Monograph:

Owens, D.H. and Chotai, A. (1983) Robust Sampled Regulators for Stable Systems From Plant Step Data. Research Report. ACSE Report 213 . Department of Control Engineering, University of Sheffield, Mappin Street, Sheffield

ROBUST SAMPLED REGULATORS FOR STABLE SYSTEMS FROM PLANT STEP DATA

by

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Research Report No. 213

January 1983

This work is supported by the UK Science and Engineering Research Council under Grant GR/B/23250.

Keywords: Digital control; sampled-data systems; multivariable systems; stability; robustness; system order reduction; process control.

Abstract

The discrete time integrating regulator introduced by Åstrom is extended to cover the case of multivariable systems with non-monotonic step responses. The use of approximate models to increase the possible range of sampling rates is also considered.

1. Introduction

In a recent paper (Åstrom 1980) the use of a strongly simplified process model for the design of integrating regulators for stable systems with monotone step responses was convincingly demonstrated. The technique provided a simple way of designing robust regulators by elementary graphical analysis of plant step data that avoided the need for access to extensive computer-aided-design facilities. It is therefore of great potential value where practical controllers are required in on-line design situations.

A major limitation of the work is that it does not apply in the form presented to synchronously sampled multivariable systems or to systems with non-monotonic step responses. It is the objective of this paper to extend the ideas to cover these cases using recent results developed by the authors (Owens and Chotai, 1983). The background theory is provided in section 2 with applications to design based on first and second order deadbeat models and higher order models in section 3. The use of second and higher order models allows faster sampling rates than those permitted in Astrom (1980).

2. Background Theory

In a recent paper (Owens and Chotai, 1983) the authors introduced basic theoretical results to enable the successful design of output feedback systems for an m-input, m-output multivariable plant G by off-line design based on an approximate model. A basic result obtained is given below for completeness:

Lemma 1: If a mxm controller with z-transfer function matrix (z-TFM) K(z) stabilizes a model with z-TFM $G_A(z)$ under unity negative feedback, then it will also stabilize a discrete plant with z-TFM G(z) under unity negative feedback if

(a) both the plant and model are stable,

(b) the composite system GK is controllable and observable and

(c) $\sup_{z \in D} \gamma(z) < 1$ ...(1)

where the set D consists of the two circles $|z| = 1$ and $|z| = R$ with R large enough to be regarded as infinite for all practical purposes, $\gamma(z)$ is any available real-valued function on D satisfying

$$\gamma(z) \geq r(\tilde{L}(z)) \qquad \forall \; z \in D \qquad ...(2)$$

with

$$\tilde{L}(z) \triangleq \| (I_m + K(z)G_A(z))^{-1} K(z) \|_P \Delta(z) \qquad ...(3)$$

and $\Delta(z)$ is any available matrix-valued function satisfying

$$\Delta(z) \geq \| G(z) - G_A(z) \|_P \;\; \forall \; z \in D \qquad ...(4)$$

(Note: (i) r(M) denotes the spectral radius of the mxm matrix M. The spectral radius of a scalar (ie a 1x1 matrix) is simply its absolute value.

(ii) The 'absolute value' $\|M\|_P$ of the mxm matrix $M = [M_{ij}]$ is the matrix $[|M_{ij}|]$ of moduli of the elements of M.

(iii) If A and B are two mxm real matrices, then the relation $A \leq B$ denotes the m^2 inequalities $A_{ij} \leq B_{ij}$, $1 \leq i, j \leq m$.

(iv) $A<B$ denotes the m^2 inequalities $A_{ij}<B_{ij}$, $1 \leq i, j \leq m$.

(v) the lemma follows from theorem 1 in Owens and Chotai (1983) by replacing the Nyquist contour by the set D defined above and assuming unit feedback).

In the following sections, it is assumed that plant open-loop step responses are available in the form of the mxm matrix step response sequence $Y(k)$, $k \geq 0$, with elements $Y_{ij}(k)$ defined to be equal to the value of the output y_i at time $t = kT$ (T = sample interval) in response to a unit step input in input u_j from zero initial conditions with $u_\ell(t) \equiv 0$, $\ell \neq j$, $k \geq 0$. If $Y_A(k)$, $k \geq 0$, is the corresponding step response sequence for the model G_A, then the modelling error can be characterized by the error sequence

$$E(k) \triangleq Y(k) - Y_A(k) \quad , \qquad k \geq 0 \qquad \ldots(5)$$

The modelling error can be used to form a suitable bound $\Delta(z)$ as follows:

Lemma 2: Define the mxm matrix

$$N_\infty^{\ P}(E) = \begin{pmatrix} N_\infty(E_{11}) & \cdots & N_\infty(E_{1m}) \\ \vdots & & \vdots \\ N_\infty(E_{m1}) & \cdots & N_\infty(E_{mm}) \end{pmatrix} \qquad \ldots(6)$$

where $N_\infty(E_{ij})$ is the 'total variation' of the sequence $E_{ij}(k)$, $k \geq 0$,

$$N_\infty(E_{ij}) \triangleq \sum_{k=1}^{\infty} |E_{ij}(k) - E_{ij}(k-1)|$$

$$\equiv \sum_{\ell\geq 1} |E_{ij}(k_{ij\ell}) - E_{ij}(k_{ij\ell-1})| \qquad \ldots(6a)$$

and $0 = k_{ijo} < k_{ij1} < k_{ij2} < \ldots.$ are the local maxima and minima of $E_{ij}(k)$, $k\geq 0$, in the extended half axis $k\geq 0$. Then

$$||G(z) - G_A(z)||_P \leq N_\infty^P(E) \qquad \forall \quad z \in D \qquad \ldots(7)$$

(Remark: The result is the discrete equivalent to lemma 2 in Owens and Chotai (1983). It is important to note (i) that local maxima and minima are identified on the extended half-line (ie the point $k = +\infty$ maybe a local maximum or minimum) and (ii) that $N_\infty(E_{ij})$ can be assessed graphically by identification of the maxima and minima of E_{ij}).

Proof: Elementary considerations yield the characterization

$$G(z) - G_A(z) = \sum_{k=1}^{\infty} z^{-k}(E(k) - E(k-1)) \qquad \ldots(8)$$

and hence, for $|z| \geq 1$, we have

$$|G_{ij}(z) - (G_A(z))_{ij}| \leq \sum_{k=1}^{\infty} |E_{ij}(k) - E_{ij}(k-1)|$$

$$= \sum_{\ell\geq 1} \sum_{r=k_{ij\ell-1}+1}^{k_{ij\ell}} |E_{ij}(r) - E_{ij}(r-1)|$$

$$= \sum_{\ell\geq 1} |E_{ij}(k_{ij\ell}) - E_{ij}(k_{ij\ell-1})| \qquad \ldots(9)$$

as the definition of $k_{ij\ell}$, $\ell\geq 1$, ensures that $E_{ij}(r)-E_{ij}(r-1)$ has only one sign in the range $k_{ij\ell-1}+1 \leq r \leq k_{ij\ell}$. Equation (9) is simply

$$|G_{ij}(z) - (G_A(z))_{ij}| \leq N_\infty(E_{ij}) \quad , \quad 1 \leq i, j \leq m \qquad \ldots(10)$$

which yields (7) when put in matrix form.

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3. Robust Regulator Design

Consider the problem of robust regulator design for an m-input/m-output plant G. The plant is not necessarily assumed to be monotone (Åstrom, 1980). The applications of lemmas 1 and 2 are considered for various classes of plant model and integral control system. As in Åstrom (1980), the controller gain and sampling rate will be regarded as design variables and it is therefore necessary to have access to the continuous plant step response matrix H(t) with (i,j)th element $H_{ij}(t)$ equal to the response of $y_i(t)$ from zero initial conditions to a unit step in $u_j(t)$ with $u_r(t) \equiv 0$, $r \neq j$. If all outputs are sampled and all inputs are actuated sunchronously with frequency T^{-1}, then clearly $Y(k) = H(kT)$, $k \geq 0$, and $Y(o) = 0$.

3.1. Design Based on the Model $G_A(z) = z^{-1}B$

Following Åstrom (1980), consider the approximate model of the plant

$$y(t) = B\, u(t-T) \qquad \ldots(11)$$

where B is a nonsingular mxm matrix and T>0. This construction is equivalent to modelling the response of y_i to a unit step in u_j as shown in Fig.1(a) and reduces to the model used by Åstrom (1980) if m = 1. If T is taken to be the sample interval of synchronous input actuation and output sampling, then the model has transfer function matrix

$$G_A(z) = Bz^{-1} \qquad \ldots(12)$$

Consider the integrating process controller

$$u_k = u_{k-1} + \varepsilon B^{-1}(r_k - y_k) \qquad \ldots(13)$$

where r_k, $k \geq 0$, is the set-point sequence. This controller has transfer function matrix $K(z) = \varepsilon B^{-1} z/(z-1)$ and reduces to that of Åstrom (1980) if $\varepsilon = 1$. It stabilizes the model G_A if $0<\varepsilon<2$ but we will concentrate on the case when the input-output characteristics are also monotonic/overdamped ie

$$0 < \varepsilon \leq 1 \qquad \ldots(14)$$

Combining lemmas 1 and 2 then yields the following result and corollary relating the stability of the plant to the choice of B and T:

Theorem 1: The digital controller (13) stabilizes the real plant if (14) holds and

$$r_o \triangleq r(\|B^{-1}\|_P N_\infty^P(E)) < 1 \qquad \ldots(15)$$

Proof: The conditions of lemma 1 hold with $\Delta(z) = N_\infty^P(E)$ (lemma 2) and

$$\gamma(z) = \sup_{z\in D} \left| \frac{\varepsilon z}{z-1+\varepsilon} \right| r_o \equiv r_o < 1 \;\; \forall \; z\in D \qquad \ldots(16)$$

as is easily proved by noting that

$$\tilde{L}(z) = \left| \frac{\varepsilon z}{z-1+\varepsilon} \right| \|B^{-1}\|_P \qquad \ldots(17)$$

Corollary 1.1: The conclusions of theorem 1 hold if $\gamma_o<1$ where γ_o is any conveniently computable upper bound for r_o.

The result is easily applied by choice of T and B, evaluation of the error sequence E(k), $k \geq 0$, and its total variation $N_\infty^P(E)$ and checking condition (15). The corollary allows upper bounds γ_o for r_o to be used if they are more conveniently calculated eg. the vector induced matrix norm

$$\gamma_o = \max_{1 \leq i \leq m} \sum_{j=1}^{m} \left(\|B^{-1}\|_P \, N_\infty^P(E) \right)_{ij}$$

$$= \max_{1 \leq i \leq m} \sum_{j=1}^{m} \sum_{k=1}^{m} |(B^{-1})_{ik}| \, N_\infty(E_{kj}) \qquad \ldots(18)$$

Equation (15) can be simplified by expressing $N_\infty^P(E)$ in terms of H and B by noting that

$$N_\infty^P(E) = N_\infty^P(Y) + \|H(T) - B\|_P - \|H(T)\|_P \qquad \ldots(19)$$

which relates the total variation of E to the total variation of the sampled plant step response Y, B and H(T). In general terms, B and T should be chosen to make the combined contribution of the second two terms as 'negative' as is necessary to satisfy (15). This turns out to be particularly simple if we choose $\|B\|_P \geq \|H(T)\|_P$ in the sense that

$$\begin{array}{lll} B_{ij} \geq H_{ij}(T) & \text{if} & H_{ij}(T) > 0 \\ B_{ij} = \text{arbitrary} & \text{if} & H_{ij}(T) = 0 \\ B_{ij} \leq H_{ij}(T) & \text{if} & H_{ij}(T) < 0 \end{array} \qquad \ldots(20)$$

when (19) reduces to

$$N_\infty^P(E) = N_\infty^P(Y) + \|B\|_P - 2\,\|H(T)\|_P \qquad \ldots(21)$$

and (15) to

$$r_o = r(\|B^{-1}\|_P(N_\infty^P(Y) + \|B\|_P - 2\|H(T)\|_P)) < 1 \quad \ldots(22)$$

In the case of m = 1, this is simply the relation

$$\left| 1 + \frac{N_\infty^P(Y) - 2|H(T)|}{|B|} \right| < 1 \quad \ldots(23)$$

which reduces to that of Åstrom (1980) with $N_\infty^P(Y)$ replacing $|H(\infty)|$. In fact, if G is monotonic in the sense (Owens and Chotai, 1982) that each element of Y is monotonic, we have

$$N_\infty^P(Y) = \|H(\infty)\|_P \quad \ldots(24)$$

when (23) reproduces Åstroms work for scalar systems and (22) extends it to the multivariable case. Theorem 1 therefore represents a true generalization of his result to multivariable systems with non-monotonic step responses.

An implicit constraint in the work of Åstrom (1980) and the above is that the sampling rate must be relatively slow. This can be illustrated by letting $T \to 0+$ when $H(T) \to 0$ and, substituting (19) into (15), we obtain

$$r_o \to r(\|B^{-1}\|_P(N_\infty^P(Y) + \|B\|_P)) \geq 1 \quad \ldots(25)$$

due to the fact that $\|B^{-1}\|_P(N_\infty^P(Y) + \|B\|_P) \geq \|B^{-1}\|_P \|B\|_P \geq I_m$. The condition of theorem 1 cannot therefore be satisfied under fast sampling conditions. In contrast, it is frequently possible to satisfy the condition under slow sampling. Consider, for example, the choice of B = H(T) and $T \to +\infty$ when $H(T) \to H(\infty)$ and hence (15) reduces to the requirement that

$$r_o \to r(\|H(\infty)^{-1}\|_P (N_\infty^P(Y) - \|H(\infty)\|_P)) < 1 \quad \ldots(26)$$

which is satisfied provided that $N_\infty^P(Y)$ does not differ from $\|H(\infty)\|_P$ by too much. Equivalently the plant should not be too oscillatory! Note, for example, that $r_o \to 0$ if the plant is monotonic by (24) and hence (15) can always be satisfied by choice of slow sampling conditions.

The possibility of using faster sampling rates can be achieved by using other plant models as illustrated below.

3.2. Design Based on the Model $G_A(z) = (az^{-1}+(1-a)z^{-2})B$, $0<a\leq 1$

Consider the use of a sample interval of length T and the deadbeat model with step response modelled as shown in Fig.1(b) or, equivalently by the equation

$$y(t) = B(au(t-T) + (1-a)u(t-2T)), \quad 0<a\leq 1 \quad \ldots(27)$$

and the consequent z-transfer function matrix

$$G_A(z) = B(az^{-1} + (1-a)z^{-2}) \quad \ldots(28)$$

in the design of an integrating control system of the form of (13).

It is easily verified (see appendix) that K stabilizes G_A producing an overdamped closed-loop system in the gain range

$$0 < \varepsilon \leq \varepsilon^*(a) \triangleq a^{-2} ((2-a) - 2\sqrt{1-a}) \quad \ldots(29)$$

where $\varepsilon^*(a)$ is monotonically increasing in the interval $0<a\leq 1$ and satisfies

$$\tfrac{1}{4} = \varepsilon^*(0+) < \varepsilon^*(a) \leq 1 = \varepsilon^*(1) \quad \ldots(30)$$

In this case, using $\Delta(z) = N_\infty^P(E)$,

$$\tilde{L}(z) = \left| \frac{\varepsilon z^2}{z^2+(\varepsilon a-1)z+\varepsilon(1-a)} \right| \|B^{-1}\|_P \quad \ldots(31)$$

and, choosing

$$\gamma(z) = \sup_{z\in D} \left| \varepsilon z^2 \Big/ (z^2+(\varepsilon a-1)z+\varepsilon(1-a)) \right|_{r_o} \equiv r_o \quad , \qquad \ldots(32)$$

it is trivially verified that theorem 1 holds with (14) replaced by (29).

The result can be applied in design studies for any values of a and B using the relation

$$N_\infty^P(E) = N_\infty^P(Y) + \|H(T) - aB\|_P - \|H(T)\|_P + \|H(2T) - H(T) - (1-a)B\|_P - \|H(2T) - H(T)\|_P \qquad \ldots(33)$$

A particularly simple result holds however if we choose a and B such that, for all i,j,

$$\begin{aligned} aB_{ij} &\geq H_{ij}(T) && \text{if} \quad H_{ij}(T) > 0 \\ aB_{ij} &= \text{arbitrary} && \text{if} \quad H_{ij}(T) = 0 \\ aB_{ij} &\leq H_{ij}(T) && \text{if} \quad H_{ij}(T) < 0 \end{aligned} \qquad \ldots(34)$$

and

$$\begin{aligned} (1-a)B_{ij} &\geq H_{ij}(2T) - H_{ij}(T) && \text{if} \quad H_{ij}(2T) - H_{ij}(T) > 0 \\ (1-a)B_{ij} &= \text{arbitrary} && \text{if} \quad H_{ij}(2T) - H_{ij}(T) = 0 \\ (1-a)B_{ij} &\leq H_{ij}(2T) - H_{ij}(T) && \text{if} \quad H_{ij}(2T) - H_{ij}(T) < 0 \end{aligned} \qquad \ldots(35)$$

In this case, $N_\infty^P(E)$ simplifies to

$$N_\infty^P(E) = N_\infty^P(Y) + \|B\|_P - 2\,\|H(T)\|_P - 2\,\|H(2T) - H(T)\|_P \qquad \ldots(36)$$

which is independent of a.

Further simplification is possible if, for all i,j, we also choose T such that

$$H_{ij}(2T) \geq H_{ij}(T) \quad \text{if} \quad H_{ij}(T) > 0$$

$$H_{ij}(2T) \leq H_{ij}(T) \quad \text{if} \quad H_{ij}(T) < 0 \qquad \ldots(37)$$

when (36) reduces to

$$N_\infty^P(E) = N_\infty^P(Y) + \|B\|_P - 2\|H(2T)\|_P \qquad \ldots(38)$$

which is independent of the detailed value of H(T). The stability condition (15) now takes the form

$$r_o = r(\|B^{-1}\|_P(N_\infty^P(Y) + \|B\|_P - 2\|H(2T)\|_P)) < 1 \qquad \ldots(39)$$

Comparing with (22), it is seen that the construction defined by (34) and (35) for the second order model (28) plays the same role as the construction (20) for the model (12) and yields a stability criterion of identical form with H(T) replaced by H(2T). The second order model clearly therefore allows the possibility of faster sampling rates than those allowed by the first order model. If, for example, (22) is satisfied for T = 2T' and (34), (35) and (37) are satisfied for T replaced by T' and some a, then the sampling interval T used by Astrom can be halved to T' = T/2. Further increases in sampling rate must use a more complex process model.

3.3. Design Based on the Model $G_A(z) = g(z)B$

Given any choice of sample interval T, examination of the step sequence Y(o) = H(o), Y(1) = H(T),.... could be used to suggest a simplified model

$$G_A(z) = g(z)B \qquad \ldots(40)$$

where g(z) is chosen so that $g(z)B_{ij}$ is a simplified representation of the step response sequence $Y_{ij}(k)$, $k \geq 0$, but where g(z) is independent of (i,j). Consider the choice of scalar compensator k(z) so that the control system

$$K(z) = k(z)B^{-1} \qquad \text{...(41)}$$

stabilizes the real plant G.

Theorem 2: If k(z) is specified so that the scalar feedback system with closed-loop transfer function

$$h(z) = \frac{g(z)k(z)}{1 + g(z)k(z)} \qquad \text{...(42)}$$

is stable, then the controller (41) stabilizes the real plant G if

(a) both G and g(z) are stable,

(b) the composite system GK is both controllable and observable,

and (c) $\gamma_o \sup_{z \in D} \left| \frac{k(z)}{1+g(z)k(z)} \right| < 1$ ...(43)

where γ_o is any available upper bound for

$$r_o \triangleq r(\|B^{-1}\|_P N_\infty^P(E)) \qquad \text{...(44)}$$

Proof: Conditions (a) and (b) are simply those of lemma 1 whilst (c) reduces to (1) by taking $\Delta(z) = N_\infty^P(E)$ and

$$\gamma(z) \triangleq \left| \frac{k(z)}{1+g(z)k(z)} \right| \gamma_o$$

$$\geq r(\left| \frac{k(z)}{1+g(z)k(z)} \right| \|B^{-1}\|_P N_\infty^P(E))$$

$$= r(\tilde{L}(z)) \qquad \text{...(45)}$$

Condition (a) is easily achieved if the plant G is stable. Condition (b) cannot be checked in general unless a model of the plant is available, the designer relying on the fact that it holds generically and hence is unlikely to be violated in practice. If it is violated then the implemented scheme contains uncontrollable and/or unobservable stable modes that do not affect stability but may lead to undesirable system state dynamics.

Condition (c) can be checked analytically (if feasible) or numerically. The authors prefer the graphical technique described by the following corollary:

Corollary 2.1: The conclusions of theorem 2 remain valid if (43) is replaced by the equivalent conditions that

(i) $$\gamma_o \lim_{|z|\to\infty} \left| \frac{k(z)}{1+g(z)k(z)} \right| < 1 \qquad \ldots(46)$$

(ii) the plot of the inverse transfer function $(g(z)k(z))^{-1}$ with $z = e^{-i\theta}$, $0 \le \theta \le \pi$, with superimposed 'confidence circles' of radius

$$d(z) \triangleq |g^{-1}(z)| \; \gamma_o \qquad \ldots(47)$$

at each frequency generates a 'confidence band' that does not touch or contain the (-1,0) point of the complex plane.

Proof: Condition (i) follows from (43) by considering $|z| = R \to +\infty$. Condition (ii) follows by writing (43) in the form

$$|1 + (g(z)k(z))^{-1}| > d(z) \quad , \quad |z| = 1 \qquad \ldots(48)$$

The graphical interpretation of (ii) is illustrated in Fig.2 and has the same structure as some of the frequency domain results in Owens and Chotai (1983). The result holds for any choice of g and k including those of sections 3.1 and 3.2. It can therefore be used to widen the range of gains specified by (14) and (30) and to include proportional action to improve response speeds.

4. Conclusions

It has been demonstrated that the important results of Åstrom (1980) concerning the design of integrating regulators for process plant with monotone step responses are a special case of recent results on approximation due to Owens and Chotai (1983). These results have been used to extend his work to the multivariable case and to systems with non-monotonic step responses. It has been shown how the sample rates possible in his design can be partially increased by the use of a second order deadbeat approximate plant model. As in Åstrom (1980), the integrating regulator is designed using simple graphical operations on the plant step response. Further increases in sampling rates may be possible if an improved process model is used, but theorem 2 and its corollary imply that more detailed analysis is necessary in this case.

Acknowledgement

This work is supported by SERC under contract GR/B/23250.

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Appendix

The characteristic polynomial of the closed-loop system is

$$z^2 + (\varepsilon a-1)z + \varepsilon(1-a) \qquad \ldots(49)$$

which indicates that the closed-loop system is overdamped if

$$(\varepsilon a-1)^2 > 4\varepsilon(1-a) \qquad \ldots(50)$$

or, equivalently,

$$a^2\varepsilon^2 + (2a-4)\varepsilon + 1 \geq 0 \qquad \ldots(51)$$

This certainly holds if

$$0 < \varepsilon \leq \varepsilon^*(a) \triangleq a^{-2}((2-a) - 2\sqrt{1-a}) \qquad \ldots(52)$$

by considering roots and, taking $0<a<1$, we can expand $\varepsilon^*(a)$ in a power series

$$\varepsilon^*(a) = a^{-2}((2-a) - 2(1 - \frac{a}{2} - \frac{a^2}{8} - \ldots)$$

$$= \frac{1}{4} + \sum_{j\geq 1} c_j a^j \qquad \ldots(53)$$

where $c_j>0$, $j\geq 1$. Clearly $\varepsilon^*(0+) = \frac{1}{4}$ and $\varepsilon^*(a)$ is monotonic in $0<a<1$ with $\varepsilon^*(1-) = \varepsilon^*(1) = 1$ by continuity and (52).

The stability of the closed-loop system follows by noting that $G_A K$ has poles at $z = 0$ and $z = 1$ with one zero at $-(a^{-1}-1)<0$ and considering the structure of the root-locus of the system.

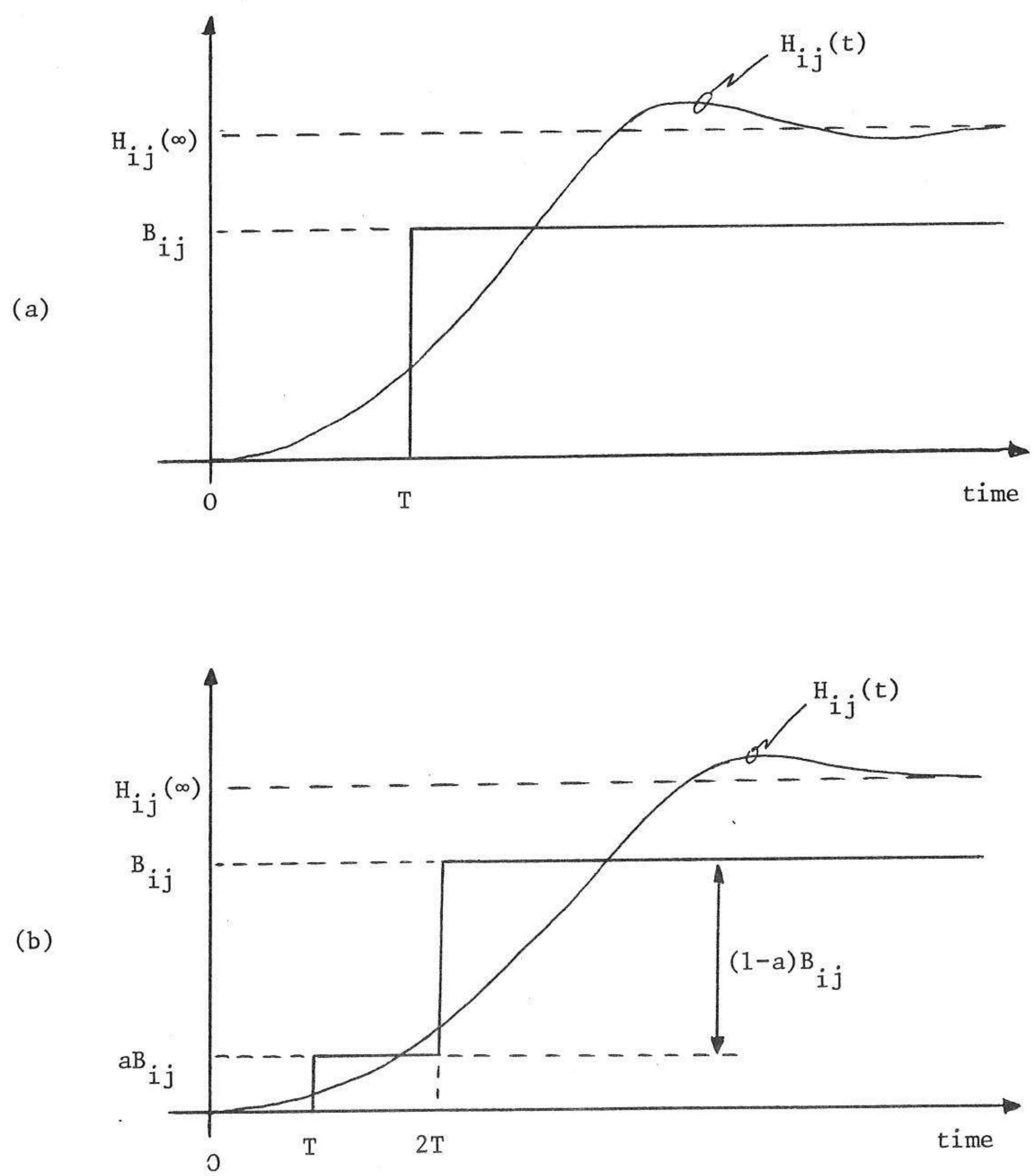

Fig. 1. First and Second Order Deadbeat Plant Models

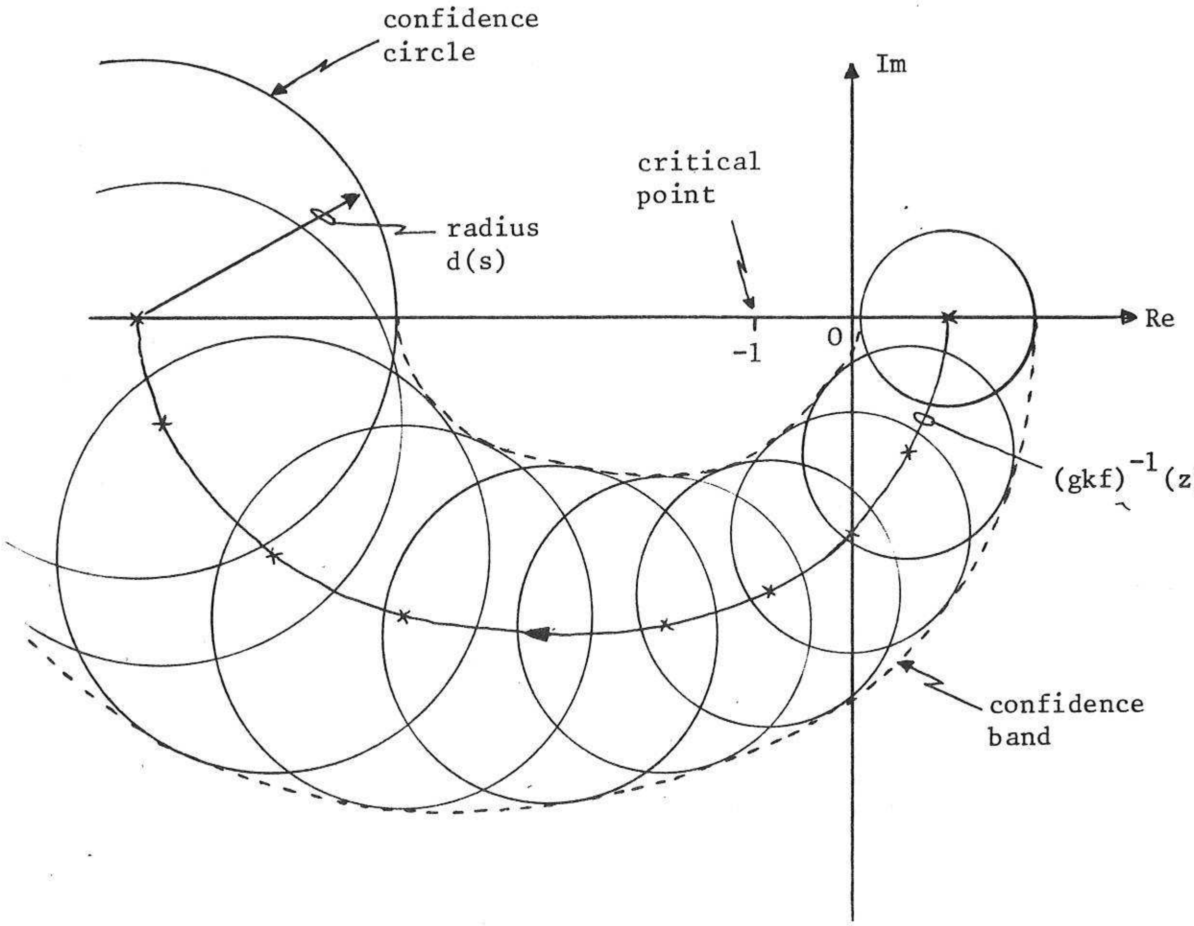

Fig. 2. Stability check using confidence bands