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IDENTIFICATION OF NON-LINEAR SYSTEMS USING THE WIENER MODEL

by

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ABSTRACT

An algorithm for the identification of non-linear systems which can be described by a Wiener model consisting of a linear system followed by a single-valued non-linearity is presented. Cross-correlation techniques are employed to decouple the identification of the linear dynamics from the characterization of the non-linear element.

INTRODUCTION

Although the functional series expansion of Volterra provides an adequate representation for a large class of non-linear systems, practical identification schemes based on this description often result in an excessive computational burden. It is for this reason that several authors[1,2] have considered the identification of specific configurations of non-linear systems, notably cascade systems composed of linear subsystems with memory and continuous zero-memory non-linear elements.

The Wiener model, illustrated in Fig.1, consists of a linear system followed by a continous no-memory non-linear element. The model is a much simplified version of Wiener's original non-linear system characterization[3] and belongs to the class of models studied by Cameron and Martin[4], and Bose[5]. In the present study, correlation analysis is used to decouple the identification of the linear and non-linear component subsystems when the input is a white Gaussian process. The results of a simulation study are included to illustrate the validity of the algorithm.

IDENTIFICATION OF THE LINEAR SUBSYSTEM

Consider the Wiener model, Fig.1, where the linear time-invariant system has an impulse response $h(t)$ and the continuous single-valued non-linear element can be represented by a finite polynomial of the form

$$y(t) = \gamma_1 q(t) + \gamma_2 q^2(t) + \ldots + \gamma_k q^k(t) \qquad (1)$$

The measured system output, $z(t)$, can then be expressed as

$$z(t) = \gamma_1 \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau + \gamma_2 \iint_{-\infty}^{\infty} h(\tau_1)h(\tau_2)x(t-\tau_1)x(t-\tau_2)\,d\tau_1 d\tau_2$$

$$+ \ldots + \int_{-\infty}^{\infty} \ldots \int h(\tau_1)\ldots h(\tau_k)x(t-\tau_1)\ldots x(t-\tau_k)\,d\tau_1 \ldots d\tau_k$$

$$+ n(t) \qquad (2)$$

which has the form of a Volterra series with the special property that the kernals are separable.

If the input signal $x(t)$ is a zero mean white Gaussian process with a spectral density of 1 watt/cycle, then its i'th dimensional autocorrelation function is given by[6]

$$\overline{x(t_1)x(t_2)\ldots x(t_i)} = 0 \qquad \text{for } i \text{ odd}$$

$$= \sum_i \prod_{n \neq m} \delta(t_n - t_m) \qquad \text{for } i \text{ even} \qquad (3)$$

where the summation is over all ways of dividing 'i' objects into pairs.

If the input to the Wiener model comprises a Gaussian white process $x(t)$ with a mean level 'b', then from eqn (1) the measured system output $z(t)$ is given by

$$z(t) = w_1(t) + w_2(t) + \ldots + w_k(t) + n(t) \qquad (4)$$

where

$$w_i(t) = \gamma_i \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} h(\tau_1)\ldots h(\tau_i)\{x(t-\tau_1)+b\}$$

$$\ldots \{x(t-\tau_i)+b\}\,d\tau_1 \ldots d\tau_i \qquad (5)$$

The first order cross-correlation function between the Gaussian white input $x(t)$ and the measured system output is defined as

$$\phi_{xz}(\sigma) = E\{z(t)x(t-\sigma)\}$$

$$= \overline{w_1(t)x(t-\sigma)} + \overline{w_2(t)x(t-\sigma)} + \ldots$$

$$\ldots + \overline{w_k(t)x(t-\sigma)} + \overline{n(t)x(t-\sigma)} \qquad (6)$$

Evaluating the first term on the rhs of eqn (6)

$$\overline{w_1(t)x(t-\sigma)} = \gamma_1 \int_{-\infty}^{\infty} h(\tau_1)\{\overline{x(t-\tau_1)+b\}x(t-\sigma)}\, d\tau_1$$

$$= \gamma_1 h(\sigma) \qquad (7)$$

Considering the second term on the rhs of eqn (6)

$$\overline{w_2(t)x(t-\sigma)} = \gamma_2 \iint_{-\infty}^{\infty} h(\tau_1)h(\tau_2)\overline{\{x(t-\tau_1)+b\}\{x(t-\tau_2)+b\}x(t-\sigma)}\, d\tau_1 d\tau_2$$

$$= 2b\gamma_2 h(\sigma) \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1 \qquad (8)$$

Similarly, for the third term

$$\overline{w_3(t)x(t-\sigma)} = 3\gamma_3 h(\sigma)\{ \int_{-\infty}^{\infty} h^2(\tau_1)\, d\tau_1 + b^2 \iint_{-\infty}^{\infty} h(\tau_2)h(\tau_3)\, d\tau_2 d\tau_3 \qquad (9)$$

Higher order terms are evaluated in a similar manner.

Collecting terms

$$\phi_{xz}(\sigma) = h(\sigma)\{\gamma_1 + 2b\gamma_2 \int_{-\infty}^{\infty} h(\tau_1)\, d\tau_1 + 3\gamma_3 \int_{-\infty}^{\infty} h^2(\tau_1)\, d\tau_1$$

$$+ 3\gamma_3 b^2 \iint_{-\infty}^{\infty} h(\tau_2)h(\tau_3)\, d\tau_2 d\tau_3 + \ldots\} + \overline{n(t)x(t-\sigma)} \qquad (10)$$

Assuming that the input signal and the noise process are statistically independent, and providing the linear subsystem is stable, bounded-inputs bounded outputs, the first order cross-correlation function eqn (10) becomes directly proportional to the impulse response of the linear system

$$\phi_{xz}(\sigma) = \beta h(\sigma) \qquad (11)$$

This represents an application of a result due to Nuttall[7], who showed that for a wide class of signals the input-output cross-correlation function for a non-linear no-memory device is proportional to the input autocorrelation function.

If the identification is performed with the aid of a digital computer, the cross-correlation function eqn (11) will be in sampled data form and estimates of the coefficients in the pulse transfer function representation of the linear system

$$Z\{\beta h(\sigma)\} = \frac{B(z^{-1})}{A(z^{-1})} \qquad (12)$$

can be obtained using a least squares algorithm.

IDENTIFICATION OF THE NON-LINEAR ELEMENT

Consider the schematic diagram of the identification procedure illustrated in Fig.1. The error e(i) between the sampled process output z(i) and the output of the Wiener model $\hat{y}(i)$ can be defined as

$$e(i) = z(i) - \hat{y}(i) \qquad (13)$$

where $$\hat{y}(i) = \gamma_1{}^{1}\hat{q}(i) + \gamma_2{}^{1}\hat{q}^2(i) \; \ldots + \gamma_k{}^{1}\hat{q}^k(i) \qquad (14)$$

$$\hat{q}(i) = -a_1\hat{q}(i-1) \; \ldots \; -a_\ell\hat{q}(i-\ell) + b_1x(i-1) \; \ldots \; +b_\ell x(i-\ell) \qquad (15)$$

and $\gamma_t = \beta^t \gamma_t{}^1$, $t = 1,2..k$. Combining eqn's (13), (14) and (15), and considering $(N+\ell)$ measurements of the sampled process input and output gives the matrix equation

$$\begin{pmatrix} Z(\ell+1) \\ Z(\ell+2) \\ \vdots \\ Z(\ell+N) \end{pmatrix} = \begin{pmatrix} \hat{q}(\ell+1), \hat{q}^2(\ell+1) & \ldots & \hat{q}^k(\ell+1) \\ \hat{q}(\ell+2), \hat{q}^2(\ell+2) & \ldots & \hat{q}^k(\ell+2) \\ \vdots \quad \vdots & & \vdots \\ \hat{q}(\ell+N), \hat{q}^2(\ell+N) & \ldots & \hat{q}^k(\ell+N) \end{pmatrix} \begin{pmatrix} \gamma_1^{\,1} \\ \gamma_2^{\,1} \\ \\ \gamma_k^{\,1} \end{pmatrix} + \begin{pmatrix} e(\ell+1) \\ e(\ell+2) \\ \vdots \\ e(\ell+N) \end{pmatrix}$$

or $$Z = \phi\theta + E \tag{16}$$

Since all the elements of the matrices Z and ϕ can either be measured or estimated, a least squares estimate of the coefficients $\gamma_j^{\,1}$, $j = 1,2\ldots k$ associated with the non-linear element can be readily computed

$$\hat{\theta} = (\phi^T\phi)^{-1}\phi^T Z \tag{17}$$

and the identification is complete.

SIMULATION RESULTS

The identification procedure outlined above was used to identify the parameters in a Wiener model consisting of a linear system with transfer function

$$G(s) = \frac{1}{s^2 + 6s + 25} \tag{18}$$

in cascade with a non-linear element of the form

$$y(t) = 5.0q(t) + 50.0q^2(t) + 500.0q^3(t) \tag{19}$$

To provide a realistic simulation study, the model was simulated on an Applied Dynamics 4 analogue computer with an input signal produced from the summation of a dc level and the output of a white Gaussian noise generator. Samples of the input-output signals were processed using a CONPAC 4020 process computer to provide an estimate of the sample cross-correlagram illustrated in Fig.2.

Least squares estimates of the parameters in the linear pulse transfer function model and the polynomial representation of the non-linear element are summarised in Table 1.

Parameter	a_1	a_2	b_1	b_2	γ_1	γ_2	γ_3
True value	-1.58	0.67	0.215	0.0	5.0	50.0	500.0
Estimate	-1.57	0.66	0.216	-0.004	5.14	52.42	518.12

Table 1 A Summary of the Identification Results

CONCLUSIONS

A procedure for the identification of systems having the structure of the Wiener model has been presented. Provided the non-linear system can be excited by a Gaussian white process with a non-zero mean, the impulse response of the linear system can be identified independently of the non-linear element. This effectively decouples the identification procedure and simplifies considerably the identification of this class of non-linear system.

REFERENCES

1. NARENDRA, K.S., GALLMAN, P.G.: 'An iterative method for the identification of non-linear systems using a Hammerstein model', IEEE Trans. Automat. Contr., 1966, AC-11, pp.546-550.

2. GARDINER, A.B.: 'Identification of processes containing single-valued non-linearities', Int. J. Control, 1973, 18, (5), pp.1029-1039.

3. WIENER, N.: 'Nonlinear problems in random theory', John Wiley, New York, 1958.

4. CAMERON, R.H., MARTIN, W.T.: 'The orthogonal development of non-linear functions in series of Fourier-Hermite functionals', Ann. Math., 1947, 48, pp.385-389.

5. BOSE, A.G.: 'A theory of non-linear systems', Tech. Report 309, Research Lab. of Electronics, M.I.T., 1956.

6. SCHETZEN, M.: 'Average of the product of Gaussian variables', Quart. Progress Report, 60, Research Lab. of Electronics, M.I.T., 1960, pp.137-141.

7. NUTTALL, A.H.: 'Theory and application of the separable class of random processes', Tech. Report 343, Research Lab. of Electronics, M.I.T., 1958.

Figure Captions

FIG. 1. Schematic diagram of the identification procedure for the Wiener model.

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o Estimated

\- Theoretical

FIG. 2. A comparison of impulse responses.

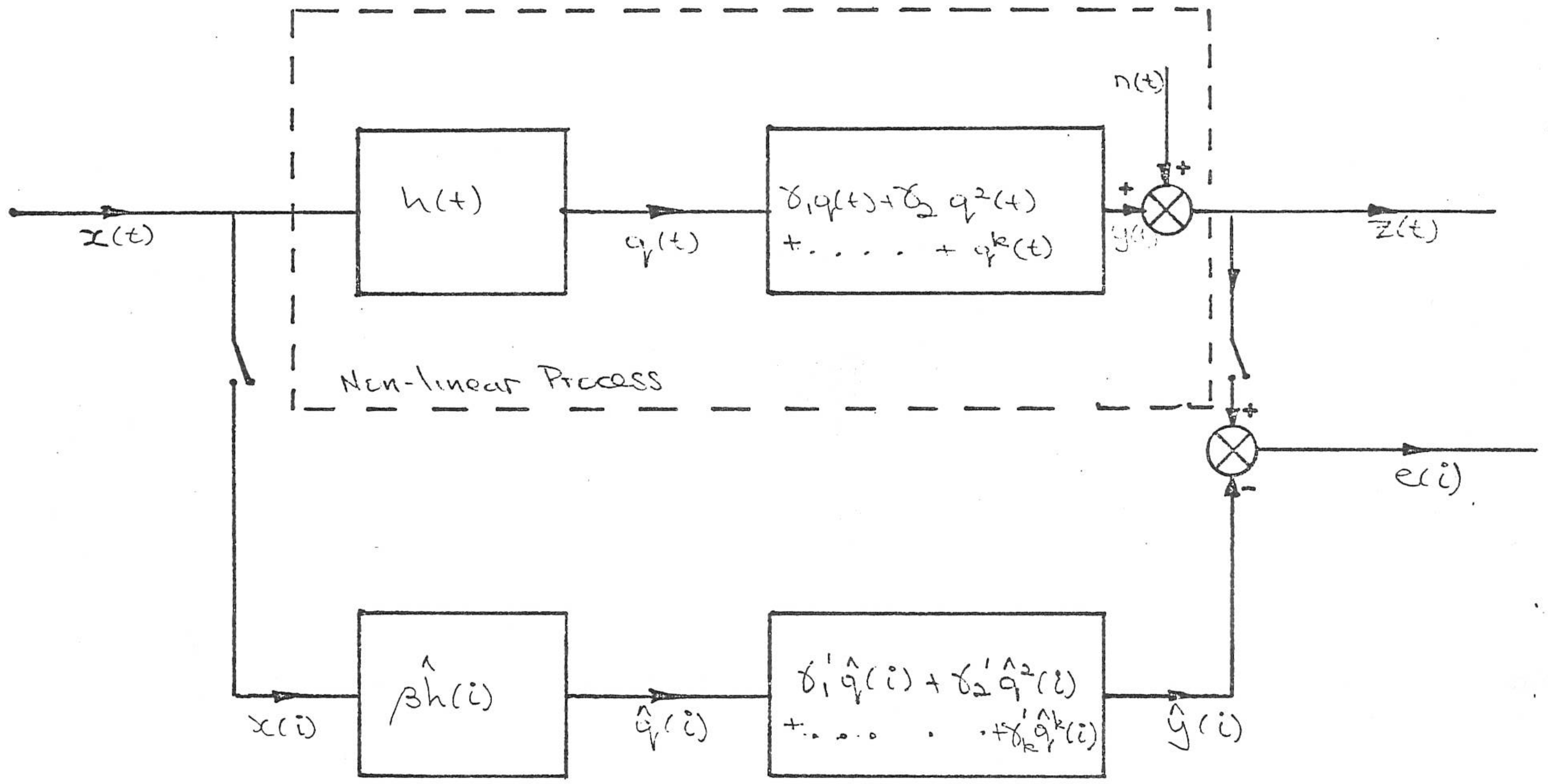

FIG 1. Schematic diagram of the identification procedure for the Wiener model

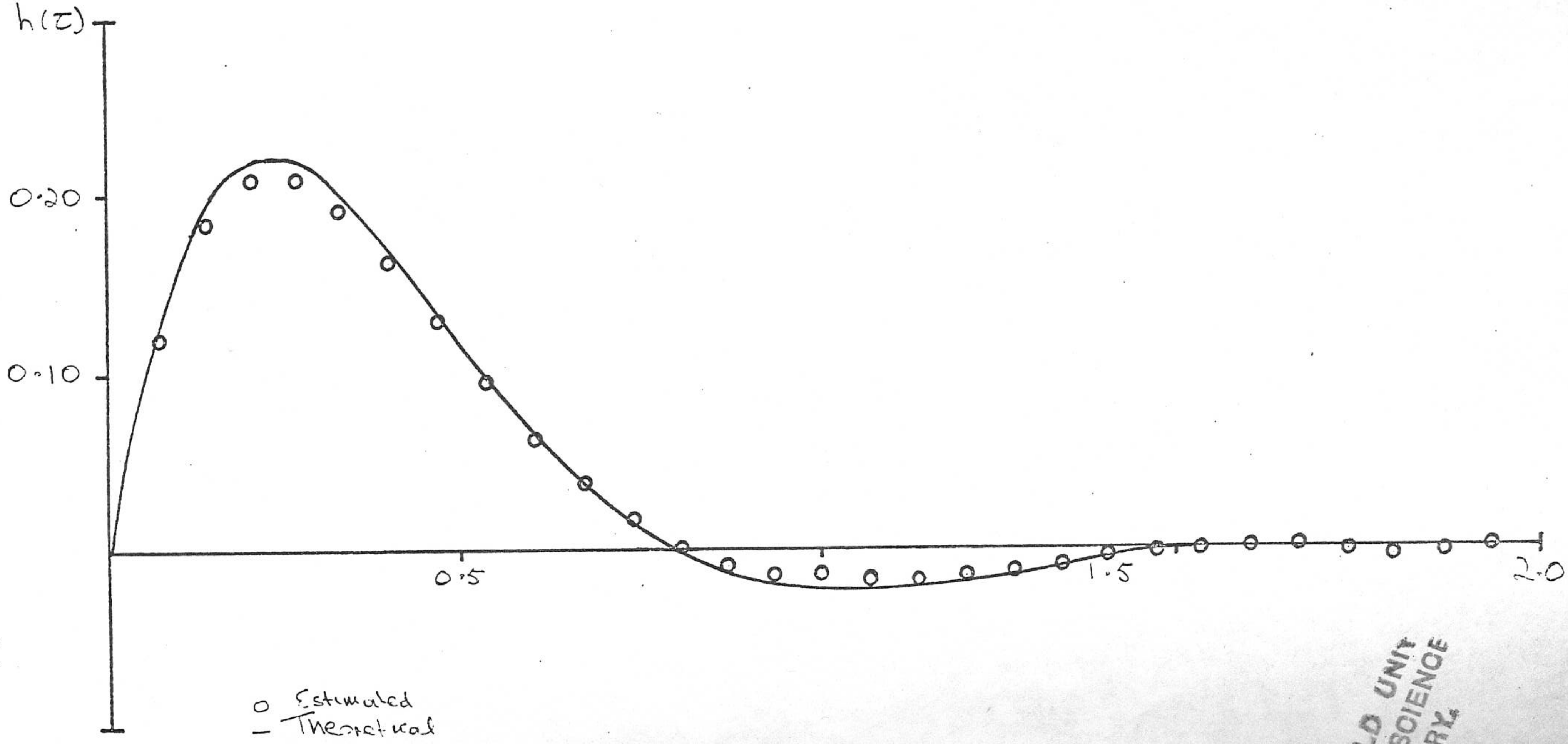

FIG 2 A Comparison of Impulse Responses.