[n], \cdots, b_K[n]]^T$, with $b_k[n]$ being the complex-valued reflection coefficient of the kth target,

$$\mathbf{A} = [\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \cdots, \mathbf{a}_t(\theta_K) \otimes \mathbf{a}_r(\theta_K)] \quad (4)$$

is the transmit-receive or virtual array manifold, and $\mathbf{n}[n]$ is the received complex-valued white noise with a power σ^2. Assume that all reflected target signals and noises are uncorrelated with each other. Then the data covariance matrix can be expressed as

$$\mathbf{R}_x = E[\mathbf{x}[n]\mathbf{x}[n]^H] = \mathbf{A}\mathbf{R}_b\mathbf{A}^H + \sigma^2\mathbf{I} \quad (5)$$

where $E[\cdot]$ and $[\cdot]^H$ denote the expectation operation and the Hermitian transpose, respectively, and $\mathbf{R}_b = E[\mathbf{b}[n]\mathbf{b}[n]^H]$ is the covariance matrix of the reflection coefficients vector. In practice, the sample covariance matrix of (5)

$$\hat{\mathbf{R}}_x = \frac{1}{L}\sum_{n=1}^{L} \mathbf{x}[n]\mathbf{x}[n]^H \quad (6)$$

is used, where L is the number of snapshots or data length.

3 Proposed Method

Due to the existence of array model errors, the exact knowledge of the manifold of the receive array in (4) is unknown. If we directly apply the traditional subspace-based methods or the existing DOA estimation methods for MIMO array for DOA estimation, their performance will degrade.

To solve the problem, define $\mathbf{A}_1$ and $\mathbf{A}_2$ as the first and last N rows of $\mathbf{A}$, respectively, which are given by

$$\begin{aligned} \mathbf{A}_1 &= [\mathbf{a}_r(\theta_1), \cdots, \mathbf{a}_r(\theta_K)], && (7) \\ \mathbf{A}_2 &= [e^{-j2\pi d \sin(\theta_1)/\lambda}\mathbf{a}_r(\theta_1), \\ &\quad \cdots, e^{-j2\pi d \sin(\theta_K)/\lambda}\mathbf{a}_r(\theta_K)] \\ &= \mathbf{A}_1\mathbf{Q} && (8) \end{aligned}$$

where $\mathbf{Q}$ is an $N \times N$ diagonal matrix, with $e^{-j2\pi d \sin(\theta_k)/\lambda}$ being its kth main diagonal element.

We observe that, although there are model errors in $\mathbf{A}_1$, a rotational invariance property between $\mathbf{A}_1$ and $\mathbf{A}_2$ is still maintained, which enables the use of ESPRIT for DOA estimation. Let $\mathbf{U}_s$ be the signal subspace composed of the principal eigenvectors corresponding to the K largest eigenvalues of $\hat{\mathbf{R}}_x$. Then $\mathbf{A}$ and $\mathbf{U}_s$ have a relationship which can be determined by a unique nonsingular matrix $\mathbf{T}$ as

$$\mathbf{A} = \mathbf{U}_s\mathbf{T}. \quad (9)$$

Define $\mathbf{U}_1$ and $\mathbf{U}_2$ as the first and last N rows of $\mathbf{U}_s$, respectively. We have

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{U}_1\mathbf{T}, && (10) \\ \mathbf{A}_2 &= \mathbf{U}_2\mathbf{T} = \mathbf{A}_1\mathbf{Q}. && (11) \end{aligned}$$

Using (10) and (11), the relationship between $\mathbf{U}_1$ and $\mathbf{U}_2$ is given by

$$\mathbf{U}_2 = \mathbf{U}_1\mathbf{T}\mathbf{Q}\mathbf{T}^{-1}. \quad (12)$$

Now using the traditional ESPRIT technique, the main diagonal elements of $\mathbf{Q}$ can be obtained via the eigendecomposition of $(\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H\mathbf{U}_2$. Since the two transmit sensors have been well calibrated, $\{\theta_k\}_{k=1}^K$ can be obtained easily from $\mathbf{Q}$.

It should be noted that unlike the traditional ESPRIT estimator and existing ESPRIT estimators for MIMO arrays, the rotational invariance property exploited here depends only on the calibrated transmit sensors and is not related to the array manifold of the receive array. Thus, the proposed method still works well without any knowledge of the receive array model errors. Note that two ESPRIT-type DOA estimators were presented without the location information of any of the receive sensors in [13, 20]. However, they are based on velocity receive sensors, which yields a high cost. Moreover, each pair of identical velocity sensors of the receive array still needs to be well calibrated to keep orthogonal orientation between each other. Furthermore, as mentioned already there are not only position errors in practice but also gain and phase errors, which were not addressed by the methods in [13, 20].

A traditional MIMO array provides several important attributes such as larger virtual spatial aperture and more degrees of freedom (DOFs). Since these attributes have a direct impact on the performance of a DOA estimator, it is important to have a discussion of the related issues for the proposed method in the following.

3.1 DOFs

In our proposed method, we only use the receive array instead of the entire virtual array for DOA estimation. As a result, the DOFs of the proposed method stay the same as the traditional phased array. Thus, the proposed method is able to identify the same number of sources as the traditional phased array. Because of the waveform diversity, however, the maximum number of sources that can be unambiguously identified by the traditional MIMO array can be up to $M = 2$ times that of its phased-array counterpart and the proposed method.

3.2 Spatial aperture or spatial resolution

Since the proposed method imposes less constraint on the receive sensor spacing, its receive sensor spacing can be arranged to be much larger than the half wavelength used in both the traditional phased array and the traditional MIMO array to achieve an aperture that is larger than those of the traditional phased array and the traditional MIMO array for high resolution DOA estimation.

3.3 Array geometry

Since the invariance property used by the proposed method is not related to the manifold of the receive array, its geometry is not limited to that of a uniform linear array. It can be a nonuniform one such as the minimum redundancy array [21], and other sparse arrays [22], and we can obtain various benefits provided by those nonuniform arrays.

4 Simulations

In this section, simulations are carried out to investigate the performance of the proposed method compared with the traditional MIMO ESPRIT estimator [17], and the MUSIC estimator [1]. We consider a MIMO array configuration where an array with $M = 2$ sensors and half-wavelength spacing is used for transmitting and a linear array with $N = 8$ sensors is used for reception. The receive array spacing is 0.5λ for the proposed method, except for Simulations 4, and 0.5λ for other algorithms in all simulations. There are $K = 2$ non-coherent targets located at $\theta_1 = 10^\circ$ and $\theta_2 = 20^\circ$, respectively. The additive noise is spatially white complex Gaussian. 100 simulation runs are performed to assess the performance of the tested algorithms. Define root mean squared error (RMSE) as

$$\frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{100}\sum_{n=1}^{100}(\theta_k - \hat{\theta}_{n,k})^2}, \tag{13}$$

where $\hat{\theta}_{n,k}$ is the estimate of DOA θ_k at the nth run. In all simulation runs, the number of snapshots $L = 100$ is used.

4.1 Simulation 1: Exactly known manifold of the receive array

We consider a scenario where the actual manifold of the receiving array is known exactly. Fig. 1 shows the RMSEs of the estimation algorithms versus the input SNR. As shown, the proposed method has a lower estimation accuracy than the other two algorithms due to less information employed in its operation.

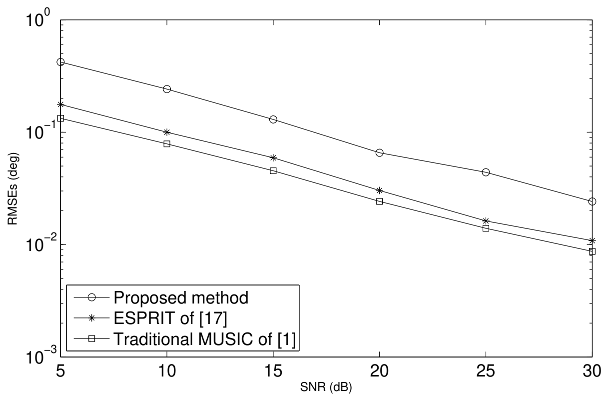

Figure 1: RMSEs of DOA estimation versus input SNR.

4.2 Simulation 2: Effect of sensor position errors

Fig. 2 shows the effect of sensor position errors of the receiving array on the performance of the algorithms tested. In this case, the sensor position error is assumed to be random and uniformly distributed within the range of $[-0.2\lambda, 0.2\lambda]$. From Fig. 2, we see that the performance of both the MUSIC and the ESPRIT based methods has degraded severely compared with the results of Fig. 1. However, the performance of the proposed method is not affected by sensor position errors and outperforms the other two clearly.

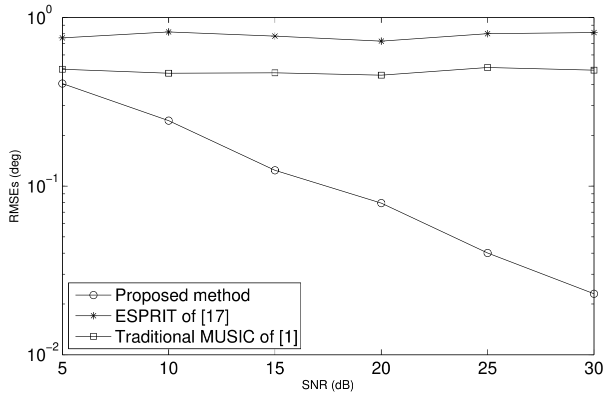

Figure 2: RMSEs of DOA estimation versus input SNR.

4.3 Simulation 3: Effect of gain and phase errors

In the third example, the effect of gain and phase errors on the performance of the algorithms tested is demonstrated. The gain and phase errors are assumed to have a uniform distribution: $\alpha_k \in [0.8, 1.2]$ and $\phi_k \in [-\pi/36, \pi/36]$. Note that α_k and ϕ_k change from run to run while remaining constant for all snapshots. Fig. 3 shows the result. As shown, the gain and phase errors have significantly degraded the performance of the MUSIC and ESPRIT methods. However, the proposed method has achieved robustness against both the gain and phase errors with a much better performance.

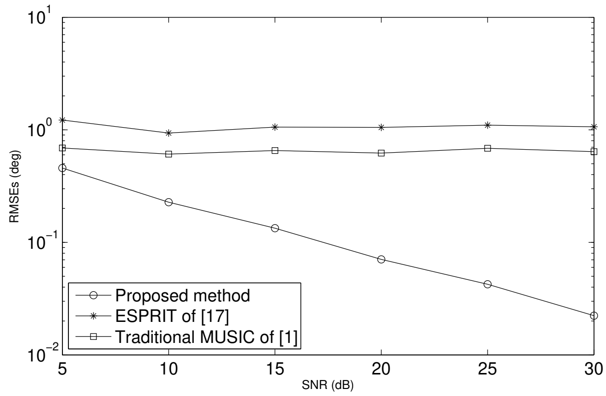

Figure 3: RMSEs of DOA estimation versus input SNR.

4.4 Simulation 4: Effect of receiving array spacing

In the last example, we study the effect of sensor spacing of the receiving array on the performance of the proposed method. The sensor spacing of the proposed method is set to 2λ, and the spacing for the two other algorithms remains 0.5λ to avoid the aliasing problem. All other parameters remain the same as in Simulation 3. From Fig. 4, we see that the estimation accuracy of the proposed method is higher compared to the results of Fig. 3, due to the increased aperture size.

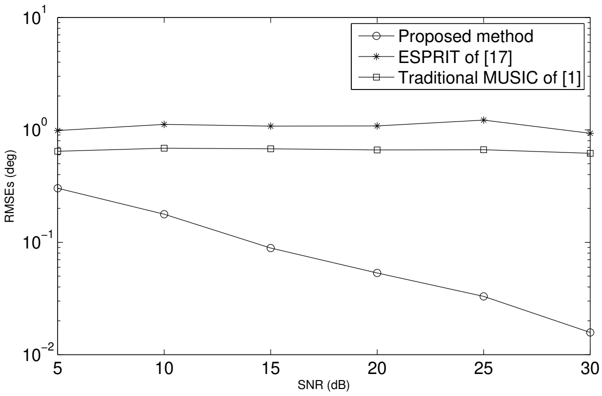

Figure 4: RMSEs of DOA estimation versus input SNR.

5 Conclusions

A robust DOA estimation method based on a MIMO array configuration has been proposed with only two well-calibrated transmit sensors. Due to orthogonality of the transmitted waveforms, the rotational invariance property between the two sets of received data associated with the two transmit sensors is still maintained without knowledge of the array manifold of the receiving side. As a result, the ESPRIT algorithm can be employed for the following DOA estimation, leading to a robust solution. The effectiveness and advantage of the proposed method has been demonstrated by extensive simulations.

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