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A NEW METHOD FOR THE DESIGN OF ENERGY TRANSFER FILTERS

Xiaofeng Wu, Z Q Lang and S. A. Billings
Department of Automatic Control and Systems Engineering
The University of Sheffield
Sheffield, S1 3JD, UK
Email: x.wu@sheffield.ac.uk

Abstract

This paper is concerned with the development of a new method for the design of Energy Transfer Filters (ETFs). ETFs are a new class of nonlinear filters recently proposed by the authors, which employ nonlinear effects to transfer signal energy from one frequency band to a different frequency location. The new method uses the powerful Orthogonal Least Squares (OLS) algorithm to solve the Least Squares problem associated with the design and compared with previous methods achieves much better filtering performance.

Keywords: Frequency Domain Analysis; Energy Transfer Filters; Orthogonal Least Squares Algorithm; Parsimonious Models; NARX Model

1. Introduction

Filtering is an operation which is concerned with processing a signal so that a new signal with improved characteristics can be produced. Conventional linear filter design is based on the principle that signal energy in unwanted frequency bands is attenuated. The traditional lowpass and bandpass filters such as Butterworth filters, Chebyshev filters, etc. (Zelniker and Taylor 1994) are examples of conventional designs, and are widely used in electrical and electronic, communication and control engineering areas.

There have been many studies which include the term nonlinear filter in the title. However the majority of these investigations relate to designing low order Volterra series models that minimize a cost function or which implement channel equalization or other similar time domain objectives (Mathews 1991, Sicuranza 1992, Zelniker and Taylor 1994, Heredia and Arce 2000). There appears to have been very few if any attempts to design nonlinear filters based on frequency domain objectives.

Recently, a totally new filtering concept known as energy transfer filtering has been proposed and an algorithm for the design of a class of energy transfer filters (ETFs) has been developed (Billings and Lang 2002). Energy transfer filtering is based on the principle that signal energy in one frequency band can be moved or transferred to other frequency locations by exploiting the properties of nonlinear dynamic effects.

It is well known that, in contrast to the case of linear systems where the possible output frequencies are exactly the same as the frequencies of the input, the possible output frequencies of nonlinear systems are much richer than the frequencies of the input. Consequently, a desired frequency domain energy transfer effect can be realized by a proper design of a nonlinear system. This nonlinear system is referred to as the energy transfer filter (ETF) by Billings and Lang (2002). By using a ETF, signal energy can be moved or transferred to higher frequencies or lower frequencies, or it can be focused around one frequency location. There are many design possibilities, and subject to realizability constraints, these general principles can be applied in many areas to provide extra degrees of freedom and additional benefits to filter design. Currently a series of research studies are underway at Sheffield to investigate the application of the principles of energy transfer filters in different engineering areas.

In Billings and Lang (2002), the design of energy transfer filters, which can be described by the NARX (Nonlinear AutoRegressive with eXogenous input) model with input nonlinearities, was investigated. The algorithm proposed for the design consists of three steps and was successfully applied to the design of many energy transfer filters dealing with both one and two input signals. However, as pointed out in the very original paper on the study of ETF designs, there is a upper limit to the maximum lag K_u for the input in the NARX model. If K_u has reached that realistic limit, but the performance of the designed ETF is still not satisfactory, the design has to stop without a satisfactory solution. Consequently, an improved design procedure is needed to overcome this problem.

Motivated by the requirement to improve the original ETF design approach in Billings and Lang (2002), in the present study, a new algorithm is proposed for the design of ETF filters. The new method uses the Orthogonal Least Squares (OLS) approach to determine both an optimal structure and the parameters of the nonlinear part of a NARX model for the design. This novel idea circumvents the difficulties using the original design approach. Compared with the previous designs, energy transfer filters designed using the new approach can not only have significantly improved performance but also considerably reduced complexity. This could make the designs easier to implement using DSP chips, dedicated processors, or in electronic circuits and communication systems for signal processing purposes.

2. Energy Transfer Filters

2.1 The Concept

In conventional linear filtering unwanted energy is attenuated, whereas in energy transfer filters the unwanted energy is moved or transferred to new frequency locations which can be different from the frequency range of the input. This is achieved by exploiting the frequency domain properties of nonlinear dynamic systems where the possible output frequencies are normally richer than the frequencies of the corresponding input.

Consider the case of the generation of harmonics and intermodulations in nonlinear systems. When a nonlinear system is subject to a sinusoidal input of frequency ω, the output could contain frequencies at the harmonics ω, 2ω, 3ω. Some of the input signal energy is therefore moved by the system to the second, third, and higher order harmonics. When the frequency components of the input are ω_1, ω_2, ω_3, the output frequencies of the nonlinear system could include the original input frequencies ω_1, ω_2, ω_3; harmonics $2\omega_1$, $2\omega_2$, $2\omega_3$; intermodulation frequencies: $\omega_1 - \omega_2$, $\omega_3 - \omega_2$, $\omega_1 - \omega_3$ and many others. Consequently, a more complicated frequency domain energy transfer phenomenon could be observed. The basic idea of energy transfer filters is to exploit these well known nonlinear effects to achieve a desired frequency domain energy transfer via an appropriate design of a nonlinear dynamic system in the frequency domain.

2.2 Energy Transfer Filters Of A NARX Model With Input Nonlinearity

The NARX model with input nonlinearity is a specific case of the NARMAX model (Chen and Billings 1989; Pearson 1999) and is given by:

$$\begin{cases} y(k) = \sum_{n=1}^{N} y_n(k) \\ y_n(k) = \begin{cases} \sum_{l_1,l_n=1}^{K_{nu}} c_{0n}(l_1,...,l_n) \prod_{i=1}^{n} u(k-l_i) & n \geq 2 \\ \sum_{l_1=1}^{K_y} c_{10}(l_1) y(k-l_1) + \sum_{l_1=1}^{K_{1u}} c_{01}(l_1) u(k-l_1) & n = 1 \end{cases} \end{cases} \tag{1}$$

where $y(k)$ and $u(k)$ are the output and input of the model at discrete time k, $c_{0n}(l_1,...,l_n)$ and $c_{10}(l_1)$ are the model coefficients, $K_{nu}, n = 1,...,N$, and K_y are the maximum lags with respect to the model input and output respectively. Under certain conditions, the NARX model with input nonlinearity (1) is an equivalent description to the well-known discrete possibly infinite Volterra systems of the form (Kotsios 1997)

$$y(k) = \sum_{n=0}^{\infty} \sum_{i_1=0}^{\infty} \cdots \sum_{i_n=0}^{\infty} a_{i_1 i_2,...i_n} u(k-i_1) \ldots u(k-i_n) \tag{2}$$

Compared with a truncated Volterra series approximation to equation (2), the advantage of the model (1) is obvious. First, stability can be checked, due to the existence of a number of useful theorems, which is very important in filter design. Second, the finite expression equation (1) can easily be transformed to a linear-in-the-parameters form.

More specifically, model (1) can be written in the form

$$y(k) - \sum_{l_1=1}^{K_y} c_{10}(l_1)y(k-l_1) = \sum_{n=N_0}^{N}\sum_{l_1=1}^{K_{nu}}\cdots\sum_{l_n=l_{n-1}}^{K_{nu}} \bar{c}_{0n}(l_1,...,l_n)\prod_{i=1}^{n} u(k-l_i) \tag{3}$$

where N_0 and N are the minimum and maximum order of system nonlinearity respectively, and

$$\bar{c}_{0n}(l_1,...,l_n) = |\pi(l_1,...l_n)|\bar{\bar{c}}_{0n}(l_1,...,l_n) \tag{4}$$

where

$$\bar{\bar{c}}_{0n} = \frac{\sum_{\pi(\cdot)} c_{0n}(l_1,...,l_n)}{|\pi(l_1,...,l_n)|} \tag{5}$$

The summation in equation (5) is over all distinct permutations of the indices $l_1,...,l_n$, $|\pi(l_1,...,l_n)|$ represents the number of such permutations and is given by

$$|\pi(l_1,...,l_n)| = \frac{n!}{k_1!k_2!\cdots k_r!} \tag{6}$$

where r is the number of distinct values in a specific set of $(l_1,...,l_n)$, $k_1,...k_r$ denote the number of times these values appear in $(l_1,...,l_n)$.

Generally speaking, the design of an energy transfer filter based on a NARX model with input nonlinearity involves determining the structure parameters N_0, N, K_y, K_{nu} and the coefficients $c_{10}(l_1), l_1 = 1,...,K_y$, $\bar{c}_{0n}(l_1,...,l_n), l_1 = 1,...,K_{nu},...,l_n = l_{n-1},...,K_{nu}$ in equation (3) to achieve a desired frequency domain energy transfer effect for one or several given specific input signals. In Billings and Lang (2002), an algorithm was developed for the ETF design and the effectiveness of the algorithm was demonstrated by several simulation examples, where both one and two input signal situations were considered. This original algorithm is the basis of the research studies presented in this paper.

2.3 The Original Design Algorithm

Given a specified input, the relationship between the output spectrum of the ETF model (3) and the spectrum of the input can be written as (Lang and Billings 1996):

$$\begin{aligned} Y(j\omega) = G(j\omega)\sum_{n=N_0}^{N}\frac{1/\sqrt{n}}{(2\pi)^{(n-1)}}\sum_{l_1=1}^{K_{nu}}\cdots\sum_{l_n=l_{n-1}}^{K_{nu}}\bar{c}_{0n}(l_1,...,l_n) \\ \times\int_{\omega_1+\cdots\omega_n=\omega} e^{-j(\omega_1 l_1+\cdots\omega_n l_n)}\prod_{i=1}^{n}U(j\omega_i)d\sigma_{n\omega} \end{aligned} \tag{7}$$

where $G(j\omega) = \dfrac{1}{1-\sum_{l_1=1}^{K_y} c_{10}(l_1)j\omega}$.

Based on equation (7) and other theoretical results about the output frequency characteristics of nonlinear systems (Lang and Billings 1997), the ETF design algorithm was proposed by Billings and Lang (2002). The algorithm consists of three steps and can be summarized as follows.

Step 1. Determination of N_0 and N. Given the frequency band (a, b) of the input signal to process and the desired output frequency band (c, d), N_0 is determined by evaluating the output frequency range f_{Y_n} according to (Lang and Billings 1997)

$$\begin{cases} f_{Y_n} = \begin{cases} \bigcup_{k=0}^{i^*-1} I_k & \text{when} \frac{nb}{(a+b)} - \lfloor \frac{na}{(a+b)} \rfloor < 1 \\ \bigcup_{k=0}^{i^*} I_k & \text{when} \frac{nb}{(a+b)} - \lfloor \frac{na}{(a+b)} \rfloor \geq 1 \end{cases} \\ \text{where } i^* = \lfloor \frac{na}{(a+b)} \rfloor + 1 \\ \lfloor \cdot \rfloor \text{ is an operand to take the integer part.} \\ I_k = (na - k(a+b), nb - k(a+b)) \quad \text{for } k = 0, ..., i^* - 1, \\ I_{i^*} = (0, nb - i^*(a+b)) \end{cases} \tag{8}$$

for $n = 1, n = 2$,..., until $n = \bar{n}$ such that at least part of the desired output frequency band (c, d) falls into f_{Y_n}. The value N_0 is then taken as $N_0 = \bar{n}$.

N is determined by evaluating $f_{Y_n} \bigcup f_{Y_{n-1}}$ for $n = \bar{n}, n = \bar{n} + 1$,..., until $n = \bar{\bar{n}}$ such that (c, d) completely falls into $f_{Y_n} \bigcup f_{Y_{n-1}}$. Then $N = \bar{\bar{n}}$.

Step 2. Determination of the parameters in the nonlinear part of the model (3) for *a priori* given maximum lags $K_{nu}, n = N_0, ..., N$. Given the desired output spectrum $Y^{\#}(j\omega)$, the parameters $\bar{c}_{0n}(l_1, ..., l_n), l_1 = 1, ..., K_{nu}, ..., l_n = l_{n-1}, ..., K_{nu}$; $n = N_0, ..., N$ are evaluated based on the equations

$$\begin{aligned} Y^{\#}(j\omega(p)) = \sum_{n=N_0}^{N} \frac{1/\sqrt{n}}{(2\pi)^{(n-1)}} \sum_{l_1=1}^{K_{nu}} \cdots \sum_{l_n=l_{n-1}}^{K_{nu}} \bar{c}_{0n}(l_1, ..., l_n) \\ \times \int_{\omega_1+\cdots\omega_n=\omega(p)} e^{-j(\omega_1 l_1+\cdots\omega_n l_n)} \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega} \\ p = 1, ..., M \end{aligned} \tag{9}$$

to make the right-hand side of the equations approach the desired output spectrum as closely as possible. In the equation, M is an *a priori* given integer and $\omega(p) \in (c, d), p = 1, ..., M$.

This is achieved in the algorithm by using a least squares routine to solve the group equations

$$\begin{cases} \text{Re}[Y^{\#}(j\omega(p))] = \sum_{n=N_0}^{N} \frac{1/\sqrt{n}}{(2\pi)^{(n-1)}} \sum_{l_1=1}^{K_{nu}} \cdots \sum_{l_n=l_{n-1}}^{K_{nu}} \bar{c}_{0n}(l_1, ..., l_n) \text{Re}[g_{l_1,...,l_n}(j\omega(p))] \\ \qquad p = 1, ..., M \\ \text{Im}[Y^{\#}(j\omega(p))] = \sum_{n=N_0}^{N} \frac{1/\sqrt{n}}{(2\pi)^{(n-1)}} \sum_{l_1=1}^{K_{nu}} \cdots \sum_{l_n=l_{n-1}}^{K_{nu}} \bar{c}_{0n}(l_1, ..., l_n) \text{Im}[g_{l_1,...,l_n}(j\omega(p))] \\ \qquad p = 1, ..., M \end{cases} \tag{10}$$

where

$$g_{l_1,...,l_n}(j\omega(p)) = \int_{\omega_1+\cdots\omega_n=\omega(p)} e^{-j(\omega_1 l_1+\cdots\omega_n l_n)} \prod_{i=1}^{n} U(j\omega_i) d\sigma_{n\omega} \tag{11}$$

is the Fourier transform of the time series $u(k-l_1)u(k-l_2)\cdots u(k-l_n)$, which can be readily evaluated from the given input.

Step 3. Determination of the parameters in the linear part of the model (3). Denote the results obtained in Step 2 as

$$\hat{\bar{c}}_{0n}(l_1,...,l_n), \quad l_1 = 1,...,K_{nu},...,l_n = l_{n-1},...,K_{nu}; \quad n = N_0,...,N$$

This step is to determine a stable linear filter with frequency response function $G_1(j\omega)$ to minimize the criterion

$$J(G_1) = \sum_{p=1}^{M} \{Y^{\#}[j\omega(p)] - G_1[j\omega(p)]\bar{Y}[j\omega(p)]\}^* \times \{Y^{\#}[j\omega(p)] - G_1[j\omega(p)]\bar{Y}[j\omega(p)]\} \tag{12}$$

where

$$\bar{Y}(j\omega(p)) = \sum_{n=N_0}^{N} \frac{1/\sqrt{n}}{(2\pi)^{(n-1)}} \sum_{l_1=1}^{K_{nu}} \cdots \sum_{l_n=l_{n-1}}^{K_{nu}} \hat{\bar{c}}_{0n}(l_1,...,l_n) g_{l_1,...,l_n}(j\omega(p)) \qquad p = 1,...,M \tag{13}$$

and then design a traditional bandpass filter with frequency response function $G_2(j\omega)$ to remove any unwanted residual frequency components in $G_1(j\omega)\bar{Y}(j\omega)$ which are outside the output frequency band (c,d) to make $G_1(j\omega)G_2(j\omega)\bar{Y}(j\omega)$ approach $Y^{\#}(j\omega)$ as required by the design. Consequently the linear part of the ETF model (3) is determined as

$$G(j\omega) = G_1(j\omega)G_2(j\omega) \tag{14}$$

In this algorithm, $K_{nu}, n = N_0,...,N$ are important structure parameters to be given *a priori* and were all taken as K_u in Billings and Lang (2002) for simplicity. To achieve a final design, the original algorithm uses an iterative approach to find an appropriate value for K_u. For example, K_u can initially be set as $K_u = 1$, the design using this K_u is completed and the performance of the resulting filter is checked to see if this is satisfactory or not. If the result is satisfactory, then the design is finished. Otherwise, take $K_u = 2$ and repeat the procedure. This process can be continued with $K_u = 3, 4, ...,$ until a satisfactory result is achieved. However, as indicated by Billings and Lang (2002), for a given M, the maximum value of K_u which can be taken for the design is limited by the inequality

$$M \geq \frac{[K_u^N + K_u^{N-1} + \cdots + K_u^{N_0} + (N - N_0 + 1)K_u]}{4} \tag{15}$$

If the value of K_u is close to the upper limit but the performance of the filter is still not satisfactory, the design has to stop without a satisfactory solution.

Therefore, an improved design method needs to be developed to overcome these problems with the original design algorithm. This requirement is the motivation of the development of a new method for the design of energy transfer filters in the present study.

3. The New Design Approach

It can be observed that the problem with the original ETF design algorithm in Section 2.3 is caused by the conventional least squares solution to equation (10) where $K_{nu} = K_u$ is increased to a limit such that the number of the parameters n_p

$$n_p = \frac{(K_u^{N+1} - K_u^{N_0}) + (K_u - 1)K_u(N - N_0 + 1)}{2(K_u - 1)} \tag{16}$$

become close to the number of the equations which is $2M$.

Given a particular $K_{nu} = K_u$, equation (10) actually uses a linear combination of all the terms $g_{l_1,...,l_n}(j\omega), l_1 = 1, ..., K_u, ..., l_n = l_{n-1}, ..., K_u, n = N_0, ..., N$ to approximate the desired output spectrum $Y^{\#}(j\omega)$ over the frequency set $\{\omega(1), ..., \omega(M)\}$. However, some of these terms may not be needed and some other terms which are not covered by these terms such as the terms the form of $g_{\bar{l}_1,...,\bar{l}_n}(j\omega), \bar{l}_1 > K_u, ..., \bar{l}_n > K_u, n = N_0, ..., N$ may provide significant contributions if included. The second step of the original algorithm takes the effects of all terms into account with K_u up to a limit, and can neither avoid any unnecessary terms within these choices nor include terms which are outside these choices but can provide significant contribution to the desired output spectrum. Consequently, when terms like $g_{\bar{l}_1,...,\bar{l}_n}(j\omega), \bar{l}_1 > K_u^*, ..., \bar{l}_n > K_u^*, n = N_0, ..., N$ are needed to achieve a desired approximation for $Y^{\#}(j\omega)$, where K_u^* happen to be the limit of K_u, the original algorithm often fails to produce a desired design. In order to circumvent this problem, the use of the powerful orthogonal least squares method is proposed in the present study to solve the least squares problem defined by (10) so as to improve the design results at the second step of the original algorithm. This is the basic idea of the new ETF design approach.

3.1 The Orthogonal Least Squares Algorithm

Consider a linear regression model, that is linear-in-the-parameters

$$z(k) = \sum_{i=1}^{m} \phi_i(k)\theta_i + e(k) \qquad k = 1, ..., \bar{N} \tag{17}$$

where $z(k)$ represents the k-th measurement, $\bar{N}$ is the data length, m is the number of regressors, $\phi_i(k)$ and θ_i, $i = 1, ..., m$ denote the regressors and parameters respectively, and $e(k)$ is the modelling error, assumed to be a zero mean white noise sequence.

When using the orthogonal least squares algorithm (Billings *et al* 1989 a, b), the parameters θ_i are estimated by transforming model (17) to an equivalent auxiliary model

$$z(k) = \sum_{i=1}^{m} g_i w_i(k) + e(k) \tag{18}$$

where

$$w_1(k) = \phi_1(k) \tag{19}$$

$$w_j(k) = \phi_j(k) - \sum_{i=1}^{j-1} \alpha_{ij} w_i(k) \quad j = 2, ..., m \tag{20}$$

and

$$\alpha_{ij} = \frac{\sum_{k=1}^{\bar{N}} w_i(k)\phi_j(k)}{\sum_{k=1}^{\bar{N}} w_i^2(k)} \qquad j = 2, 3, ..., m \quad i < j \tag{21}$$

Then the parameters $g_i, i = 1, ..., m$ of the auxiliary model (18) can be readily obtained as

$$\hat{g}_i = \frac{\sum_{k=1}^{\bar{N}} w_i(k)z(k)}{\sum_{k=1}^{\bar{N}} w_i^2(k)} \qquad i = 1, ..., m \tag{22}$$

Finally, the original model parameters can be calculated from $\hat{g}_i$ according to

$$\hat{\theta}_m = \hat{g}_m \tag{23}$$

$$\hat{\theta}_i = \hat{g}_i - \sum_{j=i+1}^{m} \alpha_{ij}\hat{\theta}_j \qquad i = m-1, m-2, ..., 1 \tag{24}$$

Considering $\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} w_i(k)w_j(k) = 0, i \neq j$ (Chen, Billings and Luo 1989), multiplying equation (18) by itself and taking an average over the data records gives

$$\frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} z^2(k) = \sum_{i=1}^{m} g_i^2 \frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} w_i^2(k) + \frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} e^2(k) \tag{25}$$

Equation (25) shows that the reduction in the mean squared error by including the i-th term, $g_i w_i(k)$, in the auxiliary model of equation (18) is $g_i^2 \frac{1}{\bar{N}} \sum_{k=1}^{\bar{N}} w_i^2(k)$. Expressing this as a fraction of the total model output energy yields the Error Reduction Ratio (ERR) for the i-th term as

$$ERR_i = \frac{\hat{g}_i^2 \sum_{k=1}^{\bar{N}} w_i^2(k)}{\sum_{k=1}^{\bar{N}} z^2(k)} \times 100\% \qquad 1 \leq i \leq n \tag{26}$$

The ERR values can be computed together with the parameter estimates to indicate the significance of each candidate term in the auxiliary model so as to determine the structure and associated parameters of the original model, the corresponding procedure is summarized as follows:

i) Consider all regressors $\phi_i(k), i = 1, 2, ..., m$ as possible candidates for $w_1(k)$, calculating

$$w_1^{(i)}(k) = \phi_i(k) \tag{27}$$

$$\hat{g}_1^{(i)} = \frac{\sum_{k=1}^{\bar{N}} w_1^{(i)}(k) z(k)}{\sum_{k=1}^{\bar{N}} (w_1^{(i)}(k))^2} \tag{28}$$

$$ERR_1^{(i)} = \frac{(\hat{g}_1^{(i)})^2 \sum_{k=1}^{\bar{N}} (w_1^{(i)}(k))^2}{\sum_{k=1}^{\bar{N}} z^2(k)} \tag{29}$$

find the maximum of $ERR_1^{(i)}$, such that $ERR_1^{(j)} = max\{ERR_1^{(i)}, 1 \le i \le m\}$. Then the first term selected should be the j-th term. i.e. $\phi_j(k)$, and therefore define $w_1(k) = \phi_j(k)$.

ii) Consider all the $\phi_i(k), i = 1, 2, ..., m, i \ne j$ as possible candidates for $w_2(k)$ calculating

$$w_2^{(i)}(k) = \phi_i(k) - \alpha_{12}^{(i)} w_1(k) \tag{30}$$

$$\hat{g}_2^{(i)} = \frac{\sum_{k=1}^{\bar{N}} w_2^{(i)}(k) z(k)}{\sum_{k=1}^{\bar{N}} (w_2^{(i)}(k))^2} \tag{31}$$

$$ERR_2^{(i)} = \frac{(\hat{g}_2^{(i)})^2 \sum_{k=1}^{\bar{N}} (w_2^{(i)}(k))^2}{\sum_{k=1}^{\bar{N}} z^2(k)} \tag{32}$$

where

$$\alpha_{12}^{(i)} = \frac{\sum_{k=1}^{\bar{N}} w_1(k) \phi_i(k)}{\sum_{k=1}^{\bar{N}} w_1^2(k)} \tag{33}$$

find the maximum of $ERR_2^{(i)}$, such that $ERR_2^{(l)} = max\{ERR_2^{(i)}, 1 \le i \le m, i \ne j\}$. Then the second term selected should be the l-th term. i.e. $\phi_l(k)$, and therefore define $w_2(k) = \phi_l(k) - \alpha_{12}^{(l)} w_1(k)$.

iii) Continue the process, the term with the maximum error reduction ratio is selected and the corresponding $w_i(k)$ is produced.

iv) Step iii) continues until the summation of the error reduction ratios of the selected terms $\sum ERR_i$ is larger than a required approximation accuracy or close to 100%.

The OLS algorithm allows the selection of possible regressors from the m candidates $\phi_i(k), i = 1, ..., m$ and the determination of the coefficients associated with the selected terms to be implemented at the same time. Consequently, an effective regression model with the regressors which really have considerable contribution to the model output can be obtained.

3.2 The New Method

The new ETF design method also consists of three steps. The first and the last step are the same as steps 1 and 3 in the original algorithm as described in Section 2.3. In the second step, however, the OLS replaces the conventional least squares algorithm to solve equation (10) for the filter parameters $\bar{c}_{0n}(l_1, ..., l_n), l_1 = 1, ..., K_u, ..., l_n = l_{n-1}, ..., K_u; n = N_0, ..., N$. In the application of the OLS method, $z(k)$ and $\phi_i(k)$ in the linear regression model (17) takes the specific form:

$$\begin{cases} z(k) = \mathrm{Re}[Y^{\#}(j\omega(k))], & k = 1, 2, ..., M \\ z(k) = \mathrm{Im}[Y^{\#}(j\omega(k))], & k = M+1, ..., \bar{N} \end{cases} \tag{34}$$

where $\bar{N} = 2M$.

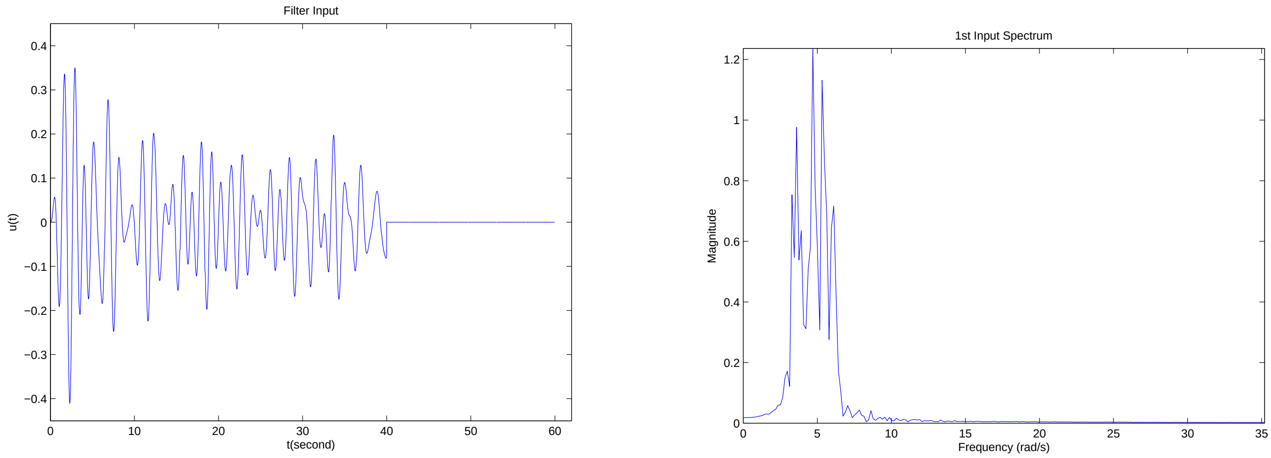
$$\begin{cases} \phi_i(k) = \mathrm{Re}[g^{(i)}_{l_1,...,l_n}(j\omega(k))], & k = 1, 2, ..., M \\ \phi_i(k) = \mathrm{Im}[g^{(i)}_{l_1,...,l_n}(j\omega(k))], & k = M+1, ..., \bar{N} \\ i = 1, 2, ..., n_p \end{cases} \tag{35}$$

where n_p is the number of parameters defined in (16), and $g^{(i)}_{l_1,...,l_n}(j\omega(k))$ is the i-th of the n_p terms of $g_{l_1,...,l_n}(j\omega(k)), l_1 = 1, ..., K_u, ..., l_n = l_{n-1}, ..., K_u; n = N_0, ..., N$, arranged in an arbitrary order. θ_i is $\bar{c}^{(i)}_{0n}(l_1, ..., l_n)$, which is the i-th of the n_p filter parameters $\bar{c}_{0n}(l_1, ..., l_n), l_1 = 1, ..., K_u, ..., l_n = l_{n-1}, ..., K_u; n = N_0, ..., N$ arranged in the same order as $g^{(i)}_{l_1,...,l_n}(j\omega(k)), i = 1, ..., n_p$.

Because of the capability of the OLS algorithm to determine both the most significant regressors and associated parameters from all the possible candidates, the lag K_u in the design can be selected to be large enough to cover the maximum lags needed. OLS normally only selects a relatively small number of terms from all the candidate regressors which make a significant contribution to the desired output spectrum $Y^{\#}(j\omega)$ over the M discrete frequency points $\{\omega(1), ..., \omega(M)\}$. As a result, only a subset of the n_p regressors $g_{l_1,...,l_n}(j\omega(k)), l_1 = 1, ..., K_u, ..., l_n = l_{n-1}, ..., K_u; n = N_0, ..., N$ may finally be selected. These selected regressors may include some terms associated with maximum lag K_u but not all the n_p regressors corresponding to the maximum K_u have to be used as in the original algorithm. Consequently, the problem with the original algorithm is overcome.

In Billings and Lang (2002), the ETF design was also considered for the cases of several specified inputs. By using the OLS method to solve the least squares problem in the second step of these more complicated ETF designs, the new algorithm can readily be extended to the several input cases. In the next section, a design example for both one and two specified input situations will be described to demonstrate the effectiveness of the new design approach.

4. Case Studies



(a) Input Signal

(b) Input Signal Spectrum

Figure 1: Input Signal in Example 1

In this section, design examples will be shown to demonstrate the effectiveness of the new method for ETF designs. The design in Section 4.1 involves a single input case to move the signal energy from a lower frequency band to a higher frequency location. The design in Section 4.2 involves a case where the signal energy from a higher frequency band is moved to a lower frequency location. The example in Section 4.3 considers a design for two specific inputs to move the energy of the two signals from a lower frequency band to a higher frequency location using a single energy transfer filter.

4.1 Example 1

Consider a continuous time signal $u(t)$ generated from a white noise uniformly distributed over $(0,4)$ and band-limited within the frequency range $(3.4, 6.2)rad/s$. The sampling interval was set as $T_s = 0.01s$. Figure 1(a) shows the signal in the time domain. Figure 1(b) shows the magnitude of the input signal spectrum. From Figure 1(b) it can be observed that the frequency range of $u(t)$ is approximately $(\bar{a}, \bar{b}) = (2.351, 7.054)rad/s$.

The objective is to design a frequency domain energy transfer filter to transfer the energy of $u(t)$ to a higher frequency band $(\bar{c}, \bar{d}) = (20.4, 30.2)rad/s$ and shape the magnitude of the filter output frequency response as specified by the desired spectrum

$$Y^d(j\omega_c) = \begin{cases} \dfrac{\exp(-500\omega_c) + j(600\omega_c^2)}{100000} & \omega_c \in (20.4, 30.2) \\ 0 & \text{otherwise} \end{cases} \tag{36}$$

where ω_c denotes the continuous frequency in radians.

The ETF design is performed using the new method as described in Section 3.2 where

$$(a, b) = T_s(\bar{a}, \bar{b}) = (0.02351, 0.07054) \tag{37}$$

$$(c, d) = T_s(\bar{c}, \bar{d}) = (0.204, 0.302) \tag{38}$$

$$Y^{\#}(j\omega) = \frac{1}{T_s} Y^d(j\frac{\omega}{T_s}) = \begin{cases} \frac{\exp(-500\omega/T_s) + j(600\omega^2/T_s^2)}{100000} & \omega \in T_s(20.4, 30.2) \\ 0 & \text{otherwise} \end{cases} \tag{39}$$

In step 1, the minimum and maximum order of the filter nonlinearity were determined as $N_0 = 3$ and $N = 5$ from the frequency range (a, b) of the input signal and the frequency range (c, d) of the desired filter output.

In step 2, M is taken as

$$M = i_d - i_c + 1 \tag{40}$$

where

$$i_c = \lfloor c\bar{M}/2\pi \rfloor \tag{41}$$

$$i_d = \lfloor d\bar{M}/2\pi \rfloor \tag{42}$$

$\bar{M} = 4008$ is the length of data used to evaluate the input spectrum $U(j\omega)$ for the design. $\omega(1), ..., \omega(M)$ are taken as

$$\omega(p) = 2(p + i_c - 1)\pi/\bar{M} \quad p = 1, 2, ..., M \tag{43}$$

With the maximum lag $K_u = 8$, the OLS method determines eight significant candidate terms for the nonlinear part of the energy transfer filter, which are given by

$$\begin{aligned}
&\sum_{n=N_0}^{N} \sum_{l_1=1}^{K_u} \cdots \sum_{l_n=l_{n-1}}^{K_u} \hat{\bar{c}}_{0n}(l_1, ..., l_n) \prod_{i=1}^{n} u(k - l_i) \\
&= \sum_{n=3}^{5} \sum_{l_1=1}^{8} \cdots \sum_{l_n=l_{n-1}}^{8} \hat{\bar{c}}_{0n}(l_1, ..., l_n) \prod_{i=1}^{n} u(k - l_i) \\
&= +(2420719.5249)u(k-8)u(k-8)u(k-8) \\
&\quad +(-609236.3234)u(k-1)u(k-8)u(k-8) \\
&\quad +(-818639.6217)u(k-7)u(k-7)u(k-8) \\
&\quad +(2198016.1539)u(k-3)u(k-7)u(k-7) \\
&\quad +(-606899.6399)u(k-5)u(k-6)u(k-8) \\
&\quad +(691317.0953)u(k-1)u(k-1)u(k-1) \\
&\quad +(-2283561.8752)u(k-7)u(k-8)u(k-8) \\
&\quad +(-982692.6294)u(k-1)u(k-2)u(k-2)
\end{aligned} \tag{44}$$

and achieves a summation of ERRs as $\sum ERR = 0.99789965595147$. The great magnitudes of the parameter values achieved above are needed to amplify the input signal magnitude. This is the same for the following examples.

In step 3, the structure of the first linear filter $G_1(j\omega)$ was chosen as

$$G_1(j\omega) = \frac{\bar{b}_1 + \bar{b}_2 q^{-1} + \bar{b}_3 q^{-2}}{\bar{a}_1 + \bar{a}_2 q^{-1} + \bar{a}_3 q^{-2}}$$

and the parameters were determined as

$$[\bar{b}_1, \bar{b}_2, \bar{b}_3] = [0.99499719798339, -1.90471481813288, 0.98082626153888]$$

$$[\bar{a}_1, \bar{a}_2, \bar{a}_3] = [1.00000000000000, -1.91469651730086, 0.98610235849883]$$

The structure of the second linear filter $G_2(j\omega)$ was configured as

$$G_2(j\omega) = G_2(j\omega)^{1/2} G_2(j\omega)^{1/2} \tag{45}$$

and $G_2(j\omega)^{1/2}$ is designed as the required bandpass filter. This is to enhance the filtering performance of $G_2(j\omega)$ (Zelniker and Taylor 1994). For $G_2(j\omega)^{1/2}$, the structure was chosen to be

$$G_2(j\omega)^{1/2} = \frac{\bar{\bar{b}}'_1 + \bar{\bar{b}}'_2 q^{-1} + \cdots + \bar{\bar{b}}'_9 q^{-8}}{\bar{\bar{a}}'_1 + \bar{\bar{a}}'_2 q^{-1} + \cdots + \bar{\bar{a}}'_9 q^{-8}}$$

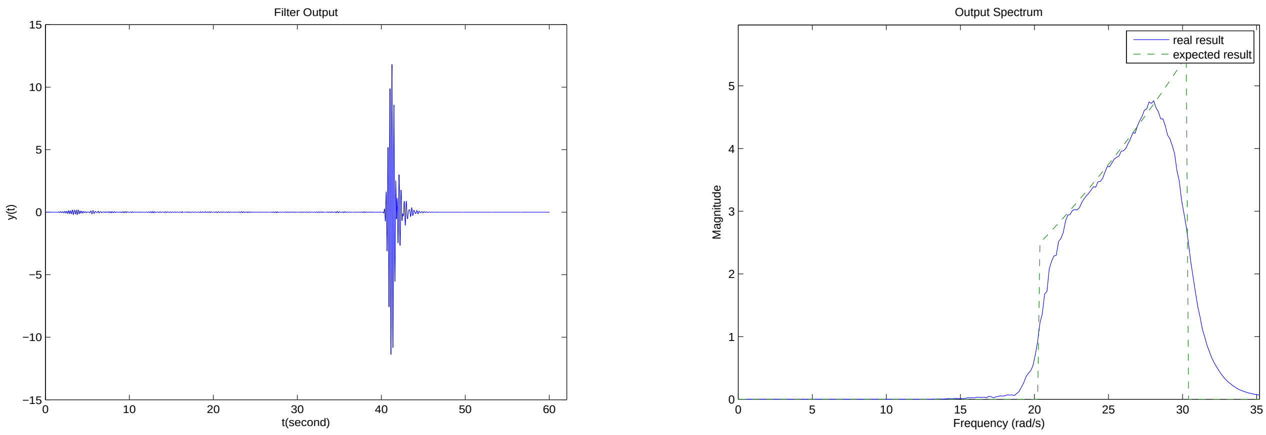
The parameters were determined as

$$[\bar{\bar{b}}'_1, ..., \bar{\bar{b}}'_9] = 1.0e - 04 * \begin{bmatrix} 0.05087927202286 \\ 0 \\ -0.20351708809145 \\ 0 \\ 0.30527563213718 \\ 0 \\ -0.20351708809145 \\ 0 \\ 0.05087927202286 \end{bmatrix}^T$$

$$[\bar{\bar{a}}'_1, ..., \bar{\bar{a}}'_9] = \begin{bmatrix} 1.00000000000000 \\ -7.50645040549444 \\ 24.88123427345764 \\ -47.55491758051167 \\ 57.31495456445121 \\ -44.60383853541906 \\ 21.88908628895510 \\ -6.19403826217024 \\ 0.77398211278076 \end{bmatrix}^T$$

Consequently,

$$G_2(j\omega) = \frac{\bar{\bar{b}}_1 + \bar{\bar{b}}_2 q^{-1} + \cdots + \bar{\bar{b}}_{17} q^{-16}}{\bar{\bar{a}}_1 + \bar{\bar{a}}_2 q^{-1} + \cdots + \bar{\bar{a}}_{17} q^{-16}}$$



(a) Output Signal

(b) Output Signal Spectrum

Figure 2: Output Signal in Example 1

where

$$[\bar{\bar{b}}_1, ..., \bar{\bar{b}}_{17}] = \text{Conv}\{[\bar{\bar{b}}'_1, ..., \bar{\bar{b}}'_9], [\bar{\bar{b}}'_1, ..., \bar{\bar{b}}'_9]\}$$
$$[\bar{\bar{a}}_1, ..., \bar{\bar{a}}_{17}] = \text{Conv}\{[\bar{\bar{a}}'_1, ..., \bar{\bar{a}}'_9], [\bar{\bar{a}}'_1, ..., \bar{\bar{a}}'_9]\}$$

$\text{Conv}(x, y)$ denotes the convolution of vectors x and y.

Figures 2(a) and 2(b) show the output response of the filter in the time and frequency domain respectively. The performance of this design can be assessed from Figure 2(b) where a comparison between the real output spectrum of the filter and the desired result can be observed. Clearly, a good result has been achieved by the design and the energy of the specified input has been moved from $(2.351, 7.054) rad/s$ in Figure 1(b) to the frequency band $(c, d) = (20.4, 30.2) rad/s$ in Figure 2(b) and the shape of the magnitude matches the desired spectrum defined by equation (36).

In order to demonstrate the improvement the new method has achieved compared with the original design method, the original method was used to conduct the same design. The maximum lag K_u for the original method in this case is 4, and the nonlinear part of the designed ETF is

$$\begin{aligned}
&\sum_{n=3}^{5} \sum_{l_1=1}^{4} \cdots \sum_{l_n=l_{n-1}}^{4} \widehat{c}_{0n}(l_1, ..., l_n) \prod_{i=1}^{n} u(k - l_i) \\
&= +(643767104418704) u(k-1) u(k-1) u(k-1) \\
&\quad +(-5.726088e+15) u(k-1) u(k-1) u(k-2) \\
&\quad +(5.743184e+15) u(k-1) u(k-1) u(k-3) \\
&\quad +(-1.947908e+15) u(k-1) u(k-1) u(k-4) \\
&\quad +(1.701482e+16) u(k-1) u(k-2) u(k-2)
\end{aligned}$$

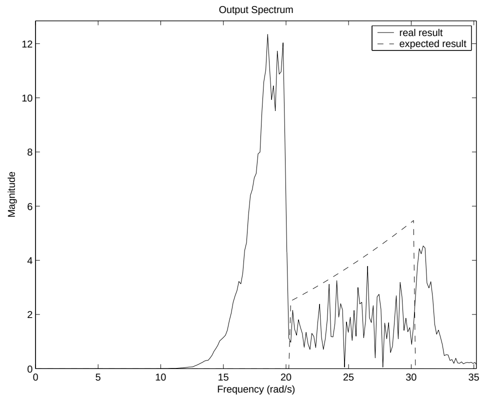


Figure 3: Output Spectrum of the Original ETF Design for Example 1

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad\qquad\qquad \cdots \qquad \cdots \qquad \cdots \\
&+(-9.987e+17)u(k-3)u(k-4)u(k-4)u(k-4)u(k-4) \\
&\quad +(1.914e+17)u(k-4)u(k-4)u(k-4)u(k-4)u(k-4)
\end{aligned} \tag{46}
$$

with altogether 111 terms, including all candidate terms under the structure parameters of $N_0 = 3, N = 5$, and $K_u = 4$. Figure 3 shows the comparison of the output frequency response of the original ETF design with the desired spectrum. Obviously, the design is not satisfactory and the newly proposed method is therefore necessary for this case to achieve a desired energy transfer effect.

4.2 Example 2

Consider a continuous time signal $u(t)$ generated from a white noise uniformly distributed over $(0, 4)$ and band-limited within the frequency range $(13.2, 20.7)rad/s$. The sampling interval was set as $T_s = 0.01s$. Figure 4(a) shows the signal in the time domain. Figure 4(b) shows the magnitude of the signal spectrum which indicates that the frequency range of $u(t)$ is approximately $(\bar{a}, \bar{b}) = (11.443, 25.553)rad/s$.

The objective is to design a frequency domain energy transfer filter to transfer the energy of the signal $u(t)$ to the lower frequency band $(\bar{c}, \bar{d}) = (2.0, 4.2)rad/s$ and shape the magnitude of the filter output frequency response as specified by the desired spectrum

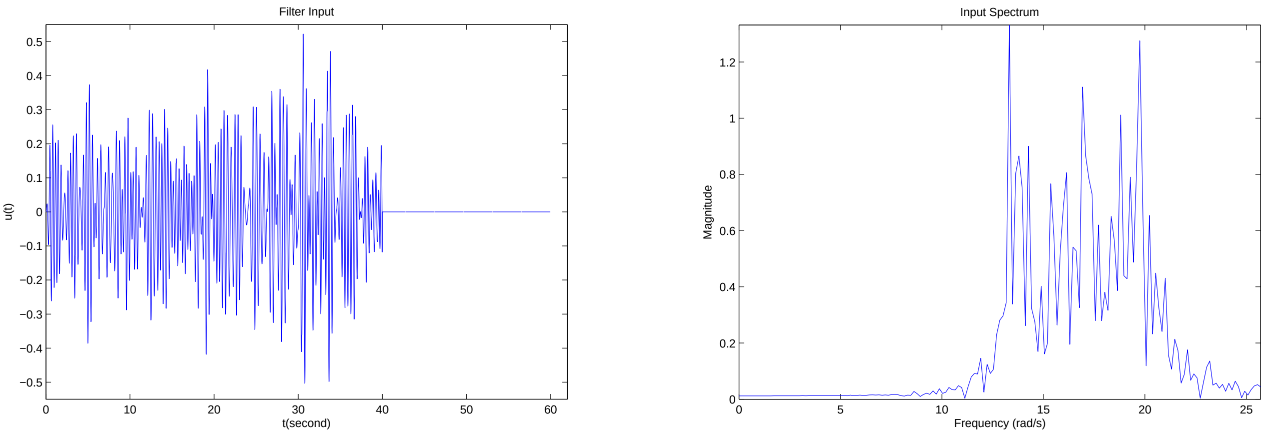
$$
Y^d(j\omega_c) = \begin{cases} \dfrac{\exp(-500\omega_c) + j(600\omega_c^2)}{100000} & \omega_c \in (2.0, 4.2) \\ 0 & \text{otherwise} \end{cases} \tag{47}
$$

The ETF design is performed using the procedure described in Section 3.2 where

$$
(a, b) = T_s(\bar{a}, \bar{b}) = (0.11443, 0.25553) \tag{48}
$$

$$
(c, d) = T_s(\bar{c}, \bar{d}) = (0.020, 0.042) \tag{49}
$$

and $Y^{\#}(j\omega)$ is given by



(a) Input Signal (b) Input Signal Spectrum

Figure 4: Input Signal in Example 2

$$Y^{\#}(j\omega) = \begin{cases} \dfrac{\exp(-500\omega/T_s) + j(600\omega^2/T_s^2)}{100000} & \omega \in T_s(2.0, 4.2) \\ 0 & \text{otherwise} \end{cases} \tag{50}$$

In step 1, The minimum and maximum of system nonlinearity were determined to be $N_0 = N = 2$.

In step 2, i_c, i_d, M, and $\omega(p), p = 1, ..., M$ were again determined using equations (40), (41), (42) and (43) while employing the c and d specified in this example.

With the maximum lag $K_u = 8$, OLS determines 18 most significant candidate terms for the nonlinear part of the energy transfer filter, which are given by

$$\begin{aligned} &\sum_{l_1=1}^{8}\sum_{l_2=l_1}^{8} \widehat{\widehat{c}}_{02}(l_1, l_2)u(k-l_1)u(k-l_2) \\ &= +(43035.0064)u(k-1)u(k-1) \\ &+(4185.4581)u(k-1)u(k-8) \\ &+(-323033.8063)u(k-2)u(k-2) \\ &+(-31390.474)u(k-1)u(k-3) \\ &+(367674.7836)u(k-3)u(k-3) \\ &+(9460.8378)u(k-7)u(k-8) \\ &+(-14282.8008)u(k-2)u(k-8) \\ &+(-386047.1)u(k-3)u(k-7) \\ &+(-86515.2327)u(k-8)u(k-8) \end{aligned}$$

$$
\begin{aligned}
&+(-1873875.0799)u(k-5)u(k-7)\\
&+(76953.9026)u(k-1)u(k-2)\\
&+(1293789.3935)u(k-4)u(k-7)\\
&+(1139147.9369)u(k-6)u(k-7)\\
&+(58923.9354)u(k-6)u(k-8)\\
&+(20038.3655)u(k-5)u(k-8)\\
&+(-193219.719)u(k-5)u(k-5)\\
&+(22838.7942)u(k-1)u(k-7)\\
&+(-127679.6638)u(k-7)u(k-7)
\end{aligned}
\tag{51}
$$

and achieves a summation of ERRs as $\sum ERR = 0.99167205522665$.

In step 3, $G_1(j\omega)$ and $G_2(j\omega)$ were chosen to be of the same structure as in Example 1. The parameters for $G_1(j\omega)$ and $G_2(j\omega)^{1/2}$ were determined as

$$[\bar{b}_1, \bar{b}_2, \bar{b}_3] = [0.94295008419603, -1.88366288231094, 0.94178381830524]$$

$$[\bar{a}_1, \bar{a}_2, \bar{a}_3] = [1.00000000000000, -1.99737222802043, 0.99852038477311]$$

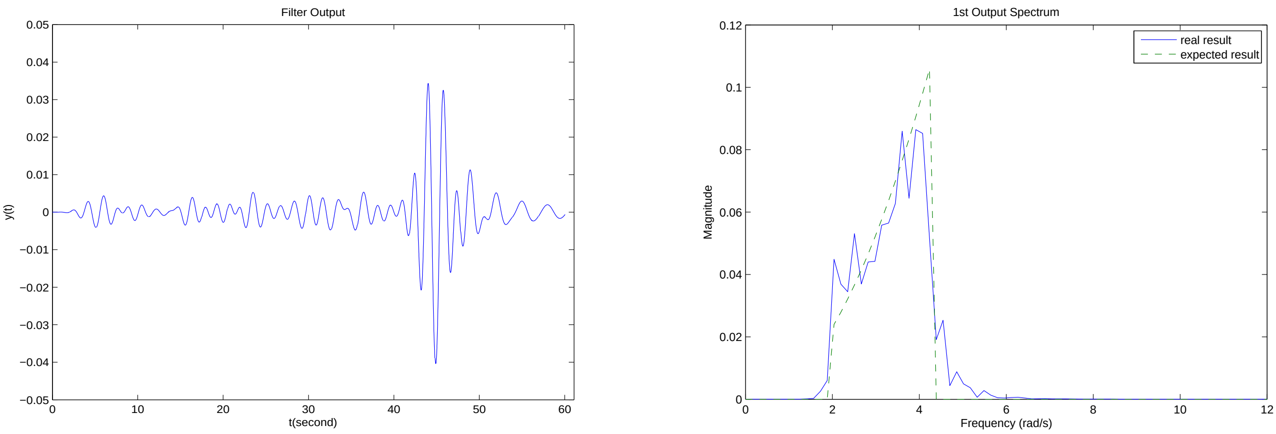
and

$$[\bar{\bar{b}}'_1, ..., \bar{\bar{b}}'_9] = 1.0e-07 * \begin{bmatrix} 0.15202442056885 \\ 0 \\ -0.60809768227540 \\ 0 \\ 0.91214652341309 \\ 0 \\ -0.60809768227540 \\ 0 \\ 0.15202442056885 \end{bmatrix}^T$$

$$[\bar{\bar{a}}'_1, ..., \bar{\bar{a}}'_9] = \begin{bmatrix} 1.00000000000000 \\ -7.93917603605685 \\ 27.57936234906552 \\ -54.75329516067795 \\ 67.94720923114389 \\ -53.97197088570229 \\ 26.79787040178942 \\ -7.60413110961782 \\ 0.94413121005658 \end{bmatrix}^T$$

The output response of the filter for the specified input is shown in figures 5(a) and 5(b) in the time and frequency domain respectively. Figure 5(b) obviously indicates that an excellent result has been achieved by the new method.

The original method was also used to conduct this design. The maximum lag K_u for this case is 6. and the nonlinear part of the designed ETF is



(a) Output Signal

(b) Output Signal Spectrum

Figure 5: Output Signal in Example 2

$$
\begin{aligned}
&\sum_{l_1=1}^{6}\sum_{l_2=l_1}^{6}\widehat{\bar{c}}_{02}(l_1,l_2)u(k-l_1)u(k-l_2)\\
&= +(19101617.0993)u(k-1)u(k-1)\\
&+(-9128864.0808)u(k-1)u(k-2)\\
&+(4556417.0233)u(k-1)u(k-3)\\
&+(-587485.6886)u(k-1)u(k-4)\\
&+(1873025.7942)u(k-1)u(k-5)\\
&+(-681926.538)u(k-1)u(k-6)\\
&+(-80647532.8118)u(k-2)u(k-2)\\
&+(14836435.1942)u(k-2)u(k-3)\\
&+(-9663476.1744)u(k-2)u(k-4)\\
&+(-8572359.2661)u(k-2)u(k-5)\\
&+(2413531.4597)u(k-2)u(k-6)\\
&+(149146990.6272)u(k-3)u(k-3)\\
&+(2740836.1858)u(k-3)u(k-4)\\
&+(24699144.0849)u(k-3)u(k-5)\\
&+(-1773388.9834)u(k-3)u(k-6)\\
&+(-150315202.0674)u(k-4)u(k-4)\\
&+(-30147544.1394)u(k-4)u(k-5)\\
&+(-5655097.4187)u(k-4)u(k-6)
\end{aligned}
$$

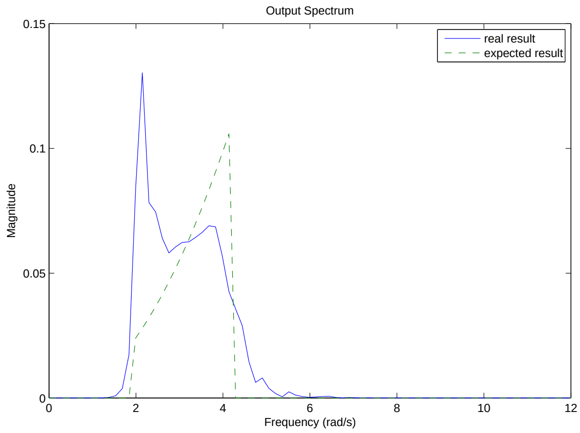


Figure 6: Output Spectrum of the Original ETF Design for Example 2

$$
\begin{aligned}
&+(85813972.9887)u(k-5)u(k-5)\\
&+(12812971.7825)u(k-5)u(k-6)\\
&+(-20822067.1606)u(k-6)u(k-6)
\end{aligned} \tag{52}
$$

which includes all candidate terms under the structure parameters of $N_0 = N = 2$ and $K_u = 6$. Figure 6 shows the comparison of the output frequency response of the original ETF design with the desired spectrum. It can clearly be seen that the result is not satisfactory.

4.3 Example 3

In this example two continuous time signals $u_1(t)$ and $u_2(t)$ are produced by the same method as in previous subsections. Figures 7(a) and 7(b) show the signals in the time and frequency domain respectively, which indicates their frequency ranges are approximately $(\bar{a}, \bar{b}) = (5.173, 8.308) rad/s$. The design objective is to move the energy of the two signals to the higher frequency band $(\bar{c}, \bar{d}) = (11.6, 13.6) rad/s$ and shape the magnitudes of the filter output frequency responses as specified by the equations

$$
Y_1^d(j\omega_c) = \frac{\exp(-500\omega_c) + j(600\omega_c^2)}{100000} \qquad \omega_c \in (11.6, 13.6) \tag{53}
$$

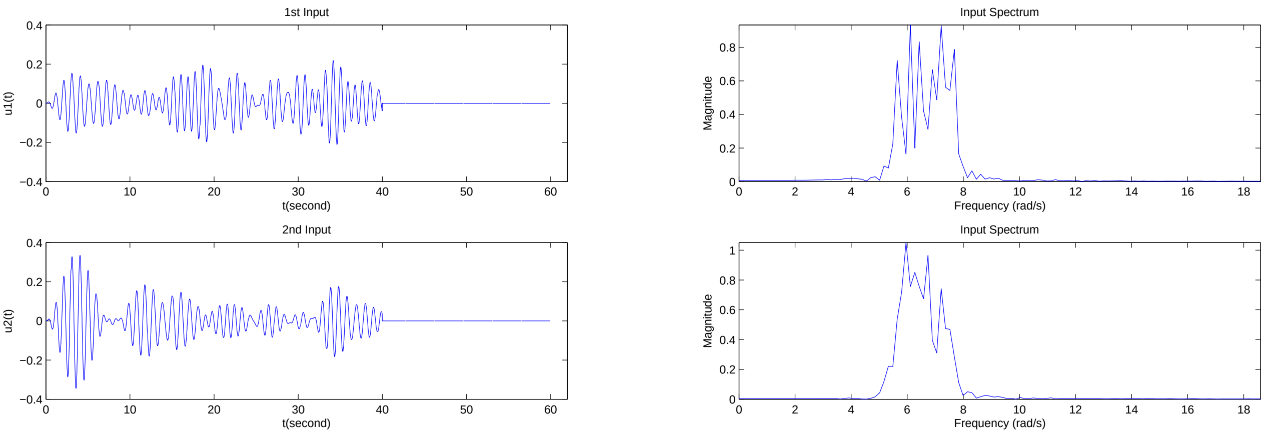
and

$$
Y_2^d(j\omega_c) = 10 * \omega_c^{-0.9} \qquad \omega_c \in (11.6, 13.6) \tag{54}
$$

respectively.

The ETF design was performed using basically the same procedure as described in Section 3.2. The nonlinear part of the design is

$$
\sum_{l_1=1}^{8} \sum_{l_2=l_1}^{8} \widehat{c}_{02}(l_1, l_2) u(k-l_1) u(k-l_2)
$$



(a) Input Signals (b) Input Signal Spectra

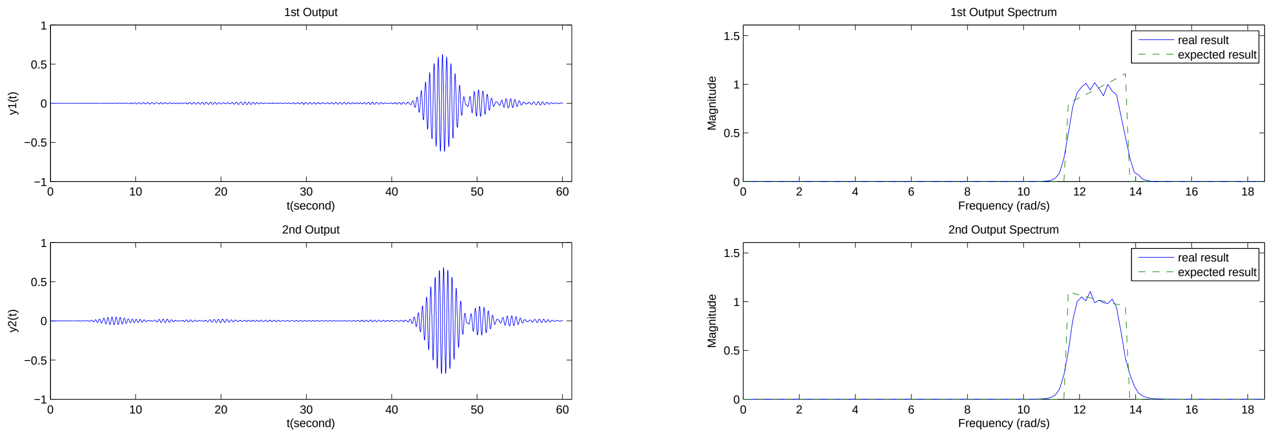
Figure 7: Input Signals in Example 3

$$
\begin{aligned}
= &+(-1441924.3292)u(k-1)u(k-1) \\
&+(285379.0137)u(k-8)u(k-8) \\
&+(18541750.5851)u(k-1)u(k-2) \\
&+(-22756229.2545)u(k-2)u(k-2) \\
&+(40355664.3173)u(k-3)u(k-3) \\
&+(-14514030.8131)u(k-1)u(k-3) \\
&+(5411325.3953)u(k-2)u(k-8) \\
&+(18274452.7226)u(k-5)u(k-6) \\
&+(-5502362.5948)u(k-1)u(k-8) \\
&+(-1964489.9017)u(k-7)u(k-7) \\
&+(-11651325.7597)u(k-4)u(k-7) \\
&+(5536975.2207)u(k-1)u(k-7) \\
&+(-30575169.6926)u(k-4)u(k-4)
\end{aligned} \tag{55}
$$

$G_1(j\omega)$ and $G_2(j\omega)$ were chosen to be of the same structure as in Example 1. The parameters for $G_1(j\omega)$ and $G_2(j\omega)^{1/2}$ were determined as

$$[\bar{b}_1, \bar{b}_2, \bar{b}_3] = [0.46296892264257, -0.91831862499173, 0.46279723749927]$$

$$[\bar{a}_1, \bar{a}_2, \bar{a}_3] = [1.00000000000000, -1.98314681365700, 0.99927891831368]$$



(a) Output Signals (b) Output Signal Spectra

Figure 8: Output Signals in Example 3

and

$$[\bar{\bar{b}}'_1, ..., \bar{\bar{b}}'_9] = 1.0e - 07 * \begin{bmatrix} 0.09739595170405 \\ 0 \\ -0.38958380681620 \\ 0 \\ 0.58437571022431 \\ 0 \\ -0.38958380681620 \\ 0 \\ 0.09739595170405 \end{bmatrix}^T$$

$$[\bar{\bar{a}}'_1, ..., \bar{\bar{a}}'_9] = \begin{bmatrix} 1.00000000000000 \\ -7.88512643636851 \\ 27.26377679465393 \\ -53.98949989228977 \\ 66.97190890674408 \\ -53.28866123207621 \\ 26.56054905178529 \\ -7.58202584692058 \\ 0.94907871450787 \end{bmatrix}^T$$

The output responses of the ETF to the specified inputs are shown in figures 8(a) and 8(b) in the time and frequency domain respectively. Figure 8(b) obviously indicates that an excellent result is again achieved by the new method for the more complicated design case.

For the same design problem, the original method produced an energy transfer filter with nonlinear part including 45 terms, which are all candidate terms under the structure parameters $N_0 = N = 2$, $K_u = 9$. Figure 9 shows the comparison of the output frequency responses

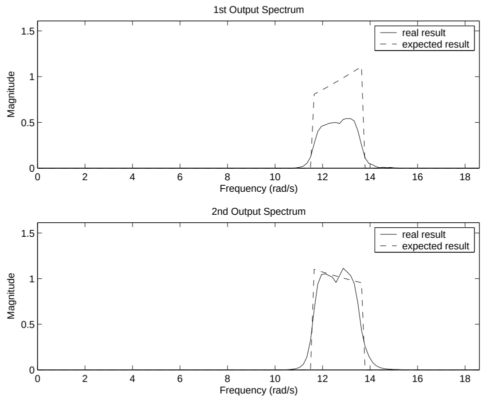


Figure 9: Output Spectrum of the Original ETF Design for Example 3

of this energy transfer filter with the desired spectra. The result again indicates that the new design method has to be used to achieve the required energy transfer performance.

5. Conclusions

A new method for the design of energy transfer filters of a NARX model with input nonlinearity has been developed in the present study. The method uses the orthogonal least squares approach to determine the nonlinear part of the energy transfer filter. This overcomes the problem with the original method which may not achieve an expected design when an upper limit for the maximum lag K_u of the filter input is reached but the filtering performance is still not satisfactory. Several design examples demonstrate the performances and advantage of the new method. The new OLS based design should make energy transfer filters easier to implement applications in electronic and communication systems.

Acknowledgments

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