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THE MODELLING OF SEMIFLEXIBLE CONVEYOR STRUCTURES

FOR COAL-FACE STEERING INVESTIGATIONS

PART 2: SPATIALLY CONTINUOUS MODELS

by

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One of a pair of companion papers submitted to the
Institution of Mechanical Engineers for publication
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# The modelling of semiflexible conveyor structures for coal-face steering investigations

## PART 2 Spatially Continuous models

Edwards J.B. [*]

## 1. Introduction

Armoured flexible conveyor (a.f.c) structures used on longwall coal-faces comprise some 100 - 200 robust steel trays loosely joined end-to-end to permit vertical floor undulations to be followed, at least approximately. This allows a degree of vertical steering to be attempted by control of the height of the floor cut by the cutter-loader machine which generally rides on the a.f.c structure. Play in the joints is also incorporated to permit the a.f.c to be snaked horizontally onto the newly cut floor after each pass of the cutter-loader and this permits also a degree of lateral steering to be attempted. The word 'attempted' rather than 'achieved' is used in the cases of both lateral and vertical steering since successful steering often proves to be extremely difficult in practice, either manually in the absence of considerable human skill, or automatically in the absence of highly intelligent computer controllers.

The vertical steering problem was clearly demonstrated in a companion paper[1] where it was shown that, contrary to expectation, rigid trays and hard limits on play, do not appear to stabilise a simple analogue steering system. In that paper it was shown that the overall multipass system dynamics may be considered to be a simple loop comprising two inter-connected subsystems $G_s(D)$, the steering system proper and $G_c$ the a.f.c model, separated by a face-length delay L as indicated in Fig. 1. In Fig. 1, h represents the a.f.c height, y the cut-floor height, $\ell$ the distance measured from, say, the L.H. face-end, n the cut-, or pass-number and $D \equiv d/d\ell$. {Preset controller reference $y_r$ and seam

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undulation $z(n,\ell)$ are omitted from Fig. 1 since, being external

disturbances, these do not affect the stability of the system when

modelled linearly: as is our intention here}.

## 1.1 Simplest linear model of the a.f.c

The effect of the vague concept of the a.f.c acting as some form

of floor-smoothing filter can be demonstrated quantitively by substi-

tution of some reasonably appropriate linear transfer-operator  for

$G_c$. After the elementary rubber-conveyor model, which merely assumes

$$h(n+1,\ell) = y(n,\ell) \tag{1}$$

so that    $G_c = 1.0$ (2)

$$(2)$$

the simplest reasonable linear model is

$$\{1-(X_cD)^2\}h(n+1,\ell) = y(n,\ell) \tag{3}$$

giving    $G_c(D) = 1/\{1-(X_cD)^2\}$ (4)

Such a model would respond to a step or impulse in the cut-floor in the

manner shown in Fig. 2 which clearly bears some resemblance to the

behaviour expected from the rigid-tray model, provided a suitable ratio

$X_c/X_p{}^*$, in the region of unity, is chosen.  Although not derived on a

rigorous physical basis, the model is at least reasonable in as much as

its response is two-sided i.e. the profile $h(n+1,\ell)$ is lifted either

side the disturbance, as would occur with a real a.f.c.  (The behaviour

of the real a.f.c would, of course, only be symmetrical, as shown in

Fig. 2, if the floor-disturbance were applied at the centre or end of

a tray).   The model transfer-operator must involve only even-powers of

D to produce the essential characteristic of two-sidedness  in its tran-

sient response, as will become clear later and, for this reason, the

transfer-operator $G_c(D)$ in equation (4) is the simplest spatially-

dynamic model that could be formulated.

* $X_p$ = tray length.

## 1.2   Predicted effect on steering system stability

To demonstrate its effect we couple conveyor model, $G_c(D)$, as specified by equation (4), to the simple steering system $G_s(D)$ developed in the previous paper, viz:

$$G_s(D) = (1+X_1 D)/\{1+X_1 D+k_h \text{ Del}(X)\} \tag{5}$$

where X is the coal-sensor delay, $X_1$ the smoothing lag of the sensor's filter and $k_h$ is the height gain (i.e. sensitivity) of the automatic controller. Del ( ) denotes the delay operation.   Thus, the gain around the multipass system loop of Fig. 1 at any frequency $\omega$ is $|G_s(j\omega)||G_c(j\omega)|$ and to avoid the unlimited buildup of energy in the system over repeated passes, at any frequency then, as previously stated,

$$|G_s(j\omega)||G_c(j\omega)| < 1.0 \text{ for all } \omega \tag{6}$$

Here, of course

$$G_s(j\omega) = (1+X_1 j\omega)/\{1+X_1 j\omega + k_h \exp(-Xj\omega)\} \tag{7}$$

and

$$G_c(j\omega) = 1/\{1+X_c^2\omega^2\} \tag{8}$$

so that, for multipass stability:

$$|G_s(j\omega)||G_c(j\omega)| = \sqrt{\left(\frac{1+X_1^2\omega^2}{\{1+X_1^2\omega^2 + k_h^2 + 2k_h(\cos\omega X - X_1\omega\sin\omega X)\}}\right)} \frac{1}{(1+X_c^2\omega^2)}$$

$$< \quad 1.0 \quad \text{ for all } \quad \omega \tag{9}$$

Fig. 3 (based on computations of $|G_s(j\omega)||G_c(j\omega)|$ calculated from the expression above) illustrates how increasing the a.f.c decrement distance $X_c$ would stabilise the system by bringing the resonance peak below unity, so satisfying condition (9).   As might be expected, $X_c$ is relatively ineffective until its value approaches delay distance X in order of magnitude.   These predictions based on frequency domain calculations are readily confirmed by transient response simulations computed   in the manner of Section 3.

### 1.3   Contrasting microscopic-and macroscopic-model predictions

Now in the preceeding paper, simulation based on a detailed, rigid-tray chain-model predicted the persistence, not the elimination of multipass instability following the introduction of a.f.c. stiffness. These contradictory forecasts based on microscopic- and macroscopic-models are therefore worrying and indicate that, despite our modelling efforts to date, a much deeper appreciation of a.f.c dynamics is needed. Since the simple macroscopic model $G_c(D)$ calculated from equation (4) was intuitively - guessed rather than physically-derived this should clearly be our first candidate for re-examination and in the following Section of the paper we develop an improved operator $G_c(D)$ based on a sound physical structure rather than behavioural fitting i.e. we model causes rather than isolated effects, and then examine predicted effects in the light of experience.

### 2.   A continuous elastic beam model

The model is based on the idealising assumptions that

(i) the side-channels of the a.f.c structure may be regarded as a pair of continuous, parallel, uncoupled elastic beams (so that deck-plate stiffness is again ignored),

(ii) the a.f.c rests on a bed of fine material (fines) overlying a solid floor and the reaction of the 'fines' is proportional to the compression of the bed

and (iii) the uncompressed bed if fines, produced by cut-floor degradation or uncleared cuttings, has a constant depth along the entire face.

Fig. 4 illustrates a beam settling to a height $h(\ell)$ above the flat datum into the bed of fines, the uncompressed depth of which is denoted by the constant $d_f$. After floor-degradation, the solid floor profile changes from the original height $y(\ell) + z(\ell)$ cut by the power-loader to $y(\ell) + z(\ell) - d_s$, where $d_s$ is the constant depth of solid material

degraded in the pushover process.  The height above datum of the

uncompressed surface of fine material is therefore $y(\ell) + z(\ell) - d_s + d_f$

$= y(\ell) + z(\ell) + d_f(r-1)/r$ since

$$d_s = d_f/r \tag{10}$$

where r is the bulk-density ratio of the solid/fine material.  Hence,

the bed compression under the weight of the beam will be $y(\ell) + z(\ell) +$

$d_f(r-1)/r - h(\ell)$.

Now if, $k_f$ is defined as the bed reaction force p.u. length p.u.

bed-compression (i.e. a parameter closely related to a conceptual

compressibility factor for the fines.), then the net upward load $F(\ell)$

p.u. length  exerted on the beam is given by

$$F(\ell) = k_f\{y(\ell) + z(\ell) + d_f(r-1)/r - h(\ell)\} - \omega \tag{11}$$

where $\omega$ is the constant weight p.u. length of the beam.  As stated in

assumption (ii), we shall regard $k_f$ as a constant.  For simplicity

furthermore we assume that

$$k_f\, d_f(r-1)/r = \omega \tag{12}$$

which implies that on a level floor, the beam would settle back to the

height of the solid floor originally produced by the power loader.  In

this way we are merely assuming that floor degradation causes no upward

biassing effect on the a.f.c.  Any such biassing caused by contravention

of equation (12) would have no destabilisng effect however, being only

an external process disturbance and not a dependent process variable.

For stability studies of this linear system therefore, assuming equation

(12) to hold is in no way restrictive.

From elementary beam theory we have that

$$E\, I\, d^4 h(\ell)/d\ell^4 = F(\ell) \tag{13}$$

where E is Young's Modulas and I  the second moment of area of the beam

about its neutral axis so that combining (11) ,. (12) and (13) we obtain,

$$E \ I \ d^4 \ h(\ell)/d\ell^4 = k_f\{y(\ell) + z(\ell) - h(\ell)\} \tag{14}$$

For multipass investigations we must of course reinstate argument

n (the pass number) and since the variable h($\ell$) in equation (14)

pertains to the a.f.c profile <u>after</u> the cut floor profile y($\ell$) + z($\ell$)

has been produced we obtain

$$E \ I \ d^4 h(n+1,\ell)/d\ell^4 = k_f\{y(n,\ell)+z(n,\ell) - h(n+1,\ell)\} \tag{15}$$

so that, dropping external disturbance z(n,$\ell$) for stability studies

as before, we deduce finally that

$$\{1+ (X_c D)^4\}h(n+1,\ell) = y(n,\ell) \tag{16}$$

where
$$X_c^4 = E \ I/k_f \tag{17}$$

or
$$G_c(D) = 1/\{1 + (X_c D)^4\} \tag{18}$$

in this case. This model is therefore simple, its order of complexity

being only one greater than that of the simplest possible two-sided

dynamic model postulated previously {equation (4)} and has the advantage

that it has been properly deduced from precisely defined premises. We

now examine its behaviour by simulation and analysis.

3. <u>Simulation method for the beam-model (and alternative analytic models)</u>

Transfer-operator $G_c$(D) defined by equation (18) may be readily

partial-fractioned into the form

$$G_c(D) = \frac{0.5(1+X_c D/\sqrt{2})}{X_c^2 D^2 + \sqrt{2}X_c D+1} + \frac{0.5(1 - X_c D/\sqrt{2})}{X_c^2 D^2 - \sqrt{2} \ X_c D + 1} \tag{19}$$

or
$$G_c(D) = G_{c1}(D) + G_{c1}(-D) \tag{20}$$

where
$$G_{c1}(D) = 0.5(1+X_c D/\sqrt{2})/\{X_c^2 D^2 + \sqrt{2} \ X_c D + 1\} \tag{21}$$

From inspection of (21) we immediately recognise $G_{c1}$(D) to be a cascade

of a simple first-order phase-advancer (the numerator) and a second-

order lag (the denominator) of undamped natural frequency, $\omega_o$, and

damping ratio, $\xi$, given by

$$\omega_o = X_c^{-1} \qquad (22)$$

and

$$\xi = 1/\sqrt{2} = 0.707 \qquad (23)$$

This is in contrast to the second-order model {equation (4)} which may be partial-fractioned into the form of equation (20) but where $G_{c1}(D)$ would be a first-order lag given by

$$G_{c1}(D) = 0.5/(1+X_c D) \qquad (24)$$

Indeed all linear models generating symmetrical two-sided impulse-responses may be decomposed in the manner of equation (20) and may be simulated in the following sequence

(a) Pass input data $y(n,\ell)$ through filter $G_{c1}(D)$ and store output data = $h_1(n,\ell)$.

(b) Reverse input data sequence to form $y(n,\ell-L)$ and pass this through $G_{c1}(D)$ to form output = $h_2(n,L-\ell)$, and store.

(c) Reverse sequence of data $h_2(n,L-\ell)$ to form $h_2(n,\ell)$ and store.

(d) Form a.f.c profile $h(n,\ell)$ by setting

$$h(n,\ell) = h_1(n,\ell) + h_2(n,\ell) \qquad (25)$$

(The data-sequence reversals in steps (b) and (c) are required to effect the reverse-time derivatives implied by argument -D in the component transfer-operator $G_{c1}(-D)$ of equation 20).

Unlike the simple model of equation (4), the beam model is slightly oscillatory in its step response as indicated by the simulation response of Fig. 5 and as would be expected analytically since $\xi < 1.0$ {equation (23)}.

4. <u>Analytical predictions of the multipass system behaviour using the beam model</u>

The analytic criterion for multipass system stability remains that previously stated in inequality (6). Retaining the same steering-system model (5) as used earlier but using equation (18) i.e. the elastic beam

model for $G_c(j\omega)$ this criterion now becomes

$$\left|G_s(j\omega)\right|\left|G_c(j\omega)\right| = \sqrt{\left(\frac{1+X_1^2\omega^2}{\{1+X_1^2\omega^2+k_h^2+2k_h(\cos\omega X - X_1\omega\sin\omega X)\}}\right)}\;\frac{1}{\{1+(X_c\omega)^4\}}$$

$$< 1.0 \text{ for all } \omega \tag{26}$$

Spectra computed from equation (26) are illustrated in Figs. 6 and 7 for $k_h = 0.5$ with $X_1 = 0$ and $0.5X$ respectively for various characteristic distances $X_c$. From these curves it is clear that stability may be achieved by increasing $X_c$ (i.e. increasing the effective beam stiffness) sufficiently. More precisely, as shown in Appendix 1, it is readily deduced that, provided

$$X_1 < X \tag{27}$$

then multipass stability may be expected with some confidence by setting

$$X_c > {}_m\sqrt{k_h/(1-k_h)}\;(X+X_1)/\pi \tag{28}$$

if $G_c(D)$ is given by

$$G_c(D) = 1/\{1+(X_cD)^m\}\;,\quad m = 2,4,5\ldots$$

so that, for the elastic beam model

$$X_c > {}_4\sqrt{k_h/1-k_h})\;(X+X_1)/\pi \tag{29}$$

The rules of thumb (28) and (29) are only approximate since they rely on the assumption that lag $X_1$ has little effect on the resonance-peak amplitude and may be lumped with delay $X$ for calculation of the resonant frequency, $\omega_r$, of system $G_s(j\omega)$. Furthermore it is assumed that the resonant frequency, $\omega_{r1}$, of the composite system $G_s(j\omega)G_c(j\omega)$ is such that

$$\omega_{r1} \simeq \omega_r = \pi/(X+X_1) \tag{30}$$

which is valid provided the resonance peak of $G_s(j\omega)$ is fairly sharp compared to the rate of cut-off of $G_c(j\omega)$. It is interesting to note that in the special, but very practical case of $k_h = 0.5$, the critical value

of $X_c$ obtained with the 4th-order a.f.c model will be the same as that for any other model-order, m, since $k_h/(1-k_h) = 1.0$ in this case.

5.  Simulation results for the elastic beam model

Multipass simulations were conducted by modelling the a.f.c in the manner described in Section 3. The results obtained are typified by those shown in Figs. 8 and 9 which illustrate the attempted recovery of the system from an initial unit-step-disturbance in the cut-floor at mid face. The system parameters were $X_1 = 0.5 X$, $k_h = 0.5$, $X_2 = 0$ in both cases, Fig. 8 shows the response with beams of insufficient stiffness to ensure stability, i.e. $X_c = 0.30X$ while Fig. 9 demonstrates the stable performance of a system satisfying condition (29) i.e. $X_c = 0.5 X$. The results are in accord with the analytical predictions and the inclusion of a small actuator lag $X_2$ is found in simulation to have little effect on the critical value of $X_c$.

6.  Discussion & Conclusions

In the preceeding companion paper it was demonstrated that an a.f.c composed of truly rigid trays freely linked and supported on a rigid undulating floor does not appear to stabilise simple analogue steering systems based on delayed roof or floor-sensor measurements. This observation is in sharp contrast to the predictions, in the present paper, of a linear analytic model based on a continous elastic beam representation of the a.f.c. This has demonstrated that provided a sufficiently high ratio of effective beam-stiffness to floor-compressibility can be achieved {see equations (17) and (29)} for given steering system parameters, then stable steering over repeated passes is attainable.

The original object of choosing an elastic a.f.c model was to allow approximate stability-and performance-predictions to be made analytically rather than by exhaustive simulation, (which is expensive in view of the

numerous steering system and a.f.c parameters$\left(k_h, k_g, X, X_1, X_2, \Delta\gamma, \Delta\alpha, X_p\right)$
and structures available for variation and the relatively large size

and complexity of the nonlinear a.f.c fitting routine.  It was hoped

that elastic yield of the beam and the powdered floor might represent

the free play in the piecewise rigid structure sufficiently closely to

allow the substitution of the simple linear model for the detailed

fitting program, at least for reducing the field of sensible parameter

change.  Because of the contraction in the predictions, however, this

naive hope would appear to be unrealisable at this point in time.  A

number of deductions of practical importance may nevertheless be made

on the basis of the work reported in these papers.

As regards reconciling the behaviour of the discrete and continuous-

linear, models, it is clear that a measure of agreement will only be

achieved when some elasticity is incorporated into the tray-by-tray

model described in the first paper.  Now when all the free play in the

joints is taken up then some elastic bending and twisting at the joints

will occur in practice despite the trays themselves remaining rigid.

It follows logically therefore that we might expect oscillations in

the cut-floor, and hence in the real a.f.c profile, to grow in an

unstable fashion, as predicted by the first paper, until the free play

is taken up and elastic yield begins.  (To model this effect the dynamic

programming must now minimise the sum of the total potential-plus-strain

energy instead of potential energy alone, the strain energy per joint

being proportional to the square of the overbending-and/or overtwisting-

angle between the trays in question.  Indeed work is now in hand to

incorporate such changes to the a.f.c fitting routine).  Once elastic

yield occurs, at sufficient oscillation amplitude, we may expect the

system to stabilise, i.e. perhaps to limit-cycle, because of the fairly

stiff nature of the angular-end-stops.  We may therefore view our discrete

rigid-tray model as a small-signal model of existing a.f.c structures

and the elastic-beam model as an approximate large-signal model which

does not reproduce the small-amplitude free-play oscillations.   An

evaluation of an equivalent E,I and $k_f$ for a given angular joint

stiffness and floor compressibility   would, of course, need to be

made and the uncertainty of parameter $k_f$ might pose problems.

Rather than being preoccupied with trying to force the models

into agreement by changing the discrete model it may, however, be

more profitable practically to adapt the a.f.c structure itself to

more closely resemble the elastic beam model by perhaps replacing

existing free joints with spring joints of suitable stiffness.   The

adjective semiflexible, applied to the conveyor would thus come to

imply elastic flexibility rather than completely free movement within

hard constraints.   The stabilising power of an elastic structure has

been convincingly demonstrated in this second paper and, with hindsight,

its effect is obvious in that such a structure will have a natural ten-

dency to straighten itself (i.e. to spring straight) whereas present

structures do not, so relieving the task of the steering system to a

large extent. The springy  conveyor would therefore be a largely self-

regulating process and it is well known  in control engineering that

such processes are the easiest to stabilise.

For some time, similarities between longwall coal-cutting systems

and machine-tools have been used to support the argument for completely

rigid a.f.c's: on the basis that a rigid machine bed-frame is a first [3]

essential for the avoidance of chatter (i.e. instability), in metal

cutting.  A rigid 200m bed-frame for a coal-face is, of course, an

impossibility and therefore recent attempts towards realising the machine

tool analogy have compromised by lengthening and strengthening individual

machine-support sections but still retaining a degree of freeplay, (so-

called articulation) at the joints between sections.  The basic principle

of construction of the original a.f.c structure has therefore not changed

with these developments. Indeed the concept of a completely rigid bed-frame along the entire-face and the requirement for vertical steering, (be it for manoeuvring, following natural seam undulations or merely regulating to a flat seam) are completely unreconcilable. The natural solution, allowing steering to take place and a semi-rigid full-face bed-frame is to adopt a continuous elastic beam type of construction for the a.f.c., i.e. to build a bridge rather than a string of pontoons.

It is interesting to also note that there is, in practice, an increasing tendency at collieries to experiment with existing a.f.c's by bolting various items of a.f.c furniture (e.g. face-side ramp plates and goaf-side cable-trays) not to each a.f.c tray individually, but across the tray-joints, thus eliminating much of the free play and creating something of a continuous structure as proposed above. Unfortunately, of course, the stiffness of such structures is not a controlled-i.e. designed-parameter and bolts tend to shear if snaking distances are kept short. Nevertheless it is reassuring to note the convergence of theoretical and practical ideas in this respect.

Finally, on the subject of elasticity, it is worth noting that machine haulage chains and the conveyor chains themselves provide the a.f.c with some self-straightening tendency if deviations are not excessive and the current changeover to rack-and pinion haulages or the loose-mounting of a.f.c's on larger support structures (mentioned earlier in this section) is clearly a step in the wrong direction as regards elasticating the a.f.c. This factor naturally points to the possibility of active tensioning of an a.f.c structure to achieve literally controlled elasticity. This is, of course, receiving serious consideration for wider application in bridge construction and may therefore provide a basis of a.f.c stabilisation also. An investigation of its feasibility would seem to be worthwhile in future research.

7. References

(1) Edwards, J.B., Wolfenden, R.A. and Yazdi, A.M.S.R. "The modelling of semiflexible conveyor structures for coal face steering investigations. Part I: Spatially discrete models", Proc. I.Mech.E. (companion to present paper).

(2) Edwards, J.B. and Greenberg, J.M. "Longitudinal interaction in multipass processes", Proc. I.E.E., Vol.124, No.4, 1977, pp.385-392.

(3) Robson, E.M. "Thick seam mining", Trans. of I.Min.E. Symposium on Productivity through Technology, Harrogate, Oct. 1978, paper No.8, pp.8.1 to 8.10.

## Appendix 1

#### Development of rule of thumb for calculation of the critical a.f.c

#### decrement distance $X_c$

If $G_s(D)$ is given by equation (5) then, if $X_1 = 0$, it is clear that the peak value of $|G_s(j\omega)|$ occurs at resonant frequency

$$\omega_r = \pi/X \qquad (A1.1)$$

and $\qquad |G_s(j\omega_r)| = 1/(1-k_h) \qquad (A1.2)$

If $\qquad 0 \leq X_1 < X \qquad (A1.3)$

then lag $X_1$ may be regarded as additional delay, so making

$$\omega_r \simeq \pi/(X+X_1) \qquad (A1.4)$$

and having little effect on $|G_s(j\omega_r)|$ if $\omega_r$ is now given by (A1.4).

If $\omega_{r1}$ is the resonant frequency of the composite steering and conveyor system $G_s(j\omega)\ G_c(j\omega)$ then, for critical stability

$$|G_s(j\omega_{r1})|\,|G_c(j\omega_{r1})| = 1.0 \qquad (A1.5)$$

and if the resonance peak of $G_s(j\omega)$ is sharp compared to the rate of cut-off of $G_c(j\omega)$ then

$$\omega_{r1} \simeq \omega_r \qquad (A1.6)$$

so that, from (A1.2), (A1.5) and (A1.6), we get that for critical stability

$$|G_c(j\omega_r)| \simeq 1-k_h \qquad (A1.7)$$

Now if $G_c(d) = 1/\{1+(X_c D)^m\} \quad m = 2,4,6 \qquad (A1.8)$

$$|G_c(j\omega_r)| = G_c(j\omega_r) = 1/\{1+(X_c\omega_r)^m\} \qquad (A1.9)$$

so that from (A1.7) and (A1.9) we deduce that, for critical stability

$$(X_c\omega_r)^m \simeq k_h/(1-k_h) \qquad (A1.10)$$

and from (A1.4) therefore

$$X_c \simeq {}_m\!\sqrt{k_h/(1-k_h)}\ (X+X_1)/\pi \qquad (A1.11)$$

For reasonable assurance of stability therefore $X_c$ must be chosen such that

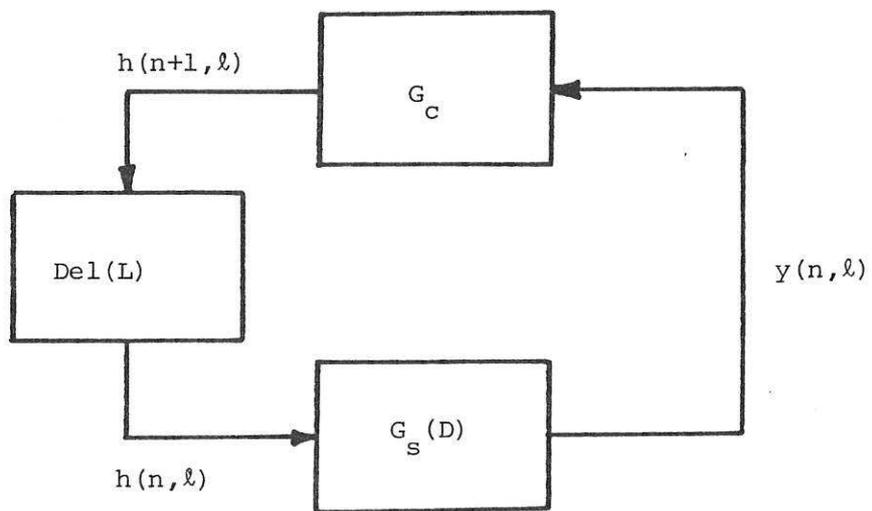$$X_c > {}_m\!\sqrt{k_h/(1-k_h)}\ (X+X_1)/\pi \qquad (A1.12)$$

Fig. 1   Representing the multipass system as two
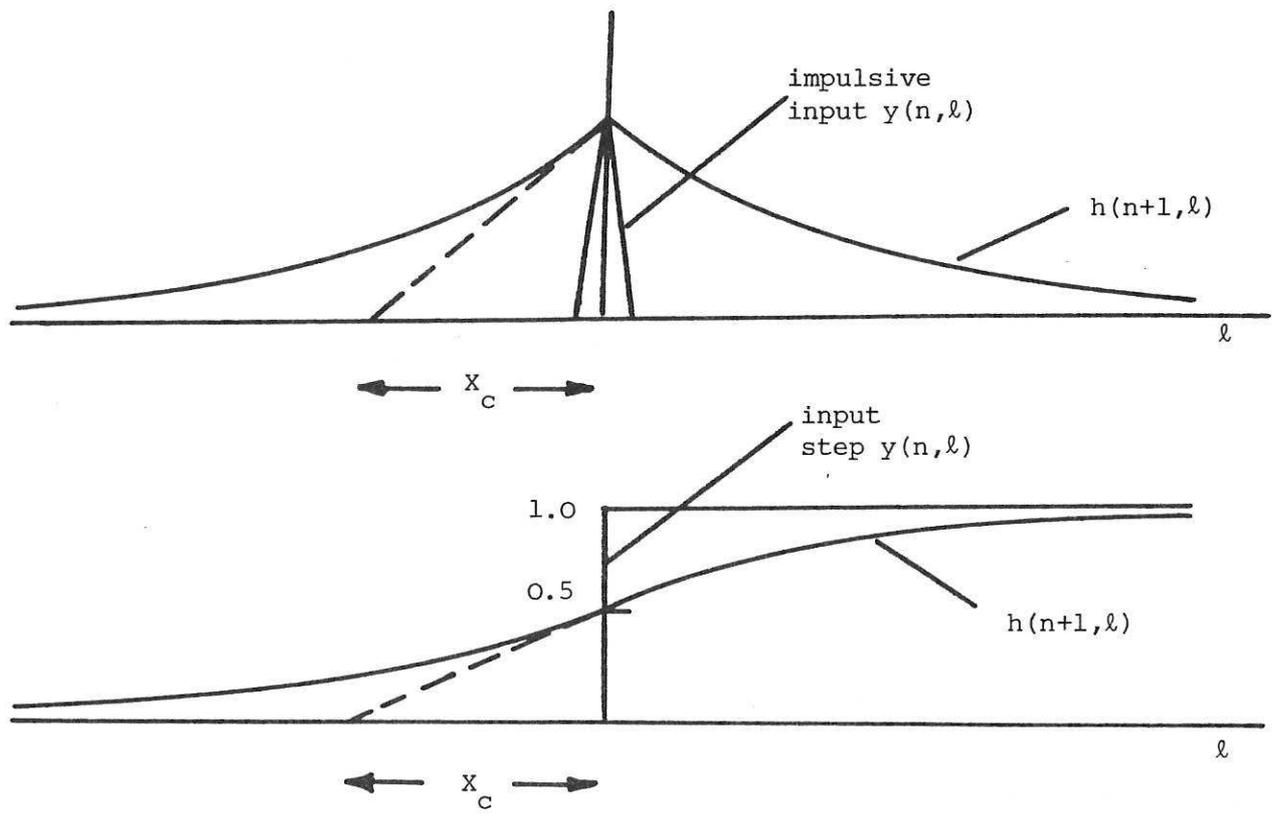loop-interconnected subsystems

FIg. 2  Impulse-and step-responses of simplest
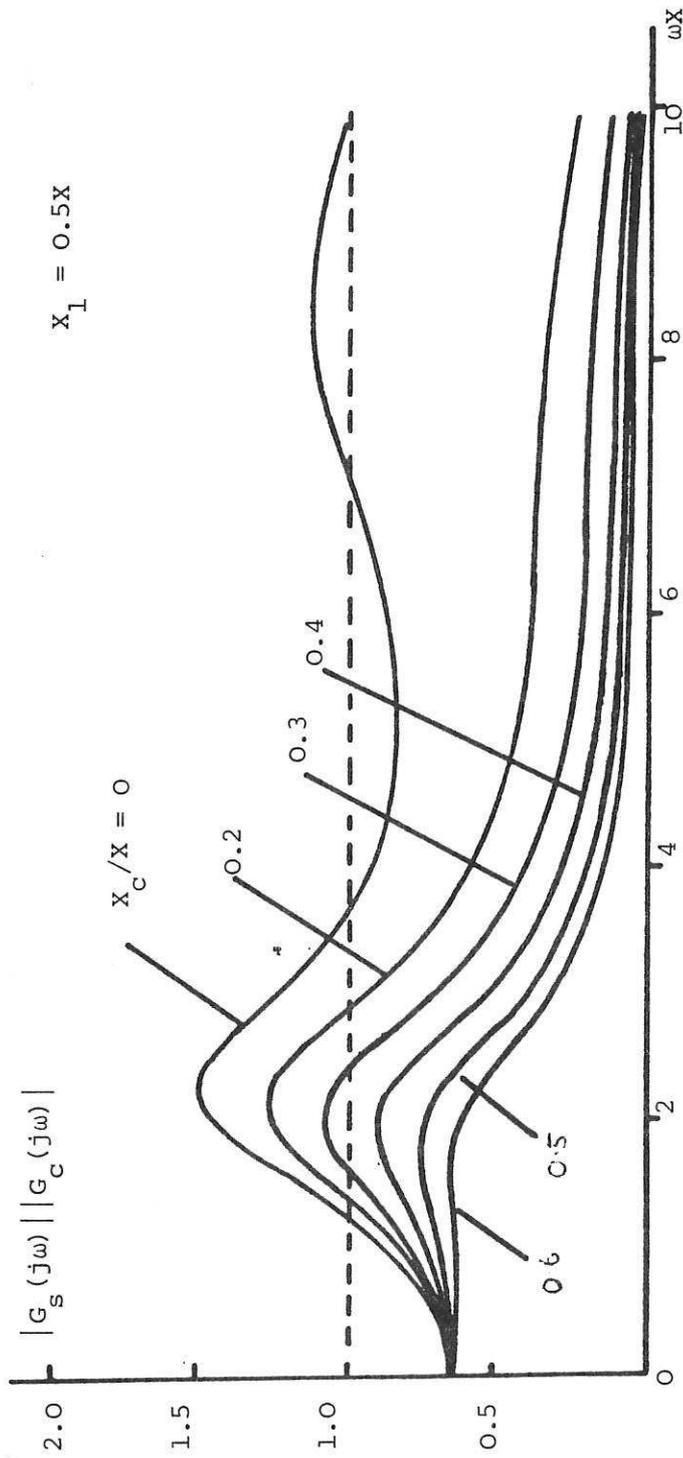        linear dynamic model of a.f.c

Fig. 3 Spectra of $\left|G_s(j\omega)\right|\left|G_c(j\omega)\right|$ for simple dynamic model of a.f.c
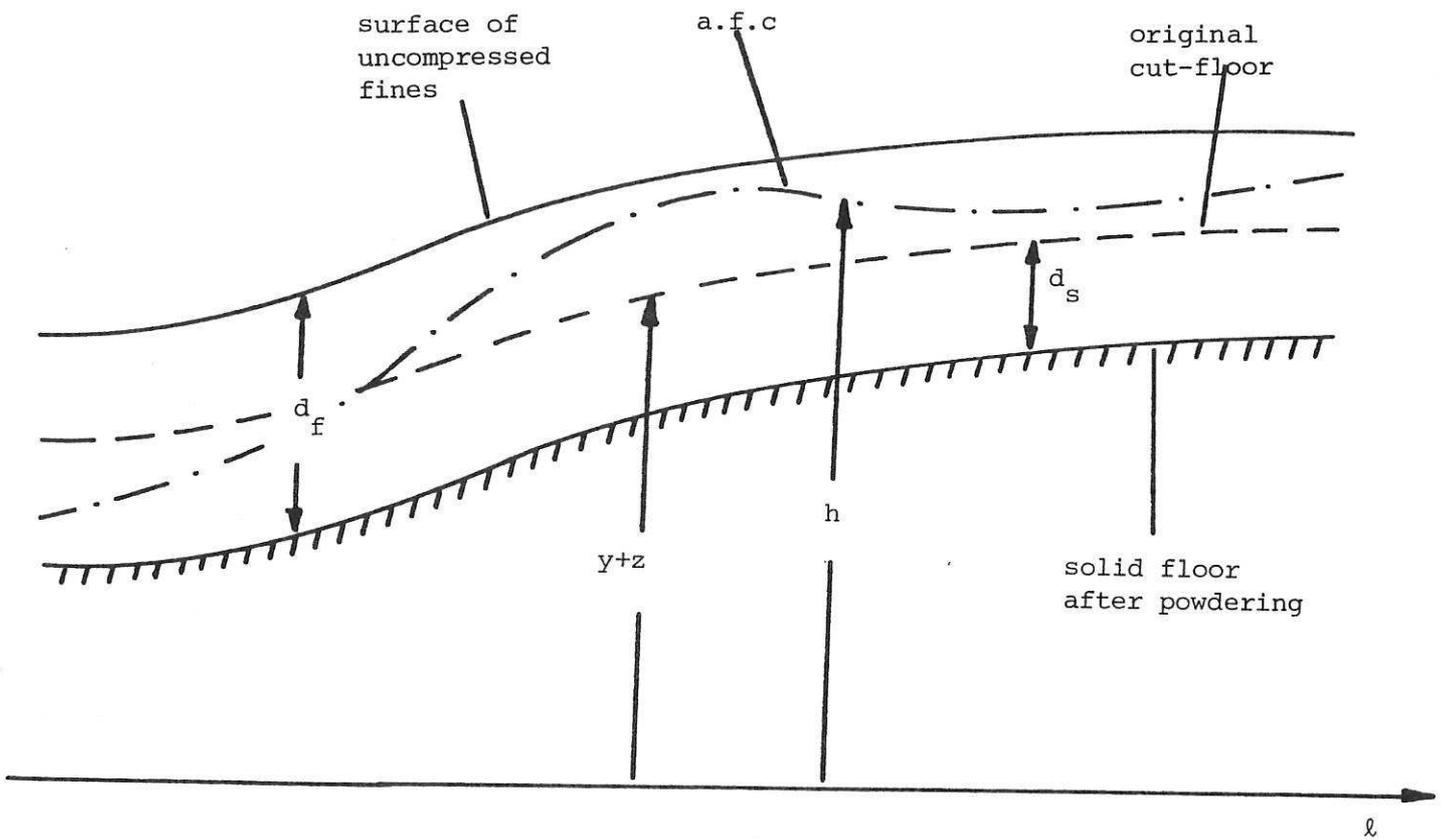
Fig. 4  Showing elastic beam model of a.f.c settling on  powdered floor
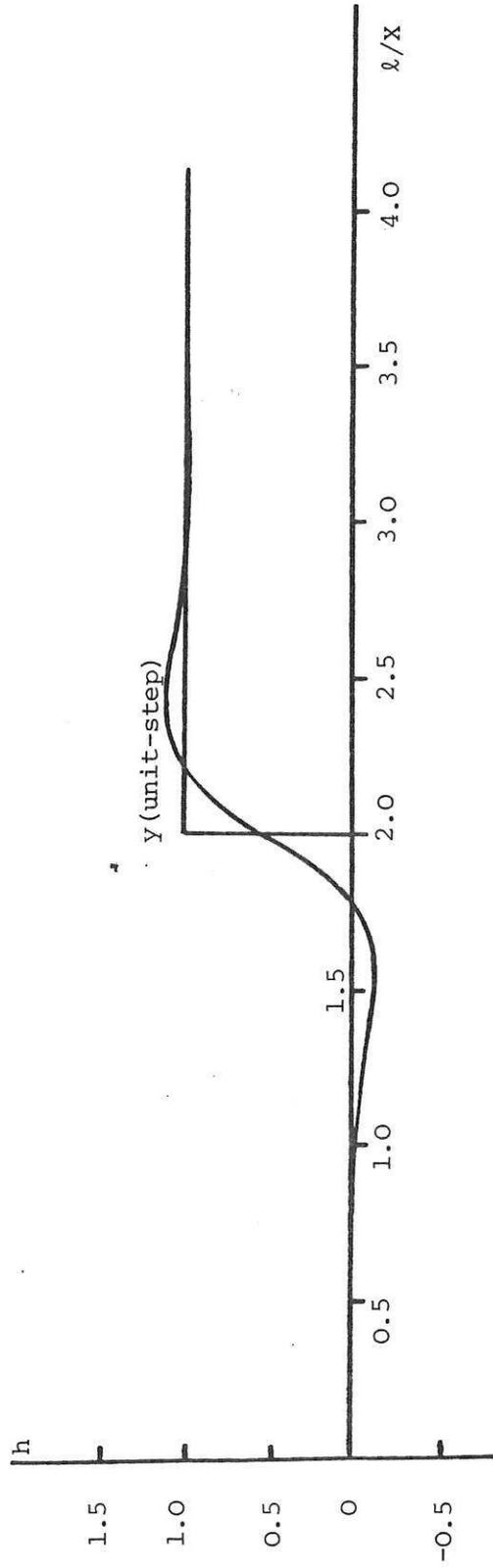
$$\frac{X}{c} = 0.5x$$

y(unit-step)

ℓ/x

h

1.5

1.0

0.5

0

-0.5

0.5   1.0   1.5   2.0   2.5   3.0   3.5   4.0

Fig. 5  Unit-step response of elastic beam model

Fig. 6 Spectra of $\left| G_s(j\omega) \right| \left| G_c(j\omega) \right|$ for elastic beam model

$\left|G_s(j\omega)\right|\left|G_c(j\omega)\right|$

2.0

1.5

1.0

0.5

0

$X_1 = 0.5x$

$x_c/x = 0$

0·2

0·3

0·4

0·5

0·6

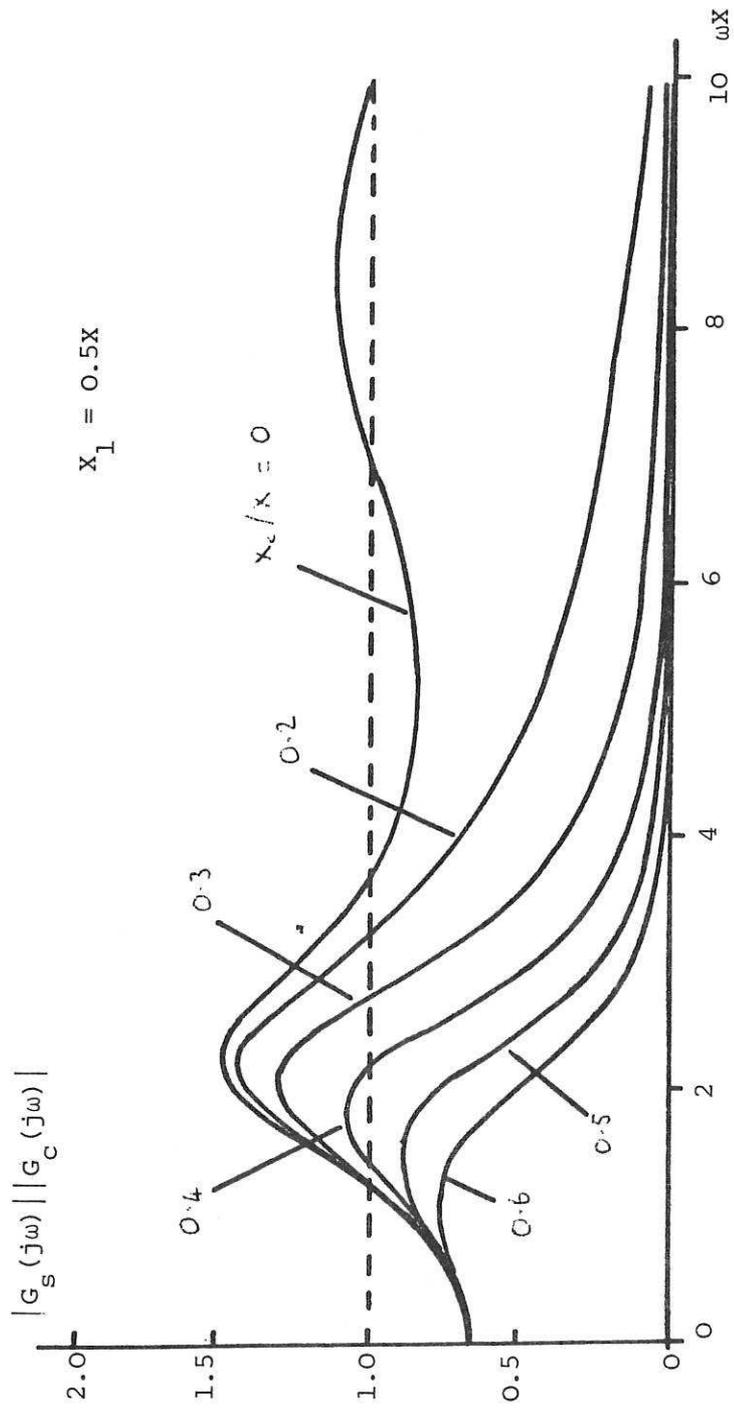0    2    4    6    8    10    $\omega X$
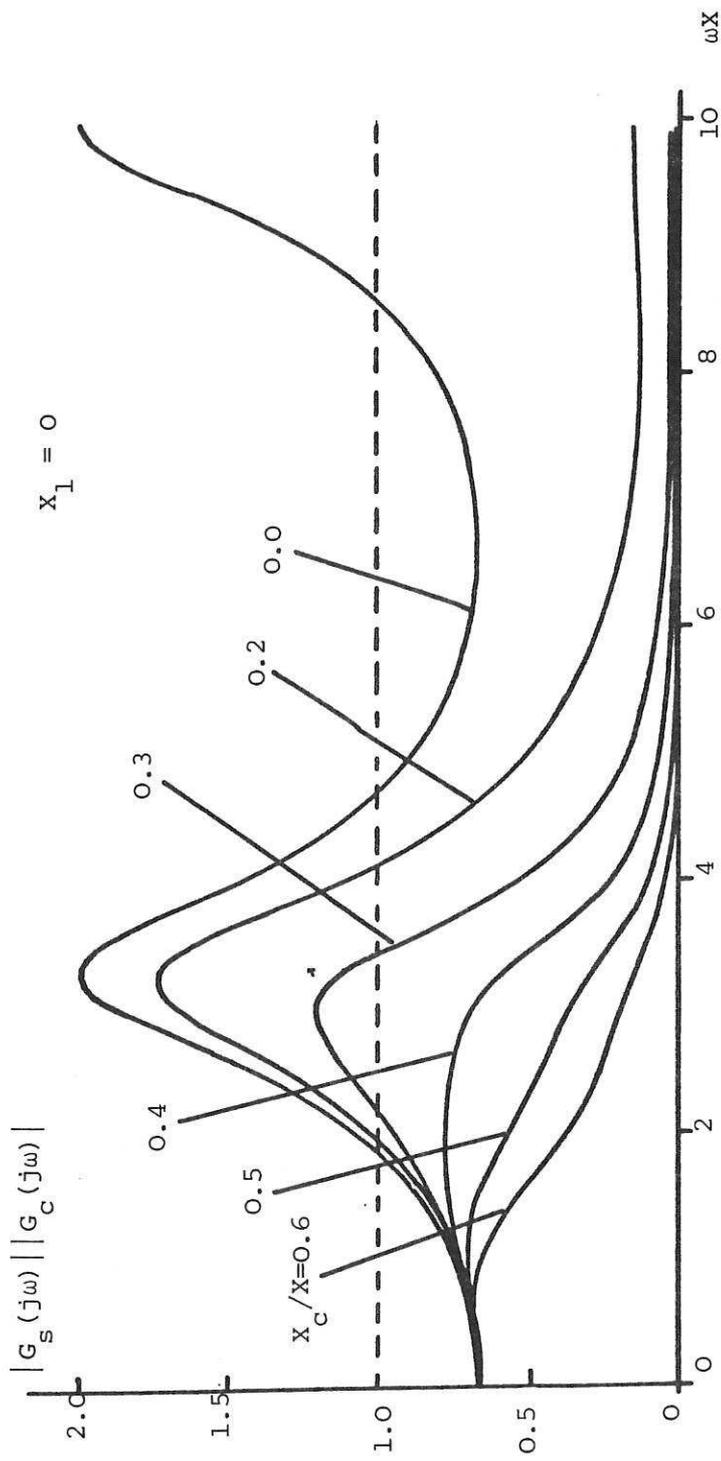
Fig. 7  Spectra of $\left|G_s(j\omega)\right|\left|G_c(j\omega)\right|$ for elastic beam model

Fig. 8  Coal-cutter
recovery from unit
initial step disturbance
(stiff conveyor)

$X_c = 0.32x$

$h(5, \ell)$

$h(4, \ell)$

$h(3, \ell)$

$h(2, \ell)$

$h(1, \ell)$

$\ell/x$

n

1.0

1.0

1.0

1.0

1.0

1.0

2.0

3.0

4.0

0

1
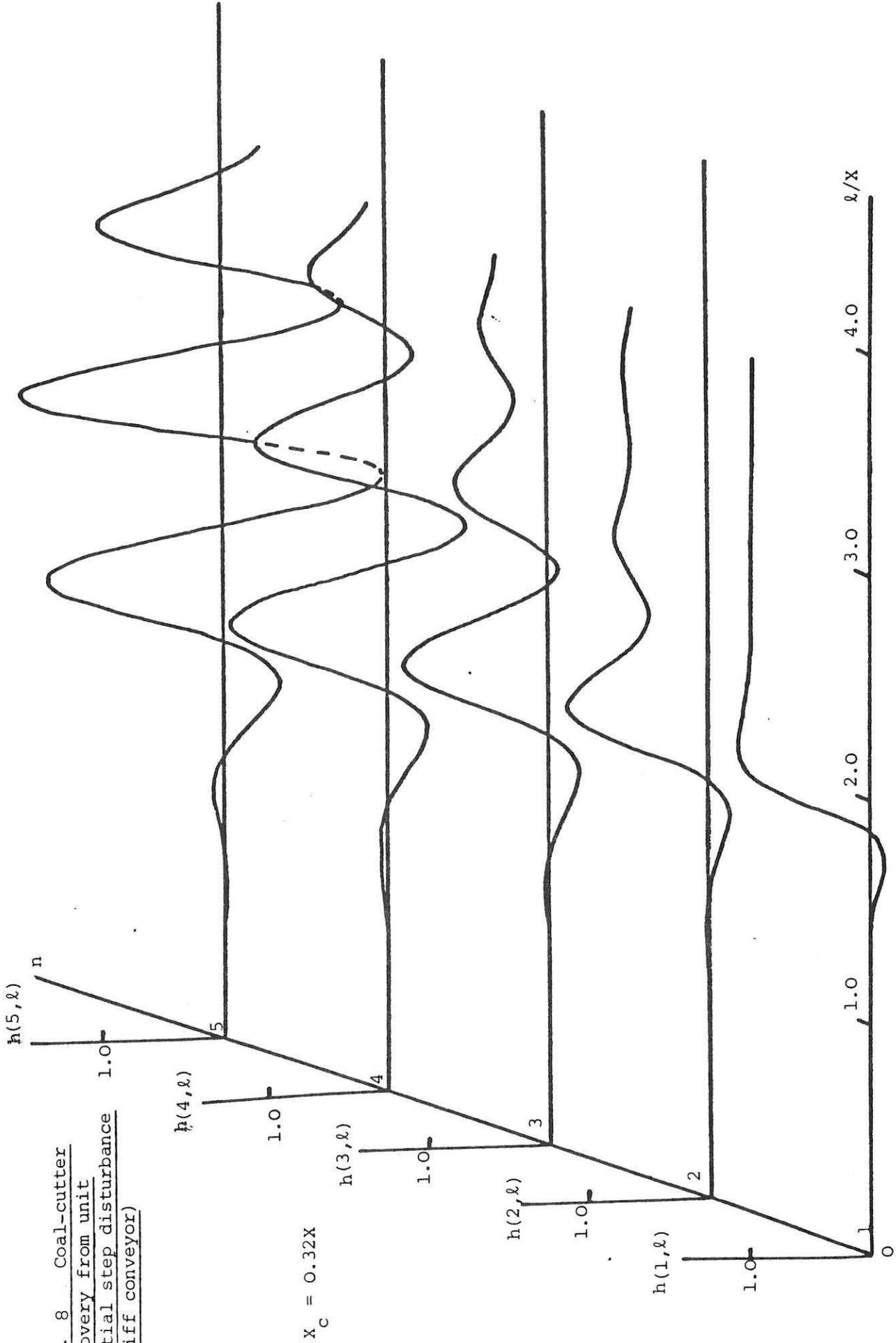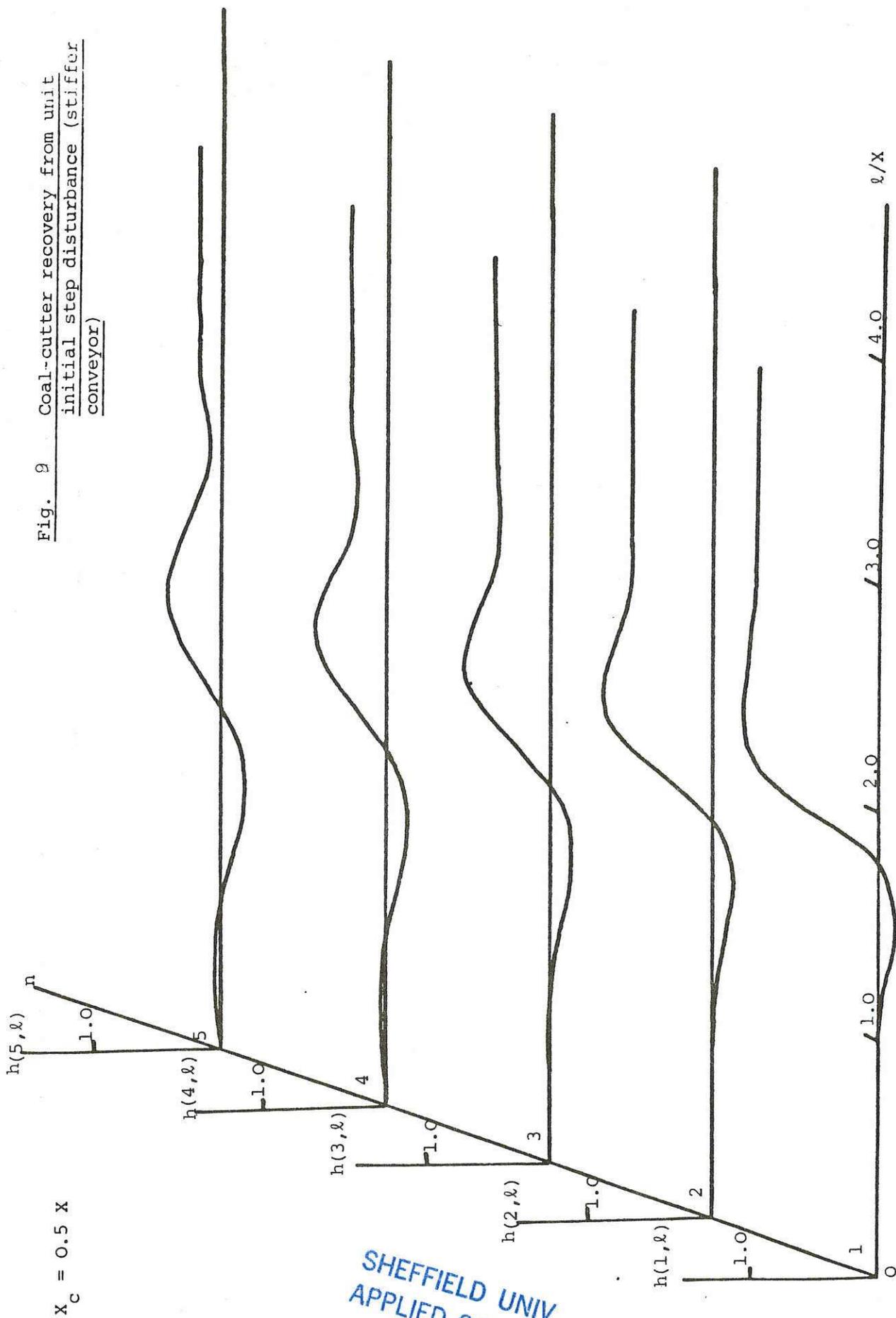
2

3

4

5

Fig. 9 Coal-cutter recovery from unit
initial step disturbance (stiffer
conveyor)

$x_c = 0.5 \, x$