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SMITH PREDICTORS IN MULTIVARIABLE CONTROL: SOME CASE STUDIES

by

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1. Introduction

The robustness of control designs is still an active research area. The work includes assessing the robustness of designed control system with respect to changes in plant dynamic characteristics and developing design techniques that guarantee a required degree of robustness. These problems are of particular importance for the case where the plant includes significant time delay in its dynamics. It is well known that the Smith predictor[1] is often used in this case. The scheme is shown in Fig.2. The design should be robust enough to cope with the observed plant/model mismatch and to retain stability in the presence of changes in plant dynamics.

In[2], a general theory describing the robustness of the Smith scheme is given. It can be summarized as follows: the Smith predictor is stable if -

1) The plant component G and its model G_A map U_o^{ℓ} into y_o^m and their restrictions to U_o^{ℓ} have finite induced norms. The delay components T, T_A map y_o^m into itself with restrictions to y_o^m of finite induced norm.

2) The restriction to y_o^m of the "delay free" mapping $r \rightarrow U_A \triangleq (I+KG_A)^{-1}Kr$ has range in U_o^{ℓ} and finite induced norm.

3) $\lambda_1 \triangleq ||(I+KG_A)^{-1}K\Delta TG_A|| < 1$ ...(1)

4) $\lambda_2 = \frac{1}{1-\lambda_1} ||(I+KG_A)^{-1}KT\Delta G|| < 1$ ...(2)

where K is the forward path controller

ΔT and ΔG are the additive perturbations of the model G_A and T_A. Which is

$$G = G_A + \Delta G \qquad ...(3)$$

$$T = T_A + \Delta T \qquad ...(4)$$

The condition 1) boils down in practice to the requirement that the plant TG and its approximate model $T_A G_A$ are open-loop stable. The condition 2) simply states that the feedback scheme of Fig.3 is stable in the normal practical sense, whilst conditions (3) and (4) put bounds on the permissible mismatch.

The result described above has great generality allowing some distributed, non-rational in G and non-delay elements in T. There are clearly an infinity of stability criteria derivable from this result. For simplicity, the author of[2] suppose that G and G_A are rational and strictly proper TFMs (transfer function matrices), that K is rational and proper and that both $T = \text{diag}\{e^{-s\tau_j}\}_{1\leq j\leq m}$ and $T_A = \text{diag}\{e^{-s\tau_{Aj}}\}_{1\leq j\leq m}$ are mxm diagonal matrices of pure delay. In this case, the theorem has the following simple form:

If the plant component G and its model G_A are asymptotically stable and the delay free feedback system of Fig.4 is input-output stable then the Smith scheme of Fig.2 is BIBO stable if

$$\lambda_1 \triangleq \max_{1\leq i\leq \ell} \sup_{s\in\partial\Omega} \sum_{j=1}^{\ell} |((I+KG_A)^{-1}K(T-T_A)G_A)_{ij}| < 1 \qquad \ldots(5)$$

$$\lambda_2 = \frac{1}{1-\lambda_1} \max_{1\leq i\leq \ell} \sup_{s\in\partial\Omega} \sum_{j=1}^{\ell} |((I+KG_A)^{-1}KT(G-G_A))_{ij}| < 1 \ldots(6)$$

where $\partial\Omega$ is the boundary of Nyquist contour.

In this report, we will first discuss the problem of expressing the plant into separate form TG in different cases. Then, by looking at a process control example we will illustrate the choice of approximate model G_A, T_A and their controller K such that the stability of scheme Fig.4 can be easily guaranteed. In addition, we will analyse the necessity and necessary condition of including integral action in the controller. Finally, the robustness of the design in some cases is discussed.

2. Real plant and their expression

In this report, we assume that the TFM of the real plant is known and that the time delay of some elements are big (somewhat near the time constant of the same element). The performance of the normal feedback control scheme in this case is necessarily bad and to cope with this problem the Smith predictor is assumed to be used.

The underlying assumption made is that the plant can be expressed in the separable form TG (see Fig.1), where T is a pure delay diagonal matrix, G is a rational and strictly proper TFM but does not have to be a "delay free" component.

As an example, we look at ℓ input-ℓ output system, the TFM of which is

$$\left(\frac{g_{ij}}{1+T_{ij}S} e^{-\tau_{ij}S} \right)_{\ell \times \ell} \qquad \ldots(7)$$

A lot of plants can be described approximately by such a TFM. The value of g_{ij}, T_{ij} and τ_{ij} may be very different for different plant. They can be expressed in a separable form by such a means: express T as a diagonal matrix of pure delays

$$T = \operatorname{diag} \left\{ e^{-\tau_{im}S} \right\}_{1 \leq im \leq \ell} \qquad \ldots(8)$$

where the time delay τ_{im} is the smallest time delay in the ith row of the plant TFM. Leave the difference of the time delay in the component G, defined by

$$G = \left(\frac{g_{ij}}{1+T_{ij}S} e^{-(\tau_{ij}-\tau_{im})S} \right)_{\ell \times \ell} \qquad \ldots(9)$$

For the 2x2 TFM:

$$\begin{bmatrix} \frac{g_{11}}{1+T_{11}S} e^{-\tau_{11}S} & \frac{g_{12}}{1+T_{12}S} e^{-\tau_{12}S} \\ \frac{g_{21}}{1+T_{21}S} e^{-\tau_{21}S} & \frac{g_{22}}{1+T_{22}S} e^{-\tau_{22}S} \end{bmatrix} \qquad \ldots(10)$$

we can divide the decomposition into 6 kinds and give the separable forms as in table 1.

In the 6th line of table 1, G has diagonal form but T is non diagonal. This is a very particular situation and describes a plant with large interaction.

We wish to design a forward path controller K for the Smith control scheme such that the scheme

1) is stable
2) has small steady-state errors (e.g. less than 10%) in response to step demands
3) has small overshoot (e.g. less than 20%)
4) has acceptable interaction (e.g. less than 20%)
5) is robust in the sense that if the plant component G changes to $\tilde{G}$ and T to $\tilde{T}$ over a period of time, stability will be retained provided that the changes $\tilde{G}$-G and $\tilde{T}$-T are small enough.

Situation	T	G
$\tau_{11} = \tau_{12}$ $\tau_{21} = \tau_{22}$	$\begin{pmatrix} e^{-\tau_{11}S} & 0 \\ 0 & e^{-\tau_{22}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S} & \frac{g_{12}}{1+T_{12}S} \\ \frac{g_{21}}{1+T_{21}S} & \frac{g_{22}}{1+T_{22}S} \end{pmatrix}$
$\tau_{11} \geq \tau_{12}$ $\tau_{21} \geq \tau_{22}$	$\begin{pmatrix} e^{-\tau_{12}S} & 0 \\ 0 & e^{-\tau_{22}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S}e^{-(\tau_{11}-\tau_{12})S}, & \frac{g_{12}}{1+T_{12}S} \\ \frac{g_{21}}{1+T_{21}S}e^{-(\tau_{21}-\tau_{22})S}, & \frac{g_{22}}{1+T_{22}S} \end{pmatrix}$
$\tau_{11} \leq \tau_{12}$ $\tau_{21} \leq \tau_{22}$	$\begin{pmatrix} e^{-\tau_{11}S} & 0 \\ 0 & e^{-\tau_{21}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S}, & \frac{g_{12}}{1+T_{12}S}e^{-(\tau_{12}-\tau_{11})S} \\ \frac{g_{21}}{1+T_{21}S}, & \frac{g_{22}}{1+T_{22}S}e^{-(\tau_{22}-\tau_{21})S} \end{pmatrix}$
$\tau_{11} \geq \tau_{12}$ $\tau_{21} \leq \tau_{22}$	$\begin{pmatrix} e^{-\tau_{12}S} & 0 \\ 0 & e^{-\tau_{21}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S}e^{-(\tau_{11}-\tau_{12})S}, & \frac{g_{12}}{1+T_{12}S} \\ \frac{g_{21}}{1+T_{21}S}, & \frac{g_{22}}{1+T_{22}S}e^{-(\tau_{22}-\tau_{21})S} \end{pmatrix}$
$\tau_{11} \leq \tau_{12}$ $\tau_{21} \geq \tau_{22}$	$\begin{pmatrix} e^{-\tau_{11}S} & 0 \\ 0 & e^{-\tau_{22}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S}, & \frac{g_{12}}{1+T_{12}S}e^{-(\tau_{12}-\tau_{11})S} \\ \frac{g_{21}}{1+T_{21}S}e^{-(\tau_{21}-\tau_{22})S}, & \frac{g_{22}}{1+T_{22}S} \end{pmatrix}$
$\frac{g_{11}}{1+T_{11}S} = \frac{g_{21}}{1+T_{21}S}$ $\frac{g_{12}}{1+T_{12}S} = \frac{g_{22}}{1+T_{22}S}$ for all $s \in D$	$\begin{pmatrix} e^{-\tau_{11}S} & e^{-\tau_{12}S} \\ e^{-\tau_{21}S} & e^{-\tau_{22}S} \end{pmatrix}$	$\begin{pmatrix} \frac{g_{11}}{1+T_{11}S} & 0 \\ 0 & \frac{g_{22}}{1+T_{22}S} \end{pmatrix}$

Table 1

In section 3 and 4, we will discuss the control design problem. The first stage of doing that is the choice of an approximate model G_A and T_A. As described previously, T_A is a diagonal matrix of pure delays and can be chosen as

$$T_A = \text{diag}\,\{e^{-\tau_{Aj}S}\} \qquad \ldots(11)$$

where τ_{Aj} have somewhat different values to τ_{jm}.

It is self-evident that G_A should be of low order and simple form in order to be easy to realize and choose the form of controller. In this report, we confine our attention to two kinds of approximate model G_A i.e.the pseudo-diagonal model and the first order model[3].

3. Proportional Control

3.1. Pseudo-diagonal model

For the plant with TFM (7), the model G_A can be chosen as

$$G_A = \text{diag}\left\{\frac{1}{1+\alpha_i S}\right\}_{1\le i\le \ell} P \qquad \ldots(12)$$

(When the time constants in the same line are similar, the $\alpha_i>0$ should be near to the average value of T_{ij}, $j = 1,\ell$)

or

$$G_A = P\,\text{diag}\left\{\frac{1}{1+\alpha_j S}\right\}_{1\le j\le \ell} \qquad \ldots(13)$$

(When the time constants in the same column are similar, the $\alpha_j>0$ should be near to the average value of T_{ij}, $i = 1,\ell$).

In other words, the dynamic of the plant component is represented by a diagonal matrix and the interaction is represented by a constant state interaction matrix P. P can be chosen as G(o) in general to match steady statesbut it can also be chosen by other means. Then let the controller be of form

$$K = P^{-1} \operatorname{diag} \{k_j\}_{1 \le j \le \ell} \quad \text{(relative to form 12)} \qquad \ldots(14)$$

or

$$K = \operatorname{diag} \{k_j\}_{1 \le j \le \ell} \; P^{-1} \quad \text{(relative to form 13)} \qquad \ldots(15)$$

The closed-loop TFM of scheme Fig.4 is expressed by (16) or (17) respectively.

i.e. $$H_c(s) = (I+G_A K)^{-1} G_A K = \operatorname{diag} \left\{ \frac{k_j}{1+k_j+\alpha_j S} \right\}_{1 \le j \le \ell} \qquad \ldots(16)$$

(for form 12 and 14)

or

$$H_c(s) = (I+G_A K)^{-1} G_A K = P \operatorname{diag} \left\{ \frac{k_j}{1+k_j+\alpha_j S} \right\} P^{-1} \qquad \ldots(17)$$

(for form 13 and 15)

It is very clear that in both cases the scheme Fig.4 is stable if and only if $k_j > -1$.

The closed-loop TFM of equivalent Smith scheme Fig.3 is

$$H_s(s) = T(I+GK^* T)^{-1} GK^* \qquad \ldots(18)$$

where K^* is a uniquely defined linear mapping of Y^m into U^ℓ, which is [2]

$$K^* = (I+KG_A - KT_A G_A)^{-1} K \qquad \ldots(19)$$

The final value of the response of Smith scheme to a step demand, the Laplace transform of which is $\frac{1}{S}R$, is

$$Y(\infty) = T(o)(I+G(o)K^*(o)T(o))^{-1} G(o)K^*(o)R \qquad \ldots(20)$$

Because of

$$K^*(o) = (I+K(o)G_A(o)-K(o)T_A(o)G_A(o))^{-1} K(o) = K(o) \qquad \ldots(21)$$

and $T(o) = T_A(o) = I$,

$$Y(\infty) = (I+G(o)K(o))^{-1}G(o)K(o)R \qquad \ldots(22)$$

In the pseudo-diagonal and proportional control case, if we choose $P = G(o)$, then the final value is

$$Y(\infty) = \text{diag}\left\{\frac{k_j}{1+k_j}\right\}_{1\leq j\leq \ell} R \qquad \text{(relative to form 12 and 14)} \qquad \ldots(23)$$

or

$$Y(\infty) = G(o)\text{diag}\left\{\frac{k_j}{1+k_j}\right\}_{1\leq j\leq \ell} G(o)^{-1}R \qquad \text{(relative to form 13 and 15)} \qquad \ldots(24)$$

We next check condition (5) and (6) to decide the gain that can be used. It is intuitive that for robustness of the design, the norms λ_1 and λ_2 should be less than unity and the smaller the norms, the more robust the design will be. But on the other hand, from (23) and (24), the greater the norms and hence the higher the gain k_j, the smaller the steady-state errors. For simplicity, we suggest that choose the controller gain to ensure that $\lambda_1 \leq 0.8$, $\lambda_2 \leq 0.8$.

We now do an example as follows.

The plant has been expressed as

$$T = \begin{pmatrix} e^{-35x22.8S} & 0 \\ 0 & e^{-35x3S} \end{pmatrix}, \quad G = \begin{pmatrix} \frac{119.3}{1+812.8S} & \frac{-62.3}{1+904S} \\ \frac{55.3}{1+776S} & \frac{-109.7}{1+715S} \end{pmatrix} \qquad \ldots(25)$$

and we choose

$$T_A = \begin{pmatrix} e^{-30x22.8S} & 0 \\ 0 & e^{-30x3S} \end{pmatrix}, \quad G_A = \begin{pmatrix} \frac{1}{1+850S} & 0 \\ 0 & \frac{1}{1+750S} \end{pmatrix} G(o) \qquad \ldots(26)$$

and $K = G(o)^{-1}k$.

Choosing a value of k, we can check the validity of the condition (5) and (6) at a selection of frequency points covering the bandwidth of interest. If λ_1 or $\lambda_2 > 0.8$, we could then reduce k in an attempt to reduce λ_1 and λ_2. Repeat this procedure until $\lambda_1 < 0.8$ and $\lambda_2 < 0.8$. By this means we find that $k = 3.0$ is a suitable value and the norms λ_1 and λ_2 are shown in Fig.6 as a function of frequency. The closed-loop response of the Smith scheme are shown in Fig.7. From these we can see that the performance is acceptable except the steady-state error.

For comparison purposes, we also give the closed-loop response of normal feedback control scheme of Fig.5 using the same controller. The benefits of the Smith scheme are self-evident - much less oscillation, much shorter setting times and smaller interaction than those of normal feedback control. In other words, by using Smith predictor the performance can be much improved.

3.2. First order model

The first order model and its proportional controller suggested by Owens[3] is of form:

$$\left.\begin{aligned} G_A &= (A_o S + A_1)^{-1} \\ K &= kA_o - A_1 \end{aligned}\right\} \qquad \ldots(27)$$

where A_o and A_1 are constant matrices

k is a scalar.

For plant which is of TFM (7), we suggest two methods to decide A_o and A_1[4] that leads relative small plant/model mismatch. One method is as

$$\left.\begin{aligned} A_o^{-1} &= \lim_{s\to\infty} SG(s) \\ A_1^{-1} &= \lim_{s\to 0} G(s) \end{aligned}\right\} \qquad \ldots(28)$$

The other is as follows:

$$\text{Let} \quad \left.\begin{aligned} A_1^{-1} &= P \qquad \text{(a } \ell x \ell \text{ constant matrix)} \\ A_o &= \text{diag}\,\{\alpha_j\}_{1\leq j\leq \ell}\, A_1 \end{aligned}\right\} \qquad \ldots(29)$$

and hence

$$G_A = (\text{diag}\{\alpha_j\}A_1 S + A_1)^{-1} = P\ \text{diag}\{\frac{1}{1+\alpha_j S}\}_{1\leq j\leq \ell} \qquad \ldots(30)$$

It is clear that (30) is identical with (13), so it is suitable for the case where the time constants T_{ij} in the same line are similar, and the $\alpha_j>0$ should be near to the average value of T_{ij}, $j = 1,\ell$.

The little bit different form with (29) is

$$\left.\begin{aligned} A_1^{-1} &= P \\ A_o &= A_1\ \text{diag}\,\{\alpha_j\}_{1\leq j\leq \ell} \end{aligned}\right\} \qquad \ldots(31)$$

and

$$G_A = \text{diag}\{\frac{1}{1+\alpha_j S}\}_{1\leq j\leq \ell}\ P \qquad \ldots(32)$$

The form (32) is identical with (12) and can be analysed in the same way. Like in subsection 3.1, P in general can be chosen as G(o), when

$$G_A = G(o)\ \text{diag}\{\frac{1}{1+\alpha_j S}\}_{1\leq j\leq \ell} \qquad \ldots(33)$$

or

$$G_A = \text{diag}\{\frac{1}{1+\alpha_j S}\}_{1\leq j\leq \ell}\ G(o) \qquad \ldots(34)$$

It is trivially verified that the closed-loop TFM of scheme Fig.4 is[3]

$$H_c(s) = (I+G_A K)^{-1} G_A K = \frac{k}{s+k} \{I - \frac{1}{k} A_o^{-1} A_1\} \quad \ldots(35)$$

So the scheme Fig.4 is stable if and only if $k>0$.

The closed-loop TFM of Smith scheme Fig.3 is of course the same form as (18) and (19). The final value of the response of Smith scheme to a step demand R is

$$Y(\infty) = (I+G(o)K(o))^{-1} G(o)K(o)R$$

$$= (I+kG(o)A_o - G(o)A_1)^{-1} (kG(o)A_o - G(o)A_1)R \quad \ldots(36)$$

If A_1 is chosen as $G(o)^{-1}$, then

$$Y(\infty) = (I+kA_1^{-1}A_o - I)^{-1} (kA_1^{-1}A_o - I)R$$

$$= (I - \frac{1}{k} A_o^{-1} A_1)R \quad \ldots(37)$$

The validity of condition (5) and (6) can then be checked by the similar means with section 3.1 to decide the largest gain k that can be used.

For plant (25), for example, we use formula (28) and find

$$A_o = \begin{pmatrix} 8.79 & -3.95 \\ 4.128 & -8.37 \end{pmatrix}, \quad A_1 = \begin{pmatrix} 0.01137 & -0.00646 \\ 0.00573 & -0.01237 \end{pmatrix}$$

The suitable k is 0.005 to ensure $\lambda_1<0.8$, $\lambda_2<0.8$ and the norms λ_1 and λ_2 are illustrated in Fig.8 as a function of frequency. The closed-loop response of Smith scheme and normal feedback scheme are shown in Fig.9. It is of the similar manner with pseudo-diagonal model (Fig.7).

From Fig.7 and 9, the common defect of the Smith scheme with proportional control is that the steady-state errors are too big. This is because the theorem is a "small gain theorem"[2]. In other words, norms λ_1 and λ_2 will tend to become small as the gains in K are reduced to zero. It is clear from (23), (24) and (37) that when the k becomes small the steady-state error is necessarily big.

To offset the steady-state error, the integral action should be included in the controller. This is discussed in the next section.

4. Proportional Plus Integral Control

4.1. Necessary conditions

In this subsection we will indicate that the possibility of including integral action in the controller is related to the model/ plant mismatch and the steady-state performance of the model. To achieve this, we make the following observation. When integral is included in the controller, $\lim_{s\to 0} K = \infty$, and

$$\lim_{s\to 0} \lambda_1 = \max_{1\le i\le \ell} \lim_{s\to 0} \sum_{j=1}^{\ell} |((I+KG_A)^{-1}K(T-T_A)G_A)_{ij}|$$

$$= \max_{1\le i\le \ell} \sum_{j=1}^{\ell} |(G_A^{-1}(o)(T(o)-T_A(o))G_A(o))_{ij}|$$

$$= 0 \qquad \ldots(38)$$

$$\lim_{s\to 0} \lambda_2 = \max_{1\le i\le \ell} \lim_{s\to 0} \sum_{j=1}^{\ell} |((I+KG_A)^{-1}KT(G-G_A))_{ij}|$$

$$= \max_{1\le i\le \ell} \sum_{j=1}^{\ell} |(G_A^{-1}(o)(G(o)-G_A(o))_{ij}| \qquad \ldots(39)$$

To satisfy condition (6), the necessary condition is

$$\max_{1\leq i\leq \ell} \sum_{j=1}^{\ell} |(G_A^{-1}(o)(G(o)-G_A(o)))_{ij}| < 1 \qquad \ldots(40)$$

In other words, this is the necessary condition to include integral action in the controller. When we choose G_A such that $G_A(o) = G(o)$ (for example, choose P as G(o) in formula 12, 13, 29 and 31, or use formula 28), the condition (40) is always satisfied and hence the integral action can always be included in the controller.

The final value of response of the Smith scheme to a step demand R is

$$Y(\infty) = \lim_{s\to 0} (I+GK)^{-1}GKR = R \qquad \ldots(41)$$

because $\lim_{s\to 0} K(s) = \infty$. This means that no steady-state error exists if integral action is included.

4.2. Pseudo-diagonal model

When model G_A is chosen as (12), the controller can be of form

$$K = P^{-1} \operatorname{diag} \{ k_j + \frac{c_j}{s} \} \qquad \ldots(42)$$

The closed-loop TFM of scheme Fig.4 is

$$H_c(s) = (I+G_AK)^{-1}G_AK$$

$$= \operatorname{diag}\{ \frac{k_jS + c_j}{\alpha_jS^2+(1+k_j)S+c_j} \}_{1\leq j\leq \ell} \qquad \ldots(43)$$

When we choose G_A as (13), then controller can be chosen as

$$K = \operatorname{diag} \{k_j + \frac{c_j}{s} \}_{1\leq j\leq \ell} P^{-1} \qquad \ldots(44)$$

The closed-loop TFM of scheme Fig.4 is

$$H_c(s) = P \operatorname{diag}\{ \frac{k_j S+c_j}{\alpha_j S^2+(1+k_j)S+c_j} \}_{1\leq j\leq \ell} P^{-1} \qquad \ldots(45)$$

In both (43) and (45), the scheme Fig.4 is stable if and only if

$$k_j > -1 \quad , \quad c_j > 0$$

Then the validity of condition (5) and (6) are checked by giving some value of k_j and c_j in a similar manner to 3.1. For plant (25), using model G_A (12) and $P = G(o)$, we find that $k_1 = k_2 = 1.5$, $c_1 = c_2 = 0.003$ are the suitable gain values. The norms λ_1 and λ_2 are shown in the Fig.10 and the closed-loop response of Smith scheme is illustrated in Fig.11. The response of normal feedback scheme using same controller is shown in the same graph for comparison.

The performance of the Smith scheme is good enough and just as we expect, the steady-state error becomes zero. The overshoot and interaction are both acceptable. Comparing two responses in Fig.11 we see again the benefits of Smith scheme over the normal feedback control.

4.3. First order model

The model G_A can be chosen by the same means with subsection 3.2. As suggested by Owens[3], the controller is of form

$$K = (k + c + \frac{kc}{S})A_o - A_1 \qquad \ldots(46)$$

The closed loop TFM of scheme Fig.4 is

$$H_c(s) = \frac{1}{(s+k)(s+c)} \{kcI+S((k+c)I-A_o^{-1}A_1)\} \qquad \ldots(47)$$

It is clear for any k>0, c>0 the scheme of Fig.4 is stable.

For plant (25), using model (28) and controller (46), the norms λ_1 and λ_2 are checked in the same way as section 3.1. A suitable gain value can then be found which is k = 0.004, c = 0.00085. The corresponding norms λ_1 and λ_2 are shown in Fig.12 as a function of frequency. In this case, the response of Smith scheme and normal feedback control are illustrated in Fig.13.

The performance of Smith scheme is good enough to satisfy all specifications described in section 2.

Summarizing subsections 4.2 and 4.3, we can say that the design of proportional plus integral controls for the Smith scheme using pseudo-diagonal and first order models is successful.

In Fig. 7, 9, 11 and 13, the output of predictor Z_A are shown (by dotted line) in the meantime. That indicates physically how the Smith scheme can improve the performance.

5. Robustness

The design described previously should be robust in the sense that, over a period of time, the real plant TG changes its dynamic characteristics to $\tilde{T}\tilde{G}$, the Smith scheme is still stable if the change is small enough. Reference (2) gives a theorem, which is:

If both $\tilde{T}$ and $\tilde{G}$ are BIBO stable, then the Smith scheme will retain its BIBO stability if

$$||\tilde{T}(\tilde{G}-G)|| + ||(\tilde{T}-T)G|| < \frac{(1-\lambda_1)(1-\lambda_2)}{||(I+KG_A)^{-1}K||} \qquad \ldots(49)$$

Even though this theorem provides an upper bound on the change of plant dynamic characteristics, it is too conservative sometimes. For example, if the plant changes such that $\tilde{G} = G_A$ and $\tilde{T} = T_A$, it is well known that the Smith scheme is stable. But in this situation, condition (47) might not be satisfied when model/plant mismatch is big in controller design. In this report we prefer recheck condition (5) and (6) to investigate the robustness because it is less conservative than condition (47).

5.1. When G changes

The change of G may often occur either because the change of gains or of time constants. But we here look at another case where G changes such that one element of which is of additive time delay. We still use example (25) and suppose G changes to be

$$\tilde{G} = \begin{pmatrix} \frac{119.3}{1+812.8S} e^{-(m-1)x35x22.8S} & \frac{-62.3}{1+904S} \\ \frac{55.3}{1+776S} & \frac{-109.7}{1+715S} \end{pmatrix} \qquad \ldots(50)$$

where $m \geq 1$, is a scalar constant used to represent the change of time delay. In other words, the plant changes are such that they are of different time delays in the same line and can be expressed as 2nd line of table 1. Suppose also that no change occurs in component T. The approximate model G_A, T_A are exactly the same as (26) and the controller is designed as in subsection 4.2, i.e. $k_1 = k_2 = 1.5$, $c_1 = c_2 = 0.003$. We then recheck the condition (5) and (6) and illustrate norm λ_1 and λ_2 in Fig.14 regarding m as a parameter. It is expected that no change will occur in norm λ_1 and that, the greater the m value, the more evident the change of λ_2. From Fig.14 we can see that the design allows 15% increase of time delay in the (1,1) element.

When G changes by changing of either gains or time constant, we can check condition (5) and (6) by similar means and determine if the change is allowed or not.

If the robustness of design is not enough for application, we can increase it by reducing the gains of controller or improving the model G_A and T_A.

5.2. When T changes

We consider the situation where the time delay of the plant changes. The example is the same as (25) and (26). It is clear that if the time delay of T decreases, then $(T-T_A)$ will become 'smaller' until $|\tilde{\tau}_j| < |\tau_{Aj}|$ and hence λ_1 will decrease and the stability still holds. So we observe the other case where τ_j increases to be $\tilde{\tau}_j$.

Let us suppose that no change occurs in G and that T changes to be $\tilde{T}$, where

$$\tilde{T} = \begin{bmatrix} e^{-mx35x22.8S} & 0 \\ 0 & e^{-mx35x3S} \end{bmatrix}, \quad T_A = \begin{bmatrix} e^{-30x22.8S} & 0 \\ 0 & e^{-30x3S} \end{bmatrix}$$

The norm λ_1 and λ_2 are shown in Fig.15 regarding m as a parameter. Because the change of τ_j effects both T and ΔT, so both λ_1 and λ_2 changes as expected. The greater the value of m, the more evident the change of λ_1 and λ_2. From Fig.15 the limitation of change of T can be found.

As stated previously, the robustness of design can be increased by reducing the gain of controller or improving the model.

6. Conclusion

The report has considered the robustness of design problems for Smith control scheme for multivariable cases. Because the theorem of reference (1) is of great generality, the plant can be expressed into separate form in most cases. Some technique of choosing model G_A, T_A and controller K has been given for multivariable process control. The technique has an evident benefit of simplicity and hence is easy to use. By looking at an example, we illustrate that the technique of design is successful. Compared with the normal feedback control scheme, the benefits of the Smith scheme are self-evident.

References

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(3) D.H.Owens, 'Feedback and multivariable systems', Peter Peregrinus, Stevenage, 1978.

(4) D.H.Owens and H.M.Wang and A.Chotai, 'Some case studies in approximate models in multivariable process control', Research Report No. 203, Dept. of Control Engineering, University of Sheffield, 1982.

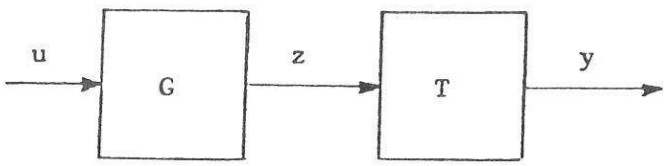


Fig.1. Plant decomposition

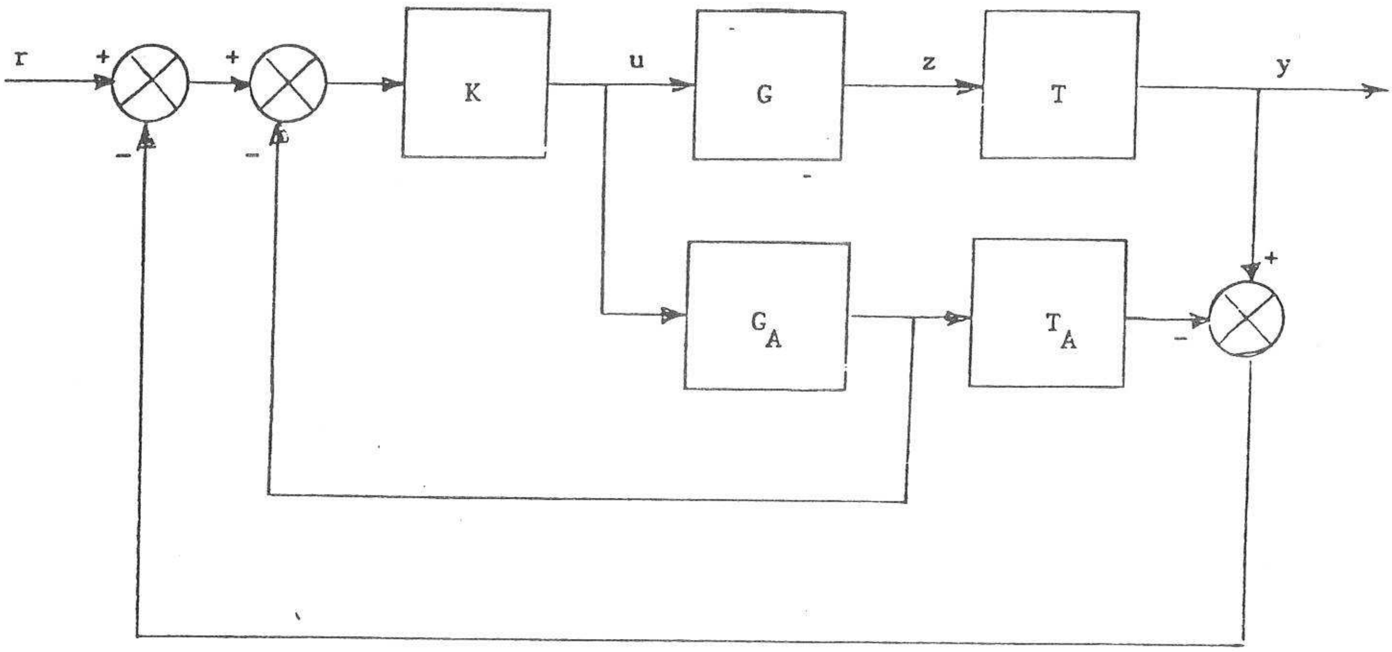


Fig.2. Smith Control Scheme

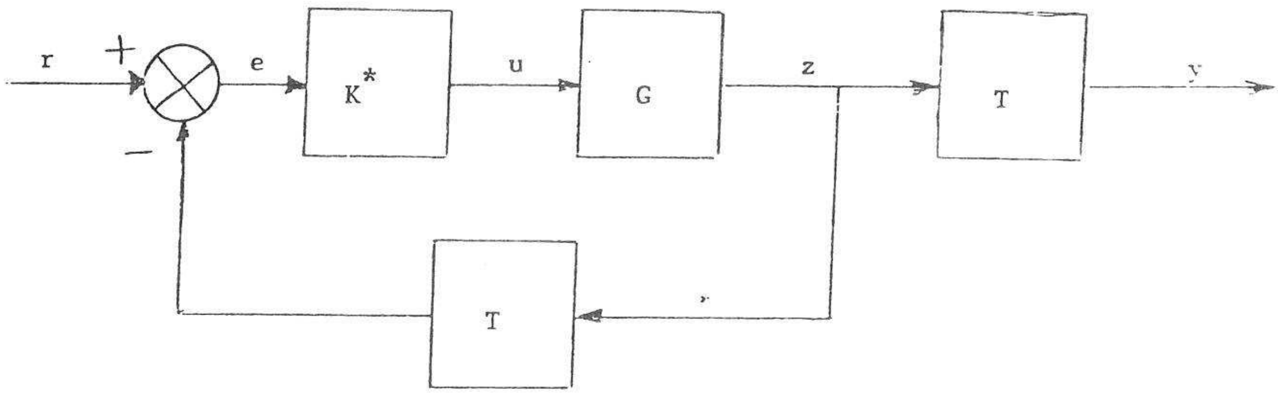


Fig.3. Equivalent Smith Scheme

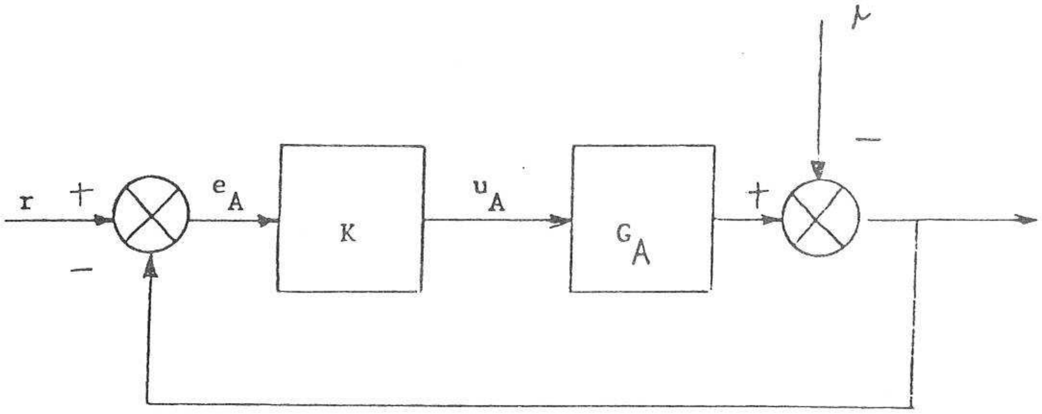


Fig.4. Delay-Free Control Scheme

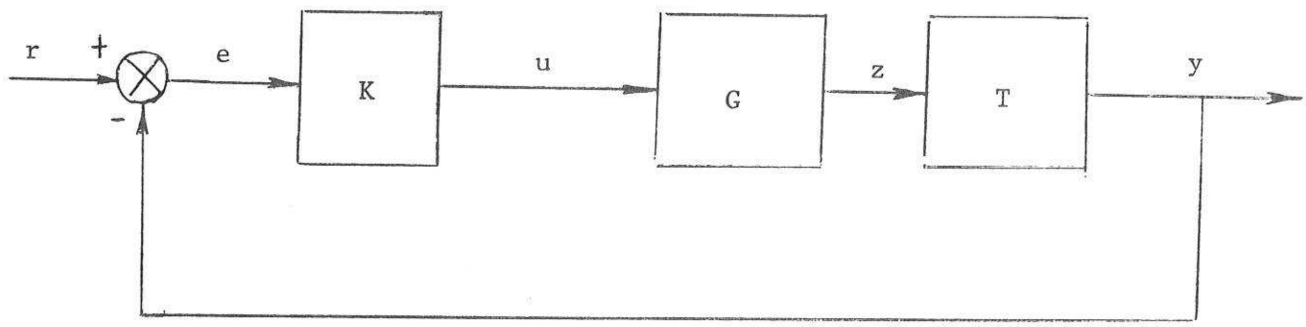
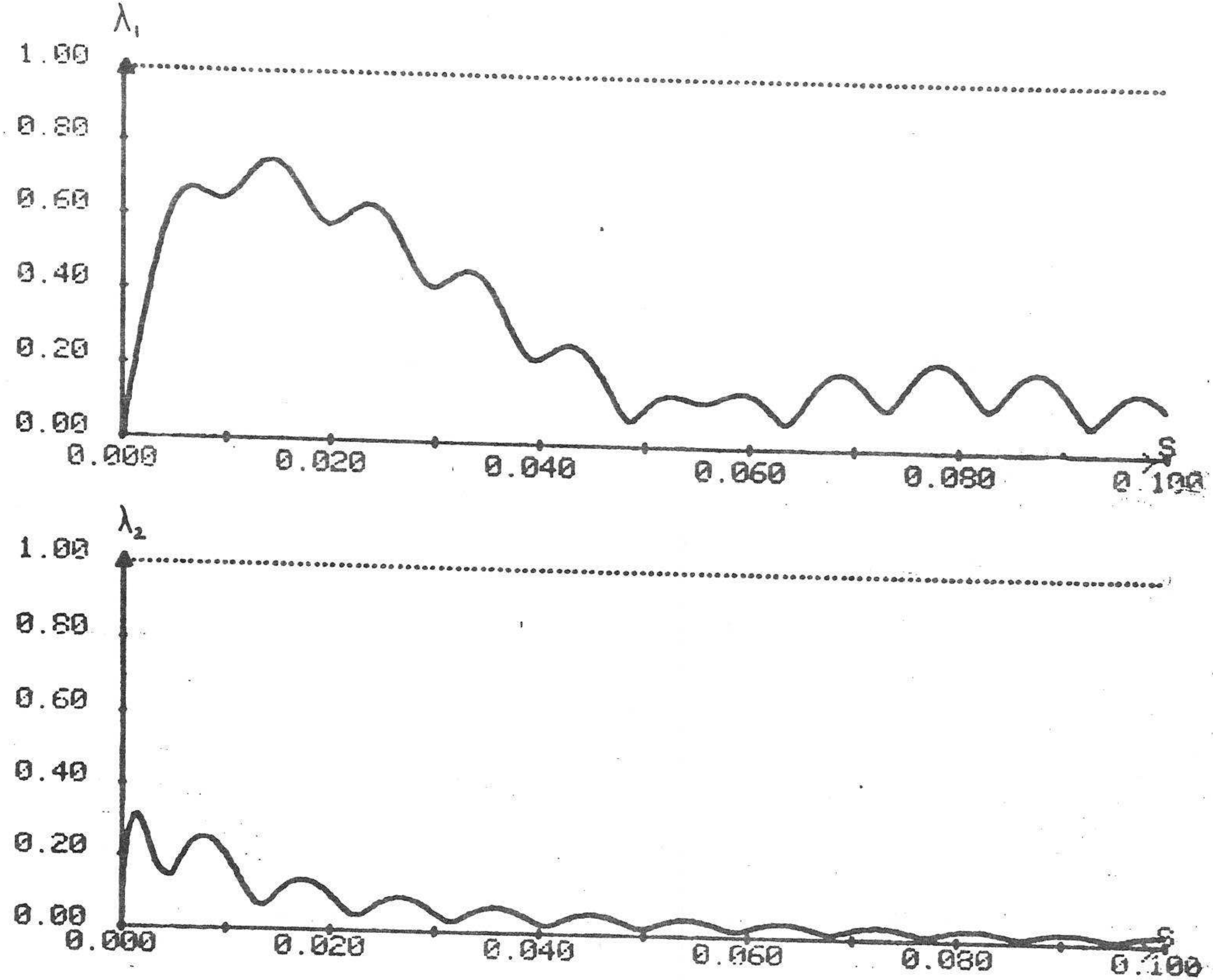


Fig.5. Normal Feedback Control Scheme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0.80
0.60
0.40
0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S
λ_2
1.00
0.80
0.60
0.40
0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S

Fig. 6

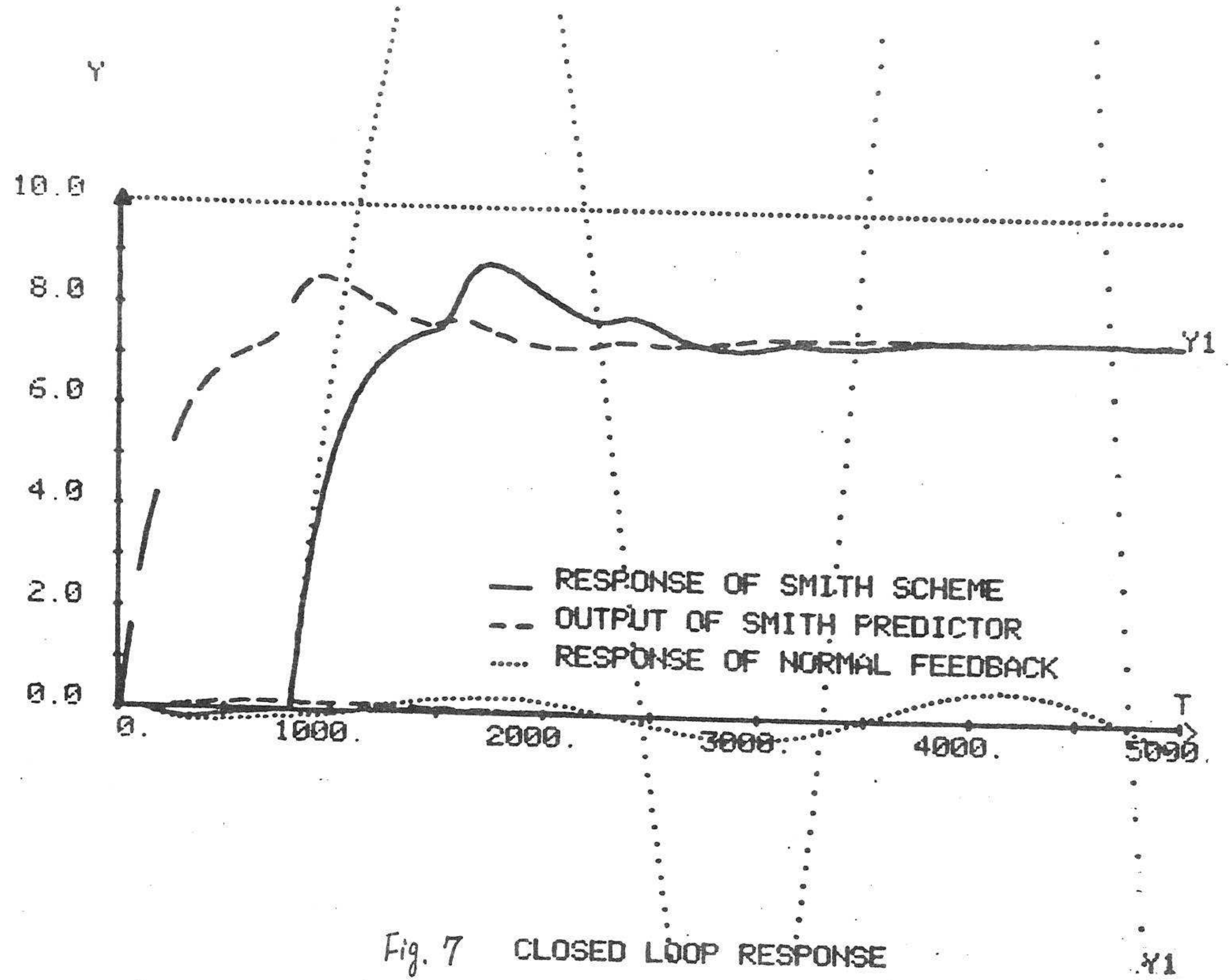
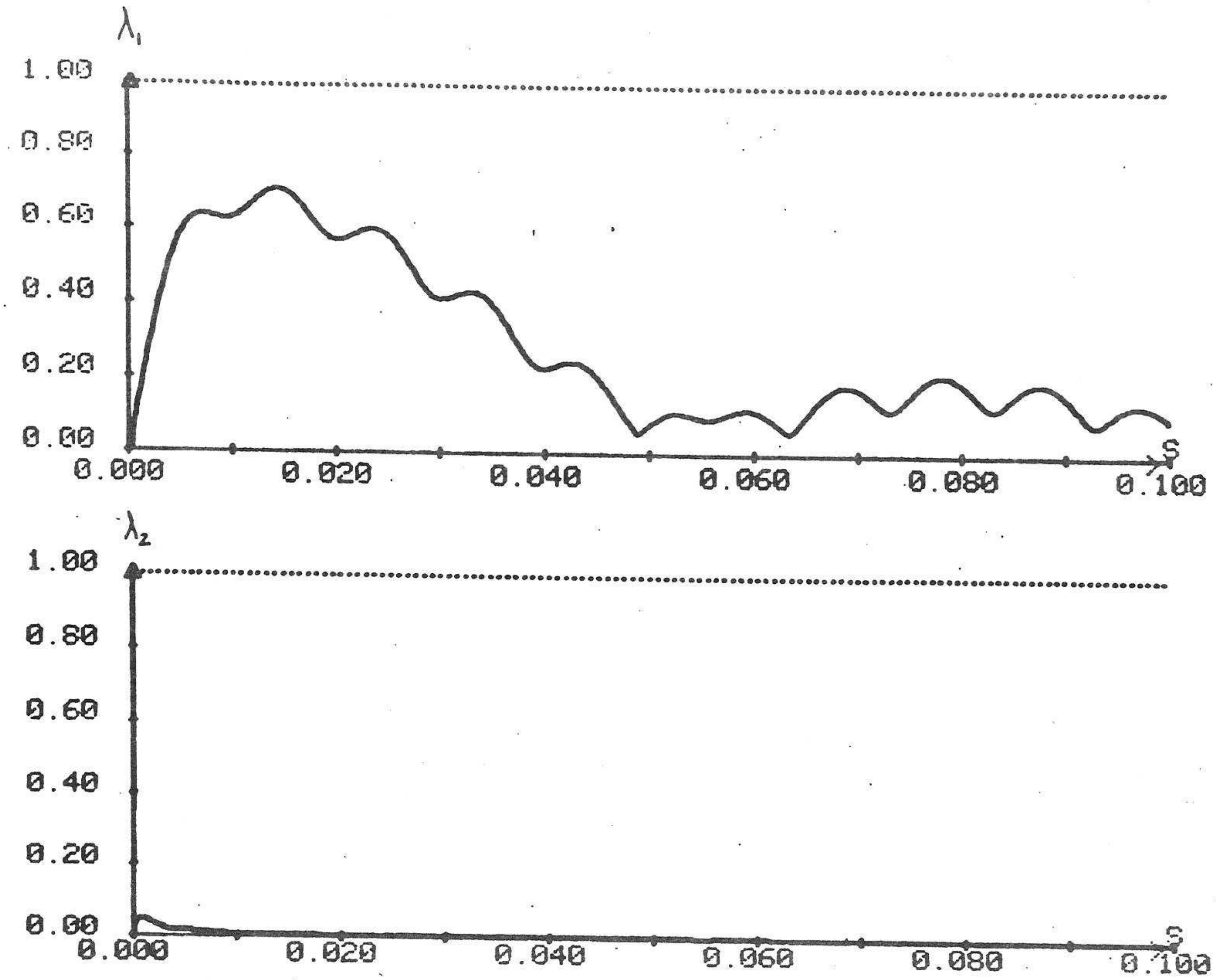


Fig. 7 CLOSED LOOP RESPONSE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0.40
0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S
λ2
1.00
0.80
0.60
0.40
0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S

Fig. 8

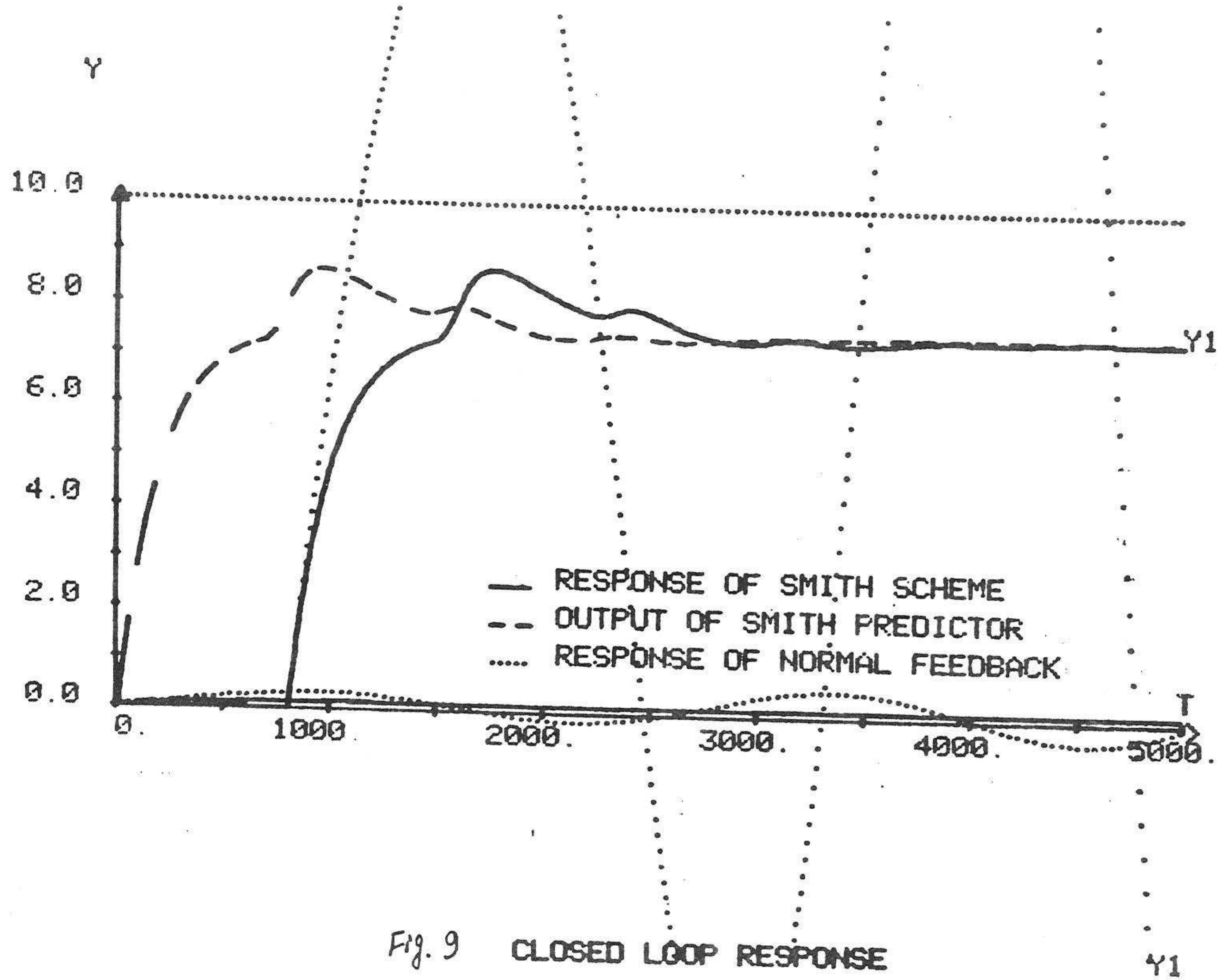
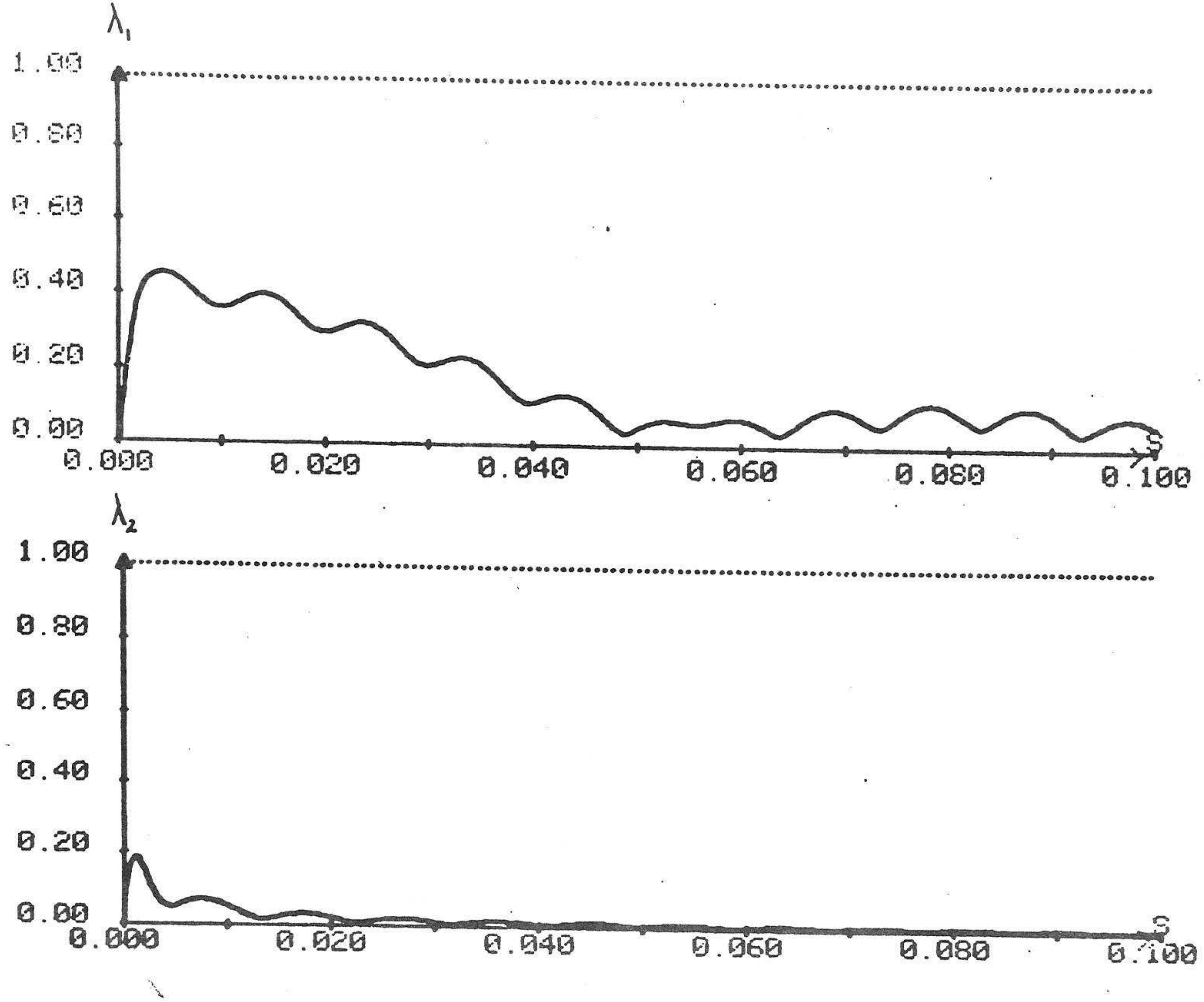


Fig. 9 CLOSED LOOP RESPONSE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0.000
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0.060
0.080
0.100
S
λ2
1.00
0.80
0.60
0.40
0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S

Fig. 10

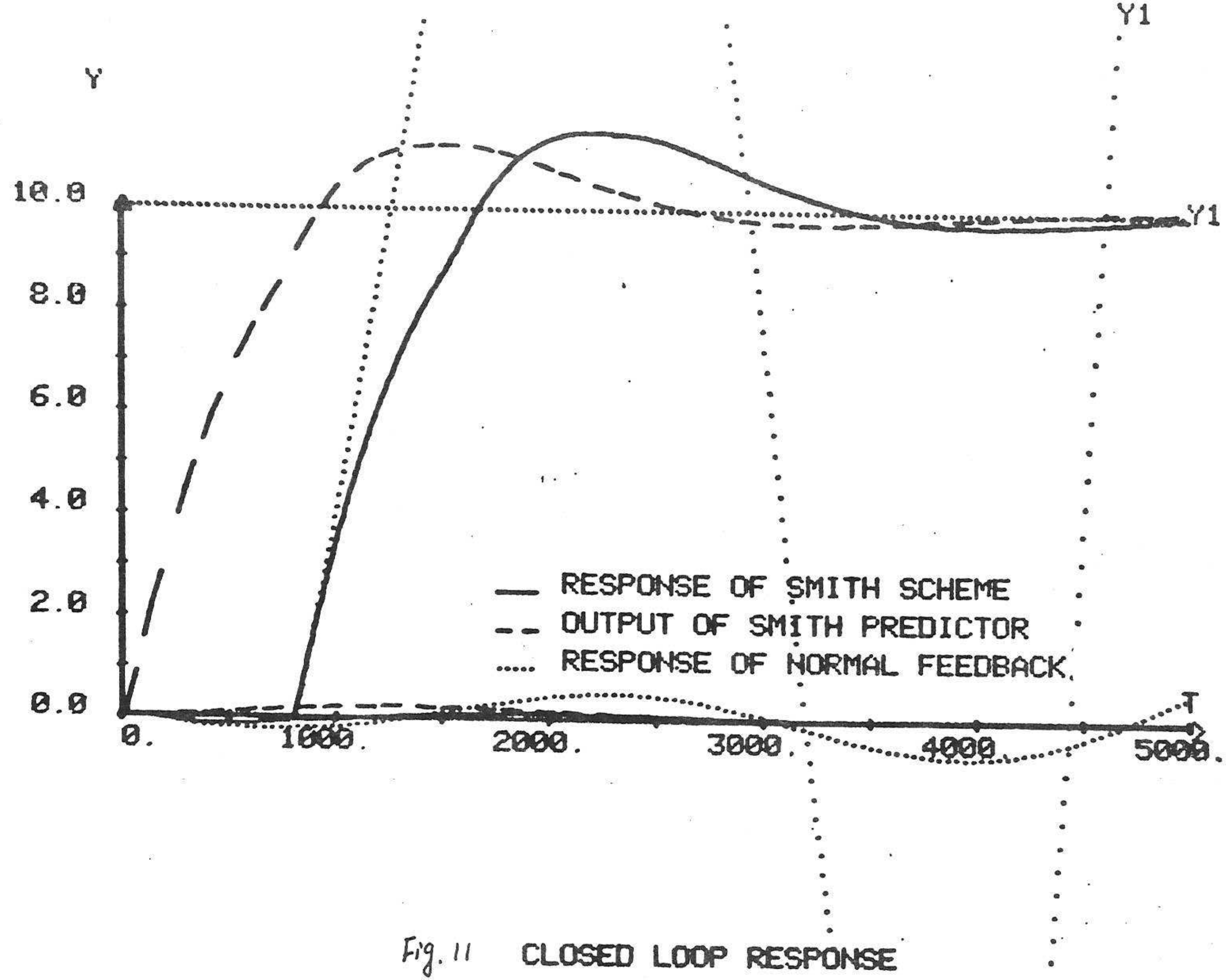
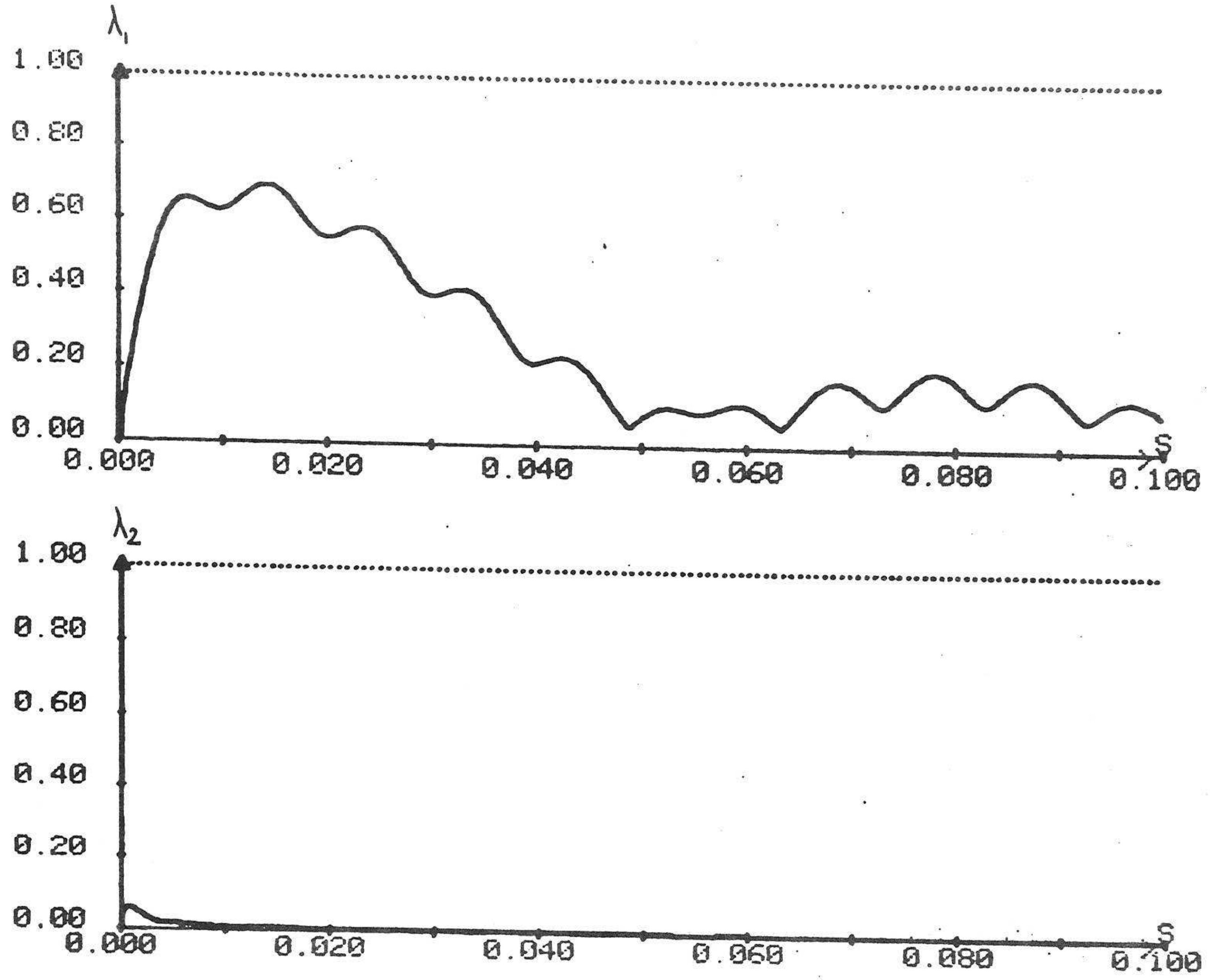


Fig. 11 CLOSED LOOP RESPONSE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0.40
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0.000
0.020
0.040
0.060
0.080
0.100
S
λ2
1.00
0.80
0.60
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0.20
0.00
0.000
0.020
0.040
0.060
0.080
0.100
S

Fig. 12

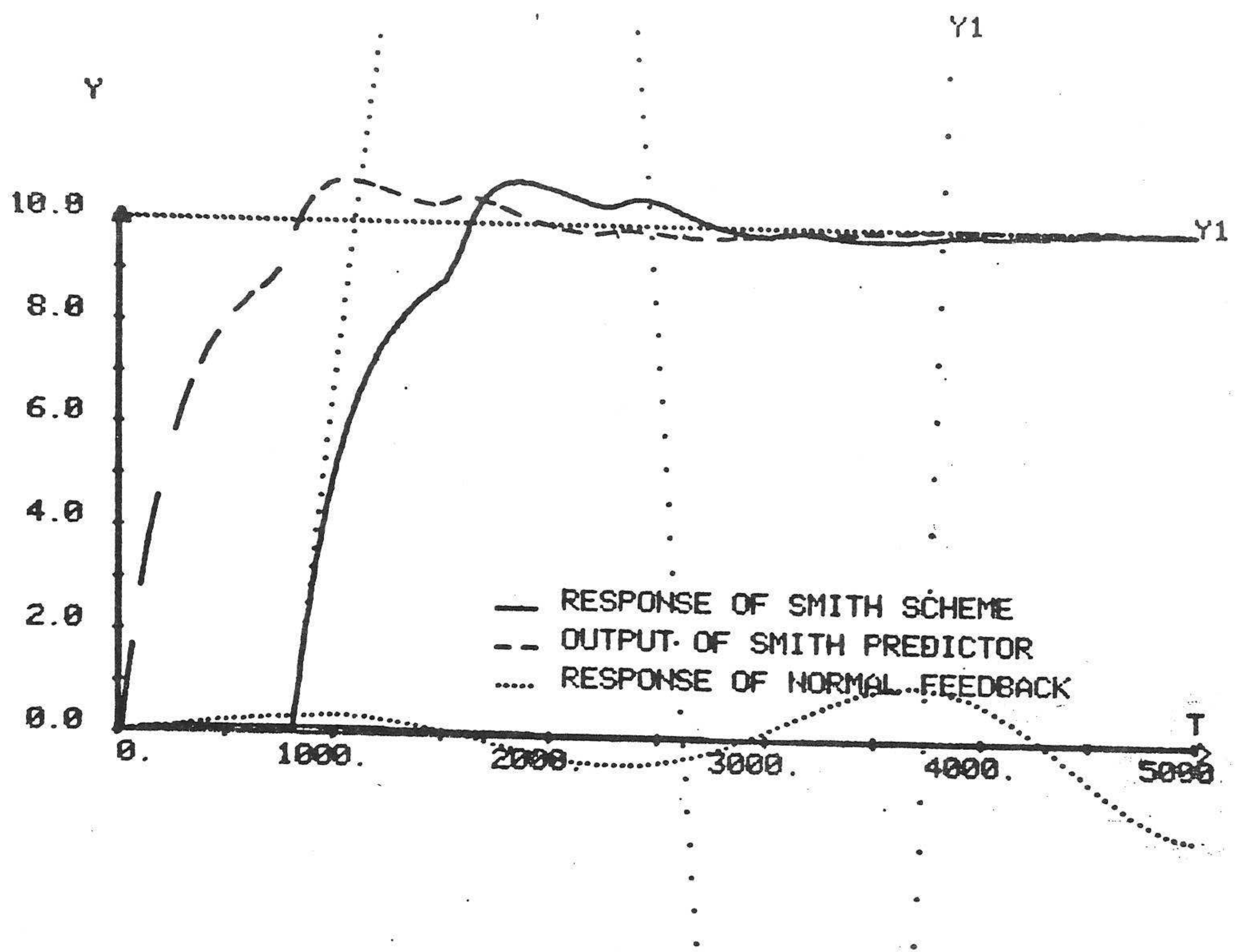
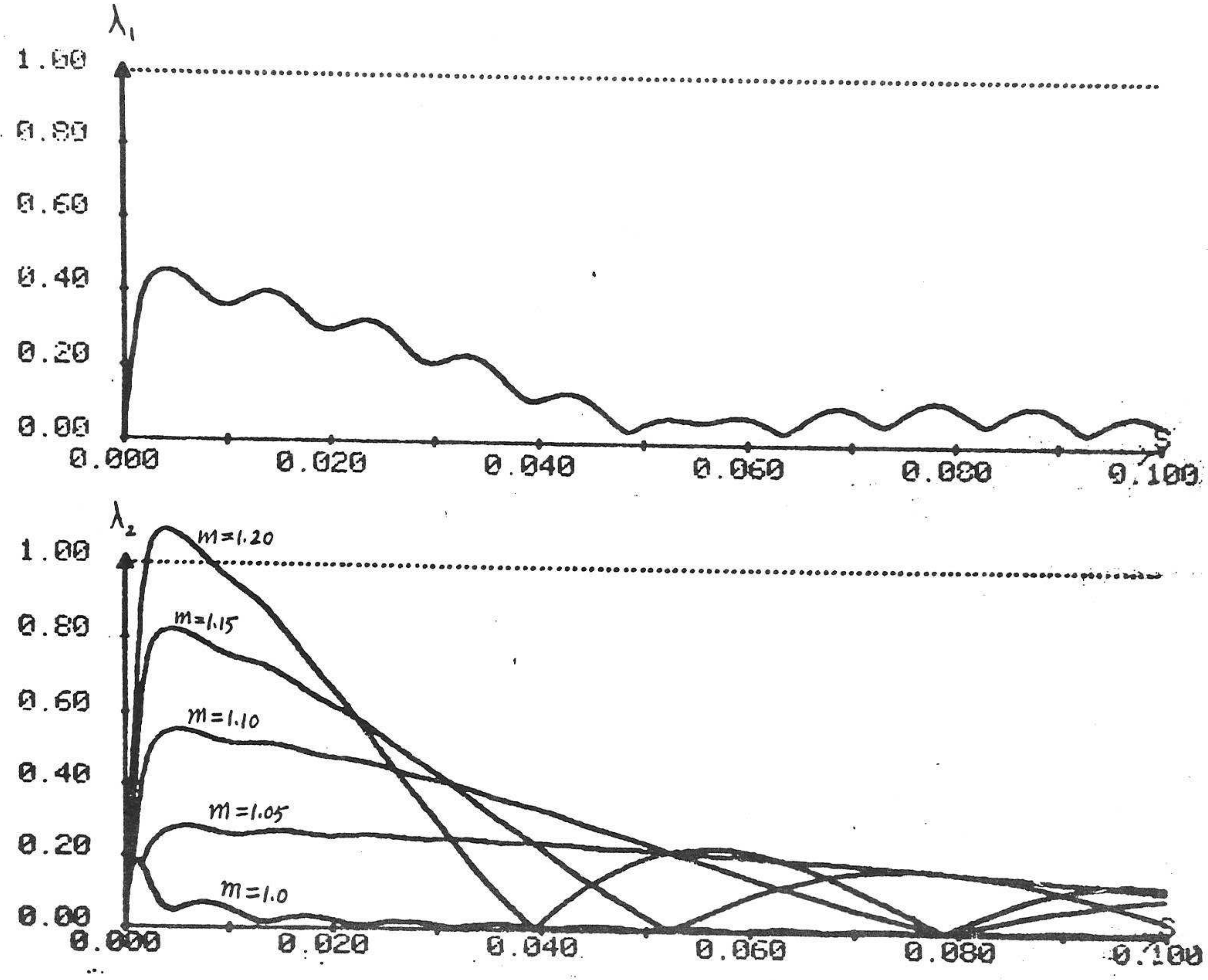


Fig. 13 CLOSED LOOP RESPONSE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S
λ2
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m=1.15
m=1.10
m=1.05
m=1.0
1.00
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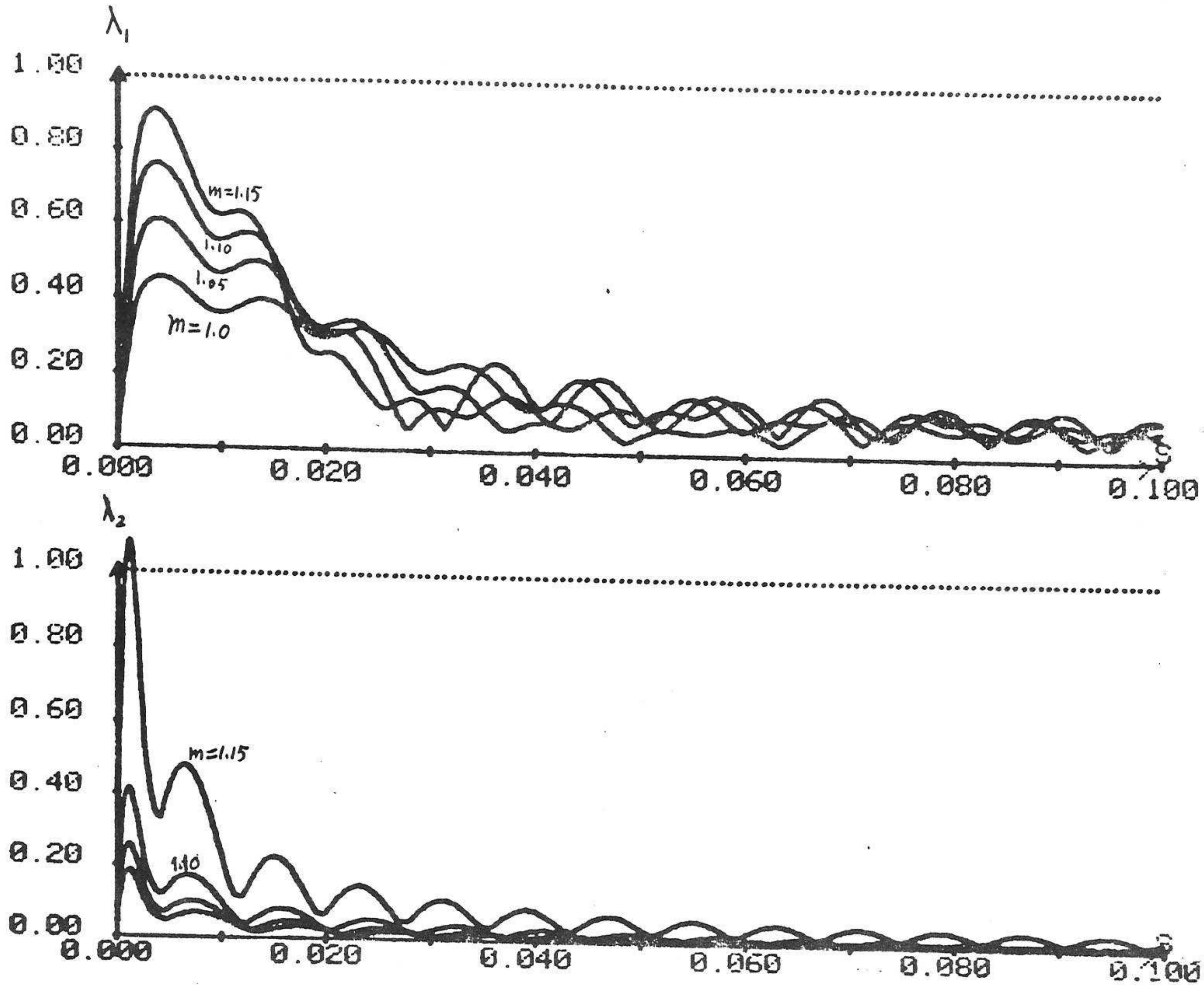
0.080
0.100
S

Fig 14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0.020
0.040
0.060
0.080
0.100
S
λ2
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0.80
0.60
0.40
0.20
0.00
m=1.15
1.10
0.000
0.020
0.040
0.060
0.080
0.100
S

Fig. 15