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Nonlinear System Identification using the Hammerstein Model

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ABSTRACT

An algorithm for the identification of non-linear systems which can be described by a Hammerstein model consisting of a single-valued non-linearity followed by a linear system is presented. Cross-correlation techniques are employed to decouple the identification of the linear dynamics from the characterization of the non-linear element. These results are extended to include the identification of the component subsystems of a feedforward process consisting of a Hammerstein model in parallel with another linear system.

1. INTRODUCTION

The Hammerstein model, illustrated in Fig.1, consists of a zero memory non-linear element followed by linear dynamics. The model represents a realization of the Hammerstein operator

$$H[x(t)] = \int h(t,\tau)F[\tau,x(\tau)]d\tau$$

and was originally proposed by Narendra and Gallman (1966), and later studied by Hsia and Bailey (1968), Chang and Luus (1971).

The present study represents an extension of previous results by Billings and Fakhouri (1977 and 1978) to non-linear systems which can be described by a Hammerstein model. It is shown that an estimate of the linear system impulse response can be obtained directly from the input-output cross-correlation function when the input has the properties of a white Gaussian process. The coefficients associated with the polynomial representation of the non-linear element can then be estimated using a least squares routine.

The algorithm is extended to include the identification of the component subsystems in a feedforward process which was originally considered by Brown (1969) and later studied by Simpson and Power (1970a & b). The system consists of a Hammerstein model in parallel with another linear system. The results of a simulation study are included to illustrate the validity of the algorithms.

2. IDENTIFICATION OF THE HAMMERSTEIN MODEL

Consider the Hammerstein model illustrated in Fig.1, where the linear time invariant system has an impulse response h(t) and the continuous single-valued non-linear element can be represented by a finite polynomial of the form

$$y(t) = \gamma_1 x(t) + \gamma_2 x^2(t) + \ldots + \gamma_k x^k(t) \qquad (1)$$

The measured system output z(t) can then be expressed as

$$z(t) = \int_{-\infty}^{\infty} h(\tau) F[x(t-\tau)] d\tau + n(t)$$

$$= \sum_{j=1}^{k} \gamma_j \int_{-\infty}^{\infty} h(\tau) x^j(t-\tau) d\tau + n(t) \qquad (2)$$

If the input signal x(t) is the practical realization of a zero mean white Gaussian process, then its n'th dimensional autocorrelation function is given by

$$E[x(t_1)x(t_2)\ldots x(t_n)] = 0 \qquad \text{for n odd}$$

$$= \frac{1}{2^P P!} \frac{\delta^n}{\delta S_1 \delta S_2 \ldots \delta S_n} \left(\sum_{i=1}^{n} \sum_{k=1}^{n} m_{ik} S_i S_k \right)^P$$

$$= \frac{1}{2^P P!} \sum (m_{i_1 k_1} m_{i_2 k_2} \ldots m_{i_p k_p}) \qquad (3)$$

for n even

where the summation is over all sets of indices $(i_1, k_1 \ldots i_p, k_p)$ such that only one of the indices is unity, only one is two etc. $P = n/2$ and

$$m_{ik} = E[x(t_i)x(t_k)] = \phi_{xx}(t_k - t_i)$$

which approximates to a delta function of area μ centred on $t_i = t_k$.

When the input signal comprises the summation of a white Gaussian signal, with the properties of equation (3), and a mean level b the measured system output z(t) is given by

$$z(t) = \sum_{j=1}^{k} \omega_j(t) + n(t) \qquad (4)$$

where $\omega_j(t) = \gamma_j \int_{-\infty}^{\infty} h(\tau)\{x(t-\tau)+b\}^j d\tau$ (5)

Defining

$$z'(t) = z(t) - \overline{z(t)}$$

$$\omega_i'(t) = \omega_i(t) - \overline{\omega_i(t)}$$

the first order cross-correlation function is given by

$$\phi_{xz'}(\sigma) = E\left[z'(t)x(t-\sigma)\right]$$

$$= \overline{\omega_1'(t)x(t-\sigma)} + \overline{\omega_2'(t)x(t-\sigma)}$$

$$\ldots + \overline{\omega_k'(t)x(t-\sigma)} + \overline{n(t)x(t-\sigma)} \quad (6)$$

where —— indicates time average.

Evaluating the first term on the rhs of equation (6)

$$\overline{\omega_1'(t)x(t-\sigma)} = \overline{\{\omega_1(t)-\overline{\omega_1(t)}\}x(t-\sigma)}$$

$$= \overline{\omega_1(t)x(t-\sigma)}$$

$$= \gamma_1 \int_{-\infty}^{\infty} h(\tau)\overline{\{x(t-\tau)+b\}x(t-\sigma)}d\tau$$

$$\overline{\omega_1'(t)x(t-\sigma)} = \gamma_1 \int_{-\infty}^{\infty} h(\tau)\phi_{xx}(\tau-\sigma)d\tau \quad (7)$$

Providing the signal x(t) has the properties of a white Gaussian process, then $\phi_{xx}(\tau-\sigma)$ approximates to a delta function at $\tau = \sigma$ and equation (7) reduces to

$$\overline{\omega_1'(t)x(t-\sigma)} = \gamma_1\mu h(\sigma) \quad (8)$$

Considering the second term on the rhs of equation (6)

$$\overline{\omega_2'(t)x(t-\sigma)} = \overline{\{\omega_2(t)-\overline{\omega_2(t)}\}x(t-\sigma)}$$

$$= \overline{\omega_2(t)x(t-\sigma)}$$

$$= \gamma_2 \int_{-\infty}^{\infty} h(\tau)\overline{\{x(t-\tau)+b\}^2 x(t-\sigma)}\,d\tau$$

$$= \gamma_2 \int_{-\infty}^{\infty} h(\tau)\{2b\phi_{xx}(\tau-\sigma)\}d\tau$$

$$\overline{\omega_2'(t)x(t-\sigma)} = 2b\gamma_2\mu h(\sigma) \qquad (9)$$

Similarly, for the third term on the rhs of eqn (6)

$$\overline{\omega_3'(t)x(t-\sigma)} = \gamma_3 \int_{-\infty}^{\infty} h(\tau)\{3\phi_{xx}(0)\phi_{xx}(\tau-\sigma)+3b^2\phi_{xx}(\tau-\sigma)\}d\tau$$

$$= 3\gamma_3\mu h(\sigma)\{\phi_{xx}(0)+b^2\} \qquad (10)$$

Although, theoretically $\phi_{xx}(0)$ would be infinite, in practice $x(t)$ can only approximate to a white noise process and $\phi_{xx}(0) = \varepsilon^2$, the variance of $x(t)$ which is finite. Equation (10) can therefore be written as

$$\overline{\omega_3'(t)x(t-\sigma)} = 3\gamma_3\mu h(\sigma)\{\varepsilon^2+b^2\} \qquad (11)$$

Higher order terms are evaluated in an analogous manner.

Collecting terms

$$\phi_{xz'}(\sigma) = h(\sigma)\mu\{\gamma_1+2b\gamma_2+3\gamma_3(\varepsilon^2+b^2)+\ldots\}+\overline{n(t)x(t-\sigma)} \qquad (12)$$

Providing the signal $x(t)$ and the noise process $n(t)$ are statistically independent equation (12) becomes directly proportional to the impulse response of the linear system

$$\phi_{xz'}(\sigma) = \alpha h(\sigma) \qquad (13)$$

Thus computing the cross-correlation function $\phi_{xz'}(\sigma)$ provides an estimate of the impulse response of the linear subsystem and this effectively decouples the identification of the linear and non-linear components in the Hammerstein model.

A Gaussian signal with a non-zero mean level is used as the input signal to ensure that all the terms in equation (6) contribute to the cross-correlation function.

If the identification is performed with the aid of a digital computer, the cross-correlation function equation (13) will be in sampled data form and estimates of the coefficients in the pulse transfer function representation of the linear system

$$Z\{\alpha h(\sigma)\} = \frac{B(z^{-1})}{A(z^{-1})} \tag{14}$$

can be obtained using a least squares algorithm Isermann et al (1974).

Once an estimate of $\alpha h(\sigma)$ is available from equation (13), the parameters in the polynomial representation of the non-linear element can be estimated. Consider the schematic diagram of the identification procedure illustrated in Fig.2. The error e(i) between the sampled process output z(i) and the output of the Hammerstein model can be defined as

$$e(i) = z(i) - \hat{v}(i)$$

where

$$\hat{v}(i) = \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}\hat{y}(i-j)$$

$$= \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}\{\gamma_1'u(i-j)+\gamma_2'u^2(i-j)+\ldots+\gamma_k'u^k(i-j)\} \tag{15}$$

and $\gamma_t = \alpha\gamma_t'$, $t = 1,2\ldots k$.

If $(N+\ell)$ measurements of the sampled process input and output are available the matrix equation

$$\begin{bmatrix} Z(\ell+1) \\ Z(\ell+2) \\ \vdots \\ Z(\ell+N) \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u(1+\ell-j), & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^2(1+\ell-j) \ldots & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^k(1+\ell-j) \\ \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u(2+\ell-j), & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^2(2+\ell-j) \ldots & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^k(2+\ell-j) \\ \vdots & & \\ \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u(N+\ell-j), & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^2(N+\ell-j) \ldots & \sum_{j=0}^{\ell} \{\alpha\hat{h}(j)\}u^k(N+\ell-j) \end{bmatrix}$$

$$\cdot \begin{bmatrix} \gamma_1' \\ \gamma_2' \\ \vdots \\ \gamma_k' \end{bmatrix} + \begin{bmatrix} e(\ell+1) \\ e(\ell+2) \\ \vdots \\ e(\ell+N) \end{bmatrix}$$

or $$Z = \psi\theta + E \qquad (16)$$

can be formulated. The least squares estimate of the coefficients γ_j', $j = 1,2\ldots k$ can be readily computed as

$$\hat{\theta} = \{\psi^T\psi\}^{-1}\psi^T Z \qquad (17)$$

and the identification is complete.

3. IDENTIFICATION OF A FEEDFORWARD SYSTEM

The algorithm derived in Section 2 for the Hammerstein model can be readily extended to include the identification of a feedforward system consisting of a Hammerstein model in parallel with another linear system. The identification of this system was originally studied by Brown (1968) who used a four level test signal to identify the linear channel. Simpson and Power (1970a and b) modified Brown's scheme such that the impulse responses of both linear systems could be identified from two cross-correlation experiments, and later extended this technique to include memory-type non-linearities, Simpson and Power (1973). However both these schemes assumed that the non-linear element was known a priori. In general this will not be the case and it is shown that the result of the previous section can be extended to include the identification of all the elements in this system.

Consider the feedforward system illustrated in Fig.3. When the input signal comprises the summation of a zero mean white Gaussian process and a non-zero mean level b the measured system output z(t) can be expressed as

$$z(t) = \int_{-\infty}^{\infty} h_1(\tau_1)\{x(t-\tau_1)+b\}d\tau_1$$

$$+ \sum_{j=1}^{k} \omega_j \int_{-\infty}^{\infty} h_2(\tau_2)\{x(t-\tau_2)+b\}^j d\tau_2 + n(t) \quad (18)$$

The first order cross-correlation function is given by

$$\phi_{xz'}(\sigma) = \overline{(z(t)-\overline{z(t)})x(t-\sigma)}$$

$$= \int_{-\infty}^{\infty} h_1(\tau_1)\overline{\{x(t-\tau_1)+b\}x(t-\sigma)}d\tau_1$$

$$+ \sum_{j=1}^{k} \omega_j \int_{-\infty}^{\infty} h_2(\tau_2)\overline{\{x(t-\tau_2)+b\}^j x(t-\sigma)}\,d\tau_2 + \overline{n(t)x(t-\sigma)} \quad (19)$$

Applying the results derived in the previous section for the Hammerstein model gives

$$\phi_{xz'}(\sigma) = \mu h_1(\sigma) + \beta h_2(\sigma) = h_3(\sigma) \quad (20)$$

where $\beta = \mu\{\omega_1+2b\omega_2+3\omega_3(\varepsilon^2+b^2)+\ldots\}$ is a constant. The first order cross-correlation function is therefore directly proportional to the sum of the impulse responses of the two linear elements.

In order to completely characterise the feedforward system, an estimate of the individual linear subsystem impulse responses is required. This can be achieved by computing the second order cross-correlation function which provides an estimate of $\lambda h_2(t)$ and permits the two linear systems in equation (20) to be separated.

Define the second order cross-correlation function as

$$\phi_{xxz'}(\sigma) = \overline{z'(t)x^2(t-\sigma)} = \overline{\{z(t)-\overline{z(t)}\}x^2(t-\sigma)}$$

$$= \overline{SS'(t)x^2(t-\sigma)} + \sum_{j=1}^{k} \overline{S_j'(t)x^2(t-\sigma)} + \overline{n(t)x^2(t-\sigma)} \quad (21)$$

where $SS(t) = \int_{-\infty}^{\infty} h_1(\tau_1)\{x(t-\tau_1)+b\}d\tau_1$

$SS'(t) = SS(t)-\overline{SS(t)}$

$S_i(t) = \omega_i \int_{-\infty}^{\infty} h_2(\tau_2)\{x(t-\tau_2)+b\}^i d\tau_2$

$S_i'(t) = S_i(t)-\overline{S_i(t)}$

Evaluating the first term on the rhs of eqn (21)

$$\overline{SS'(t)x^2(t-\sigma)} = \overline{\{SS(t)-\overline{SS(t)}\}x^2(t-\sigma)}$$

$$= \int_{-\infty}^{\infty} h_1(\tau_1)\overline{\{x(t-\tau_1)+b\}x^2(t-\sigma)}\,d\tau_1$$

$$- \phi_{xx}(0)b \int_{-\infty}^{\infty} h_1(\tau_1)\,d\tau_1 = 0 \;\forall\; \sigma \qquad (22)$$

Evaluating the second term on the rhs of eqn (21)

$$\overline{S_1'(t)x^2(t-\sigma)} = \overline{(S_1(t)-\overline{S_1(t)})x^2(t-\sigma)}$$

$$= \omega_1 \int_{-\infty}^{\infty} h_2(\tau_2)\overline{\{x(t-\tau_2)+b\}x^2(t-\sigma)}\,d\tau_2$$

$$- \omega_1\phi_{xx}(0)b \int_{-\infty}^{\infty} h_2(\tau_2)\,d\tau_2$$

$$= 0 \;\forall\; \sigma \qquad (23)$$

Considering the third term on the rhs of eqn (21)

$$\overline{S_2'(t)x^2(t-\sigma)} = \overline{(S_2(t)-\overline{S_2(t)})x^2(t-\sigma)}$$

where

$$\overline{S_2(t)} = \omega_2\phi_{xx}(0) \int_{-\infty}^{\infty} h_2(\tau_2)\,d\tau_2+\omega_2 b^2\int h_2(\tau_2)\,d\tau_2 \qquad (24)$$

Thus

$$\overline{S_2'(t)x^2(t-\sigma)} = \omega_2 \int_{-\infty}^{\infty} h_2(\tau_2)\overline{\{x(t-\tau_2)+b\}^2x^2(t-\sigma)}\,d\tau_2$$

$$- \omega_2\phi_{xx}(0)\int h_2(\tau_2)\,d\tau_2\{\phi_{xx}(0)+b^2\}$$

$$= \omega_2 \int_{-\infty}^{\infty} h_2(\tau_2)\{\overline{x^2(t-\tau_2)x^2(t-\sigma)}\}d\tau_2$$

$$- \omega_2\phi_{xx}^{\ 2}(0)\int h_2(\tau_2)\,d\tau_2$$

$$= \omega_2 \int_{-\infty}^{\infty} h_2(\tau_2)\{\phi_{xx}^{\ 2}(0)+2\phi_{xx}^{\ 2}(\tau_2-\sigma)\}d\tau_2$$

$$- \omega_2\phi_{xx}^{\ 2}(0) \int_{-\infty}^{\infty} h_2(\tau_2)\,d\tau_2$$

$$\overline{S_2'(t)x^2(t-\sigma)} = 2\omega_2 \int_{-\infty}^{\infty} h_2(\tau_2)\phi_{xx}^{\ 2}(\tau_2-\sigma)\,d\tau_2 \tag{25}$$

If $x(t)$ is the practical realisation of a zero mean white Gaussian process, $\phi_{xx}^{\ 2}(\tau_2-\sigma)$ can be considered as an impulse centred at $\tau_2 = \sigma$ with area

$$\lambda = \int_{-\infty}^{\infty} \phi^2_{\ xx}(\tau)\,d\tau$$

Equation (25), therefore reduces to

$$\overline{S_2'(t)x^2(t-\sigma)} = 2\omega_2\lambda h_2(\sigma) \tag{26}$$

Similarly, for the fourth term on the rhs of equation (21)

$$\overline{S_3'(t)x^2(t-\sigma)} = 6b\omega_3\lambda h_2(\sigma) \tag{27}$$

Finally, it can readily be proved that if the input signal and noise process are statistically independent, $\overline{n(t)x(t-\sigma)} = 0\forall\sigma$, then $\overline{n(t)x^2(t-\sigma)} = 0\forall\sigma$. Thus collecting terms

$$\phi_{xxz'}(\sigma) = h_2(\sigma)\ \{2\lambda\omega_2+6b\lambda\omega_3 + \ldots\}$$

$$= \kappa h_2(\sigma) \tag{28}$$

where κ is a constant. The second order cross-correlation function is therefore directly proportional to the impulse response of the linear system in the non-linear path.

The first and second order cross-correlation function estimates, equations (20) and (28) respectively, can now be used to estimate the coefficients ω_i, i = 1,2...k in the polynomial representation of the non-linear element.

Converting to discrete time and considering N measurements of the sampled process input and output permits the formulation of the matrix equation

$$\begin{bmatrix} Z(1) \\ Z(2) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ Z(N) \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{\ell} \hat{h}_3(j)u(1-j), & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u(1-j) & \cdots & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u^k(1-j) \\ \sum_{j=0}^{\ell} \hat{h}_3(j)u(2-j), & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u(2-j) & \cdots & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u^k(2-j) \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \sum_{j=0}^{\ell} \hat{h}_3(j)u(N-j), & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u(N-j) & \cdots & \sum_{j=0}^{\ell} \{\kappa\hat{h}_2(j)\}u^k(N-j) \end{bmatrix} \begin{bmatrix} p \\ \omega_1' \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \omega_k' \end{bmatrix}$$

$$+ \begin{bmatrix} n(1) \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ n(N) \end{bmatrix}$$

or $$Z = \phi\theta + E \qquad (29)$$

Since all the elements of the matrices Z and ϕ can either be measured or estimated, an unbiased least squares estimate of the coefficients $p = 1/\mu$, ω_i' (i = 1,2...k) can be readily computed

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T Z \tag{30}$$

where $\omega_1 = \rho\beta + \kappa\omega_1'$

$$\omega_t = \kappa\omega_t' \qquad t = 2,\ldots k \tag{31}$$

Combining the results of equations (20) and (28) the linear components of the feedforward system can be estimated as

$$\hat{\rho}\,\hat{h}_3(\sigma) + \hat{\omega}_1'\{\kappa\hat{h}_2(\sigma)\}$$

$$= h_1(\sigma) + \omega_1 h_2(\sigma) \tag{32}$$

and the identification is complete.

Thus from equations (20), (28), (29) and (31) the identified system illustrated in Fig.4 can be synthesised. Although the identification of $h_2(\sigma)$ and the coefficients associated with the non-linear element can be decoupled, the linear system component $h_1(\sigma)+\omega_1 h_2(\sigma)$ can only be decomposed if $\omega_1 = 0$. This corresponds to the situation that would arise if the system was linear $\omega_t = 0$, $t = 2,\ldots k$.

To summarise the identification algorithm for the feedforward system consists of the following steps

1) Compute $\phi_{xz'}(\sigma) = h_3(\sigma) = \mu h_1(\sigma)+\beta h_2(\sigma)$

   $\phi_{xxz'}(\sigma) = \kappa h_2(\sigma)$

2) Apply least squares to fit pulse transfer functions to $\phi_{xz'}(\sigma)$ and $\phi_{xxz'}(\sigma)$ and use these to generate smoothed estimates of $\kappa h_2(\sigma)$ and $h_3(\sigma)$.

3) Insert the smoothed estimates into equation (29) and estimate ρ and ω_i', $(i = 1,2,\ldots k)$.

4) Estimate the linear component using the results of steps (2) and (3)

$$\hat{\rho}\ \hat{h}_3(\sigma)+\hat{\omega}_1'\{\kappa\hat{h}_2(\sigma)\}$$

$$= h_1(\sigma)+\omega_1 h_2(\sigma)$$

and fit a pulse transfer function model if required.

4. SIMULATION RESULTS

The identification procedure outlined above was used to identify the parameters in a feedforward system consisting of a linear element with pulse transfer function

$$H_1(z^{-1}) = \frac{8.0z^{-1}}{1-1.69z^{-1}+0.77z^{-2}} = \frac{n_1z^{-1}+n_2z^{-2}}{1+d_1z^{-1}+d_2z^{-2}} \tag{33}$$

in parallel with the Hammerstein model comprising a non-linear element

$$q(t) = 0.2u^2(t) + 0.3u^3(t) + 0.1u^4(t) \tag{34}$$

in cascade with the linear system

$$H_2(z^{-1}) = \frac{0.1z^{-1}}{1-1.58z^{-1}+0.63z^{-2}} \tag{35}$$

The model was simulated on an ICL 1906S digital computer and 10,000 data points were generated by recording the response to a Gaussian white input sequence N{2.0,4.0}. The first and second order cross-correlation functions were computed and the component subsystems estimated using the algorithm described in the previous section. Because $\omega_1 = 0$ in this example the two linear systems can be decomposed using the results of equations (28) and (32). A comparison of the estimated impulse response and the theoretical weighting sequences of the linear subsystems are illustrated in Figs. 5 and 6 respectively. Least squares estimates of the parameters in the pulse transfer function models and the polynomial representation of the non-linear element are summarised in Table 1.

5. CONCLUSIONS

A procedure for the identification of systems having the structure of a Hammerstein model has been presented. Providing the non-linear system can be excited by a white Gaussian system with non-zero mean the first order cross-correlation function provides an estimate of the impulse response of the linear element. This effectively decouples the identification procedure into two distinct subproblems; parameterization of the linear impulse response and estimation of the coefficients in the polynomial series representation of the non-linear characteristic.

The algorithm has been extended to include the identification of a feedforward system. It has been shown that computation of the first and second order cross-correlation functions permits the linear subsystems to be identified independently of the non-linear element thus simplifying considerably the identification of this class of non-linear systems.

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FIGURE CAPTIONS

Fig 1 The Hammerstein Model

Fig 2 Schematic Diagram of the Identification Procedure for the Hammerstein Model

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Fig 3 A Feedforward System

Fig 4 A Schematic Diagram of the Identification Procedure for the Feedforward System

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Fig 5 A Comparison of Impulse Responses for $h_1(t)$

- - - Experimental values

* * * Theoretical values

Fig 6 A Comparison of Impulse Responses for $h_2(t)$

- - - Experimental values

* * * Theoretical values

PARAMETER	$\hat{d}_1$	$\hat{d}_2$	$\hat{n}_1$	$\hat{n}_2$	γ_2	γ_3	γ_4
Estimated parameters for $H_1(z^{-1})$	-1.687	0.762	9.73	-1.92	0.29	0.284	0.10
Estimated parameters for $H_2(z^{-1})$	-1.61	0.65	0.095	0.002			

TABLE 1 A summary of the identification results

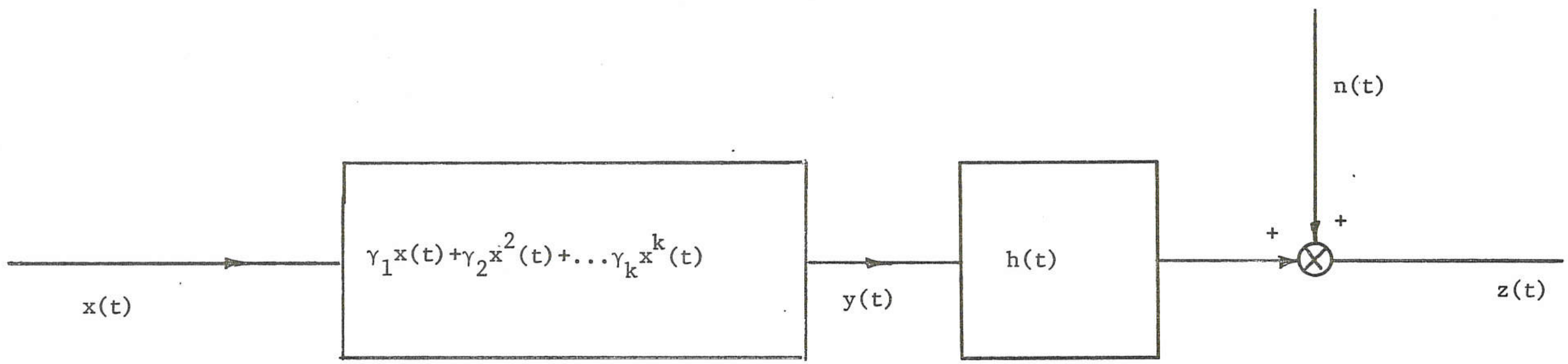

Fig. 1 The Hammerstein Model

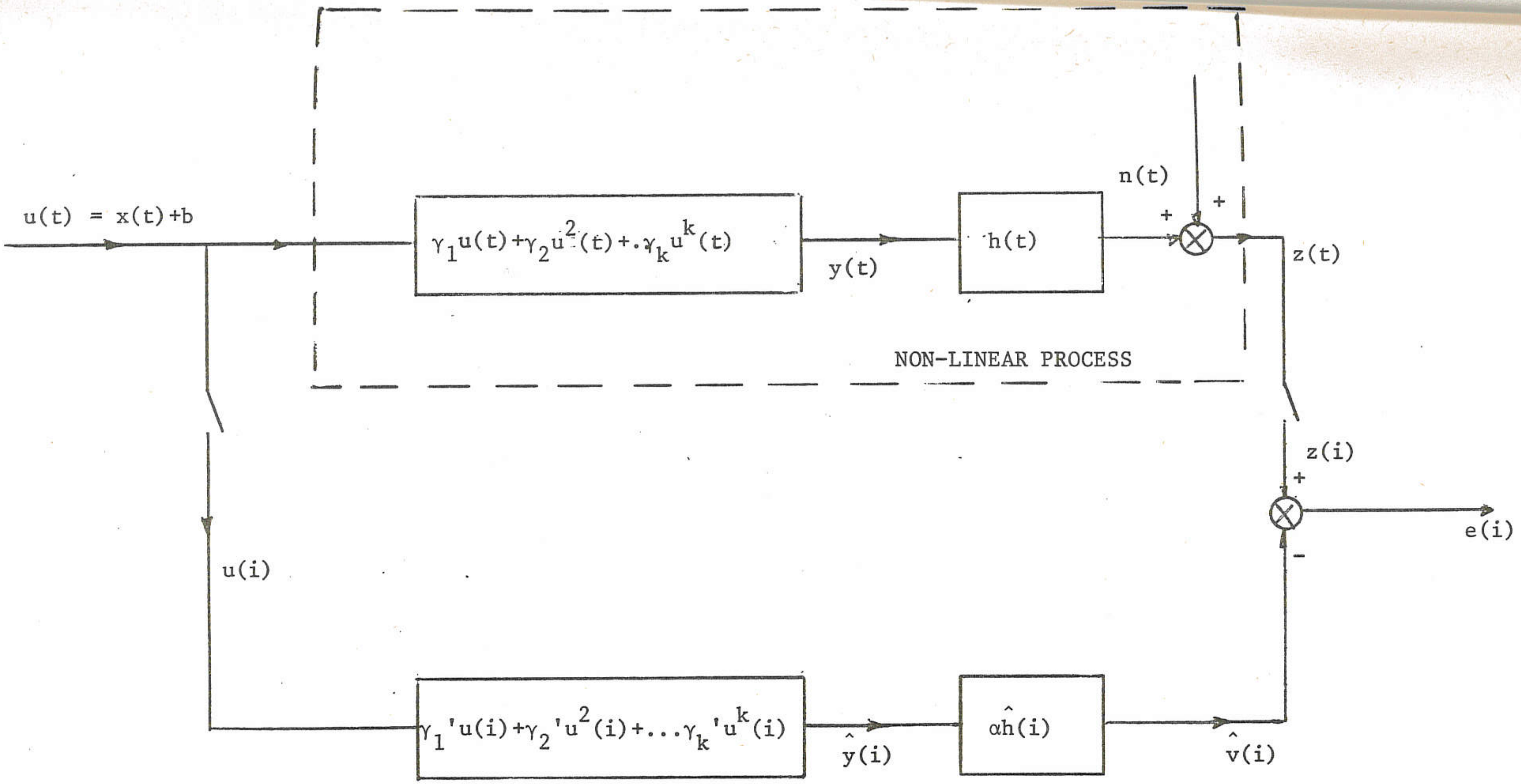

Fig. 2 Schematic diagram of the identification procedure for the Hammerstein Model

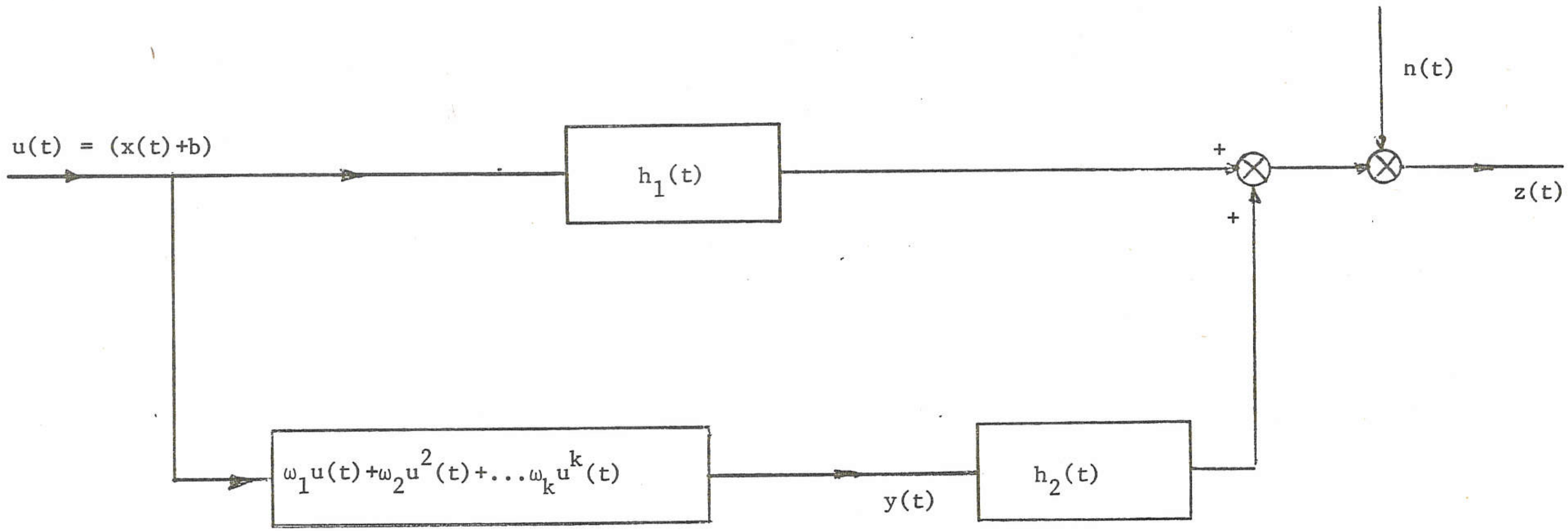

Fig. 3 A Feedforward System

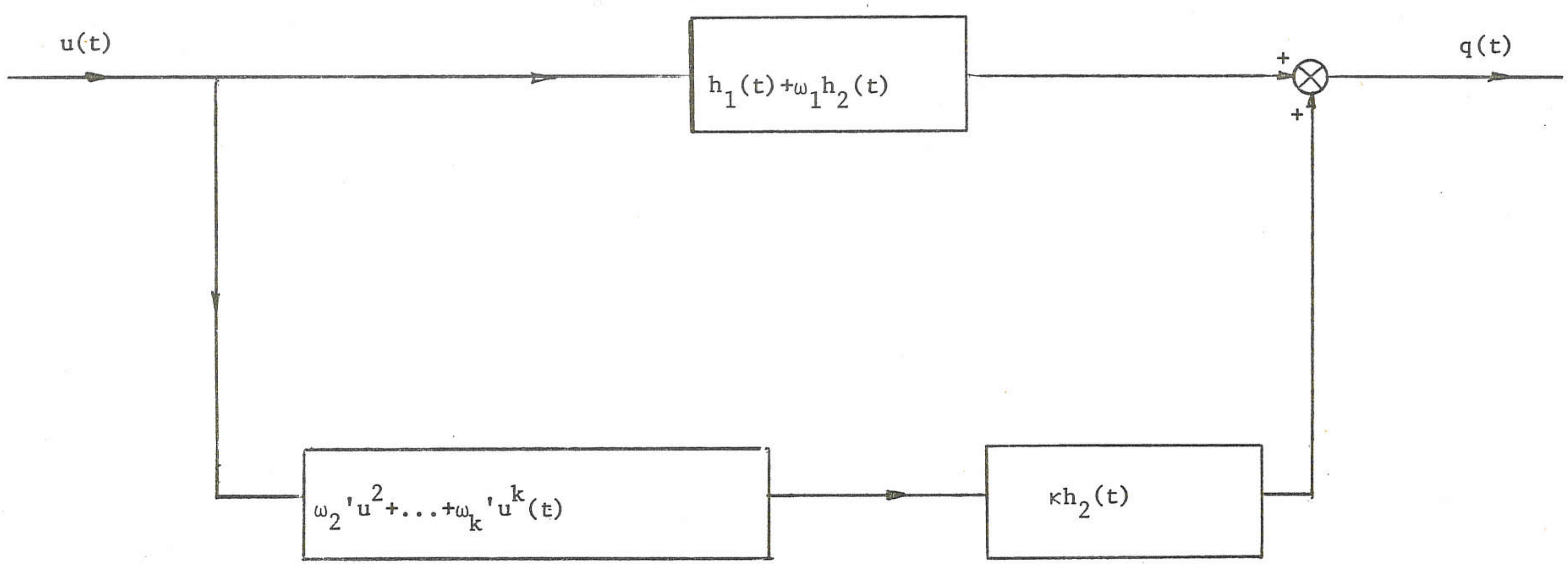

Fig. 4 A Schematic Diagram of the Identification Procedure for the Feedforward System

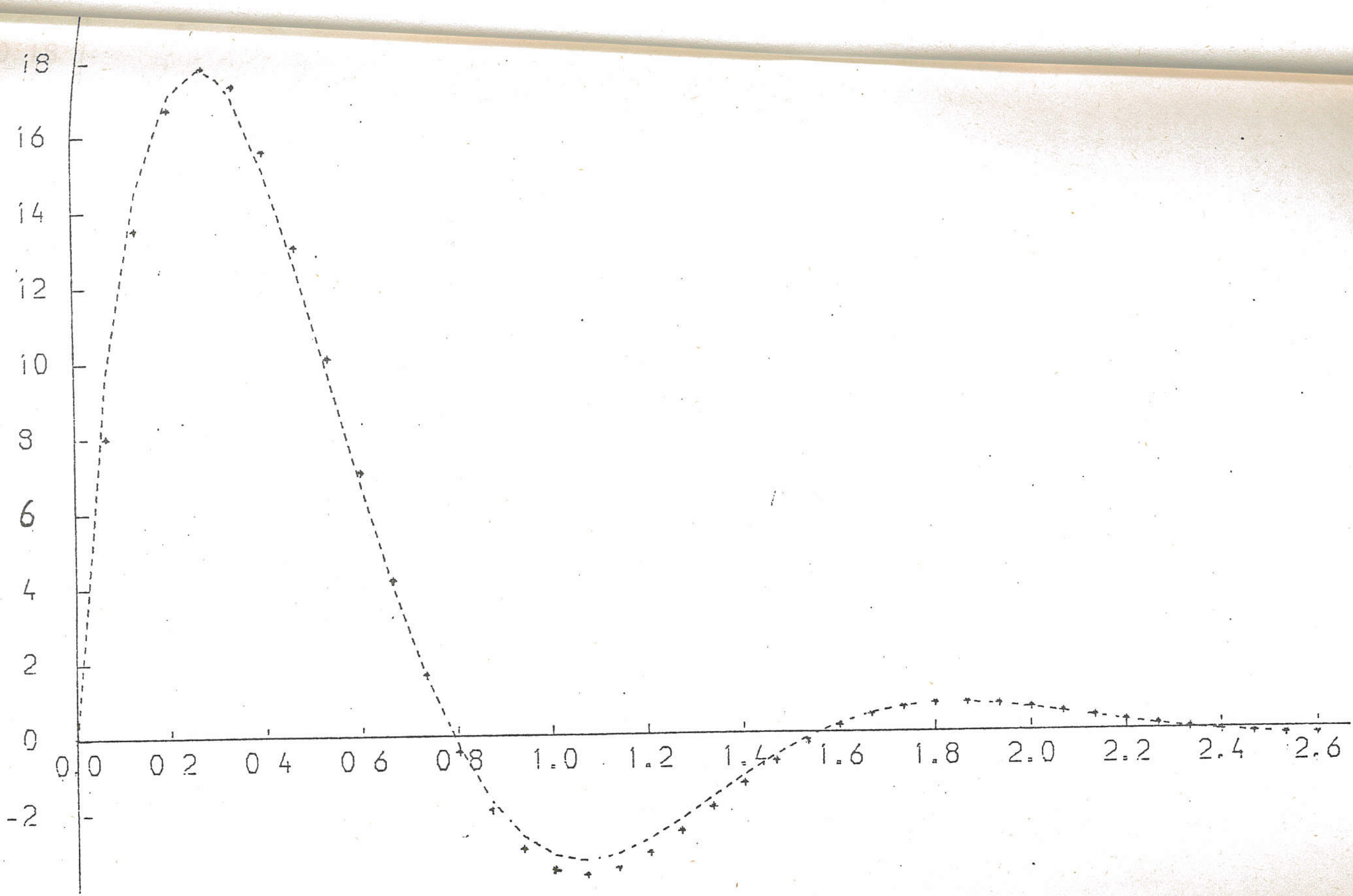

Fig. 5 A Comparison of Impulse Responses for $h_1(t)$

\- - - Experimental values

* * * Theoretical values

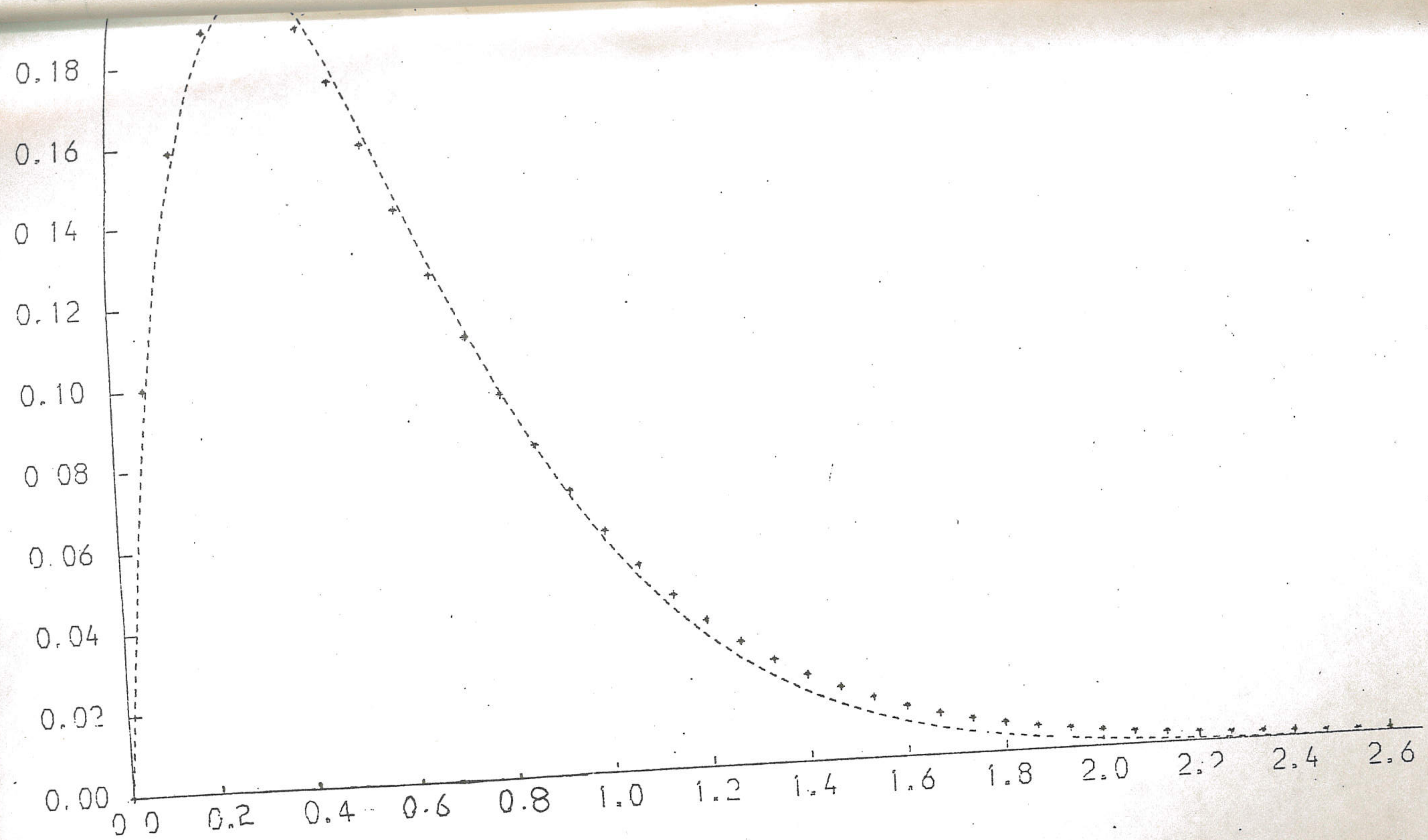

Fig. 6 A Comparison of Impulse Responses for $h_2(t)$

- - - Experimental values

* * * Theoretical values