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HIGH PERFORMANCE CONTROLLERS FOR UNKNOWN MULTIVARIABLE SYSTEMS

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Abstract

Recent work on the design of robust proportional plus integral process controllers for unknown minimum-phase (but possibly unstable) multivariable systems is compared and contrasted with recent work of Penttinen and Koivo (1980) and Astrom (1980) and illustrated by application to an open-loop unstable batch process.

1. Introduction

In recent papers Astrom (1980) and Penttinen and Koivo (1980) have considered specific examples of the general problem of constructing simple process controllers for unknown systems (unknown in the sense that its model is unknown or is too high order to make normal design calculations feasible) using only elementary computations based on inspection of graphical system open-loop step response data. Both techniques do require that the unknown plant has defined structural properties (such as monotonicity, stability...) but, when they do apply, the techniques are capable (in principle) of generating simple process controllers that are easily tuned on-line. Both techniques can, in fact, be regarded as attempts to provide rigorous alternatives or generalizations of simple tuning methods such as that due to Ziegler and Nichols (1942). Other alternatives can be found in Davison (1976) and in the following sections. In the authors opinion, all of these methods lay on important foundation to help bridge the gap between control theory and engineering practice based on experience and intuition.

It is the purpose of this paper to propose and validate an alternative philosophy and approach to the control of unknown (possibly multivariable) engineering systems that is being developed in the UK based upon the use of approximate plant models deduced from a large-order plant model or plant step-response data (Edwards and Owens 1977 , Owens 1978,1979). The approach is, in a well-defined sense, 'inverse' to those of Penttinen and Koivo (1980) and Davison (1976) and has the advantage that the effect of measurement nonlinearities can be estimated during the design exercise (see Boland and Owens (1980) and Owens (1981a)).

In the following sections the approaches described above are compared and contrasted, some indications of the limitations of the theory are outlined and a generalization of the techniques of Edwards and Owens (1977) (see also Owens (1978)) is derived and illustrated by application to the open-loop unstable multivariable systems previously considered by Munro (1972) and Rosenbrock (1974). The design method is the natural 'inverse' of that proposed by Penttinen and Koivo (1980).

2. Alternative Approaches to Unknown Systems Control

It would appear to be a general principle that, in any attempt to design a controller for an unknown system with m-inputs and ℓ-outputs described by (say) the (unknown) continuous linear time-invariant model in R^n

$$\dot{x}(t) = A\,x(t) + B\,u(t)$$
$$y(t) = C\,x(t) \qquad (1)$$

it is necessarily true that the structure of the design and the limitations encountered should reflect this uncertainty and (hopefully) generate a closed-loop system that is robust in that it is insensitive to the unknown dynamics. Of course the details will vary from situation to situation and will depend upon the physical nature of the plant. It is possible however, to propose two distinct situations where general statements can be made about stability as outlined in the following sections.

2.1 The Low Gain Philosophy for Stable Unknown Plant

Suppose that the system (1) is stable and, for simplicity, consider the case of proportional output feedback control

$$u(t) = K_1(r(t) - y(t)) \tag{2}$$

where K_1 is a constant ℓxm matrix and r(t) is the mxl demand vector. It is trivially verified that the closed-loop characteristic polynomial is

$$\rho_c(s) = |s I_n - A + BKC| \tag{3}$$

and, applying a continuity argument (or, following Koivo (1980), a Lyapunov argument), it is clear that the closed-loop system is stable if all elements of K_1 (or, equivalently, any norm of K_1 are small enough. These arguments will also carry through for proportional plus integral output feedback described by the transform relation

$$u(s) = (K_1 + \frac{1}{s} K_2)(r(s) - y(s)) \tag{4}$$

provided the elements of both matrices K_1 and K_2 are small enough and the plant (1) satisfies certain structural constraints (see Koivo (1980) or Davison (1976)). In summary we have the following simple result:

Theorem 1: An unknown multivariable plant that is known to be stable will, in general, retain its stability under low gain proportional plus integral output feedback!

The work of Davison, Penttinen and Koivo and (in the discrete case) Astrom represent specific cases of the application of this result.

2.2 The High Gain Philosophy for Minimum Phase Unknown Plant

Suppose now that m = ℓ and that the system (1) is invertible and minimum-phase in the sense that all its invariant zeros have strictly negative real parts (see, for example, Owens (1978)). Consider the general case of the proportional plus integral output feedback controller

$$u(s) = p\ (K_1 + \frac{1}{s} K_2)\ (r(s) - y(s)) \tag{5}$$

which is invertible and minimum-phase with scalar gain p. It is a simple exercise in the theory of multivariable root-loci (see, for example, Owens (1978), (1981b)) to prove the following 'inverse' to theorem 1:

Theorem 2: An unknown square invertible multivariable system that is known to be minimum phase will be stable under high enough gain proportional plus integral output feedback if

(a) the controller is minimum phase and

(b) all asymptotes of the root-locus lie in the open-left-half complex plane

(Note: (i) condition (b) is equivalent to the requirement that the system has only first order infinite zeros and/or second order infinite zeros with pivots possessing strictly negative real parts.

(ii) alternative stability analyses of minimum phase plant can be found in Owens (1974) and Edwards and Owens (1977)).

Of course, if the system is totally unkown, condition (b) could be difficult to check. Noting however (Edwards and Owens (1977) and Koivo (1980)) that the matrix CB can be estimated directly from open-loop plant step responses, the following proposition identifies situations when the conditions of the theorem can be checked quite easily:

Proposition 1: Condition (b) of theorem (2) is satisfied if CB is nonsingular and all eigenvalues of CBK_1 have strictly positive real parts.
(Note: the condition on CB ensures the existence of only first order infinite zeros!) In effect, if a system is unknown but it is known (or conjectured) to be minimum-phase and CB can be estimated from plant step responses obtained from plant tests or simulations of a complex dynamic model, then it is quite easy to find a proportional plus integral controller to stabilize the plant provided high enough gains can be used. Unfortunately the results (as stated above) neither suggest specific choices of K_1 and K_2 nor do they say how high the gain must be. This will be discussed further in section 3.

2.3 Low-gain versus High-gain: inverse approaches

In order to highlight the relationships between the high and low gain philosophies consider the class of all possible square invertible linear systems divided into four subclasses:

(a) Stable and minimum phase systems,

(b) Stable and non-minimum phase systems,

(c) Unstable and minimum phase systems and

(d) Unstable and non-minimum phase systems.

If an unknown system belongs to (a) or (b) it is clearly true (theorem 1) that, in general, it can be regulated using low-gain proportional plus integral control. It is also clear that the low gain philosophy will not apply if (c) or (d) pertains! If, however an unknown system belongs to (a) or (c) it follows from theorem 2 that, provided some simple structural constraints are satisfied, the system can be regulated using high-gain proportional plus integral controller. Note that both the low gain and high gain philosophies are applicable to an unknown system in class (a) whereas neither philosophy applies to an unknown system in class (d) and that the low-gains philosophy alone applies to (b) whereas the high-gain philosophy alone applies to (c)! Clearly the two approaches have distinct but overlapping areas of applicability and hence must be regarded as distinct alternatives. In particular, although stability and asymptotic tracking are fundamentally important design specifications, many applications demand higher performance specifications including fast rise-times, small overshoot, small transient interaction etc. In this area the applicability of the low and high-gain philosophy differ markedly as it is generally true that high performance systems require tight (i.e. high gain) control loops!

Finally, we note that the low and high gain philosophies can be pictured as 'inverses' of each other in the sense that

(i) the inverse of a high gain is a low gain, and

(ii) a minimum-phase system has a stable inverse.

3. High Performance Controllers based on Approximate Plant Models

Although theorem 2 is a useful conceptual beginning for a theory of control of minimum-phase unknown multivariable systems, many more details must be filled in. More precisely, specific choices of K_1 and K_2 must be made to ensure, not only stability, but excellent time responses for the closed-loop system. The approach taken by the authors is to base controller design upon a very simple model of plant behaviour, to ensure that the derived controller produces the derived stable, high performance responses from the approximate model and to demonstrate that, at high gains, the real plants stability and transient characteristics are arbitrarily close to those predicted by the approximate model.

Consider a square, invertible, minimum-phase system described by the mxm transfer function matrix G(s) and a unity negative feedback system for the control of G using invertible, minimum phase proportional plus integral control. Suppose that G has the inverse structure

$$G^{-1}(s) = A_o s + A_1 + A_o H(s) \qquad , \quad |A_o| \neq 0 \qquad (6)$$

where H(s) is proper and stable. Equivalently the plant is minimum phase and $CB = A_o^{-1}$ is nonsingular (see, for example, Owens (1978) p. 130). We can also assume, without loss of generality, that H(o) = O and hence that $A_1 = G^{-1}(o)$ is the D.c. inverse gain matrix.

Let $\tilde{A}_o$ and $\tilde{A}_1$ be numerical estimates of A_o and A_1 obtained by estimation of CB and G(o) from a system model or plant step responses and approximate plant dynamics by the model $G_A(s)$ of the first order (Owens (1978)) form

$$G_A^{-1}(s) = s\,\tilde{A}_o + \tilde{A}_1 \qquad (7)$$

It is seen that $G_A(s)$ approximates the high frequency and steady state plant characteristics only.

Consider now the two-term parametric controller

$$K(s) = \tilde{A}_o \operatorname{diag}\{k_j + c_j + \frac{k_j c_j}{s}\} - \tilde{A}_1 \tag{8}$$

generalising previous work of Owens (1978 p. 120). A simple calculation yields the identity

$$(I_m + G_A(s)K(s))^{-1} G_A(s)K(s) \equiv (G_A^{-1}(s) + K(s))^{-1}K(s)$$

$$\equiv \operatorname{diag}\{\frac{1}{(s+k_j)(s+c_j)}\}_{1\leqslant j\leqslant m} \; (\operatorname{diag}\{(k_j + c_j)s + k_j c_j\}_{1\leqslant j\leqslant m} - s\tilde{A}_o^{-1}\tilde{A}_1)$$

and hence the approximate system is stable in the closed-loop situation iff $k_j > 0$ and $c_j \geqslant 0$, $1 \leqslant j \leqslant m$. If we identify the k_j's with fast modes and c_j's with slower modes then a simple pole-residue calculation yields the results

(a) responses in loop j have time-constants of the order of k_j^{-1}, zero steady state errors in response to unit step demands if $c_j \neq 0$ and reset times of the order of c_j^{-1}, and

(b) defining $k = \min_j k_j$, then the system response speeds in response to unit step demand can be made to be arbitrarily fast and transient interaction effects arbitrarily small as the 'gain' k becomes large.

Clearly the controller (8) generates a high performance closed-loop system for the approximate plant (7) if loop gains are high! Suppose that we now apply the controller to the real (unknown) system (6)! Applying the results given in Edwards and Owens (1977), a <u>sufficient</u> condition for closed-loop stability is that the approximate closed-loop system is stable and that

$$\lambda \triangleq \max_{1\leqslant i\leqslant m} \sup_{s\in D} \sum_{j=1}^{m} \left| (\{I_m + Q_A^{-1}(s)\}^{-1}\{Q_A^{-1}(s) - Q^{-1}(s)\})_{ij} \right| < 1 \tag{10}$$

where $Q = GK$, $Q_A = G_A K$ and D is the usual Nyquist Contour in the complex plane. Substituting from the data indicates that

$$(I_m + Q_A^{-1})^{-1}(Q_A^{-1} - Q^{-1}) \equiv (G_A^{-1} + K)^{-1}(s(\tilde{A}_o - A_o) + \tilde{A}_1 - A_1 - A_o H)$$

$$\equiv \text{diag}\left\{\frac{s}{(s+k_j)(s+c_j)}\right\}_{1\leq j\leq m}(s\tilde{A}_o^{-1}(\tilde{A}_o - A_o) + \tilde{A}_o^{-1}(\tilde{A}_1 - A_1) - \tilde{A}_o^{-1}A_o H(s)) \tag{11}$$

and we can prove a natural 'inverse' to the results of Penttinen and Koivo in the form of the following main result of this paper.

Theorem 3: An unknown multivariable plant that is known to be minimum phase with CB nonsingular will be stable in the presence of unity negative feedback with forward path proportional plus integral controller of the form of equation (8) if

(i) the tuning parameters $k_j > 0$, $c_j \geq 0$ $(1\leq j\leq m)$,

(ii) the inequality

$$\lambda_\infty = \max_{1\leq i\leq m} \sum_{j=1}^{m} \left| (\tilde{A}_o^{-1}(\tilde{A}_o - A_o))_{ij} \right| < 1 \tag{12}$$

is satisfied, and

(iii) the gain parameter $k = \min_j k_j$ is sufficiently large.

Proof: Condition (i) is required to ensure the stability of the approximate feedback system . Conditions, (ii) and (iii) together also guarantee the truth of (10) as, noting that H is proper and stable, we obtain the inequality

$$\lim_{k\to+\infty} \lambda = \lim_{k\to\infty} \max_{1\leq i\leq m} \sup_{s\in D} \sum_{j=1}^{m} \left| (\text{diag}\left\{\frac{s^2}{(s+k_j)(s+c_j)}\right\}_{1\leq j\leq m} \tilde{A}_o^{-1}(\tilde{A}_o - A_o))_{ij} \right|$$

$$< \lambda_\infty < 1 \tag{13}$$

This proves the theorem as (10) is clearly satisfied at high gains.

The application of the result is illustrated in the next section. Before continuing however we note that conditions (i) and (iii) have an obvious interpretation. Condition (ii) is a little more difficult but, in essence, it provides explicit lower bounds on the permissible error between A_o and its computed estimate $\tilde{A}_o$ which, in turn, can be interpreted in terms of the permissible errors between $CB = A_0^{-1}$ and its estimate $\tilde{C}\tilde{B} = \tilde{A}_0^{-1}$ computed possibly by rough and ready analysis of open loop plant step response data. In particular, if the calculations are exact, then $A_0 = \tilde{A}_0$ yielding $\lambda_\infty = 0$ and (12) is satisfied trivially. In other cases, (12) can be interpreted as a measure of the robustness of the design at high gain. Finally, note that A_1 and $\tilde{A}_1$ play no role in the theorem and hence the results are insensitive to the choice of $\tilde{A}_1$ e.g. choosing $\tilde{A}_1 = 0$ the result is still valid and the controller structure considerably simplified.

4. Numerical Example

To illustrate the application of theorem 3, consider the unstable batch process discussed by Munro (1972) and Rosenbrock (1974) and defined by the matrices

$$A = \begin{pmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 \\ 5.679 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{pmatrix} , \quad C = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \tag{14}$$

which is known to be minimum phase and open-loop unstable. We will assume that it is required to design a high performance controller generating a closed-loop system with fast rise-times, zero steady state errors and small interaction effects in response to unit step demands. Also, although the system model is known, we will assume that the controller design must be undertaken without the aid of an interactive computing facility!

Following the procedure defined in section 3, we can use the model to compute the two matrices A_o and A_1 exactly i.e.

$$A_o = \tilde{A}_o = (CB)^{-1} = \begin{bmatrix} 0 & 0.176 \\ -0.318 & 0 \end{bmatrix}$$

$$A_1 = \tilde{A}_1 = -CA^{-1}B = \begin{bmatrix} 0.141 & 0.296 \\ 0.995 & 2.455 \end{bmatrix} \qquad (15)$$

which defines the approximate model G_A uniqely and immediately yields the fact that $\lambda_\infty = 0$. The open-loop responses of real and approximate systems to a unit step input in u_1 are shown in Fig. 1 and illustrate that the approximate, conceptual model is also unstable with significantly different time responses!

Suppose that a closed-loop time-constant of $\simeq 0.05$ is required from both loops and integral action of reset time of $\simeq 0.2$ in both loops to remove steady state errors. These considerations immediately suggest the choice of $k_1 = k_2 = 20$ and $c_1 = c_2 = .5$ and it is anticipated that the resultant controller will generate excellent responses from the approximate model if our chosen gains are high enough and also that the real system G will be stabilized as all conditions of theorem 3 will then be satisfied. These facts are verified in Fig. 2 which shows the closed-loop unit step responses of both real and approximate system and indicates that the real system responses are very close to those predicted from the approximate model.

Finally, we note that the final design above generates responses comparable with those obtained by Munro (1972) using sophisticated systematic multivariable frequency response techniques and an interactive computing facility. In contrast, the above design was achieved very rapidly and with only minimal computational requirements! Perhaps there is a moral here?

5. Conclusions

It has been demonstrated that the low-gain philosophy inherent in the work of Astrom, Koivo and Davison on the control of stable unknown plant has a natural 'inverse' namely, the high gain philosophy for the control of minimum-phase unknown plant. This second approach has been explored by constructing a generalization of the first authors previous work in the form of a parametric controller structure capable of stabilizing unknown multivariable plant at high gain. The applicability of the results has been verified by the design of a high performance regulator for an unstable multivariable process that compares favourably with a previous design obtained using model-based multivariable frequency response methods.

Finally, we point out that the principles outlined in this paper will extend naturally to the discrete case with 'high gains' and 'low gains' replaced by 'fast sampling' and 'slow sampling' respectively. The details of this extension are currently under consideration.

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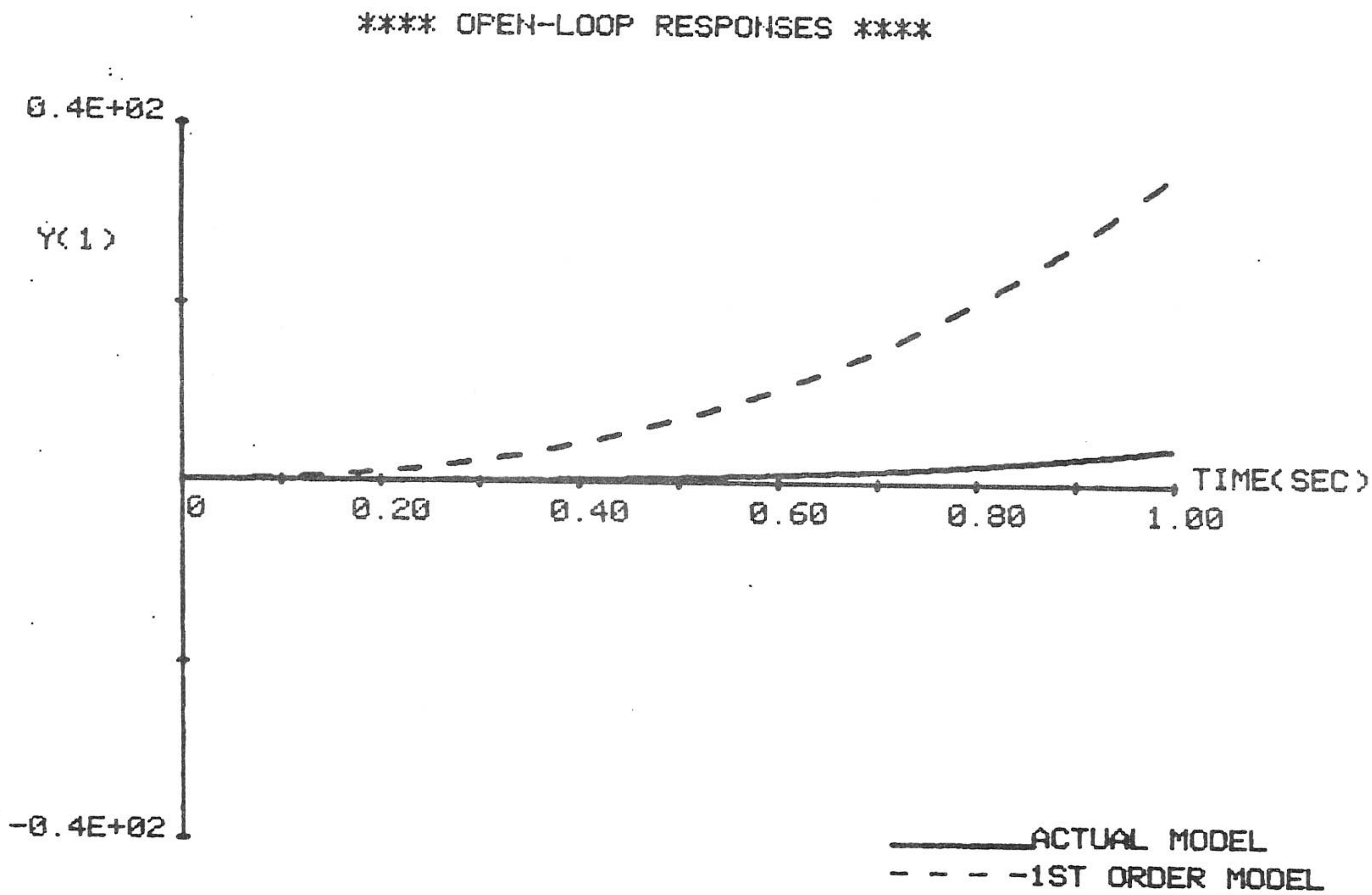

Fig. 1 (a) Response of y_1 to a unit step in u_1

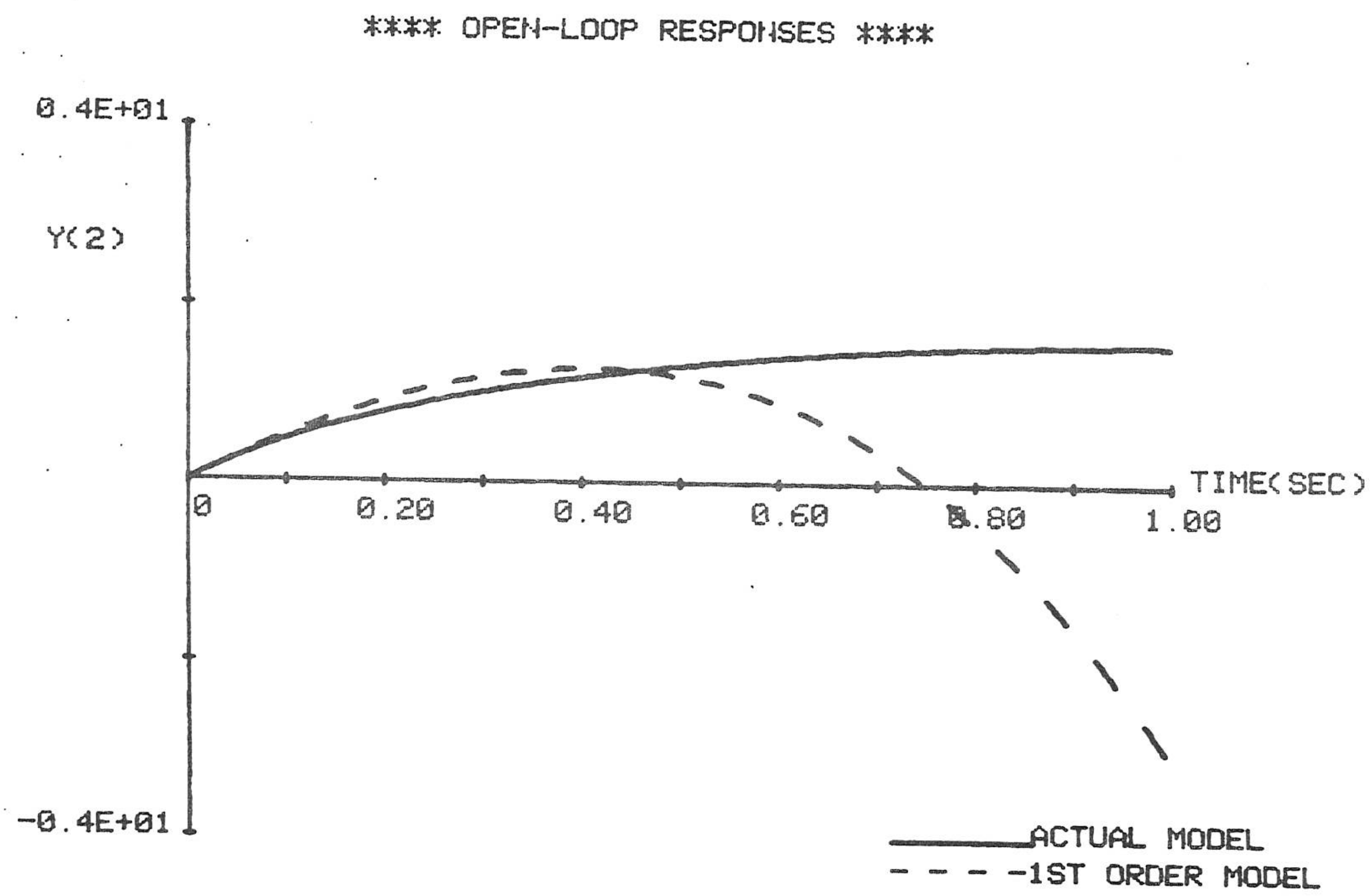

Fig. 1 (b) Response of y_2 to a unit step in u_1

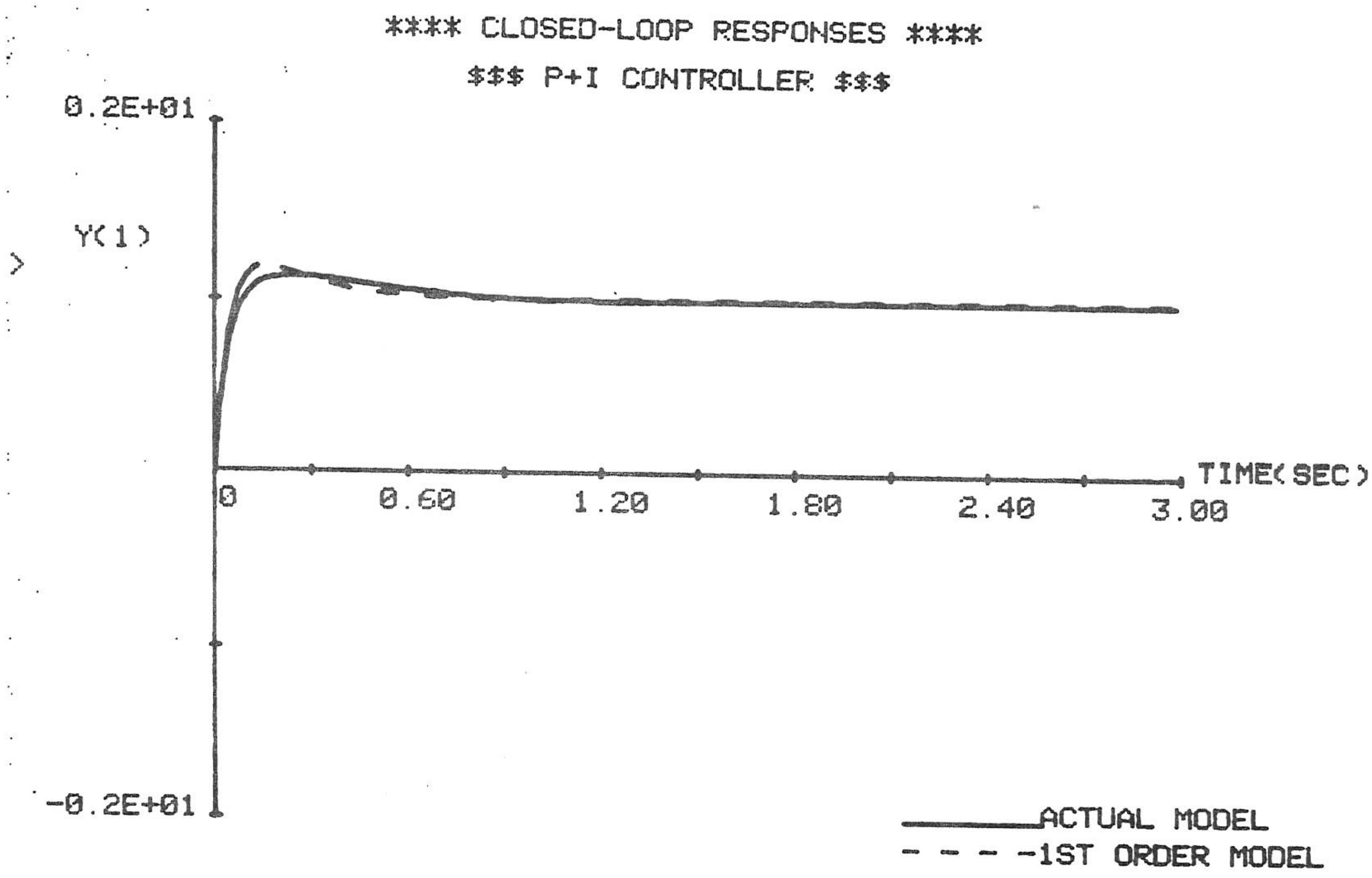

Fig. 2(a) Response of y_1 to a unit step demand in y_1

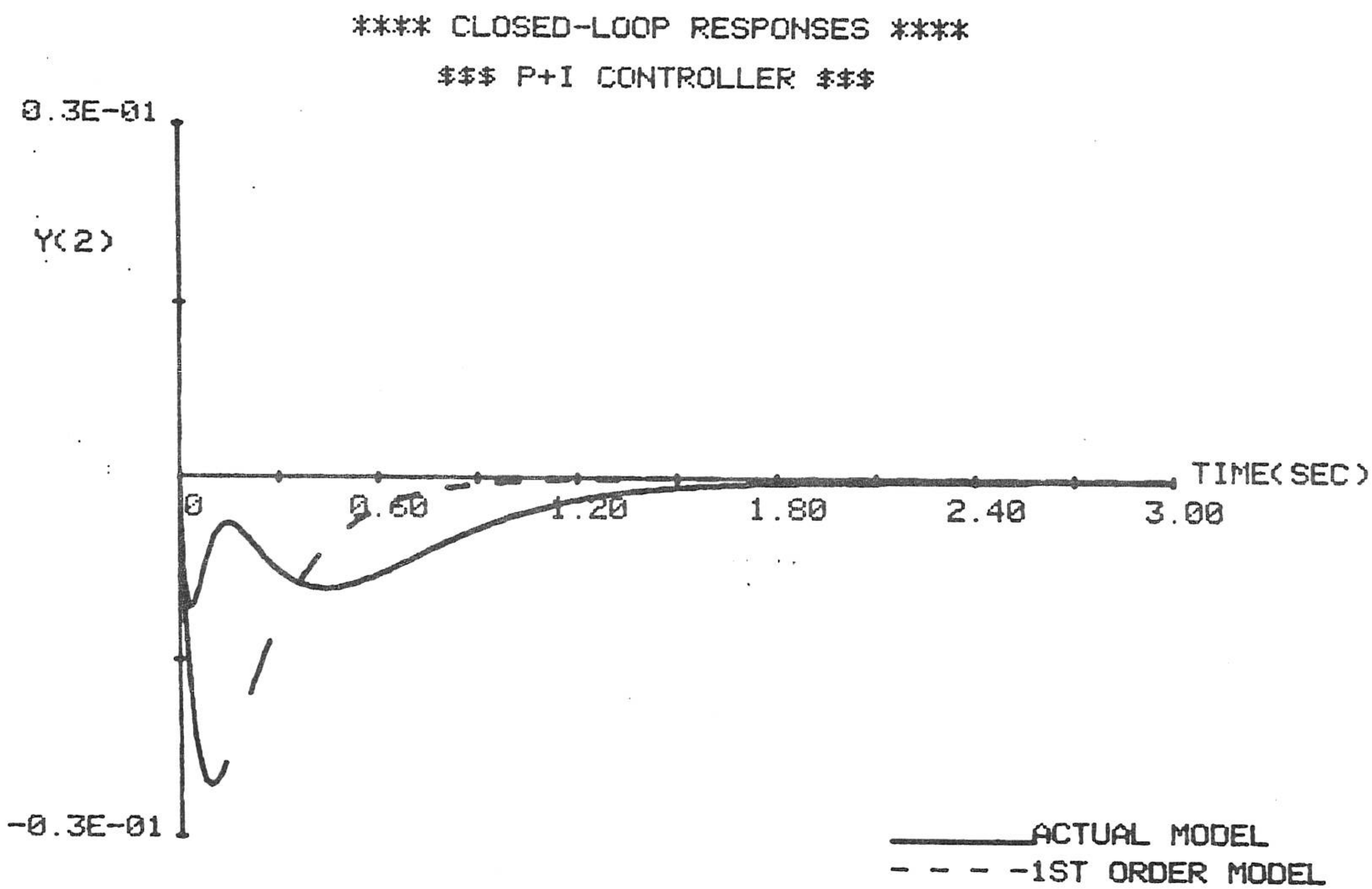

Fig. 2(b) Response of y_2 to a unit step demand in y_1

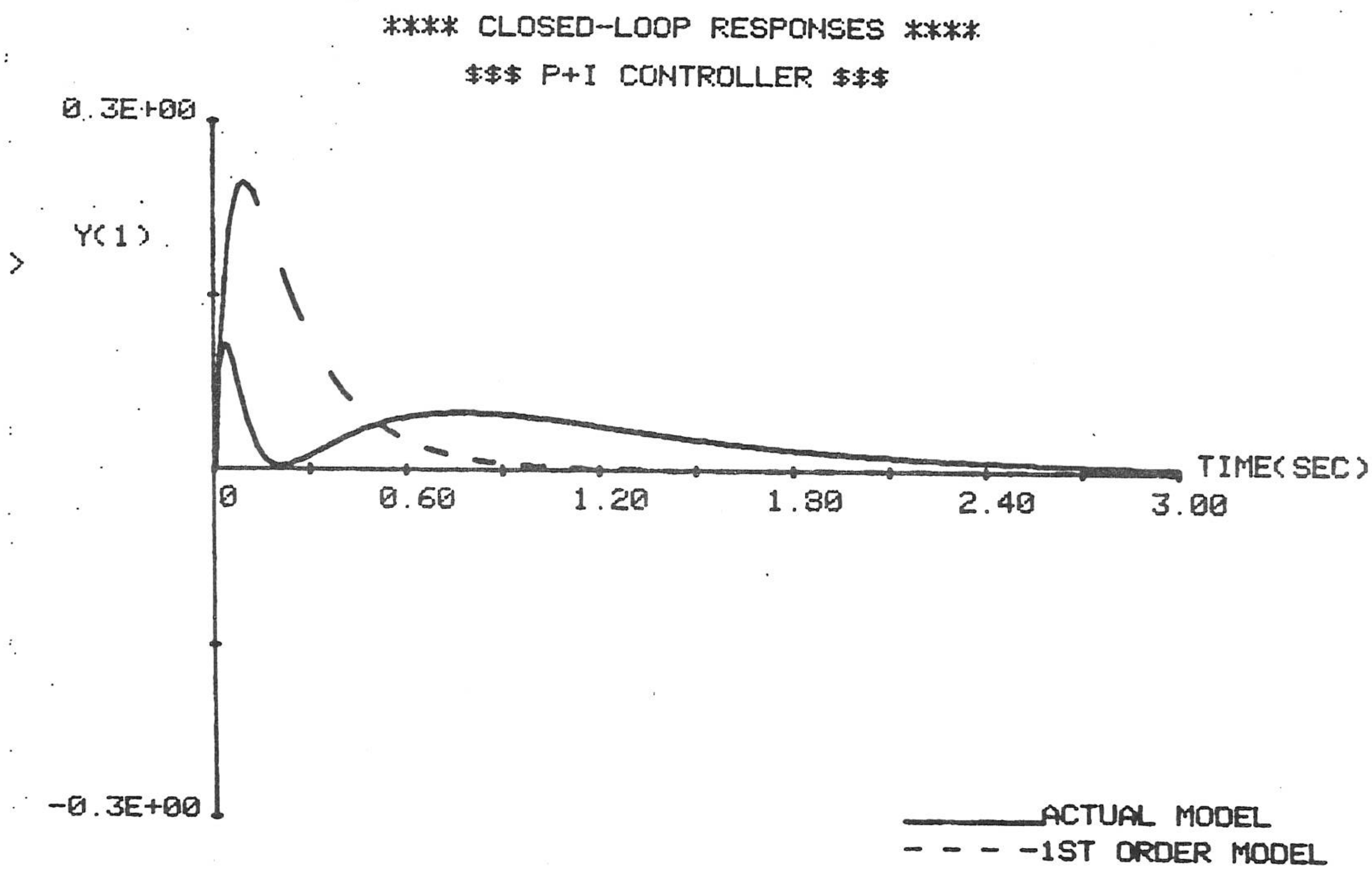

Fig. 2(c) Response of y_1 to a unit step demand in y_2

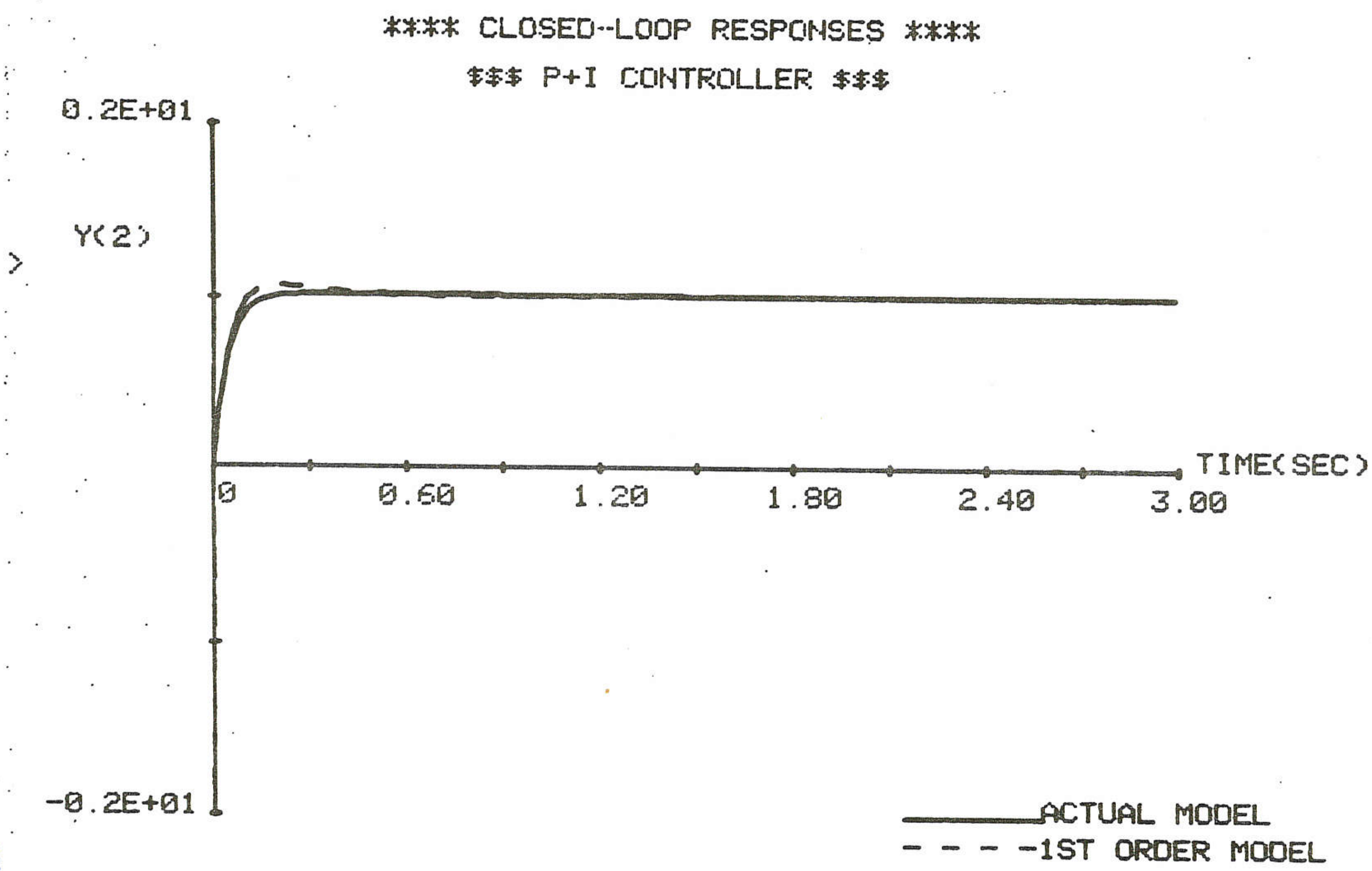

Fig. 2(d) Response of y_2 to a unit step demand in y_2

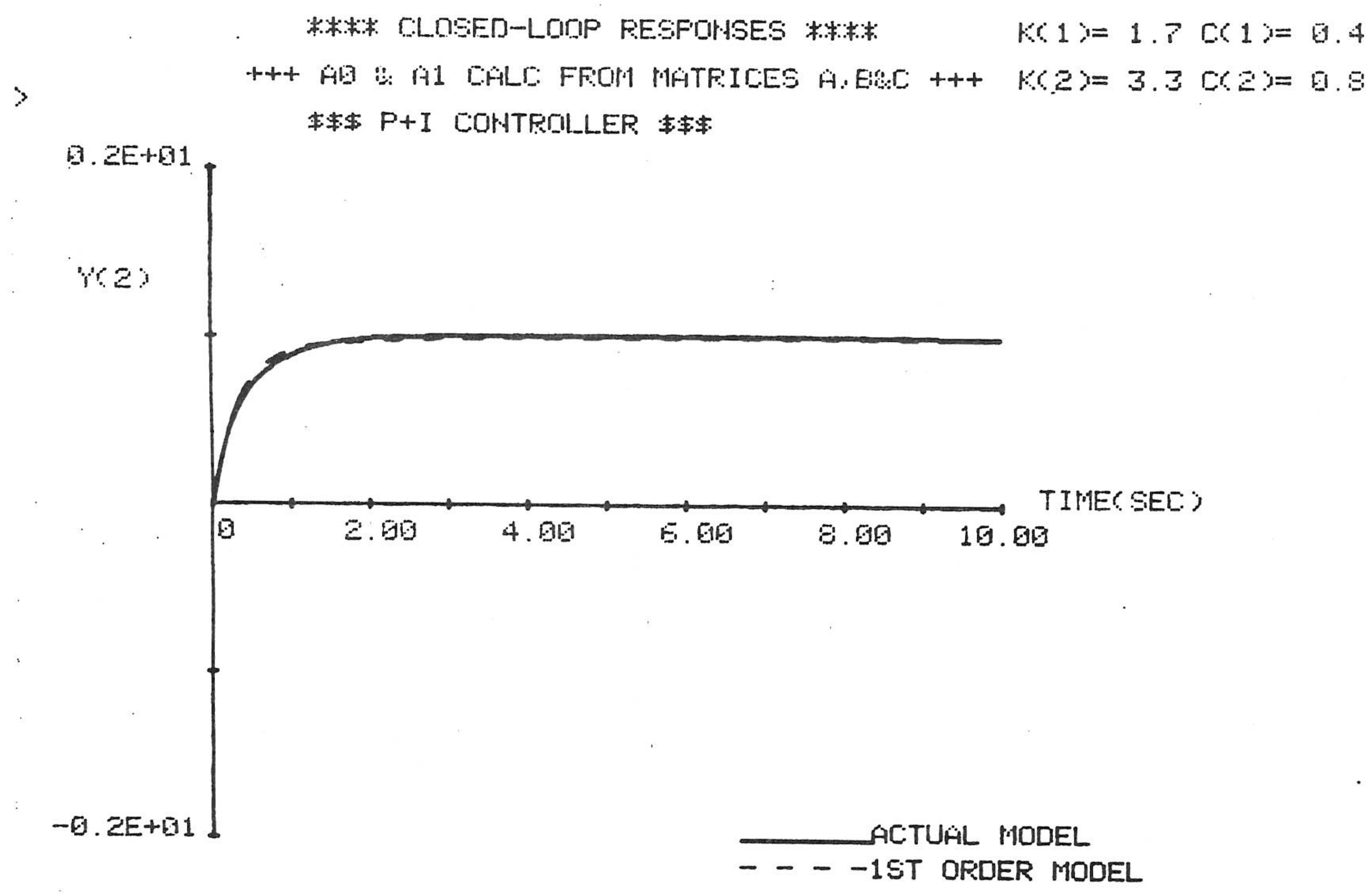

Fig. 6(b) Closed loop Respones to a Unit Step Demand in $y_2(t)$

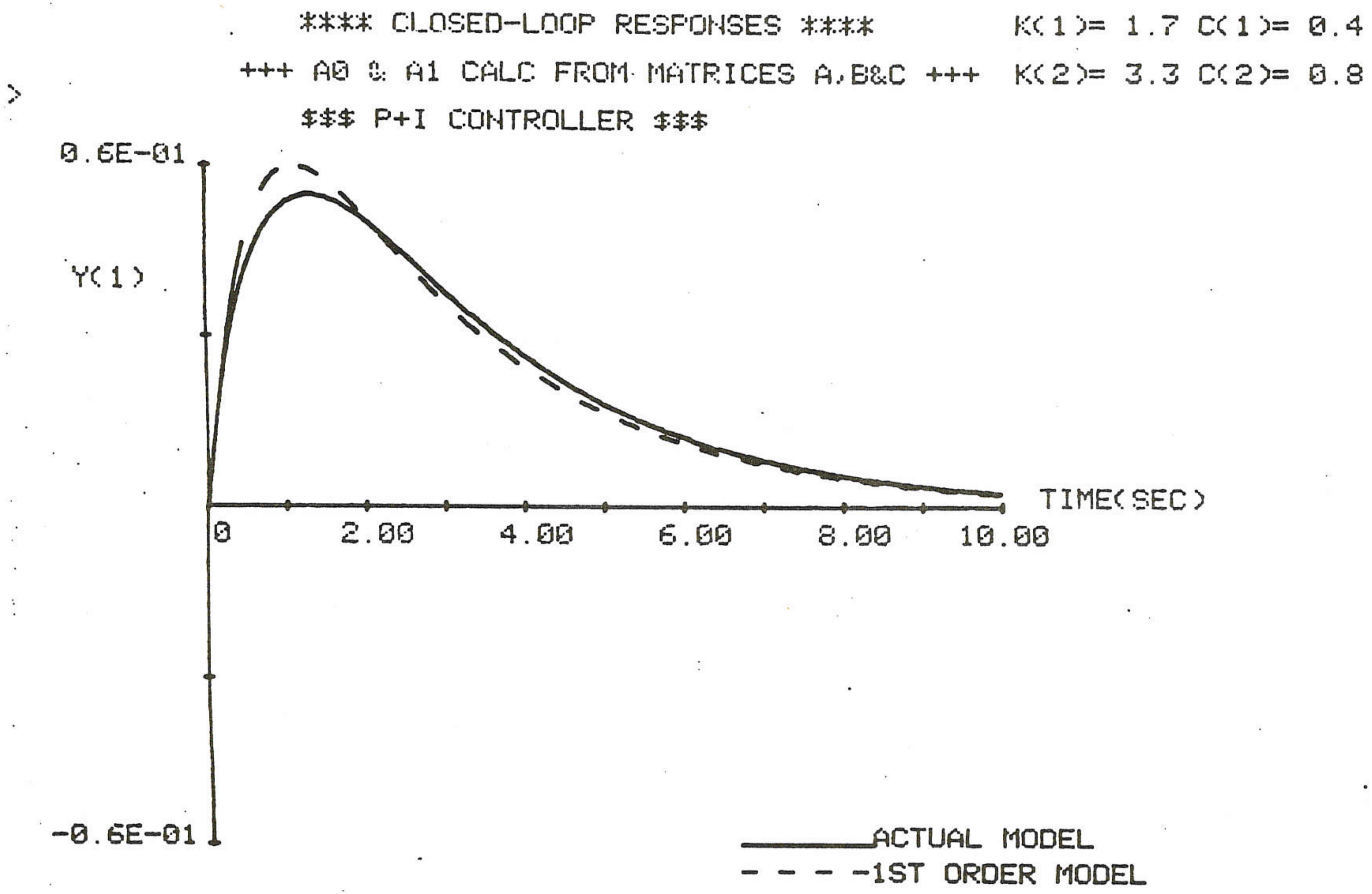

Fig. 6(a) Closed loop Responses to a Unit Step Demand in $y_2(t)$