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SIMPLE MODEL ORDER REDUCTION BY TRUNCATION WITH PERFORMANCE VERIFICATION BY FAST DIGITAL SIMULATION

by

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ABSTRACT

The availability of fast interactive digital simulation to test the accuracy of low order models has greatly increased the viability of trying simple truncation methods of model order reduction. The truncation technique is successfully applied in reducing a tenth order analytical model of an electrode position controller to third order.

Introduction

The problem of finding reduced order models of high order systems is an important one for practicing engineers, since they are a prerequisite of most modern control techniques. Many solutions to the model order reduction problem have been proposed during the past two decades, and in general these can be placed into one of two categories. The first category embraces procedures for simplifying analytically-derived, high order systems models. Other methods, involving fitting low order models directly to time or frequency response data, belong to the second category. This paper resurrects the very simple pole-truncation method of reduction[1,7], which rightfully falls into the first category of techniques. It is argued that the ready availability of digital simulation languages to test the time/frequency response of progressively lower order models greatly enhances the argument for trying this method in many systems.

The techniques promoted for simplifying analytically-derived high order models follow several paths. A common approach for systems expressed in vector-matrix form is to derive a reduced coefficient and modified forcing function matrix which retains the dominant system eigenvalues, as first proposed by Davison[4]. Other approaches in this category make use of Pade approximation techniques such as continued fraction expansion[3], Routh approximation[6], impulse response time moment matching[2], or mixed time/frequency domain matching[9].

A mandatory requirement for this first category of techniques is a realisable analytical system model and where this is not possible, a technique from the second category involving fitting a model directly to system response data[5,8] must be used. Such methods are also applied to data generated by simulation of a high order analytical model (if this exists) which overcomes noise problems often associated with real data.

First considerations suggest that the reduction methods mentioned so far ought, by nature of their sophistication, to produce better low order models than simple pole-truncation techniques. However, many problems in practical application have been discussed in the literature. These include steady-state errors in reduced models, difficulties of matrix inversion and eigenvalue/eigenvector computation and the possibility of Pade type approaches giving unstable reduced models of stable systems. Whilst ways of overcoming such problems have been proposed, the resulting algorithms are of ever increasing complexity.

Fast, interactive, digital simulation languages now readily provide a means of testing the transient and steady-state performance of reduced models. This has greatly increased the viability of trying truncation methods of order reduction, where successively lower order models are produced by neglecting progressively larger system time constants until the minimum order model with satisfactory performance is achieved. The salient point of this argument is that the percentage of situations where it is successful is high enough to justify trying a pole-truncation method of reduction, bearing in mind its speed and simplicity, and particularly because more sophisticated techniques are not guaranteed to give better results. This is illustrated here by the successful use of the method to reduce a tenth order analytical model of an electric arc furnace electrode position controller to third order.

The Electrode Position Controller

The electric arc furnace consists of a refractory lined metal vessel in which steel scrap is melted through the heat generated by electric arcs striking between the scrap and three graphite electrodes suspended above it. Maximum electric power to heat conversion occurs for a particular arc length, which must be maintained by the controller. The arc length is subject to random step charges, caused by movements of the steel scrap, which must be corrected as rapidly as possible to maximise power utilization efficiency. Control action has to be constrained to prevent overshoot of required electrode position exceeding a level where electrodes break through hitting the scrap.

Low Order Controller Model by Simple Reduction and Simulation

Table 1 shows the system equations of the tenth order analytical model. The truncation reduction method proposed consists of neglecting progressively larger system time constants and testing the characteristics of the reduced model step response at each stage by simulation. Table 2 shows the characteristics of the step response of the various lower order models produced by pole truncation, and allows the lowest order model with satisfactory performance to be readily chosen. Figure 1 illustrates graphically the step response of some of these models. The most important aspect of the controller is the response time, which is accurately maintained by all the low order models right down to order three. Hence the truncation method has successfully produced a third order reduced model of the tenth order electrode control system. As the entire procedures of reduced model derivations and simulations was completed within a time of thirty minutes, the speed of this technique is very attractive.

Conclusions

The value of interactive digital simulation languages in testing the performance of reduced order models produced by a truncation method has been shown. This is a very fast reduction technique which can often produce satisfactory low order models, as has been illustrated by its application to a high order model of an electric arc furnace electrode position control system.

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TABLE 1

Equations of analytical model of electrode control system

$$x_1 = \frac{1.689\,(d - x_{10})}{(1 + 0.013s)} \quad ; \quad x_2 = \frac{6.5(0.5x_1 - x_4)}{(1 + 0.056s)}$$

$$x_3 = \frac{57.96\,x_2}{(1 + 0.27s)} \quad ; \quad x_4 = \frac{0.004806(x_3 - 12x_5)}{(1 + 0.0015s)}$$

$$x_5 = \frac{0.6892(0.0646\,x_3 - 0.7752\,x_5 - 0.22\,x_7)}{(1 + 0.021s)}$$

$$x_6 = \frac{2.34\,x_5}{(1 + 0.052s)} \quad ; \quad x_7 = \frac{11.1\,x_6}{s}$$

$$x_8 = \frac{0.0748\,x_7}{s} \quad ; \quad x_{10} = \frac{341}{s^2 + 7.39s + 341}$$

where s is the Laplace transform variable

d represents steel scrap position

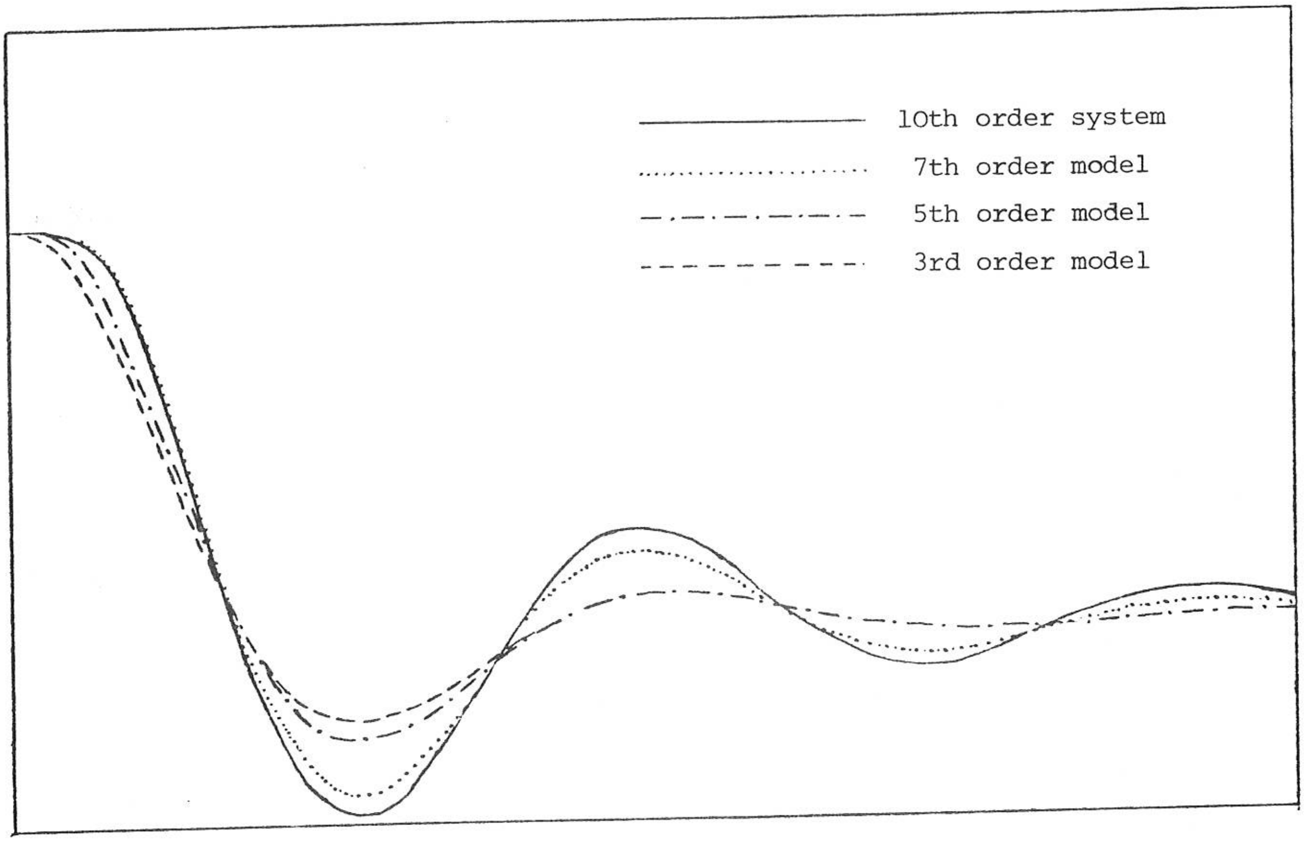
x_{10} represents electrode tip position

and $x_1 \ldots\, x_8$ represent other physical system variables.

TABLE 2

Step response of original and reduced order models

Model Order	Time Constant Omitted	Response Time(s)	Overshoot (%)
10		0.88	48.5
9	0.0015	0.88	48.5
8	0.013	0.87	45.7
7	0.021	0.87	42.1
5	0.052,0.056	0.89	27.9
3	second order t.f.	0.90	24.0



STEP RESPONSE OF ORIGINAL 10TH ORDER SYSTEM AND REDUCED ORDER MODELS

FIG 1