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SEQUENTIAL DESIGN OF LINEAR MULTIVARIABLE SYSTEMS RETAINING FULL OUTPUT FEEDBACK

by

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ABSTRACT

The sequential method for multivariable control systems design is reformulated to include full output feedback at each stage in the design process. The approach is shown to ease some of the nonminimum phase properties of the original method.

A recent method for multivariable feedback design[1,2] is based upon the idea of sequentially closing the feedback loops around a given mxm plant $G_p(s)$. As the ith loop is closed a transfer function $k_i(s)$ and an mxm precompensator matrix[1]

$$G_c^{\,i}(s) = \begin{bmatrix} I_{i-1} & 0 \\ 0 & X_i \end{bmatrix} \tag{1}$$

must be chosen such that the controller matrix $G_c^{\,1}(s)\ldots G_c^{\,i}(s)\mathrm{diag}\{k_1(s),\ldots,k_i(s),0,\ldots,0\}$ produces acceptable feedback stability and interaction properties. Physically, in the ith design stage, the controller matrix produces control signals $u_1(s),\ldots,u_m(s)$ based on the feedback errors $e_1(s),e_2(s),\ldots,e_i(s)$ only ie for $i<m$, the ith stage is a situation of partial output feedback.

An alternative approach described in this letter is to sequentially introduce the effect of each controller $u_1(s),u_2(s),\ldots,u_m(s)$, whilst, at each stage, retaining full output feedback ie at the ith stage of the design process, full output feedback information is used to generate control signals $u_1(s),u_2(s),\ldots,u_i(s)$ only. Stability relationships analogous to those due to Mayne[1] are derived and an example used to illustrate how the modification introduces extra design flexibility which may be used, for example, to eliminate any non-minimum phase difficulties[3,4] arising in the analysis.

Let $G_c(s)$ be a unit matrix and consider the general case of $m\mathrm{x}\ell$ $G_p(s)$ and $\ell\mathrm{x}m$ $K(s)$ (assumed, in general, to be full). Define the following $m\mathrm{x}\ell$ and $\ell\mathrm{x}m$ matrices, $1\leqslant i\leqslant \ell$,

$$G_p^{\,i}(s) = \sum_{j=1}^{i} g_j(s)e_j^{\,T} \tag{2}$$

$$K^i(s) = \sum_{j=1}^{i} e_j k_j(s) \tag{3}$$

where $\{e_j\}$ is the natural basis in R^ℓ and $\{g_k(s)\}$, $\{k_j(s)\}$ are the columns and rows of $G_p(s)$, $K(s)$ respectively. Physically $G_p{}^i(s)$ and $K^i(s)$ describe the plant and controller dynamics with full output feedback but with the controls $u_{i+1}(s),\ldots,u_\ell(s)$ suppressed. Also let $T_o(s) = I_m$ and, $1 \leqslant i \leqslant m$,

$$T_i(s) \quad = \quad I_m + G_p{}^i(s)K^i(s) \qquad (4)$$

Noting that $G_p{}^{i-1}(s)e_i = 0$, $e_i{}^T K^{i-1}(s) = 0$, then, $1 \leqslant i \leqslant m$,

$$\begin{aligned} |T_i(s)| &= |T_{i-1}(s) + g_i(s)k_i(s)| \\ &= |T_{i-1}(s)|.|I_m + T_{i-1}{}^{-1}(s)g_i(s)k_i(s)| \\ &= |T_{i-1}(s)|.\{1 + k_i(s)T_{i-1}{}^{-1}(s)g_i(s)\} \qquad (5) \end{aligned}$$

which is analogous to Mayne's result(1). The basic design algorithm(1) now becomes

(i) Set $i = 1$ and $T_o(s) = I_m$.

(ii) Choose the m components of the row vector $k_i(s)$ by examining the scalar term $1 + k_i(s)T_{i-1}{}^{-1}(s)g_i(s)$.

(iii) If $i = \ell$ stop. Otherwise compute $T_i{}^{-1}(s)$, set $i = i+1$ and go to (ii).

Quite obviously this procedure leads to a controller design $K(s) = K^\ell(s)$ and a feedback configuration which, for all i, remains stable under simultaneous failure of control actuators $i+1, i+2, \ldots, \ell$.

By ignoring system integrity and setting $\ell = m$, the technique can be extended by applying the above algorithm to $G_p(s)G_c(s)$, where $G_c(s)$ is an mxm compensation matrix and setting $G_c(s) = G_c{}^1(s)\ldots G_c{}^m(s)$ where the mxm matrices $G_c{}^i(s)$ take the form

$$G_c{}^i(s) \quad = \quad \begin{bmatrix} I_{i-1} & Y_i \\ 0 & X_i \end{bmatrix} \qquad (6)$$

It follows that the ith column of $G_p(s)G_c(s)$ is equal to the ith column of $G_p(s)G_c^{1}(s)\ldots G_c^{i}(s)$ and hence $G_c^{i}(s)$ need not be chosen until the ith design stage. The form of compensator given by eqn. 6 is more general than that quoted by Mayne(1) who assumed that $Y_i = 0$. This fact and the assumption of non-diagonal K(s) represent the extra degrees of freedom that this alternative formulation includes in the design process.

As an example, consider example 2 of Mayne(1)

$$(s+1)^2 G_p(s) = \begin{bmatrix} 1-s & 1/3-s \\ 2-s & 1-s \end{bmatrix} \qquad (7)$$

In a similar manner to Mayne, remove the non-minimum phase entries in column one by choosing

$$G_c^{1}(s) = \begin{vmatrix} 1 & 0 \\ -2 & 1 \end{vmatrix} \qquad (8)$$

so that

$$(s+1)^2 G_p(s) G_c^{1}(s) = \begin{bmatrix} 1/3+s & 1/3-s \\ s & 1-s \end{bmatrix} \qquad (9)$$

ie $$k_1(s)T_o^{-1}g_1(s) = \{\frac{k_{11}(s)}{3} + (k_{11}(s) + k_{12}(s))s\}/(s+1)^2 \qquad (10)$$

from which $k_{11}(s), k_{12}(s)$ can be chosen to ensure minimum phase and the required stability properties. Let $k_{12} = 0$ and $k_{11} > 0$ be constants and consider the second control loop. Following Mayne(1), let $G_c^{2}(s) = I_2$, from which the numerator of $k_2(s)T_1^{-1}(s)g_2(s)$ is given by

$$k_{21}(s)(s+1)(\frac{1}{3} - s) + k_{22}(s)(1 + \frac{k_{11}(s)}{3} - s^2)$$

$$= -(k_{22} + k_{21})s^2 - \frac{2k_{21}}{3}s + \frac{k_{21}}{3} + k_{22} + \frac{k_{11}k_{22}}{3} \qquad (11)$$

Examining this expression, it is easily seen that the new method allows right-half-plane zeros to be eliminated by choosing $k_{21} = -3k_{22}$ when

$$k_2(s)T_1^{-1}(s)g_2(s) = \frac{k_{22}(s)\{2s^2 + 2s + \frac{k_{11}}{3}\}}{(s+1)\{(s+1)^2 + k_{11}(\frac{1}{3}+s)\}} \qquad (12)$$

which is stable for arbitrarily high feedback gains k_{22}. The final control design is

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} k_{11} & 0 \\ -3k_{22} & k_{22} \end{bmatrix} \qquad (13)$$

References

(1) MAYNE, D.Q.: 'The design of linear multivariable systems', Automatica, 1973, 9, pp.201-207.

(2) OWENS, D.H.: 'Dyadic modification to the sequential method for multivariable-control-systems design', Department of Control Engineering, University of Sheffield, Research Report No.18, December 1973.

(3) MEE, D.H. and HANNAH, R.S.: 'Nonminimum phase difficulties in multivariable-control-system design', Electron. Lett., 1973, 9, pp.469-470.

(4) MAYNE, D.Q.: 'Comment on nonminimum phase difficulties in multivariable-control-system design', Electron. Lett., 1973, 9, pp.571-572.