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Identification of Partial Differential Equation Models for a Class of Multiscale Spatio-Temporal Dynamical Systems

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Abstract

In this paper, the identification of a class of multiscale spatio-temporal dynamical systems, which incorporate multiple spatial scales, from observations is studied. The proposed approach is a combination of Adams integration and an orthogonal least squares algorithm, in which the multiscale operators are expanded, using polynomials as basis functions, and the spatial derivatives are estimated by finite difference methods. The coefficients of the polynomials can vary with respect to the space domain to represent the feature of multiple scales involved in the system dynamics and are approximated using a B-spline wavelet multiresolution analysis (MRA). The resulting identified models of the spatio-temporal evolution form a system of partial differential equations with different spatial scales. Examples are provided to demonstrate the efficiency of the proposed method.

1 Introduction

In recent years, the modelling, analysis, and simulation of multiscale systems have been extensively studied. Multiscale systems or processes involving multiple scales are common in nature (many examples can be found in E, Engquist, Li, Ren, and Vanden-Eijnden 2006 and references therein). Alternatively, man-made multiscale processes arise by applying multiresolution analysis-type techniques to describe signals and systems (Basseville, Benveniste, Chou, Golden, Nikoukhah, and Willsky 1992, Zhang, Pan, Bao, and Zhang 2002). But no matter how the multiscale systems are generated, multiscale problems are becoming more and more important. There are several reasons for the timing of the current interest, as stated by E and Engquist (2003) " Modelling at the level of a single scale, such as molecular dynamics or continuum theory, is becoming relatively mature, and our computational capability has reached the stage when serious multiscale problems can be contemplated. ” Whilst most of the current models of multiscale dynamical systems are derived from first principles, the identification problem of such systems should not be ignored.

The identification of conventional spatio-temporal dynamical systems has received a lot of attention recently. This has mainly been driven by the need to determine high quality models, which can be used as a basis for analysis and control of this class of systems with high accuracy. Although partial differential equation (PDE) or coupled map lattice (CML) models for such systems can sometimes be derived by analytic modelling methods, often a large number of assumptions have to be made in order to obtain such models. There is a need therefore to develop identification methods to refine, update and validate these models. The identification of CML models of spatio-temporal dynamical systems has been extensively studied over the past few years. Various methods for the identification of local CML models from spatio-temporal observations have already been proposed (Billings and Coca 2002, Mandelj, Grabec and Govekar 2001, Marcos-Nikolaus, Martin-Gonzalez and Sóle 2002, Grabec and Mandeji 1997, Parlitz and Merkwirth 2000). Coca and Billings (2002a,b,c) have also investigated identifying finite element discrete time models of distributed parameter systems based on observations of the evolution of the system and the forcing function. But there are many instances where it would be valuable to be able to determine continuous models such as a system of PDEs to describe continuous spatio-temporal systems. Obviously such models may easily be related to the original system parameters that can provide a clear physical explanation. The identification of PDE models of continuous spatio-temporal systems has been studied by several authors (Coca and Billings 2000, Fioretti and Jetto 1989, Voss, Bunner, and Abel 1998, Travis and White 1985, Phillipson 1971, Niedzwecki and Liagre 2003, Guo and Billings 2006). It is worth noting that while all of the above mentioned methods are for single scale spatio-temporal dynamical systems, there are a few results about the identification and estimation problem of multiscale systems (Digalakis and Chou 1993, Daoudi, Frakt, and Willsky 1999, Le 1995). However, very little has been done for the PDE model identification problem of multiscale systems directly from observations. The objective of this paper is to tackle this problem.

Considering the variety of multiscale systems and phenomena, in this paper a class of multiscale spatio-temporal systems is studied. This class of systems involves different scales with respect to the space domain only. In this paper a novel approach is used to reconstruct the system of PDEs for the class of unknown multiscale spatio-temporal dynamical systems. This new approach represents one of the first algorithms to determine the PDE model terms, and estimate the unknown multiscale parameters, from a given spatio-temporal data set. The approach can be regarded as the inverse of the classical Adam-Moulton method for the numerical solution of differential equations, that is, the multiscale operator of the evolution is estimated from the observed values of the system variables. By using Adams integration, a system of variable coefficient algebraic equations can be obtained for the underlying continuous spatio-temporal system that is discrete in time. The advantages of the Adams-Moulton method over Euler integration is that the former should provide a better fit for less data than the latter, and the latter works well only when the sampling interval is small which might amplify any possible noise. The multiscale coefficients are then approximated using a B-spline wavelets multiresolution analysis method. By adapting system identification techniques, the continuous multiscale operator can then be estimated. This is achieved by using a polynomial estimation of the operator and an orthogonal least squares algorithm (Chen, Billings, and Luo 1989).

The paper is organised as follows. Section 2 introduces the basic idea of the proposed approach and presents the derivation of the system of algebraic equations by using Adams-Moulton formula. The identification algorithm is given in section 3. Section 4 illustrates the proposed approach, and finally conclusions are given in section 5.

2 Problem description

Consider a class of multiscale spatio-temporal dynamical system whose evolution is governed by a system of partial differential equations as follows

$$\frac{\partial y}{\partial t} = L_x(y), x \in \Omega, t \in T \tag{1}$$

where $y(x,t) \in R^n$ is the state variable of the system, $L_x(\cdot)$ is an unknown differential operator with respect to space variable x. $\Omega \subset R^d$ is the spatial domain with boundary $\partial\Omega$. Note that the subscript x in the operator L_x indicates the operator is of multiple scales with respect to x. Assume that the initial and boundary conditions for eqn.(1) are

$$g(y(0,x)) = y_i(x) \tag{2}$$

and

$$h(y(x,t)) = y_b(x,t), x \in \partial\Omega \tag{3}$$

For such a continuous spatio-temporal system, experimental measurements are often available in the form of a series of snapshots $y(x, n\Delta t)$, $n = 0, 1, 2, \cdots$, $x \in \Omega$, where Δt is the time sampling interval. In this paper, it is assumed that all the components of the vector $y(x,t) \in R^n$ at one location x are measurable. The objective is to determine the multiscale differential operator L_x in eqn. (1) from these discrete measured values and no other *a priori* knowledge. To this end, the Adams-Moulton formula (Press, Flannery, Teukolsky, and Vetterling 1992) is used to obtain a discrete representation of eqn. (1). Consider a point x in the spatial domain Ω, let $y_n(x) = y(x, n\Delta t)$, then it follows

$$y_{n+1}(x) = y_n(x) + \int_{n\Delta t}^{(n+1)\Delta t} \frac{\partial y(x,t)}{\partial t} dt = y_n(x) + \int_{n\Delta t}^{(n+1)\Delta t} L_x(y(x,t))dt \tag{4}$$

The Adams-Moulton formula of order p is obtained by integrating a polynomial that interpolates $L_{x,n+1-j}(x), j = 0, 1, \cdots, p-1$, that is

$$y_{n+1}(x) = y_n(x) + \Delta t \sum_{j=0}^{p-1} \alpha_j L_{x,n+1-j}(x) \tag{5}$$

where $L_{x,n+1-j}(x) = L_x(y_{n+1-j}(x))$.

Note that eqn. (5) reduces to Euler integration when $p = 1$. The advantages of Adams-Moulton integration over Euler integration is the former should provide a better fit for less data than the latter and the latter works well only when the sampling interval Δt is small which might amplify any possible noise.

Unlike the numerical problem, in our case $y_n(x), n = 1, 2, \cdots$, is given, and the task is to determine the unknown operator L_x in eqn. (5). If the form of L_x is known then the task is reduced to determining the multiscale parameters only. However, when the form of L_x is unknown, it is necessary to expand L_x using a known set of basis functions or regressors belonging to a given function class. In this paper, the regressor class of polynomial functions is used. Approximating the nonlinear function L_x in (1) using the polynomial approximation space

$$L_x(y(x,t)) = \sum_{i=1}^{M} \beta_i(x) p^i(x) \tag{6}$$

yields the following representation of (5)

$$y_{n+1}(x) = y_n(x) + \Delta t \sum_{j=0}^{p-1} \alpha_j \sum_{i=1}^{M} \beta_i(x) p^i_{n+1-j}(x) \tag{7}$$

where M denotes the order of the polynomial, $\beta_i(x)$ is the coefficient of the ith polynomial term, and $p^i_{n+1-j}(x) = p^i(y_{n+1-j}(x))$ is the corresponding monomial which is the product of different spatial derivatives of $y_{n+1-j}(x)$ at x. These spatial derivatives are difficult to measure in practice therefore they are replaced by their finite difference approximations when applying the identification algorithm. Due to the property of multiple scales, the coefficients $\beta_i(x), i = 1, 2, \cdots, M$ are functions of x or its scaled version $x/\epsilon, 0 < \epsilon < 1$, which need to be approximated. There are many methods can be used to approximate these coefficient functions. In this paper, a B-spline wavelet based multiresolution analysis is used. This method has the advantage that the multiresolution analysis naturally deals with signals in a multiple scale manner.

Let $V_l \subset L^2(R^d), l \in Z$ be a multiresolution analysis with a scaling function ϕ and a wavelet function ψ. In this paper, the scaling function ϕ is chosen as a B-spline function of order m. An approximation to the $\beta_i(x), i = 1, 2, \ldots, M$ in $V_{l+1} = V_l \oplus W_l$, where W_l is the complementary subspace of V_l in V_{l+1}, yields (Chui 1992)

$$\beta_i(x) = \sum_k c^{(i)}_{l,k} \phi_{l,k}(x) + \sum_k d^{(i)}_{l,k} \psi_{l,k}(x) \tag{8}$$

By the property of a multiresolution analysis, $\beta_i(x)$ can be further decomposed into the following form

$$\beta_i(x) = \sum_k c^{(i)}_{l_0,k}\phi_{l_0,k}(x) + \sum_{l\geq l_0}\sum_k d^{(i)}_{l,k}\psi_{l,k}(x) \tag{9}$$

Substituting (9) into (6) yields

$$\begin{aligned} L_x(y(x,t)) &= \sum_{i=1}^{M}(\sum_k c^{(i)}_{l_0,k}\phi_{l_0,k}(x) + \sum_{l\geq l_0}\sum_k d^{(i)}_{l,k}\psi_{l,k}(x))p^i(x) \\ &= \sum_{i=1}^{M}\sum_k c^{(i)}_{l_0,k}\phi_{l_0,k}(x)p^i(x) + \sum_{i=1}^{M}\sum_{l\geq l_0}\sum_k d^{(i)}_{l,k}\psi_{l,k}(x)p^i(x) \end{aligned} \tag{10}$$

The following algebraic equation can then be obtained

$$y_{n+1}(x) = y_n(x) + \sum_{i=1}^{M}\sum_k c^{(i)}_{l_0,k}(\sum_{j=0}^{p-1}\Delta t\alpha_j p^i_{n+1-j}(x))\phi_{l_0,k}(x) + \sum_{i=1}^{M}\sum_{l\geq l_0}\sum_k d^{(i)}_{l,k}(\sum_{j=0}^{p-1}\Delta t\alpha_j p^i_{n+1-j}(x))\psi_{l,k}(x) \tag{11}$$

Note that the k and l in eqns (8) to (11) run from $-\infty$ to $+\infty$. However, due to the property of compact supports of B-spline wavelets, the summations in these equations are always finite. In principle, both the parameters α_j, $c^{(i)}_{l_0,k}$, and $d^{(i)}_{l,k}$ should be calculated during identification. For the sake of simplicity, the values of the α_j are the ones originally dictated by the Adams-Moulton formula. Therefore $c^{(i)}_{l_0,k}$, and $d^{(i)}_{l,k}$ are the only parameters that need to be determined. For the implementation of the identification algorithm, equation (11) needs to be discretised in the space variable x. Note that $p^i_{n+1-j}(x)$ contains some spatial neighbour terms of $y(x, n+1-j)$ like $y(x-1, n+1-j)$ and $y(x+1, n+1-j)$ etc. which depend on the highest order of the spatial derivatives. Therefore, eqn. (11) can be regarded as an implicit Coupled Map Lattice (CML) model representation of the continuous spatio-temporal dynamical system (1). It follows that the orthogonal least squares algorithm proposed by Chen, Billings, and Luo (1989) can then be applied to select the suitable terms and to determine the corresponding coefficients.

3 Identification algorithm

In this section, the identification problem of (11) is considered. Given regression equation (11), all the terms $\sum_{j=0}^{p-1}\Delta t\alpha_j p^i_{n+1-j}(x)\phi_{l_0,k}(x)$, and $\sum_{j=0}^{p-1}\Delta t\alpha_j p^i_{n+1-j}(x)\psi_{l,k}(x)$ form a set of candidate terms. To obtain a simpler model, the objective of the identification algorithm is to select the significant terms from this set while discarding the other terms. In this paper, an Orthogonal

Forward Regression algorithm (OFR) (Chen, Billings, and Luo 1989) is applied, which involves a stepwise orthogonalisation of the regressors and a forward selection of the relevant terms based on the Error Reduction Ratio criterion (Billings, Chen, and Kronenberg 1988). The algorithm provides the optimal least-squares estimate of the coefficients $c_{l_0,k}^{(i)}$ and $d_{l,k}^{(i)}$.

For a given candidate regressor set $G = \{\varphi_i\}_{i=1}^{M}$, the OFR algorithm can be outlined as follows

Step 1

$$I_1 = I_M = \{1, \cdots, M\}$$

$$w_i(t) = \varphi_i(t), \hat{b}_i = \frac{w_i^T y}{w_i^T w_i} \tag{12}$$

$$l_1 = arg \max_{i \in I_1} (\hat{b}_i^2 \frac{w_i^T y}{y^T y}) = arg \max_{i \in I_1} (err_i) \tag{13}$$

$$w_1^0 = w_{l_1}, c_1^0 = \frac{w_1^{0T} y}{w_1^{0T} w_1^0} \tag{14}$$

$$a_{1,1} = 1 \tag{15}$$

Step $j, j > 1$

$$I_j = I_{j-1} \backslash l_j - 1 \tag{16}$$

$$w_i(t) = \varphi_i(t) - \sum_{k=1}^{j-1} \frac{w_k^{0T} y}{w_k^{0T} w_k^0} w_k^0, \hat{b}_i = \frac{w_i^T y}{w_i^T w_i} \tag{17}$$

$$l_j = arg \max_{i \in I_j} (\hat{b}_i^2 \frac{w_i^T y}{y^T y}) = arg \max_{i \in I_j} (err_i) \tag{18}$$

$$w_j^0 = w_{l_j}, c_j^0 = \frac{w_j^{0T} y}{w_j^{0T} w_j^0} \tag{19}$$

$$a_{k,j} = \frac{w_k^{0T} \varphi_{l_j}}{w_k^{0T} w_k^0}, k = 1, \cdots, j - 1. \tag{20}$$

The procedure is terminated at the M_s-th step when the termination criterion

$$1 - \sum_{i=1}^{M_s} err_i < \rho \tag{21}$$

is met, where ρ is a designated error tolerance, or when a given number of terms in the final model is reached.

The estimated coefficients are calculated from the following equation

$$\begin{pmatrix} \theta_{l_1} \\ \theta_{l_2} \\ \vdots \\ \theta_{l_{M_s}} \end{pmatrix} = \begin{pmatrix} 1 & a_{1,2} & \cdots & a_{1,M_s} \\ 0 & 1 & \vdots & a_{2,M_s} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} c_1^0 \\ c_2^0 \\ \vdots \\ c_{M_s}^0 \end{pmatrix} \tag{22}$$

and the selected terms are $\varphi_{l_1}, \cdots, \varphi_{l_{M_s}}$.

4 Numerical simulation and analysis

Consider the following hyperbolic model equation in one space dimension

$$\frac{\partial y}{\partial t} + a(x)\frac{\partial y}{\partial x} = f(x,t) \tag{23}$$

with $x \in \Omega = [0,1]$, and initial condition

$$y(x,0) = \begin{cases} \sin^2(4\pi x), & 0 \leq x \leq 0.25 \\ 0, & \text{otherwise} \end{cases} \tag{24}$$

and boundary condition $y(0,t) = 0$. Note that here a backward difference operator is used in a fourth-order Runge-Kutta method to obtain a numberical solution so that the other boundary condition $y(1,t)$ is not necessary.

To test the proposed identification algorithm, three cases are investigated.

Case 1. Periodic coefficient

$$a(x) = 2 - \cos(5/2\pi x) \tag{25}$$

Case 2. Coefficient with a continuum of scales

$$a(x) = 2 - \sin(\pi \tan(\pi x)) \tag{26}$$

Case 3. Random coefficient. In this case $a(x)$ is a random variable on interval $[0.1, 1]$ with a uniform distribution.

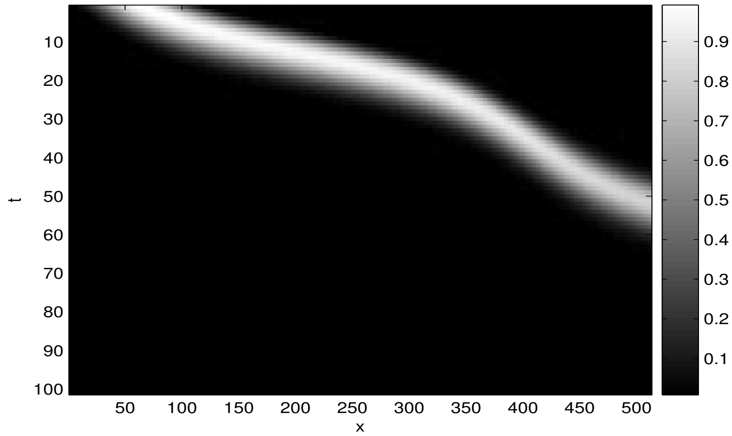

Figure 1: Data $y(x,t)$ for case 1

For the purpose of identification using the proposed approach, the PDE (23) with $f(x,t) \equiv 0$, were numerically solved for all three cases by a fourth-order Runge-Kutta method with a space step $\Delta x = 1/512$. The data with a time length 1 and a time step $\Delta t = 0.01$ are plotted in Figs.(1) to (3).

A set of 3000 spatio-temporal observations randomly selected out of 513×101 data points was used for the identification. In the simulation, the highest order of the derivatives with respect to the spatial variables was set to be 1. The 3rd Adams-Moulton integration formula was used and the polynomial expansion of order 2 was used. The order of B-spline was set to be 3, 3, 2, initial scale was 0, 0, 0, and the maximal resolution was 3, 4, 3 for the three cases, respectively. In order to obtain simple models, the number of final model terms was set to be 10. The identified terms and parameters using the orthogonal least squares algorithm for all the three cases are listed in Tables (1) to (3), where ERR denotes the Error Reduction Ratio. The corresponding approximated coefficient functions are

Case 1

$$\begin{aligned}\tilde{a}(x) = \; & 3.0446\phi_{0,0}(x) + 96.774\psi_{0,0}(x) - 63.173\psi_{1,-1}(x) + 13.015\psi_{1,0}(x) \\ & -1.9225\psi_{1,1}(x) + 0.62128\psi_{2,-3}(x) - 0.11742\psi_{2,-1}(x) + 0.031824\psi_{2,0}(x) \\ & -0.15484\psi_{2,2}(x) - 0.16752\psi_{3,2}(x)\end{aligned} \tag{27}$$

Case 2

$$\begin{aligned}\tilde{a}(x) = \; & 4.853\phi_{0,0}(x) + 60.837\psi_{0,0}(x) - 34.661\psi_{1,-1}(x) + 9.6247\psi_{1,0}(x) \\ & -0.93658\psi_{1,1}(x) - 1.1917\psi_{2,0}(x) - 1.0972\psi_{2,2}(x) + 0.62602\psi_{3,-2}(x) \\ & -0.47107\psi_{3,-1}(x) - 0.50883\psi_{4,-4}(x)\end{aligned} \tag{28}$$

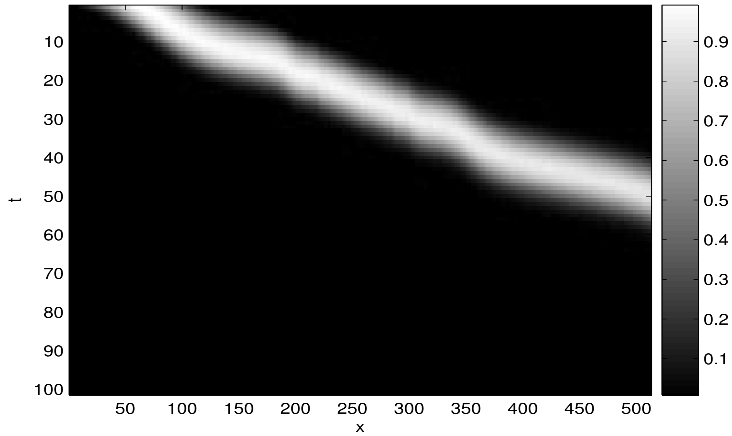

Figure 2: Data $y(x, t)$ for case 2

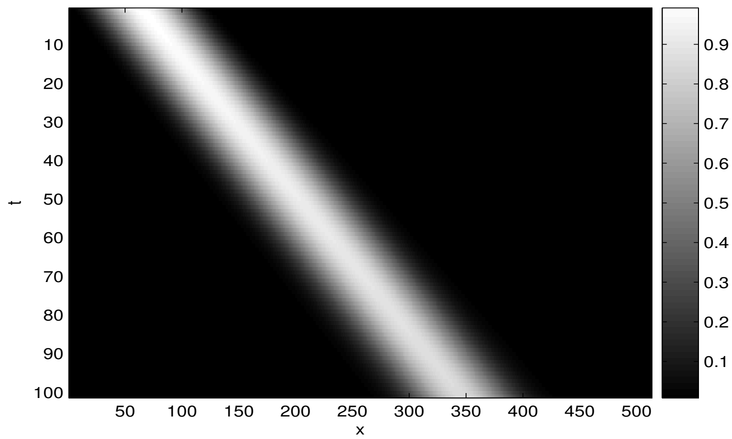

Figure 3: Data $y(x, t)$ for case 3

Variables	Terms	Estimates	ERR
$y_{n+1}(x) - y_n(x)$	$\phi_{0,0}(x)\partial y/\partial x$	-3.0446e+000	5.1278e-001
	$\psi_{0,0}(x)\partial y/\partial x$	-9.6774e+001	1.3036e-001
	$\psi_{1,-1}(x)\partial y/\partial x$	6.3173e+001	1.2241e-001
	$\psi_{1,0}(x)\partial y/\partial x$	-1.3015e+001	2.0299e-001
	$\psi_{1,1}(x)\partial y/\partial x$	1.9225e+000	3.0368e-002
	$\psi_{2,-1}(x)\partial y/\partial x$	1.1742e-001	6.0335e-004
	$\psi_{2,-3}(x)\partial y/\partial x$	-6.2128e-001	1.0961e-004
	$\psi_{2,2}(x)\partial y/\partial x$	1.5484e-001	5.2546e-005
	$\psi_{2,0}(x)\partial y/\partial x$	-3.1824e-002	1.4463e-005
	$\psi_{3,2}(x)\partial y/\partial x$	1.6753e-001	1.0272e-005

Table 1: The terms and parameters of the final model for case 1

Variables	Terms	Estimates	ERR
$y_{n+1}(x) - y_n(x)$	$\phi_{0,0}(x)\partial y/\partial x$	-4.8530e+000	6.7578e-001
	$\psi_{0,0}(x)\partial y/\partial x$	-6.0837e+001	6.6260e-002
	$\psi_{1,0}(x)\partial y/\partial x$	-9.6247e+000	4.6171e-002
	$\psi_{1,-1}(x)\partial y/\partial x$	3.4661e+001	8.8810e-002
	$\psi_{1,1}(x)\partial y/\partial x$	9.3658e-001	4.2055e-002
	$\psi_{2,0}(x)\partial y/\partial x$	1.1917e+000	1.1111e-002
	$\psi_{4,-4}(x)\partial y/\partial x$	5.0883e-001	8.8780e-003
	$\psi_{3,-2}(x)\partial y/\partial x$	-6.2602e-001	1.0917e-002
	$\psi_{2,2}(x)\partial y/\partial x$	1.0972e+000	5.0819e-003
	$\psi_{3,-1}(x)\partial y/\partial x$	4.7107e-001	4.2225e-003

Table 2: The terms and parameters of the final model for case 2

Case 3

$$\begin{aligned}
\tilde{a}(x) \; = \; & 1.0456\phi_{0,0}(x) + 1.1490\psi_{0,0}(x) - 0.31879\psi_{1,-1}(x) + 0.38996\psi_{1,0}(x) \\
& -0.39072\psi_{1,1}(x) - 0.0013772\psi_{2,0}(x) - 0.00071789\psi_{2,1}(x) + 0.00067881\psi_{3,-2}(x) \\
& +0.0020567\psi_{3,-1}(x) + 0.00084517\psi_{3,0}(x)
\end{aligned} \tag{29}$$

The identified coefficient function $\tilde{a}(x)$ and the original coefficient function $a(x)$ are shown in Figs. (4) to (6) for the three cases. From (27), it can be observed that the wavelet components with low frequencies have large wavelet coefficients while the high frequency components have small coefficients (the coefficients are all less than 1.0 for all of the 2^2- and 2^3-components). This reflects the basic feature of the original function $a(x) = 2 - \cos(5/2\pi x)$ in case 1, which is smooth, linear with a frequency 5/4. Fig.(4) clearly shows that the proposed identification algorithm can produce an excellent result for this kind of signals. For case 2, it can be seen from (28)that the identified $\tilde{a}(x)$ is a mixture of high and low frequency components. This indicates

Variables	Terms	Estimates	ERR
$y_{n+1}(x) - y_n(x)$	$\phi_{0,0}(x)\partial y/\partial x$	-1.0456e+000	8.3695e-001
	$\psi_{2,1}(x)\partial y/\partial x$	-7.1789e-004	6.1608e-002
	$\psi_{1,-1}(x)\partial y/\partial x$	3.1879e-001	3.6343e-002
	$\psi_{1,0}(x)\partial y/\partial x$	-3.8996e-001	4.2967e-002
	$\psi_{1,1}(x)\partial y/\partial x$	3.9072e-001	2.0281e-002
	$\psi_{0,0}(x)\partial y/\partial x$	-1.1490e+000	9.0303e-005
	$\psi_{3,-1}(x)\partial y/\partial x$	-2.0567e-003	5.5705e-006
	$\psi_{2,0}(x)\partial y/\partial x$	1.3772e-003	4.3753e-006
	$\psi_{3,-2}(x)\partial y/\partial x$	-6.7881e-004	2.2239e-007
	$\psi_{3,0}(x)\partial y/\partial x$	-8.4517e-004	3.5347e-007

Table 3: The terms and parameters of the final model for case 3

that the signal is essentially nonlinear which is coincident with the property of the original signal $a(x) = 2 - \sin(\pi \tan(\pi x))$. Moreover, it is interesting to notice from Fig.(5), for the fast oscillating part (in the middle of the plot) of $a(x)$ the $\tilde{a}(x)$ look like a smoothed or averaged version of the original signal. This seems to indicate that the obtained PDE can be considered as a homogenisation of the original PDE, which represents the coarse behaviour of the underlying system. This happens for case 3 as well (see Fig. (6)) while note that since $a(x)$ in this case is a random signal so that it is not possible to identify the signal itself. One of the reasons for this phenomenon may be from the selected approximation space for the $a(x)$, which is V_5 and V_4 for case 2 and case 3 while the frequency ranges of the orignal signal $a(x)$ are $256 = 2^8$Hz and ∞ for case 2 and case 3, respectively. To further verify the identified results, the hyperbolic model equation (23) were numerically simulated using a fourth-order Runge-Kutta method again but with $\tilde{a}(x)$ for case 2 and case 3. The simulation results and the errors are plotted in Figs. (7) to (10), which show good performance for all two cases. Moreover, Figs. (8) and (10) show that the evolutions from the identified models are slightly slower than the original systems, which reflects the influence of the rapidly osillatory parameters on the behavior of the systems at an coarse scale.

5 Conclusions

A new approach for the identification of PDE models of a class of multiscale continuous spatio-temporal dynamical systems has been introduced. It has been shown that by combining the Adams integration and the OFR algorithm, a system of PDEs for the underlying continuous spatio-temporal system can be obtained. It has been demonstrated that the proposed method is very effective for systems with mild oscillating parameters. For those systems with high oscillating parameters, a PDE can be identified to reflect the average behaviour of the original systems. Further studies involve dealing with noisy data and the systems with both different time and space scales.

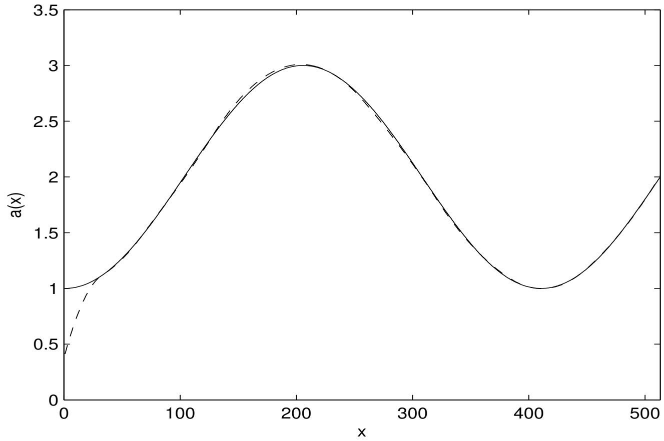

Figure 4: $\tilde{a}(x)$ (dashed) and $a(x)$ (solid) for case 1

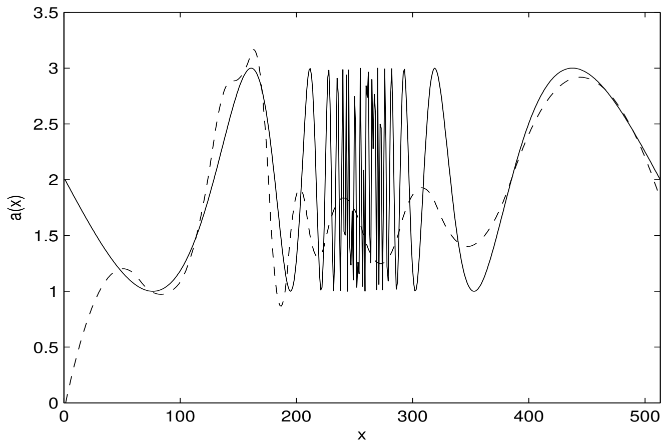

Figure 5: $\tilde{a}(x)$ (dashed) and $a(x)$ (solid) for case 2

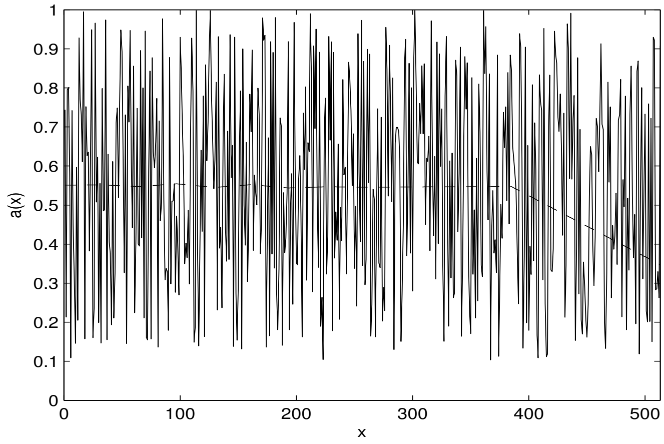

Figure 6: $\tilde{a}(x)$ (dashed) and $a(x)$ (solid) for case 3

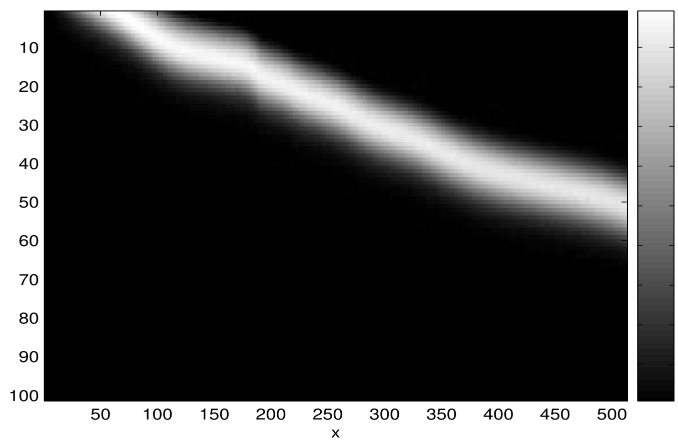

Figure 7: $y(x,t)$ calculated from (23) with $\tilde{a}(x)$ for case 2

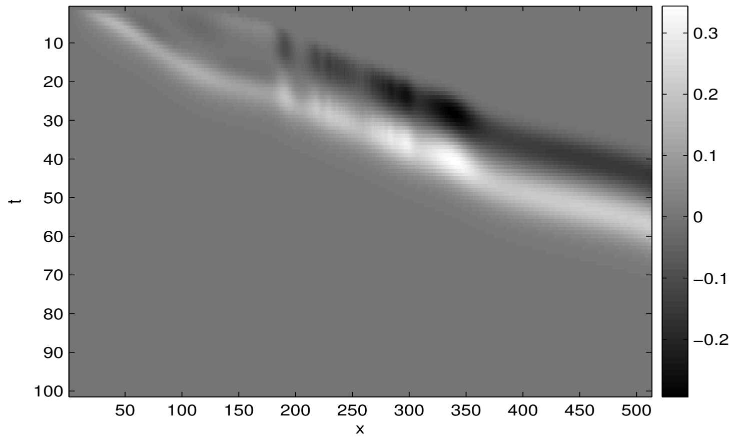

Figure 8: Error for case 2

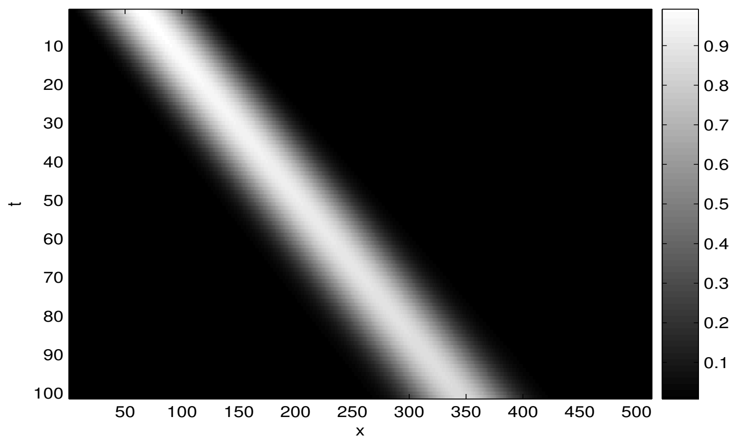

Figure 9: $y(x,t)$ calculated from (23) with $\tilde{a}(x)$ case 3

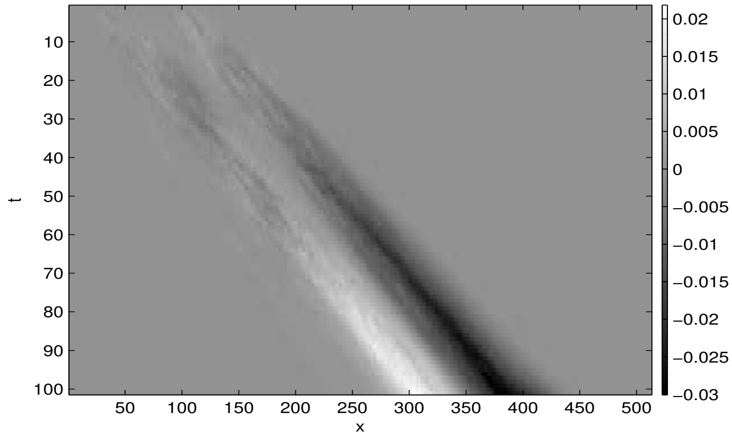

Figure 10: Error for case 3

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