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SIMPLIFIED TECHNIQUES OF MODEL ORDER REDUCTION WITH PERFORMANCE VERIFICATION BY FAST DIGITAL SIMULATION

BY

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ABSTRACT

The availability of fast interactive digital simulation to test the accuracy of low order models has greatly increased the viability of trying simple methods of model order reduction. This paper tests the performance of both a pole truncation technique and a method of concatenating poles in reducing a tenth order analytical model of an electrode position controller to third order.

1. Introduction

The problem of finding reduced order models of high order systems is an important one for practical engineers, since they are a pre-requisite of most modern control techniques. A large amount of work has been published on techniques for producing an approximate reduced order model of a system which describes the static and dynamic behaviour of the system within defined bounds of accuracy. Analysis to measure the performance of these techniques in order to decide which method is best is generally inconclusive[1,2], and the usual recommendation is to choose a technique which seems suitable for the particular circumstances prevailing.

The majority of the solutions to the model order reduction problem published in the last two decades can be placed into one of two categories. Procedures involving simplification of an analytically derived, high order system model fall into the first category whereas the second category encompasses techniques in which low order models are fitted directly to time or frequency response data.

The category of techniques promoted for simplifying analytically-derived, high order models follow several paths. A common approach for systems expressed in vector-matrix form is to derive a reduced coefficient matrix and modified forcing function matrix which retains the dominant system eigenvalues, as first proposed by Davison[3]. Other approaches in this category make use of Pade approximation techniques such as continued fraction expansion[4], Routh approximation[5], impulse response time moment matching[6], or mixed time/frequency domain matching[7]. A mandatory requirement for this class of techniques is a realisable analytical system model.

Techniques from the second reduction category are used wherever an analytical system model cannot be derived. Many versious of this method, differing in the model-fitting algorithm used, have been published[8,9,10].

This category of techniques is also often used even when an analytical system model is available. In applying these methods to data generated by simulation of a high order analytical model, the noise problems often associated with real data are avoided.

This paper investigates two alternative, simple model order reduction techniques and argues that the ready availability of digital simulation languages to test the time/frequency response characteristics of progressively lower order models greatly enhances the argument for trying this method in many systems. The first of the methods suggested is a resurrection of the simple pole-truncation procedure[11,12] whereas the second method proposed reduces the number of poles by a pole-concatenation technique.

First considerations suggest that reduction methods from the first two categories mentioned ought, by nature of their sophistication, to produce better low order models than the simple techniques proposed in this paper. However, many problems in practical application of these methods have been discussed in the literature. These include steady-state errors in reduced models, difficulties of matrix inversion and eigenvalue/eigenvector computation, and the possibility of Pade type approaches giving unstable reduced models of stable systems. Whilst ways of overcoming such problems have been proposed, the resulting algorithms are of ever increasing complexity.

Fast, interactive, digital simulation languages now readily provide a means of testing the transient and steady-state performance of reduced models. This has greatly increased the viability of trying pole-truncation and pole-concatenation methods of order reduction, where successively lower order models are produced until the minimum order model with satisfactory performance is achieved. The salient point of this argument is that the percentage of situations where such methods are successful is high enough to justify trying these methods, bearing in mind their speed and simplicity, and particularly because more sophisticated techniques are not guaranteed to give better results. This is illustrated here by the evaluation of these two simple methods in reducing a tenth order analytical model of an electric arc furnace electrode position controller to third order.

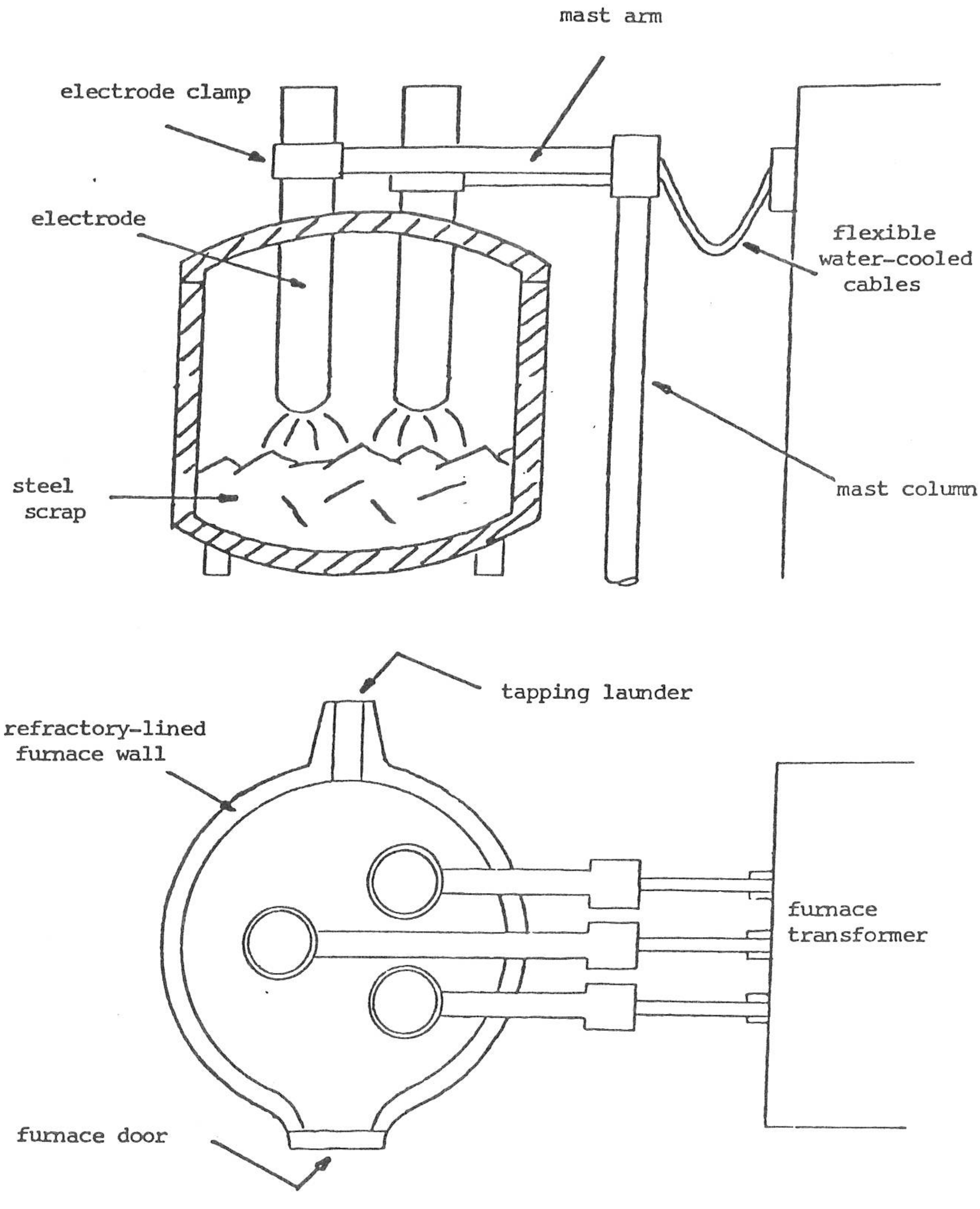
2. The Electrode Controller

The electric arc furnace produces high quality steels from a raw material of steel scrap. The volume of production is such that even very small improvements in furnace efficiency can mean large savings in steel production costs.

Fig. 1 shows a schematic diagram of a typical furnace. Steel scrap contained in a refractory-lined metal vessel is melted by the heat generated in electric arcs which strike between the scrap and three graphite electrodes suspended above the scrap. The electrodes are held in clamps at the ends of supporting mast arms, and power is supplied to them via water-cooled cables from a star-delta connected, adjustable voltage tap transformer located at the side of the furnace.

The maximum electrical power to heat conversion occurs for a particular length of electric arc, and any deviation from this optimum length impairs the power utilisation efficiency. However, the steel scrap surface is irregular by nature of the scrap and, as parts of it melt, the scrap moves about, thus changing the contours of the surface. Such disturbances in the arc length are of varying magnitude and occur continually at random points in time.

The electrode controller is responsible for moving the electrode in response to scrap movements, in order to maintain the arc length at its set-point, and needs to be fast acting to fulfil this role properly and maximise the electrical power utilisation efficiency. However, the



CONSTRUCTION OF ARC FURNACE

(Front view and plan)

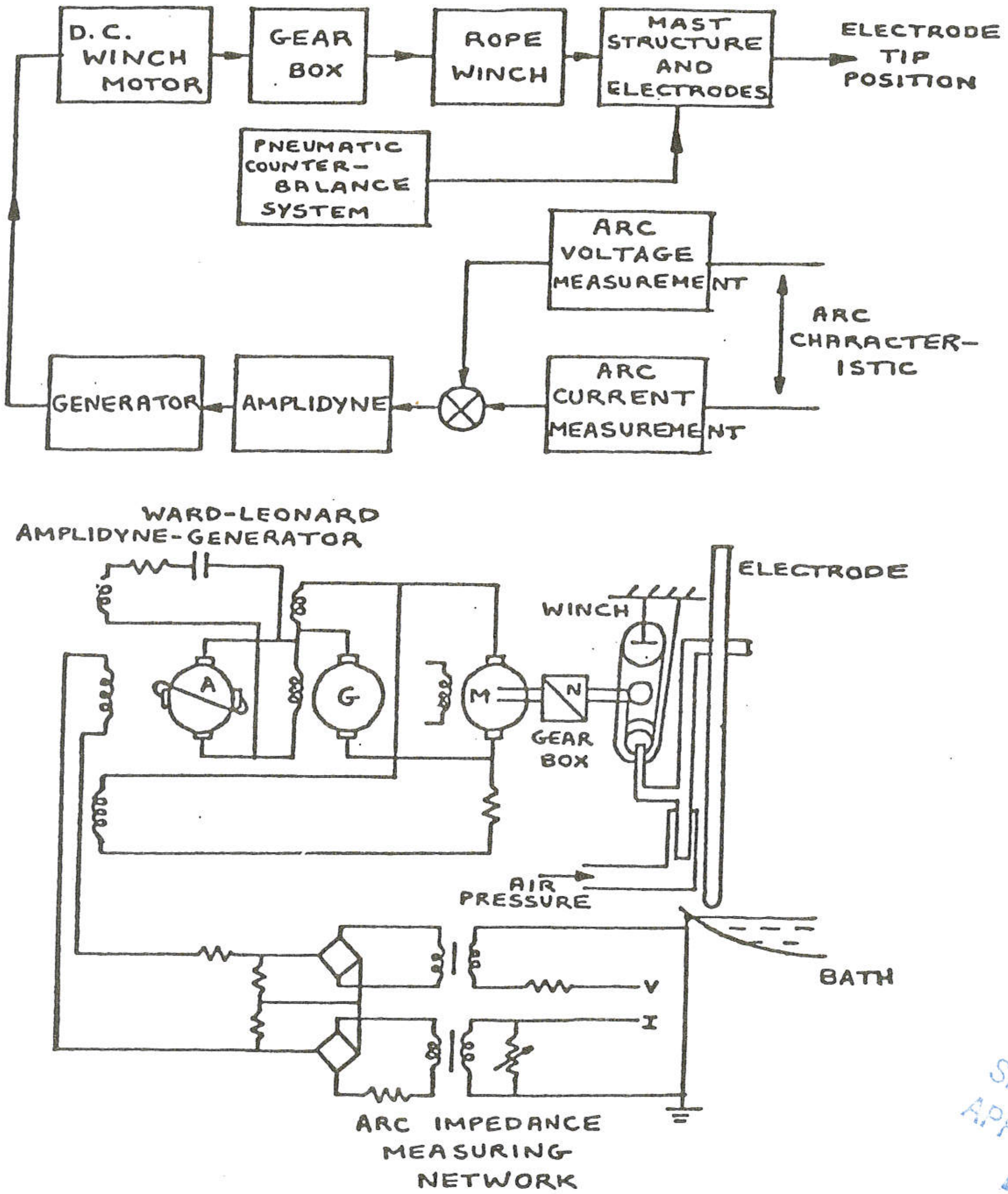
FIG 1

electrode controller performance also has an important effect on electrode and refractory wear costs, which are greatly increased if the controller allows the electrode to overshoot its desired position too far in response to a disturbance. Fast response and small overshoot are incompatible in this case, where the electrode and mast system form as oscillatory, second-order, cantilever-type structure. The best response is usually determined by extensive plant trials and operational experience.

Fig. 2 shows a schematic diagram of a typical form of electrode control system. This particular system, including a Ward-Leonard set, was chosen because the information required for modelling purposes was readily available from previous work, and it well serves the purpose of illustrating the model reduction techniques being discussed. It is not intended to imply that this is in any way 'the state of the art' in electrode controllers, and indeed most modern systems are now based on power semi-conductors[13].

In this system, an arc impedance error signal proportional to error in arc length is amplified by a Ward-Leonard amplidyne-generator set and causes a d.c. motor driving a pneumatically-counterbalanced mast to move the electrode up or down. There are normally three identical controllers, each driving one of the three electrodes in the furnace. The principle of using arc impedance as an error signal means that there is no interaction between the controllers, which can therefore be analysed in isolation[1,13].

Fig. 3 shows how the system can be represented as a sequence of blocks which can be described by a set of differential equations and state variables in the time domain. The system gain and time constants have been obtained partly from the manufacturer's specifications and partly by practical measurement in the case of components whose values are subject to change through adjustment or aging effects. The control actuator forms a seventh order open-loop system which is closed by the dynamics of the electrode carrying mast structure and the electric arc characteristic, which together



An indirect haul (counterbalanced) d.c. motor driven electrode position controller

FIG 2

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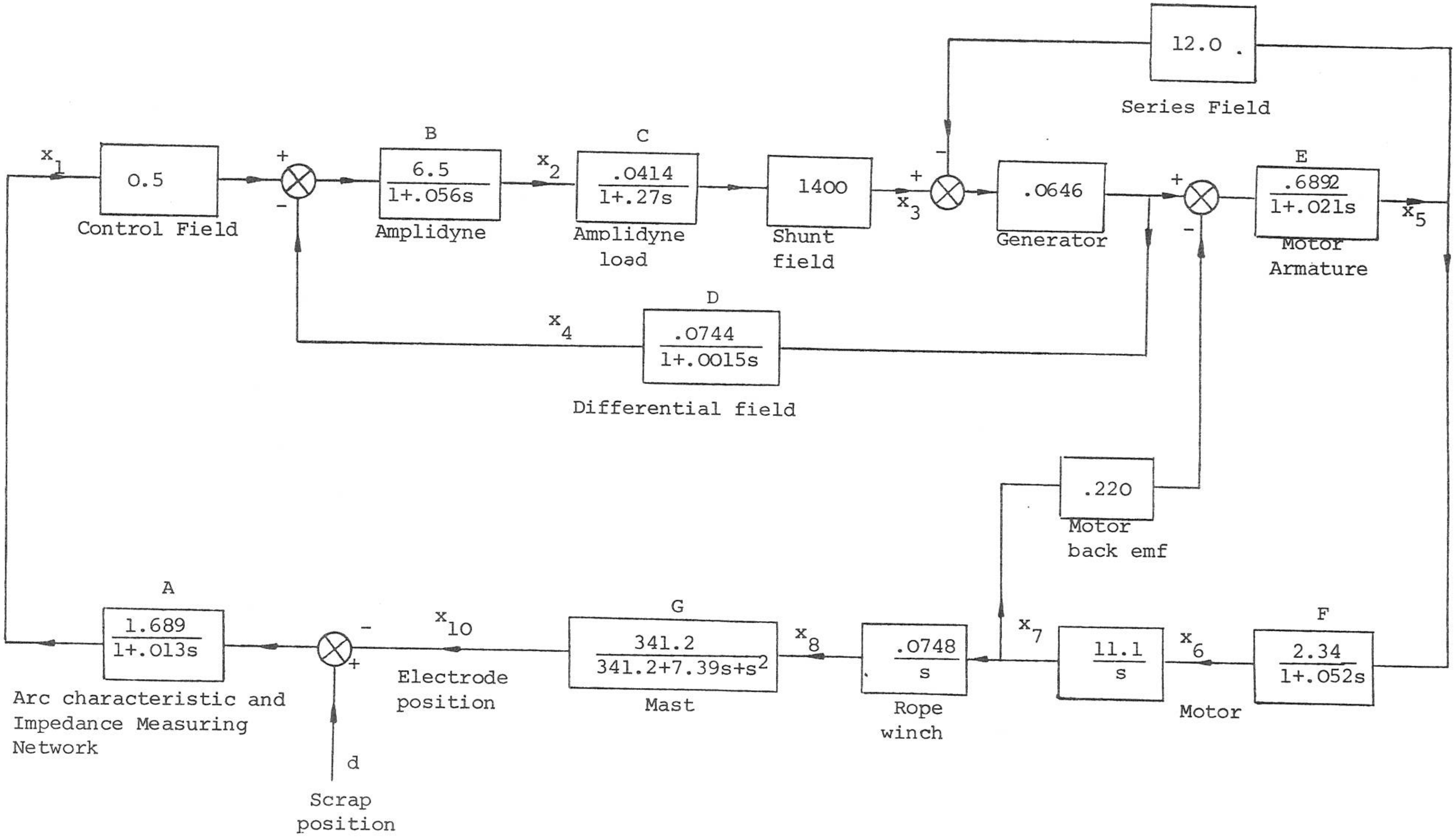


MODEL OF ELECTRODE POSITION CONTROL SYSTEM

FIG 3

can be considered as forming the controlled process. The amplidyne generator and electric arc all have non-linear characteristics. However, these characteristics are sufficiently linear over the typical operating bounds of the controller to justify the linearised approximations used to derive the following system equations.

Arc characteristics:

$$x_1 = 130.0(d - x_{10}) - 76.9x_1$$

Control actuator:

$$\dot{x}_2 = 58.0x_1 - 17.9x_2 - 116x_4$$

$$\dot{x}_3 = 214.7x_2 - 3.7x_3$$

$$\dot{x}_4 = 3.22x_3 - 671x_4 - 38.7x_5$$

$$\dot{x}_5 = 2.12x_3 - 73x_5 - 7.22x_7$$

$$\dot{x}_6 = 44.8x_5 - 19.12x_6$$

$$\dot{x}_7 = 11.1x_6$$

$$\dot{x}_8 = 0.0748x_7$$

Electrode-carrying mast:

$$\dot{x}_9 = 341.0(x_8 - x_{10}) - 7.39x_9$$

$$\dot{x}_{10} = x_9$$

These equations can be readily expressed in the general vector matrix form for a system of order n:

$$\dot{x}_n(t) = A_n x_n(t) + B_n u(t) \quad ; \quad x_n(o) = 0$$

$$y(t) = C_n x_n(t)$$

where in this case $n = 10$

3. Model Order Reduction

For a system expressed in the above vector matrix form, the procedure of model order reduction consists of finding a system of order $r (r < n)$ described by:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t) \quad ; \quad x_r(o) = 0$$

$$y(t) = C_r x_r(t)$$

and that, for a given act of inputs, the original nth order system and reduced rth order system are similar in the important aspects of their characteristics. In the case of the electrode position controller, the speed of response and magnitude of position overshoot aspects of the step response are the important performance criteria[14].

3.1 Order reduction by pole-truncation

The truncation reduction method proposed consists of neglecting progressively larger system time constants and testing the characteristics of the reduced model step response by simulation at each stage. The general transfer function $K_1/(1+T_1 s)$ of a truncated pole is thereby modified to a simple gain term K_1. Details of the simulation language used in this procedure are given in Appendix 1.

Table 1 shows the characteristics of the step response of the various low order models produced by pole-truncation, and allows the lowest order model with satisfactory performance to be readily chosen. Some of these responses are illustrated graphically in Fig. 4. This shows that the most important aspect of the controller, the response time, is accurately maintained by all low order models right down to order three. The lowest order models produced in this fashion differ in their overshoot characteristics from those of the original system, but this is not important in many applications of a low order system model, and in such cases, the pole-truncation technique therefore produces an entirely adequate low order model. The speed of the technique is extremely attractive, the results and simulations presented here having been completed within a time of thirty minutes.

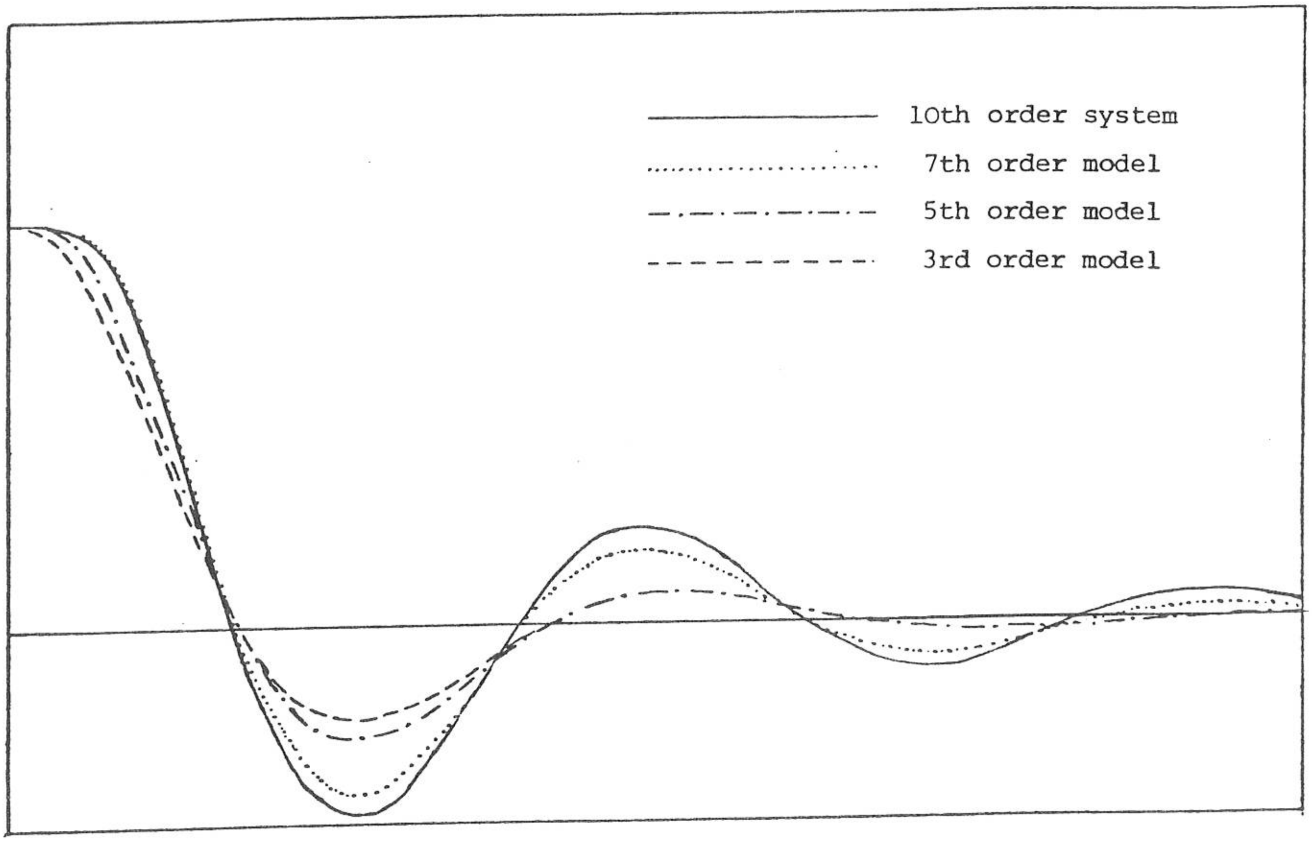
3.2 Order reduction by pole-concatenation

A simple extension to the pole-truncation technique is to combine the time constants associated with the discarded fast modes of the system with

TABLE 1

Step response of original and reduced order models produced by pole-truncation

Model Order	Time Constant Omitted	Response Time(s)	Overshoot (%)
10		0.88	48.5
9	0.0015	0.88	48.5
8	0.013	0.87	45.7
7	0.021	0.87	42.1
5	0.052,0.056	0.89	27.9
3	second order t.f.	0.90	24.0



STEP RESPONSES OF ORIGINAL 10TH ORDER SYSTEM AND REDUCED ORDER MODELS PRODUCED BY POLE-TRUNCATION

FIG 4

the time constants of slower modes. This technique of pole concatenation is a similar but much more approximate procedure to plane-projection methods[3] in which certain system modes are discarded and the retained modes are modified to take account of those discarded. Pole concatenation extends pole-truncation by modifying retained poles to compensate for truncated poles. Mathematically, this is a very simple procedure whereby the time constant of a modified transfer function is the sum of its original value and that of the discarded pole. The transfer functions of discarded modes become simple gain terms. The approximation of this approach compared with traditional, mathematically rigorous reduction methods is justified providing quick facilities to test the reduced order model by simulation are available.

For two transfer functions $\frac{K_1}{1+T_1 s}$ and $\frac{K_2}{1+T_2 s}$, where $T_2 \ll T_1$, pole-concatenation yields the two transfer functions

$$\frac{K_1}{1+(T_1+T_2)s} \quad \text{and } K_2.$$

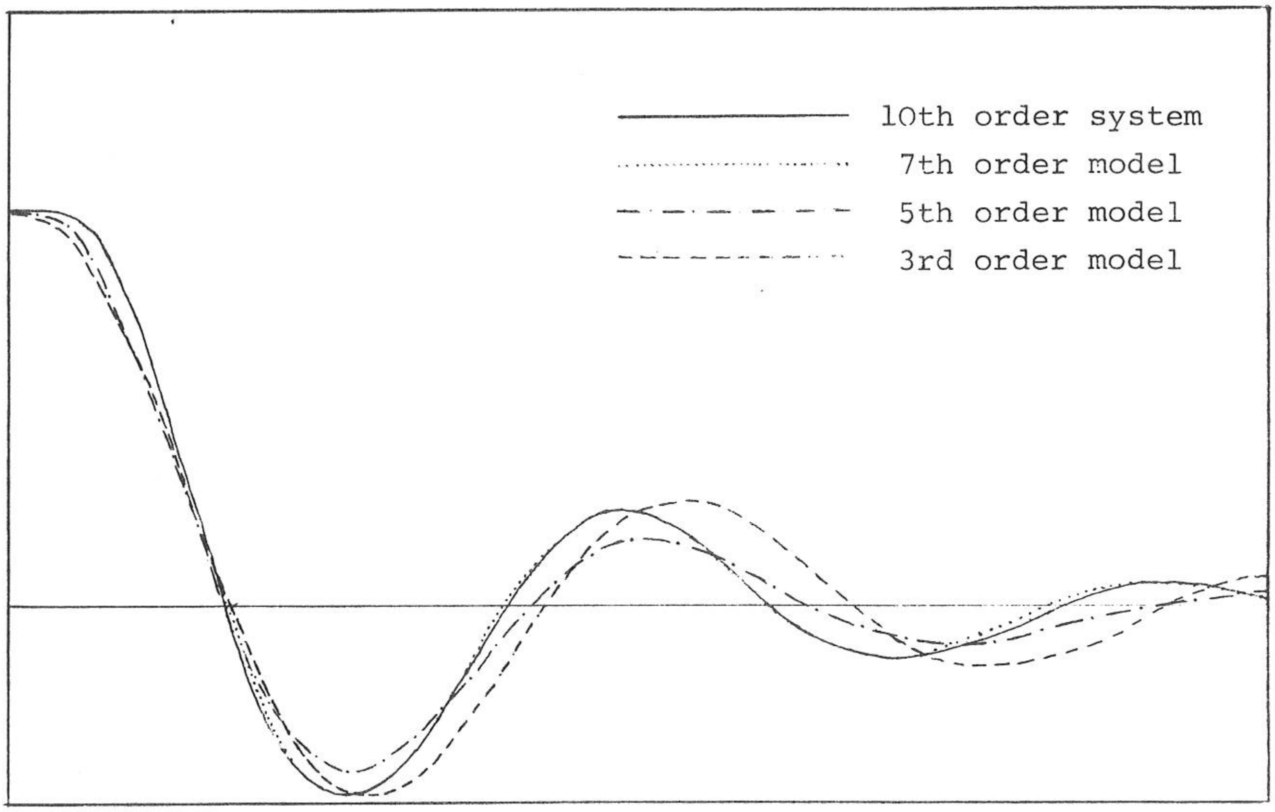
Table 2 shows the pole-truncation and pole-concatenation operations executed to derive various lower order models, and the step response characteristics of these models. These step responses are also shown graphically in Figure 5. In table 2, transfer functions are identified by letters A, B etc. as marked on Figure 3, and modified transfer functions are denoted by B' etc.

The final stage of reduction from order 5 to order 3 involves the approximation of the second order transfer function describing the dynamics of the electrode-carrying mast, by truncating terms in s^2 in the transfer function.

The transfer function $\frac{341.2}{s^2+7.39s+341.2}$ thus becomes $\frac{341.2}{7.39s + 341.2}$ which simplifies to $\frac{1}{1+0.2166s}$. This time constant of 0.2166 seconds can then be concatenated with that of the modified amplidyne-load time constant.

Operation	Modified Transfer-Function	Model Order	Response Time(s)	Overshoot (%)
		10	0.88	48.5
Truncate D	$D' = .0744$	9	0.88	48.5
Concatenate A and B	$B' = \frac{6.5}{1+.069s}$	8	0.88	47.6
Concatenate E and F	$F' = \frac{2.34}{1+.073s}$	7	0.88	48.4
Concatenate B', C and F'	$C' = \frac{.0414}{1+.412s}$	5	0.88	40.2
Approximate G	$G' = \frac{1}{1+.2166s}$			
Concatenate G' and C'	$C'' = \frac{.0414}{1+.6286s}$	3	0.89	48.8

Table 2 Step Response of original and reduced order models produced by pole-concatenation



STEP RESPONSES OF ORIGINAL 10TH ORDER SYSTEM AND REDUCED ORDER MODELS PRODUCED BY POLE-CONCATENATION

FIG 5

For puposes of clarification, the block diagram of the final third order model is shown in Figure 6.

The performance of the reduced order models produced by pole-truncation and pole-concatenation are compared in Table 3. This clearly shows the superiority of the pole-concatenation approach.

4. Conclusions

Controller performance can be improved in many processes by the implementation of digital computer based model reference and optimal control algorithms. However, in most processes with relatively short time constants, computational speed restrictions mean that control algorithms can only be implemented if a low order system model is available. Most published model reduction techniques however are mathematically complicated, and success is not guaranteed. A further difficulty with many order-reduction algorithms is that the states of the reduced system $x_1 \ldots x_r$ do not relate to the original states $x_1 \ldots x_n$. Each state $x_1 \ldots x_n$ in the system has fixed bounds corresponding to physical properties of the real process component represented. Where the states $x_1 \ldots x_r$ do not correspond directly with $x_1 \ldots x_n$, it is difficult to translate these physical constraints into the reduced order model.

This paper has introduced two very simple reduction techniques and argues that, in view of the relative ease by which reduced models can be tested by simulation, they are successful in a high enough proportion of cases to justify trying such methods as a first approach. This is supported by the successful application of the methods to produce low order models of a tenth order electric arc furnace electrode position controller.

5. Acknowledgements

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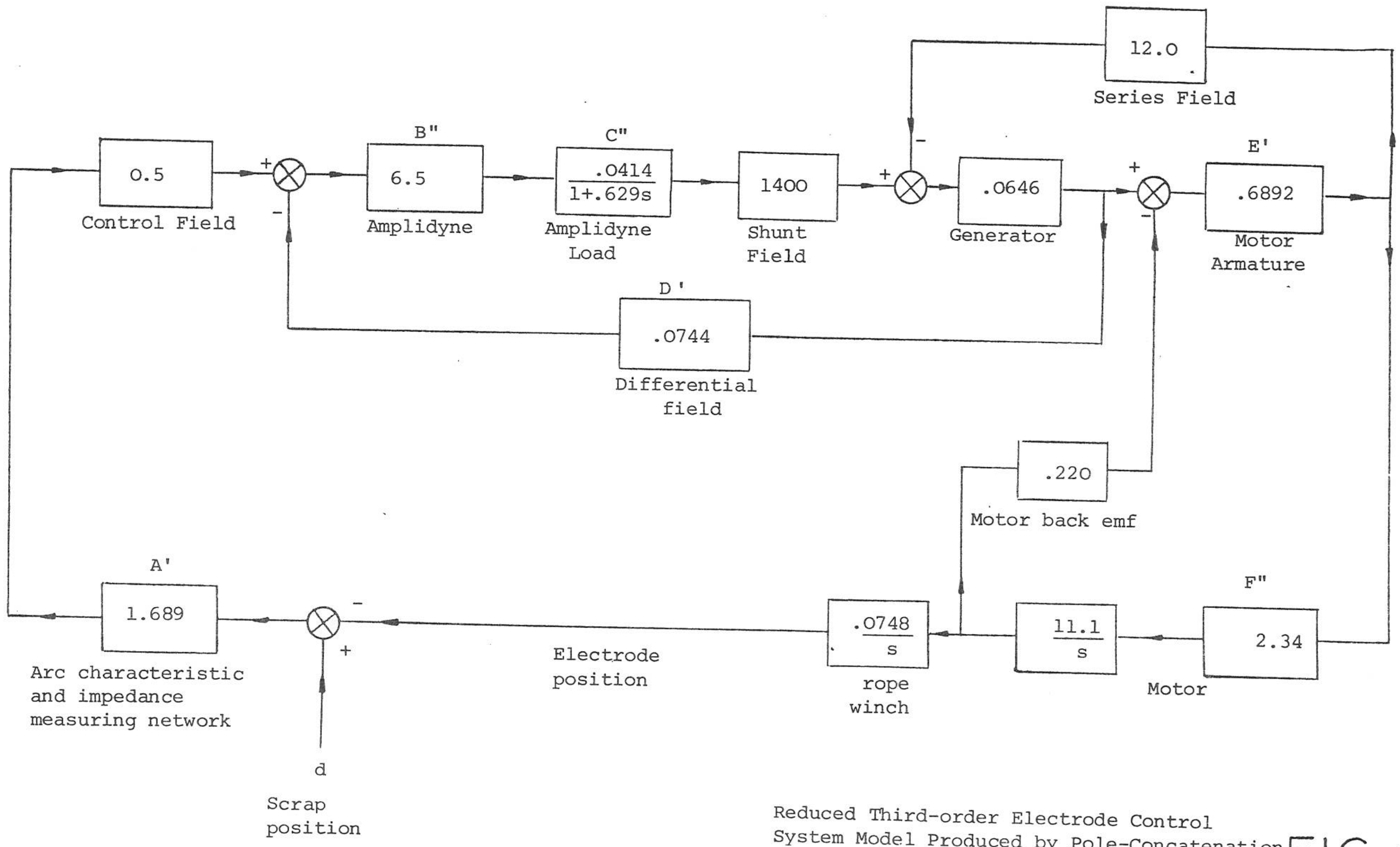

Reduced Third-order Electrode Control System Model Produced by Pole-Concatenation FIG 6

TABLE 3

Model Order	Pole-truncation		Pole-concatenation	
	Response Time (s)	Overshoot (%)	Response Time (s)	Overshoot (%)
10 (Original)	0.88	48.5	0.88	48.5
9	0.88	48.5	0.88	48.5
8	0.87	45.7	0.88	47.6
7	0.87	42.1	0.88	48.4
5	0.89	27.9	0.88	40.2
3	0.90	24.0	0.88	48.8

Comparison of step response of models produced by pole-truncation and by pole-concatenation

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APPENDIX 1

The PSI Digital Simulation Language

PSI is a block-structured interactive simulation language for studying the behaviour of dynamic, continuous and discrete systems. Although used in this example purely for simulating linear systems, PSI has facilities for simulating a variety of non-linear system components including deadspaces, saturating elements, hysteresis characteristics, time delays, logical functions and special non-linear functions defined in terms of x-y arrays. Both uniformly and normally distributed noise blocks are also available. In all, 43 different block types are available and simulation can be performed using any one of six fixed and variable step length numerical integration algorithms.

The system to be simulated is defined using a command language, and output in an appropriate format can be directed either to a graphics terminal or a lineprinter. The system structure is described in the block definition phase, where the block representing each system component is defined in terms of a unique block name, a block type code, and the block inputs. Parameters appropriate to each block are specified in the parameter definition phase. Both the system structure and parameters can be readily changed at any time be re-entering the block or parameter definition phases. When any such changes are made, it is only necessary to specify the actual blocks and parameters altered, rather than reentering the whole system description. Thus it is possible to make a large number of changes to a system model within a short period of time and observe the effect on system response for each change.