Monograph:
Wei, H.L., Liu, J. and Billings, S.A. (2008) Time-varying parametric modelling and time-dependent spectral characterisation with applications to EEG signals using multi-wavelets. Research Report. ACSE Research Report no. 977 . Automatic Control and Systems Engineering, University of Sheffield

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Research Report No. 977

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April 2008

Time-Varying Parametric Modelling and Time-Dependent Spectral Characterisation with Applications To EEG Signals Using Multi-Wavelets

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Abstract: A new time-varying autoregressive (TVAR) modelling approach is proposed for nonstationary signal processing and analysis, with application to EEG data modelling and power spectral estimation. In the new parametric modelling framework, the time-dependent coefficients of the TVAR model are represented using a novel multi-wavelet decomposition scheme. The time-varying modelling problem is then reduced to regression selection and parameter estimation, which can be effectively resolved by using a forward orthogonal regression algorithm. Two examples, one for an artificial signal and another for an EEG signal, are given to show the effectiveness and applicability of the new TVAR modelling method.

Keywords: Time-varying models, autoregressive (AR) models, system identification, model structure detection, orthogonal least squares, time-dependent spectra, wavelets, EEG.

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1. Introduction

Electroencephalography (EEG) is an important non-invasive technique for medical diagnosis in clinical neurophysiology, as well as for scientific study of brain function in cognitive neuroscience. Electroencephalographic records, or electroencephalograms, contain rich information of some aspects of brain activity associated with particularly mental processes during certain activities and task processing. Compared with other non-invasive techniques, for example, positron emission tomography (PET) and functional magnetic resonance imaging (fMRI), EEG has two main advantages. Firstly, EEG signals typically have very high temporal resolution that can often be at a level of a single millisecond; the temporal resolution of PET and fMRI, however, is often between seconds and minutes. Secondly, EEG directly measures cortical activity; while PET and fMRI record changes in blood flow or metabolic activity that are indirect measurements of neural activity. The main drawback of EEG, compared with fMRI, is perhaps the poor spatial resolution.

Conventionally, EEG analysis mostly relies on visual inspection of relevant EEG signals. In many cases, however, visual inspection of EEG signals may be subjective and insufficient because statistical information contained in EEG signals may not be adequately exploited and utilised. To obtain more relatively objective and reliable analysis results, several methods have been proposed for quantitative analysis of EEG signals. Among these, the Fourier transform based algorithms are the most commonly used tool for revealing the frequency components of EEG signals. The Fourier transform, however, has some disadvantages for dealing with non-stationary EEG signals. Therefore, other parametric and non-parametric spectral estimation methods have been proposed for EEG signal analysis (Gersch and Yonemoto, 1977; Isaksson, 1981; Pascualmarqui et al., 1988; Tseng et al., 1995; Pardey et al., 1996; Muthuswamy and Thakor, 1998; Quiroga et al., 1997, 2002, Guler et al., 2001; Panzica et al., 2003; Subasi, 2007; Zhou et al., 2008).

Autoregressive (AR) models have been successfully applied to the analysis of EEG signals including simulation (Charbonnier et al., 1987; Kaipio and Karjalainen, 1997a), spectral estimation (Madhavan et al., 1991; Medvedev and Willoughby, 1999; Guller et al., 2001; Moller et al., 2001; Subasi, 2007), classification (Wada et al. 1996; Subasi et al., 2005), and synchronization (Franaszczuk and Bergey, 1999). A common routine for dealing with non-stationary EEG signals using time-invariant AR models is to partition a long time-course of data into several segments and then apply an AR modelling approach to each of these segmentations that can be treated as stationary processes (Praetorius et al., 1977; Michael and Houchin, 1979; Barlow, 1985; Amir and Gath, 1989). Time-invariant AR models, estimated from segmented data that are treated as stationary processes, can often reveal the main underlying features of EEG signals. In many cases, however, AR models may not work well for nonstationary EEG signals, where either the data cannot simply be partitioned into several stationary time series, or the segments turn out to be too short that the estimates may be unreliable due to the fact that some segments contain too few data points (Kaipio and Karjalainen, 1997b). This has led to a growing interest in nonstationary signal processing methods for EEG data analysis (Krystal et al., 1999; Prado and Huerta, 2002; Tarvainen et al., 2004, 2006; Pachori and Sircar, 2008).

One solution when dealing with nonstationary signals is to employ time-varying parametric models, where the associated model parameters are allowed to be time-varying or time-dependent. Methods for parametric modelling of nonstationary signals can roughly be categorized into two classes: adaptive recursive estimation and deterministic basis function expansion and regression (Bohlin, 1977; Barlow, 1985). The adaptive recursive estimation methods are a stochastic approach, where the coefficients of the associated models are treated as random processes with some stochastic model structure; the most popular methods to deal with this class of models are the recursive least squares, least-mean squares and Kalman filtering algorithms (Bohlin, 1977; Barlow, 1985; Hayes, 1996). The basis function expansion and regression method is a deterministic parametric modelling approach, where the associated time-varying coefficients are expanded as a finite sequence of pre-determined basis functions; generally, these coefficients are expressed using a linear or nonlinear combination of a finite number of such basis functions. The problem then becomes time invariant, and the unknown new adjustable model parameters are those involved in the expansions. Hence, the initial time-varying modelling problem is reduced to deterministic regression selection and parameter estimation.

This paper aims to introduce a new time-varying AR (TVAR) modelling approach where the time-dependent coefficients are approximated using a finite number of multi-wavelet basis functions. Wavelets have excellent approximation properties that outperform many other approximation schemes and are well suited for approximating general nonlinear signals, even those with sharp discontinuities Wei and Billings, 2007). Wavelets have been successfully applied to EEG signal processing and analysis, see for example Schiff et al. (1994), Kalayci and Ozdamar (1995), Blanco et al. (1998), and Adeli et al. (2003), as well as have been widely used in many other fields including nonlinear signal processing and system identification, see for example Billings and Coca (1999), Liu et al. (2002), Billings and Wei (2005a, 2005b), Wei and Billings (2004a, 2004b, 2006a), and Wei, Billings and Balikhin (2004). However, not much work has been done on exploiting the attractive properties of wavelets and applying them in time-varying system identification. Based upon a multi-wavelet expansion scheme, we propose a new approach for time-dependent parameter estimation. The meaning of the term 'multi-wavelet' here is twofold. Firstly, the time-varying coefficients of the AR model are approximated using several types of wavelet basis functions, that is, the time-dependent parameter estimation involves multiple wavelets. Secondly, these wavelet basis functions are combined in a form of multiresolution wavelet decompositions. Compared with decompositions involving only a single type of wavelets, the multi-wavelet expansion scheme is much more flexible in that it exploits the properties of both smooth and non-smooth wavelet basis functions and thus can effectively track the variations of time-varying coefficients. As will be illustrated later, in comparison with traditional power spectral estimation methods and classical time-invariant AR models, the new time-varying modelling framework using multi-wavelet expansions are more effective for nonstationary EEG signal modelling.

2. The Time-Varying AR Model

The *p*-th order time-varying AR model, TVAR(*p*), is formulated as below

$$y(t) = \sum_{i=1}^{p} a_i(t) y(t-k) + e(t) \tag{1}$$

where *t* is the time instant or sampling index of the signal *y*(*t*), *e*(*t*) is the model residual that can often be treated as a stationary white noise sequence with zero mean and variance σ_e^2, and $a_i(t)$ are the time-varying coefficients.

One solution to the time-varying estimation problem (1) is to approximate the time-varying coefficients $a_i(t)$ using a set of basis functions $\{\pi_m(t): m = 1,2,\cdots,L\}$, where $\pi_m(t)$ are scalar functions, as below

$$a_i(t) = \sum_{m=1}^{L} c_{i,m} \pi_m(t) \tag{2}$$

Substituting (2) into (3), yields

$$y(t) = \sum_{i=1}^{p} \sum_{m=1}^{L} c_{i,m} \pi_m(t) y(t-k) + e(t) \tag{3}$$

Denote

$$\boldsymbol{\pi}(t) = [\pi_1(t), \pi_2(t), \cdots, \pi_L(t)],$$
$$\mathbf{x}_i(t) = y(t-k)\boldsymbol{\pi}(t),$$
$$\mathbf{x}(t) = [\mathbf{x}_1(t), \mathbf{x}_2(t), \cdots, \mathbf{x}_p(t)],$$
$$\mathbf{c}_i = [c_{i,1}, c_{i,2}, \cdots, c_{i,M}],$$
$$\mathbf{c} = [\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_p],$$

Equation (3) can then be written as

$$y(t) = \mathbf{x}(t)\mathbf{c}^T + e(t) \tag{4}$$

where the upper script '*T*' indicates the transpose of a vector or a matrix.

Equation (4) is a standard linear regression model that can be solved using linear least squares algorithms. Let $\hat{\mathbf{c}}$ be the estimate of $\mathbf{c}$, $\hat{a}_i(t)$ be the estimates of $a_i(t)$, and $\hat{\sigma}_e^2$ be the estimate of σ_e^2. The time-dependent spectral function relative to the TVAR model (1) is then given by

$$H(f,t) = \frac{\hat{\sigma}_e^2}{\left|1 - \sum_{i=1}^{p} \hat{a}_i(t) e^{-j2\pi i f / f_s}\right|^2} \tag{5}$$

where $j = \sqrt{-1}$ and f_s is the sampling frequency. Note that the spectral function (5) is continuous with respect to the frequency f and thus can be used to produce spectral estimates at any desired frequencies up to the Nyquist frequency $f_s / 2$. However, the frequency resolution is primarily not infinite, but is determined by the underlying model order and the associated parameters.

Two basic issues are encountered when the basis function expansion and regression approach is applied to general time-varying parametric modelling problems, namely, how to choose the basis functions and how to select the significant ones from the pool of the basis functions. For the first issue, while there are a number of choices and alternatives, for example, Fourier (sinusoidal) bases, Walsh and Haar functions, wavelets, discrete prolate spheroidal sequences, different types of polynomials (including the Chebyshev and Legendre types)(Niedzwiecki, 1988; Wei and Billings, 2002; Chon et al., 2005; Pachori and Sircar, 2008), there is no a guideline on how to choose the appropriate ones from these available basis functions for a specific modelling problem. In fact, each family of basis functions possess its own unique tractability and accuracy, for example, polynomial and Fourier basis functions work well for most smoothly and slowly varying coefficients; Walsh and Haar functions, however, perform well for time-varying coefficients that have sharp variations or piecewise changes.

The second issue involves regression selection and model refinement. For a high dimensional parametric regression modelling problem, the initial full regression model, produced by a basis function expansion approach, often involves a great number of regressors or model terms, whatever types of basis functions are employed. Experience and simulation results have shown that in most cases the initial full regression model may be redundant or ill-posed, meaning that many of the candidate regressors in the initial full regression equation are linearly dependent on the others and therefore can be removed from the model, and the resultant parsimonious model with just a relatively small number of regressors can often produce satisfactory results (Wei and Billings, 2002).

Biomedical signals including EEG records often involve both fast and slowly variations. In order to alleviate the dilemma that the choice of basis functions has to be highly dependent on a priori information on the signals to be studied, and also to make the modelling algorithm more flexible and able to track both fast and slowly varying trends, we propose a new TVAR modelling approach using a multi-wavelet basis function expansion scheme, where properties of different types of wavelets are exploited and combined in a form of multiresolution decompositions.

3. The Multi-Wavelet Basis Functions

From wavelet theory (Mallat, 1989; Chui, 1992), a square integrable scalar function $f \in L^2(\mathbb{R})$ can be arbitrarily approximated using the multiresolution wavelet decomposition below

$$f(x) = \sum_k \alpha_{j_0,k}\phi_{j_0,k}(x) + \sum_{j \geq j_0} \sum_k \beta_{j,k}\psi_{j,k}(x) \tag{6}$$

where the wavelet family $\psi_{j,k}(x)=2^{j/2}\psi(2^j x-k)$ and $\phi_{j,k}(x)=2^{j/2}\phi(2^j x-k)$, with $j,k\in\mathbb{Z}$ ($\mathbb{Z}$ is a set consisting of whole integers), are the dilated and shifted versions of the mother wavelet ψ and the associated scale function ϕ, $\alpha_{j_0,k}$ and $\beta_{j,k}$ are the wavelet decomposition coefficients, j_0 is an arbitrary integer representing the coarsest resolution or scale level. Also, from the properties of multiresolution analysis theory, any square integrable function *f* can be arbitrarily approximated using the basic scale functions $\phi_{j,k}(x)=2^{j/2}\phi(2^j x-k)$ by setting the resolution scale level to be sufficiently large, that is, there exists an integer *J*, such that

$$f(x)=\sum_k \alpha_{J,k}\phi_{J,k}(x) \tag{7}$$

Cardinal B-splines are an important class of basis functions that can form multiresolution wavelet decompositions (Chui, 1992). The first order cardinal B-spline is very the well-known Haar function defined as

$$B_1(x)=\chi_{[0,1)}(x)=\begin{cases}1, & x\in[0,1),\\ 0, & otherwise.\end{cases} \tag{8}$$

The second, third and fourth order cardinal B-splines $B_2(x)$, $B_3(x)$ and $B_4(x)$ are given in Table 1 (Wei and Billings, 2006b). For detailed discussions on cardinal B-splines and the associated wavelets, see Chui (1992).

One attractive feature of cardinal B-splines is that these functions are completely supported, and this property enables the operation of the multiresolution decomposition (6) to be much more convenient. For example, the *m*-th order B-spline is defined on [0, *m*], thus, the scale and shift indices *j* and *k* for the family of the functions $\phi_{j,k}(x)=2^{j/2}B_m(2^j x-k)$ should satisfy $0\le 2^j x-k\le m$. Assume that the function *f*(*x*) that is to be approximated with decompositions (6) or (7) is defined within [0, 1], then for any given scale index (resolution level) *j*, the effective values for the shift index *k*, are restricted to the collection $\{k:-m\le k\le 2^j-1\}$.

Note that while the first and second order B-splines $B_1(x)$ and $B_2(x)$ are non-smooth piecewise functions, which would perform well for signals with sharp transients and burst-like spikes, B-splines of higher order would work well on smoothly changing signals. Motivated by this consideration, this study proposes using multi-wavelet basis functions for TVAR model identification. An example of the new multi-wavelet based algorithm is given in the following.

Take the B-splines of order from 1 to 5 as an example, and consider the decomposition (7). Let $\Gamma_m=\{k:-m\le k\le 2^J-1\}$, with *m*=1,2, …, 5; let $\phi_k^{(m)}(x)=2^{J/2}B_m(2^J x-k)$, with $k\in\Gamma_m$. The time-varying coefficients $a_i(t)$ in (1) can then be approximated using a combination of functions from the families $\{\phi_k^{(m)}:m=1,\cdots,5;k\in\Gamma_m\}$. For example, one such combination can be chosen as,

Table 1 Cardinal B-splines of order from 1 to 4.

	$B_1(x)$	$B_2(x)$	$2\,B_3(x)$	$6\,B_4(x)$
$0 \le x < 1$	1	x	x^2	x^3
$1 \le x < 2$	0	$2-x$	$-2x^2+6x-3$	$-3x^3+12x^2-12x+4$
$2 \le x < 3$	0	0	$(x-3)^2$	$3x^3-24x^2+60x-44$
$3 \le x \le 4$	0	0	0	$-x^3+12x^2-48x+64$
elsewhere	0	0	0	0

$$a_i(t) = \sum_{k\in\Gamma_q} c_{i,k}^{(q)} \phi_k^{(q)}\left(\frac{t}{N}\right) + \sum_{k\in\Gamma_r} c_{i,k}^{(r)} \phi_k^{(r)}\left(\frac{t}{N}\right) + \sum_{k\in\Gamma_s} c_{i,k}^{(s)} \phi_k^{(s)}\left(\frac{t}{N}\right) \tag{9}$$

where $1 \le q < r < s \le 5$, t=1,2, …, N, and N is number of observations of the signal. Simulation results have shown that for most time-varying problems, the choice of q=3, r=4 and s=5 work well to recover the time-varying coefficients. If, however, there is strong evidence that the time-dependent coefficients have sharp changes, then the inclusion of the first and second order B-splines would work well. The decomposition (9) can easily be converted into the form of (2), where the collection $\{\pi_m(t): m = 1,2,\cdots,L\}$ is replaced by the union of the three families: $\{\phi_k^{(q)}(t): k \in \Gamma_q\}$, $\{\phi_k^{(r)}(t): k \in \Gamma_r\}$ and $\{\phi_k^{(s)}(t): k \in \Gamma_s\}$. Further derivation can then lead to the standard linear regression equation (4).

As mentioned earlier, the initial full regression equation (4) may involve a great number of free parameters; the associated regressors may be highly correlated, and the ordinary least squares algorithm may fail to produce reliable results for such ill-posed problems. These problems, however, can easily be overcome by performing an effective model refinement procedure where significant model terms or regressors can be selected one by one (Billings et al., 1989; Chen et al., 1989).

4. Model Identification and Parameter Estimation

The well-known orthogonal least squares (OLS) type of algorithms (Billings et al. 1989; Chen et al., 1989; Aguirre and Billings, 1995; Zhu and Billings, 1996; Wei et al. 2004; Billings and Wei, 2007; Wei and Billings, 2008) have been proven to be very effective to deal with multiple dynamical regression problems, which involve a great number of candidate model terms or regressors that may be highly correlated. In the present study, the OLS algorithm given in Billings et al. (2007), is used to solve the regression equation (4). This includes a model refinement procedure involving the selection of significant regressors and model parameter estimation. The resultant estimates will then be used to recover the time-varying coefficients $a_i(t)$ in the TVAR model (1).

As to the model order determination issue, this can be solved by using some model order determination criteria including the well-known Akaike information criterion (AIC) (Akaike, 1974) and Bayesian information criterion (BIC)(Schwarz, 1978; Efron and Tibshirani, 1993) below:

$$\mathrm{AIC}(p) = \ln(\hat{\sigma}_p^2) + \frac{p}{N}\ln(N) \tag{10}$$

$$\mathrm{BIC}(p) = \frac{N + p[\ln(N) - 1]}{N - p}\ln(\hat{\sigma}_p^2) \tag{11}$$

where $\hat{\sigma}_p^2$ is the variance of the model residuals calculated from the associated p-th order model.

5. Artificial Data Modelling

Prior to applying the proposed TVAR modelling approach to real EEG data analysis, a benchmark on an artificial time-varying signal was considered. The signal was defined as below:

$$y(t) = \begin{cases} 2|t|^{\alpha}\sin(2\pi f_1 t), & t \in [0,2), \\ |t|^{\alpha}\sin(2\pi f_2 t), & t \in [2,4), \\ 2|t|^{\beta}\sin(2\pi f_3 t), & t \in [4,6], \\ 0, & \text{ortherwise.} \end{cases} \tag{12}$$

where α =0.5, β =0.25, f_1 =3Hz, f_2 =8Hz, f_3 =15Hz. The above signal was sampled with a sampling interval 0.01, and thus a total of 600 observations were obtained. A Gaussian white noise sequence, with mean zero and variance of 0.04, was then added to the 600 data points.

A second order TVAR model was estimated to describe the time-varying signal. The third, fourth and fifth order B-splines, as shown by (9) where the scale index (resolution level) J was chosen to be 3, were employed to approximate the time-varying parameters $a_i(t)$ with i=1,2 and n=1,2, …, 600. An OLS algorithm (Billings et al., 2007) was then applied to estimate and refine the model including significant regressor selection and model parameter estimation.

The estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ are shown in Figure 1. The topographical map of the time-dependent spectrum estimated from the TVAR model is shown in Figure 2, and the 2-D image of the time-dependent spectrum produced from the 3-D topographical map is shown in Figure 3. The transient power spectra, calculated at different time instants from t=1 to t=600, were overlapped and these are shown in Figure 4. From these results, it is very clear that the second order TVAR model can characterize the relevant signal very well.

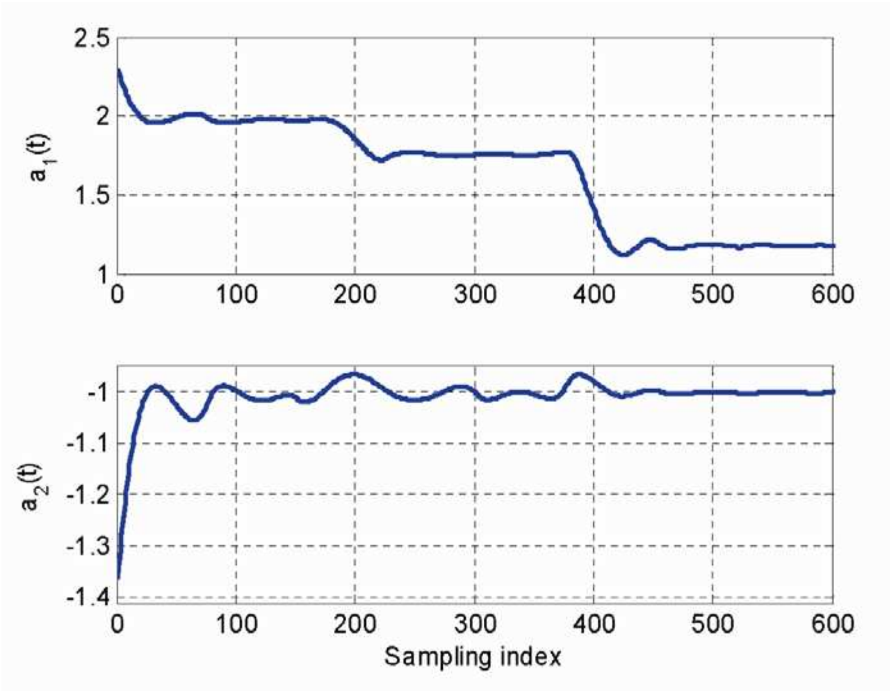


Figure 1 The estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ for the artificial signal presented by (12).

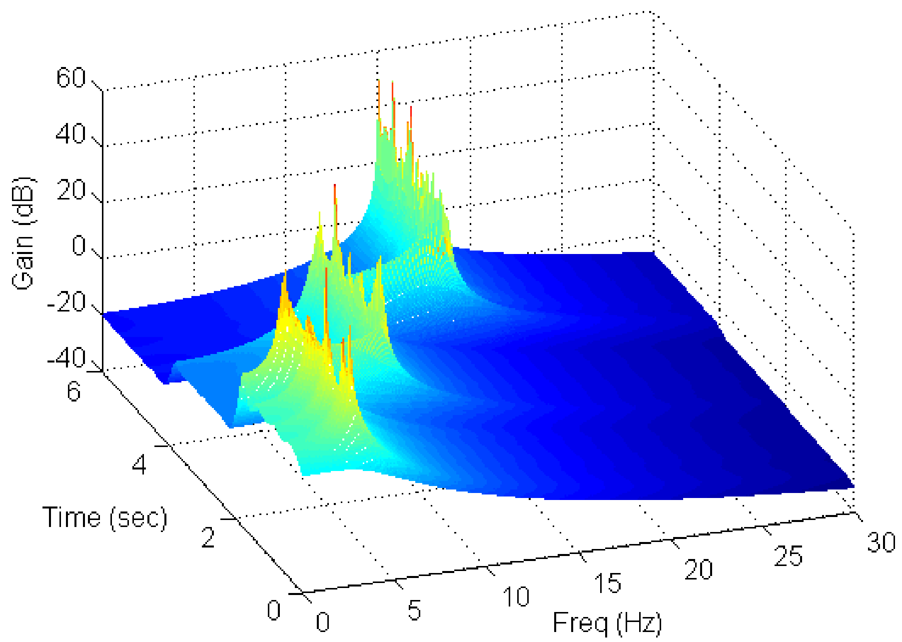


Figure 2 The 3-D topographical map of the time-dependent spectrum estimated from the TVAR(2) model for the signal presented by (12).

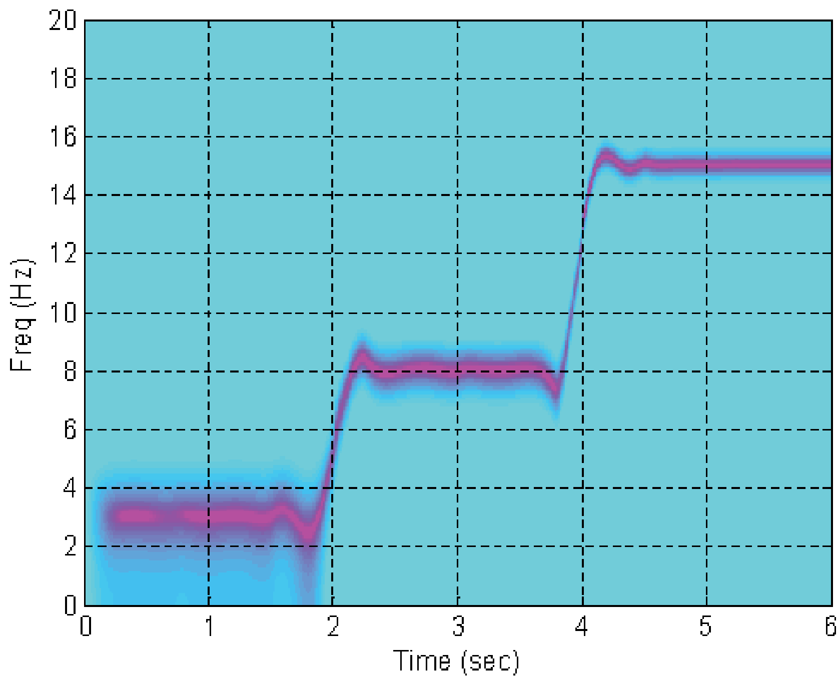


Figure 3 The 2-D image of the time-dependent spectrum produced from the 3-D topographical map shown in Figure 2.

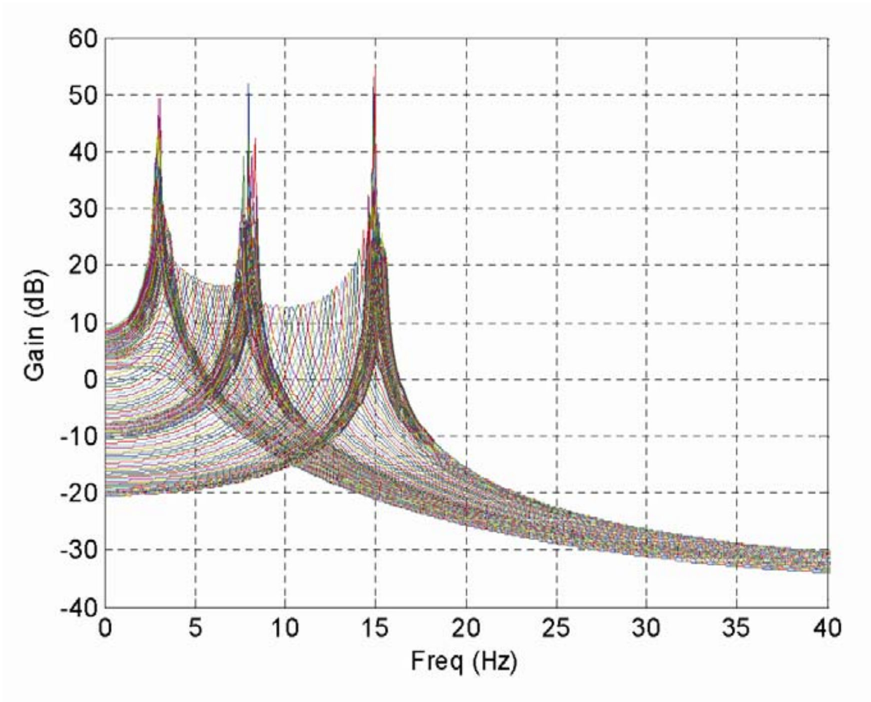


Figure 4 The overlap of the transient power spectra calculated at different time instants from t=1 to t=600 for the problem described by (12).

6. EEG Data Modelling and Analysis

The proposed TVAR modelling framework has been applied to the analysis of numerous EEG recordings. As an example, in the following, an EEG recording given and described in Andrzejak et al. (2001) was considered to illustrate the application of the proposed multi-wavelet based TVAR modelling approach. Figure 5 shows the EEG sequence of 1048 data points, recorded during 6 seconds, with an sampling rate of 173.61 Hz. This recording is for a sort of seizure activity of a patient. A detailed description can be found in Andrzejak et al. (2001).

Similar to the example given in the previous section, the third, fourth and fifth order B-splines, with the resolution level (scale index) J=3, were employed to construct TVAR models for the EEG data. Several TVAR models with different model orders were estimated using the OLS algorithm (Billings et al., 2007), and both the AIC and BIC criteria suggested that the model order can be chosen to be 4 when using these B-splines as building blocks to represent the time-varying coefficients in the TVAR model.

The estimates of the four time-varying coefficients $a_i(t)$ with i=1,2,3,4 are shown in Figure 6. The recovered signal, calculated from the TVAR model using these two time-varying coefficients, is shown in Figure 7, where only part of the data are presented for a clear visualization. The topographical map of the time-dependent spectrum estimated from the TVAR model is shown in Figure 8, and the 2-D image and the contour plot of the time-dependent spectrum produced from the 3-D topographical map are shown in Figure 9.

From Figures 8 and 9, the power spectrum of the EEG signal considered here is mainly distributed in the range from zero to 17 Hz, and three frequency bands can be obviously observed: i) the low frequency band (less than 2.5Hz); ii) the frequency band that is centralized around 6 Hz; and the high frequency band that is centralized around 17Hz. The 2-D image and the associated contour plot of the time-dependent spectrum given in Figure 9 clearly reflects how these frequencies are distributed along the time course of the signal. In other words, the variations of the time course signal can clearly be observed in this 2-D image of transient spectrum. For example, during the period from 2.7 to 3.9s the power spectrum is dominated by a high frequency component (about 17 Hz), and during the period from 4 to 5s the spectrum is dominated by a frequency component (about 6Hz), while during the period from 5 to 6s, the time course is determined by low frequency component (about 3Hz), but with higher frequency (17Hz) activity superposed to it. These properties, possessed by the proposed TVAR model, cannot be obtained using any time-invariant parametric modelling framework including the commonly used AR models.

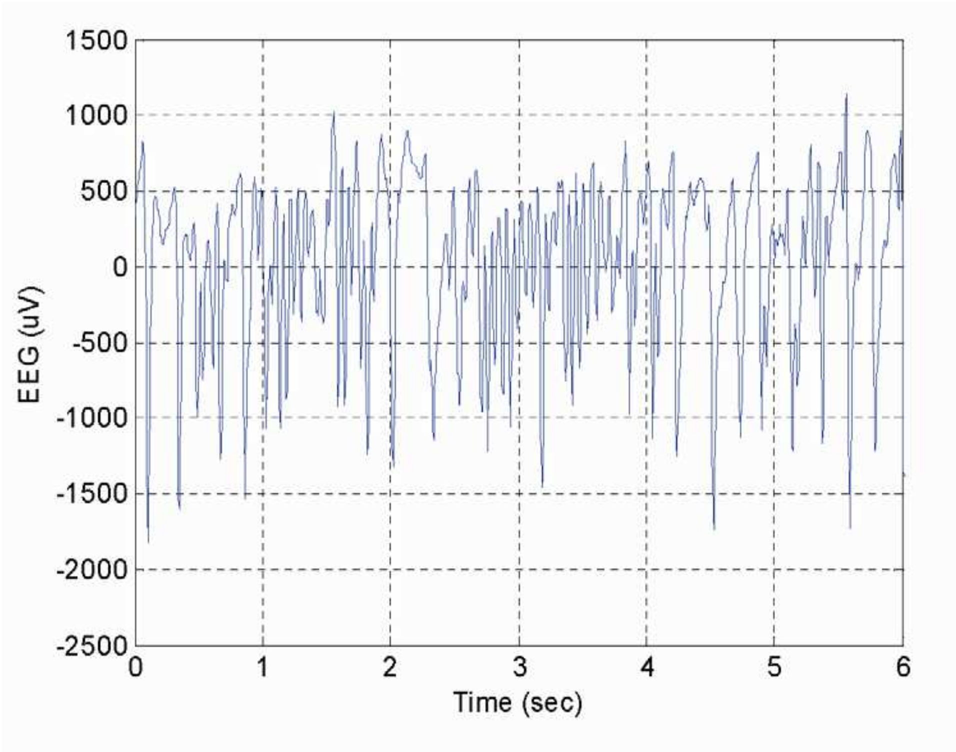


Figure 5 The EEG signal, for a sort of seizure activity of a patient, recorded during 6 seconds, with an sampling rate of 173.61 Hz.

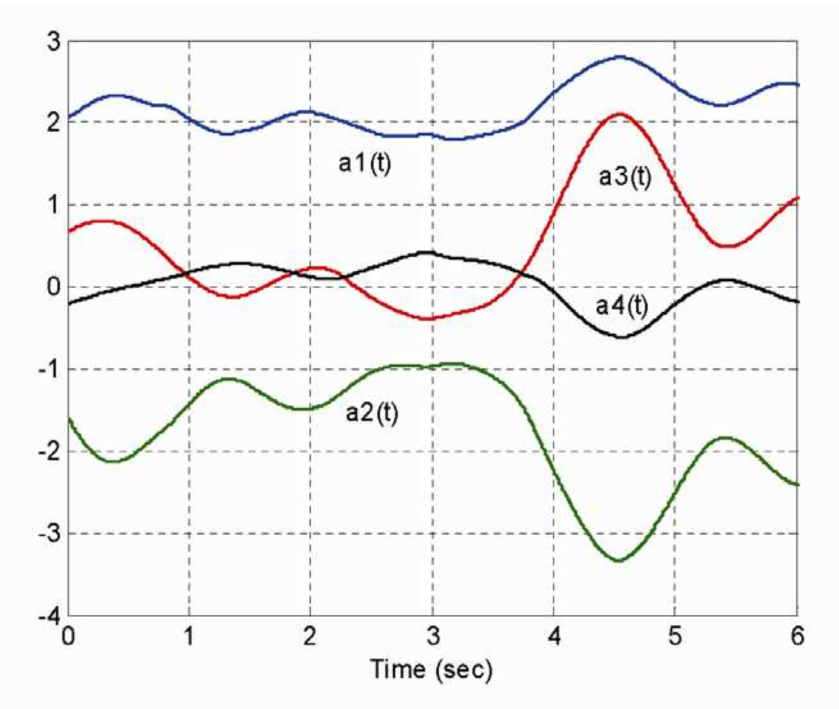


Figure 6 The estimates of the four time-varying coefficients $a_i(t)$ (i=1,2,3,4) for the EEG signal.

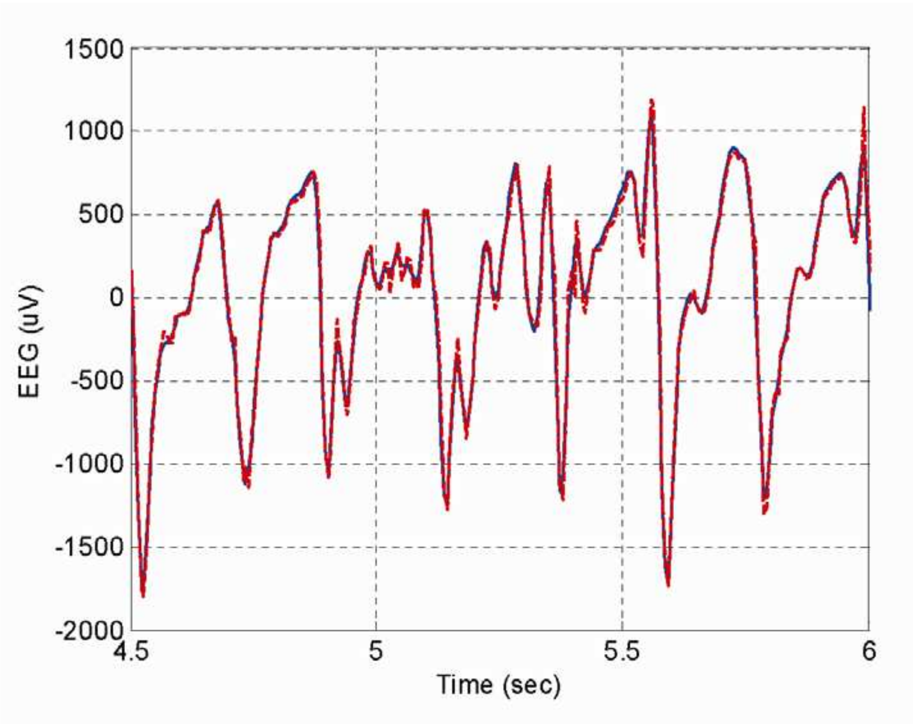


Figure 7 A comparison of the recovered signal from the identified second-order TVAR(4) model and the original observations for the EEG signal. Solid (blue) line indicates the observations and the dashed (red) line indicates the signal recovered from the TVAR(4) model. For a clear visualization only the data points of the period from 4.5 to 6s are displayed.

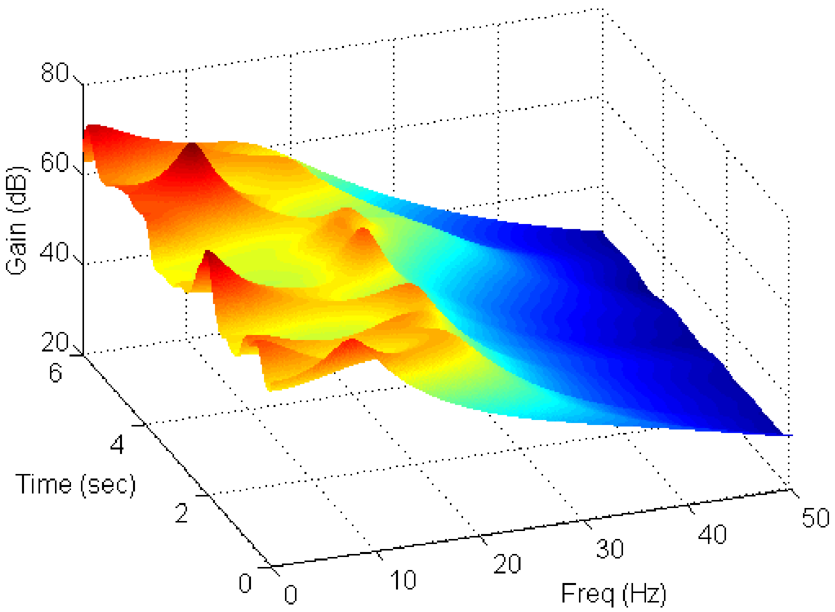
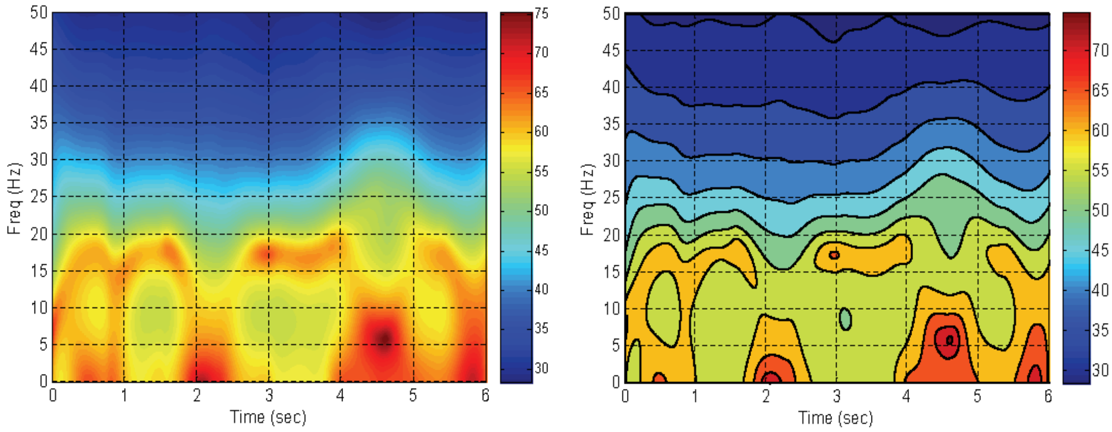


Figure 8 The 3-D topographical map of the time-dependent spectrum estimated from the TVAR(4) model for the EEG signal.



(a) (b)

Figure 9 The 2-D image and the contour plot of the time-dependent spectrum produced from the 3-D topographical map shown in Figure 9. (a) the 2-D image; (b) the contour plot.

7. Conclusions

A new time-varying parametric modelling approach has been developed in this study, where the associated time-dependent coefficients are approximated using multi-wavelet basis functions. The realization of the time-varying AR (TVAR) model here is distinguished from existing time-varying parametric models where the relevant time-dependent coefficients are represented using basis function expansions. In most existing time-varying parametric models, the basis functions used for representing the time-dependent coefficients are global, while the basis functions involved in the new proposed modelling approach are locally defined; the multi-wavelet and multiscale expansion scheme enables the time-varying models to be much more flexible and adaptable for tracking the variations of nonstationary signals including EEG recordings.

The time-dependent spectrum, calculated from the multi-wavelet based TVAR model, has a capability that not only reveals the global frequency behaviour of the signal but also reflects the local variations of the signal along the time course. In this respect, the proposed TVAR model outperforms the traditional time-invariant AR models.

A further study in this direction is to extract more features of EEG signals using the multi-wavelet based TVAR modelling method, so that these can be applied for EEG signal classification, synchronization and other diagnostic tasks.

Acknowledgements

The authors gratefully acknowledge that this work was supported by the Engineering and Physical Sciences Research Council (EPSRC), U.K.

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