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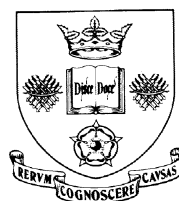


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Network Interconnectivity with Regulation and
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Network Interconnectivity with Regulation and Competition

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Abstract

A simple theoretical network model is introduced to investigate the problem of network interconnection. Prices, profits and welfare are compared under welfare maximisation, network monopoly and network monopoly with competition over one part of the network. Given that inducing actual competition may bring disbenefits such as cost duplication and co-ordination costs, we also explore the possibility of a regulator using the threat of entry on a section of the monopoly network in order to bring about the socially preferred level of interconnectivity. We show that there are feasible parameter values for which such a threat is plausible.

Keywords: Network interconnectivity, monopoly, competition, regulation

JEL codes: L14, L33, L50

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1 Introduction

Recently, there has been renewed focus on issues of competition and integration within complementary and substitute product markets following the development of computer systems and the internet, which has motivated research into complicated networks with varying degrees of connectivity. The initial technological development saw the results of Cournot (1838) concerning complementary and substitute goods being extended by Economides and Salop (1992) and Matutes and Regibeau (1992), amongst others. More recently there have been a number of studies that have applied the analysis to policy decisions, such as studies on computer operating systems (Gisser and Allen, 2001; McHardy, 2006) and video games (Clements and Ohashi, 2005). Gabszewicz *et al.* (2001) consider price equilibria where products are each indivisible but their joint consumption results in a higher utility than the sum of the utilities when the products are consumed in isolation. Denicolo (2000) considers compatibility within a bundling framework and Zhou (2003) looks at the level of access to telecommunications markets. There is also a history of such analysis in the transport literature such as Else and James (1994) who look at the railways, and McHardy and Trotter (2006) who consider airlines. Additionally, Newbery (1999) explores a number of issues regarding network utilities and Baumol and Sidak (1994) consider inter-connection and how to encourage entry, amongst other things, in local telephony.

Much of the literature deals with incentives in the network set-ups and resulting pricing. In particular, there is a great deal of research that deals with access pricing under various structures - see Laffont and Tirole (1994, 1996), Armstrong *et al.* (1996), and Armstrong and Vickers (1998). These papers tend to focus on networks traditionally seen as natural monopolies, and consider how, at what price and where it is best to introduce competition to ensure beneficial results. This paper focuses on when and where to encourage competition to bring about a network set-up that would be preferred by the end user. The model has two possible network configurations that can be selected by the firm(s): fully-connected and incomplete with the network operator choosing between the more expensive, fuller system that allows quicker access or the less

costly system with indirect connection. The type of network operator is varied and the situations where ownership regimes provide a fully-connected network are compared. Using a transport network as an example, the paper explores the effects of permitting entry by other operators when the network is operated by an incumbent monopolist subject to a regulator, and how the regulator can use entry to achieve the socially desirable provision of the network.

The model in this paper has a number of differences and similarities with other areas of network theory, not just the access pricing literature. Inter-connection is applicable but there are no specific externalities attached to either of the network configurations here. Nor are there any issues of compatibility. The network operator chooses the configuration of the network and the network will function correctly. This immediately differentiates this model from the network externalities literature - like the seminal paper by Katz and Shapiro (1985), which considers the presence of network externalities and how this impacts upon competition and compatibility. However, their finding that firms may choose complete compatibility at the detriment of consumer surplus is relevant when considering the conclusions of this paper. Additionally another paper that includes network externalities, Lambertini and Orsini (2001) that finds an oversupply of quality compared to the social optimum, is of interest when it comes to discussing the results in the final part of this paper.

The following section introduces a simple three-sector network model. Section 3 considers the benchmark scenario of the welfare-maximising social planner. Section 4 introduces a profit-maximising network monopolist, and considers the circumstances under which the network monopolist and social planners' choices over a complete or incomplete network coincide (requiring no regulation) or differ. Section 5 considers the possibility of entry on a sub-section of the network. Section 6 summarises the results and their policy implications.

2 The network

Consider the simple network shown in Figure 1. Let it represent (part of) the transport network in a circular city, and assume there are three public transport services along routes 1, 2 and 3 between the three origins and destinations r , s and t . This network can serve demands for radial travel between the city centre (s) and the perimeter of the city (points r and t) as well as non-diameter chord travel (between r and t). This simple network, therefore, allows for direct travel between points on the perimeter along route 1 as well as indirect travel between these points using routes 2 and 3. For passengers wishing to travel between r and t , the combined services along routes 2 and 3 (which are perfect complements) now provide a substitute for route 1.

We now make specific a key assumption of the model.

Assumption 1. *Services on radial routes 2 and 3 are always provided.*

In the next section we impose restrictions on key parameter values to ensure that Assumption 1 is justified within the appropriate maximising framework for each regime (so it is an equilibrium outcome of the model). However, the purpose for Assumption 1 is to frame the question of network interconnectivity in terms of whether or not the direct cross-city (chord) service (route 1) is provided. A key question is then whether or not a monopoly public transport provider would (would not) provide the complete network when it is (is not) socially optimal to do so. In the situation where there is a mismatch between the network monopoly decision and that of a social planner we consider how using entry or the threat of entry on the network may help align the objectives of the public transport provider with the society's interests.

By definition, routes 2 and 3 are of equal length (the radius of the circular city), which for simplicity is normalised to unity. Therefore, route 1 is the chord joining r and t with length x . In this framework, x lies in the open interval $0 < x < 2$.

Suppose the relevant public transport provider charges a fare, f_i , for travel on route

i ($i = 1, 2, 3$). Let the demand for direct travel along route i be given by:

$$Q_1 = \alpha - x - f_1, \quad (1a)$$

$$Q_j = \beta - 1 - f_j, \quad (j = 2, 3). \quad (1b)$$

where α and β are positive constants. As the city centre has a radius of one then to ensure that (1a) and (1b) are positive, at least at zero fares, and given $0 < x < 2$, let:

$$\alpha \geq 2, \beta > 1. \quad (2)$$

For simplicity we assume that the psychological passenger cost per unit distance is unity; thus for passengers on routes 2 and 3 (which have unit length) the relevant cost is also unity, whilst for passengers of route 1 it is x .¹ It follows that the generalised cost of direct travel along mn ($m \neq n = r, s, t$), G_{mn} , is given by:

$$G_{rt} = f_1 + x, \quad G_{rs} = f_2 + 1, \quad G_{st} = f_3 + 1. \quad (3)$$

Clearly each journey can be undertaken directly using one route or indirectly using the remaining two routes. This paper allows the provision of services along route 1 to be an option for the relevant service provider. In order to compare the gains to the relevant operator with and without services on route 1 and to incorporate the fact that pricing decisions on route 1 must be undertaken in the knowledge that too high a fare may divert passengers onto routes 2 and 3, it is necessary to consider the demands for the alternative journey rt via s . Assuming that there is no interchange penalty, the generalised cost for a passenger making indirect travel along mn via l ($m \neq n \neq l = r, s, t$), G_{mln} , is given by:

$$G_{rst} = 2 + f_2 + f_3, \quad (4a)$$

$$G_{str} = 1 + x + f_1 + f_3, \quad (4b)$$

¹It is also assumed that all services travel at an equal (constant) speed; hence, there is no need to introduce a separate time-cost parameter in the generalised travel cost.

$$G_{trs} = 1 + x + f_1 + f_3. \quad (4c)$$

If f_1 is prohibitively high or services on route 1 are not provided, then all rt travel is diverted through routes 2 and 3. For example, the operator may charge such a large fare on route 1 (or alternatively they may decide not to provide it) that rt travellers would rather journey along routes 2 and 3 - if this was the case then they would face generalised cost (4a). Using the demand densities from (1) and the generalised cost (4a) the total demand for travel on route j (own route-specific demand and indirect rst demand), \hat{Q}_j , is given by:²

$$\hat{Q}_j = \alpha + \beta - 3 - 2f_j - f_k, \quad (j \neq k = 2, 3). \quad (5)$$

In terms of the cost structure of the model, it is assumed that public transport provision on a route has a zero marginal cost per passenger; however, there is a non-zero operating cost per unit distance.³ Setting F as the operating cost per unit distance, it follows that the operating cost for routes 2 and 3 (which have unit length) is F whilst for route 1 the operating cost is Fx .

3 A first-best social planner

Before considering the conditions under which a first-best social planner would choose to provide services along route 1, we construct expressions for the consumer surplus associated with each individual route. With the social planner engaging in marginal-cost pricing, our assumption of zero marginal cost implies a fare of zero on each route:

$$f_i^S = 0, \quad (\forall i = 1, 2, 3). \quad (6)$$

Consumer surplus on each route then becomes:

$$C_1^S = \frac{1}{2}(\alpha - x)^2, \quad (7a)$$

²Throughout the paper, terms indicated with a "∧" relate to the incomplete network - i.e. excluding route 1.

³This means that there are no capital costs in the model.

$$C_2^S = C_3^S = \frac{1}{2}(\beta - 1)^2. \quad (7b)$$

In order for the framework to be consistent with route 1 being the marginal route for the purposes of this paper, we impose constraints on the parameters of the model.

Lemma 1. *The following constraints on the values α , β , and x are sufficient to ensure that at the socially optimal prices the consumer surplus on the incomplete network with routes 2 and 3 provided $\hat{C}S_{23}^S$ is always strictly greater than the level of consumer surplus on the incomplete network with routes 1 and j ($j = 2, 3$) $\hat{C}S_{1j}^S$.*

$$\alpha > \beta - x - 1. \quad (8)$$

Proof. Given the socially optimal pricing (6), consumer surplus on the incomplete network excluding route 1 is given by:

$$\hat{C}S_{23}^S = [\beta - 1 + \max(0, \alpha - 2)]^2, \quad (9a)$$

and consumer surplus on the incomplete network excluding route j ($j = 2, 3$) is given by:

$$\hat{C}S_{1j}^S = \frac{1}{2} [\alpha - x + \max(0, \beta - x - 1)]^2 + \frac{1}{2} [\beta - 1 + \max(0, \beta - x - 1)]^2. \quad (9b)$$

□

This justifies our assumption that routes 2 and 3 are always provided; implying that the demand on the radial routes is denser.⁴ Given this assumption, the decision to provide route 1 is now of particular interest. Had route 1 been an important route to the traveler then finding it to be provided would be unremarkable.

We now consider the conditions under which a social planner would provide route 1. It follows from (6) that network revenue under the social planner is zero, so welfare

⁴In the context of the transport example, this is intuitively appealing as we would expect more travelers to pass through a city centre.

is derived solely from consumer surplus. In the case where the social planner provides services on route 1, total consumer surplus across the system, C^S , is then:

$$C^S = \sum_{i=1}^3 = \frac{1}{2}(\alpha - x)^2 + (\beta - 1)^2. \quad (10)$$

Welfare under the social planner with route 1 provided, W^S , is consumer surplus minus the operating costs of the three routes:

$$W^S = C^S - (2 + x)F = \frac{1}{2}(\alpha - x)^2 + (\beta - 1)^2 - (2 + x)F. \quad (11)$$

If the social planner does not provide route 1 then the consumer surplus is measured in relation to the alternative demands in (5). Welfare, \hat{W}^S , is then:

$$\hat{W}^S = \frac{1}{2}(\alpha - x)^2 + (\beta - 1)^2 - 2F. \quad (12)$$

We are now able to consider the social planner's optimal network provision.

Proposition 1. *The welfare-maximising social planner would prefer to provide a complete network if the constant operating cost per unit distance satisfies the inequality:*

$$F < \frac{1}{2x}[(\alpha - x)^2 - (\alpha - 2)^2]. \quad (13)$$

Proof. Subtracting (11) from (10) and rearranging in terms of F gives (13). \square

Corollary 1. *If the constant operating cost per unit distance is zero, the welfare maximising social planner will always provide a complete network.*

Proof. Given (2) and $0 < x < 2$ we can see that all the elements of (13) are non-negative; that is $(\alpha - x)^2 > 0$, $(\alpha - 2)^2 \geq 0$, and $2x > 0$. We can also see from (2) and from $0 < x < 2$ that $(\alpha - x)^2 > (\alpha - 2)^2$. \square

Setting (13) as an equality, we define the social planner's threshold cost for providing a complete network i.e. the level of constant operating costs (per unit distance) which makes the social planner indifferent between providing a service on route 1 and not

providing a service, \tilde{F}^S :

$$\tilde{F}^S = \frac{1}{2x}[(\alpha - x)^2 - (\alpha - 2)^2]. \quad (14)$$

(14) provides a useful benchmark level of operating cost. If for some level of F below \tilde{F}^S a regime does not provide route 1 then the regime's network is socially suboptimal.

Corollary 2. *The welfare maximising social planner would provide a complete network for an increasing (decreasing) range of constant operating costs per unit distance as α (x) rises.*

Proof. Partially differentiating the bracketed part of the R.H.S. of (14) with respect to x and α , respectively, gives:

$$\frac{\partial \tilde{F}^S}{\partial x} = \frac{x^2 - 4\alpha + 4}{2x^2}, \quad (15a)$$

$$\frac{\partial \tilde{F}^S}{\partial \alpha} = \frac{2 - x}{x}. \quad (15b)$$

Using (2) it can be shown that (15a) is non-positive whilst (15b) is always strictly positive. \square

Corollaries 1 and 2 show that we have a well-behaved model with intuitive results. As the length of the direct route, x , falls - recall that the length of the indirect route along 2 and 3 remains constant - then the benefit that society gets from the existence of route 1 increases. Equally, an increase in α means that there are more travellers moving along route 1, so the absolute gains from providing the direct route increase.

4 Network monopoly

The case of network monopoly is more complicated than that of the social planner, since the monopolist has to select the network structure (whether or not to provide route 1) and the optimum level of fares which are interdependent, whilst the planner's price policy is independent of choice of routes. However, matters are made more straightforward by the symmetrical nature of routes 2 and 3. It follows that whatever the monopolist's choice of network configuration and fare structure, the optimal fare for route 2 will be

the same as that for route 3. Dealing with the scenario in which the network monopolist supplies a complete network, the general expression for network profit, π^M , is:

$$\pi^M = f_1(\alpha - x - f_1) + 2f_j(\beta - 1 - f_j)(2 + x)F, \quad (16)$$

where f_j is the common fare on routes 2 and 3. Profit maximisation yields the following optimal network monopoly fares under a complete network:

$$f_1 = \frac{\alpha - x}{2}, \quad (17a)$$

$$f_j = \frac{\beta - 1}{2}. \quad (17b)$$

Substituting (17) into (16) gives the reduced-form network monopoly profit:

$$\pi^M = \frac{1}{4}(\alpha - x)^2 + \frac{1}{2}(\beta - 1)^2 - (2 + x)F. \quad (18)$$

If, however, the network monopolist supplies an incomplete network (omitting route 1), the general expression for network profit is, $\hat{\pi}^M$:

$$\hat{\pi}^M = 2f_j(\alpha + \beta - 3 - 3f_j) - 2F. \quad (19)$$

In this case profit maximisation yields the following optimal network monopoly fares under an incomplete network:

$$\hat{f}_j = \frac{1}{6}(\alpha + \beta - 3), \quad (j = 2, 3). \quad (20)$$

Substituting (20) into (19) gives the reduced-form expression for maximum monopoly profit with an incomplete network:

$$\hat{\pi}^M = \frac{1}{6}(\alpha + \beta - 3)^2 - 2F. \quad (21)$$

Proposition 2. *The network monopolist would strictly prefer a complete network if the*

constant operating cost per unit distance satisfies the inequality:

$$F < \frac{\alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 12 - 4\alpha\beta}{12x}. \quad (22)$$

Proof. Subtracting (21) from (18) gives:

$$\pi^M - \hat{\pi}^M = \alpha^2 + 12\alpha - 6\alpha x + 3x^2 + 4\beta^2 - 4\alpha\beta - 12 - 12Fx. \quad (23)$$

It is straightforward to show that the right-hand side of this equation will be positive as long as F is less than the critical value in (22). \square

The expression on the R.H.S. of (22) gives us the network monopolist's threshold operating cost below which a full network will be supplied, analogous to \tilde{F}^S for the social planner in the previous section. Like the social planner, the network monopolist would always find it more profitable to supply a complete network if there was no operating cost:

Corollary 3. *If the constant operating cost per unit distance is zero, the network monopolist will always provide a complete network.*

We now consider the monopolist's decision when it faces the social planner's threshold operating cost (\tilde{F}^S).

Proposition 3. *When faced with the social planner's threshold operating cost then the monopolist's benefits from providing a complete network increase as β and x rise, and as α falls.*

Proof. The monopolist is indifferent between supplying a complete network and an incomplete network with operating cost, \tilde{F}^S , if $\vartheta^M(\tilde{F}^S) = \pi^M(\tilde{F}^S) - \hat{\pi}^M(\tilde{F}^S) = 0$. Using (14), (18) and (21) gives:

$$\vartheta^M(\tilde{F}^S) = \alpha^2 + 6\alpha x - 3x^2 + 4\beta^2 + 12 - 12\alpha - 4\alpha\beta. \quad (24)$$

Partially differentiating (24) with respect to α , β and x gives, respectively:

$$\frac{\partial \vartheta^M(\tilde{F}^S)}{\partial \alpha} = 2(\alpha + 3x - 6 - 2\beta), \quad (25a)$$

$$\frac{\partial \vartheta^M(\tilde{F}^S)}{\partial \beta} = 4(2\beta - \alpha), \quad (25b)$$

$$\frac{\partial \vartheta^M(\tilde{F}^S)}{\partial x} = 6(\alpha - x). \quad (25c)$$

Using (2) it can be seen that (25a) is always positive, so (24) is increasing in x . Given (8) then (25b) is negative and (25c) is positive, so (24) is decreasing in α and increasing in β . \square

Corollary 4. *The monopolist will provide route 1 when the social planner does; however, when the social planner does not provide a complete network it is possible that the monopolist will.*

Proof. This proof is readily verified by using simulations in the relevant range of α , β , and x . Given (2) and (8) we can show that $6\alpha x > 3x^2$ and $4\beta^2 + \alpha^2 + 12 > 4\alpha\beta + 12\alpha$ so (24) is always positive. \square

Proposition 3 establishes that the parameters for which the monopolist and social planner provide a complete network do not always match and figures 2 and 3 underline this. Figures 2 and 3 show this graphically by plotting the social planner's and the monopolist's threshold operating costs. The area on and below the lines shows values for which each would provide a complete network; any area that is below both lines represents a situation where the monopolist and social planner agree in providing a complete network.⁵ As shown in Corollary 4, Figure 2 shows the monopolist provides route 1 for some values that the social planner would not and as α increases the monopolist becomes more willing to provide route 1 for values that the social planner would prefer an incomplete network.

⁵Conversely, any area above both lines shows when the monopolist and social planner agree in providing an incomplete network.

Figure 3 shows what happens if α takes on its largest possible value in relation to β , that is (8) becomes an equality. This further underlines corollary 4, showing that as α rises (meaning the minimum allowable β also rises) then the monopolist continues to provide a complete network for a larger range of values than the social planner would prefer.

5 The impact of an entrant

This section considers the effect on a network monopolist's fare structure and network configuration when a regulator allows rival firms to enter the network.⁶ To emphasise that the network monopolist now faces the possibility of entry on part of its network we shall refer to it as an *incumbent*. Initially we look at the effect on network provision and the incumbent's behaviour when entry is allowed onto route 1. Then we move on to look at entry on route 2 - due to the symmetry of the model, the analysis would, of course, apply equally to route 3 - and pose the question "will the introduction of a rival on route 2 cause the incumbent to provide a complete network (when it was previously incomplete)?", by considering the case in which entry on route 2 leads to a Cournot quantity game on that route.⁷ We investigate whether the incumbent can use its provision of route 1 as a strategic tool to reduce profitability on route 2 and hence exclude the entrant(s).

The general expression for the profit of an incumbent providing a complete network, using (1), is:

$$\pi^I = q_1^I(\alpha - x - Q_1) + f_2(\beta - 1 - f_2) + f_3(\beta - 1 - f_3) - F(2 + x). \quad (26)$$

The profit function for an entrant k on route 1 is:

$$\pi^{E_k} = q_1^{E_k}(\alpha - x - Q_1) - Fx, \quad (27)$$

⁶This is not a free-entry model, instead the regulator is able to set the number of entrants.

⁷In both cases the analysis is restricted to when the operating costs of operation are low enough to accommodate n -firm entry under the Cournot regime.

where $q_1^{E_k}$ is the entrant's output. This assumes that the entrant faces the same operating cost as the incumbent.

If n_1^E is the total number of entrants on route 1 then total output on route 1, Q_1 , will be the sum of the quantities produced by all firms and can be defined by:

$$Q_1 = q_1^I + n_1^E q_1^{E_k}. \quad (28)$$

Profit maximisation implies the fares given in (17b) and with the Cournot assumption on route 1 the equilibrium fare is:

$$f_1 = \frac{(\alpha - x)}{(1 + n_1)}, \quad (29)$$

where $n_1 = 1 + n_1^E$. Calculating firm outputs and using (28) before substituting (29) into (26) and (27) gives profits:

$$\pi^I = \frac{(\alpha - x)^2}{(1 + n_1)^2} + \frac{(\beta - 1)^2}{2} - F(2 + x), \quad (30a)$$

$$\pi^{E_k} = \frac{(\alpha - x)^2}{(1 + n_1)^2} - Fx. \quad (30b)$$

If, on the other hand, the incumbent chooses not to provide services on route 1, but the entrant(s) do, then we do not get the switch in demands from (1) to (5) that we saw previously; the rival firms offer route 1 so the incumbent's profit becomes:

$$\hat{\pi}^I = \frac{(\beta - 1)^2}{2} - 2F. \quad (31)$$

Profit for an entrant i on route 2 is:

$$\hat{\pi}^{E_k} = q_1^{E_k}(\alpha - x - Q_1) - Fx. \quad (32)$$

As the incumbent is no longer supplying route 1 then $q_1^I = 0$, so \hat{Q}_1 can be defined as:

$$\hat{Q}_1 = n_1^E q_1^{E_k}. \quad (33)$$

Profit maximisation gives the equilibrium fare of the entrant:

$$\hat{f}_1 = \frac{\alpha - x}{1 + n_1^E}. \quad (34)$$

Calculating entrant output, then using this and (34) in (32) gives profit:

$$\hat{\pi}^{E_k} = \frac{(\alpha - x)^2}{(1 + n_1^E)^2} - Fx. \quad (35)$$

Proposition 4. *The entrant(s) will provide route 1 for higher levels of the constant operating cost per unit distance than the incumbent.*

Proof. The incumbent's threshold operating cost in the provision of route 1, calculated by subtracting (31) from (30a), is:

$$\tilde{F}^I = \frac{(\alpha - x)^2}{(1 + n_1)^2 x}. \quad (36)$$

The incumbent would thus provide route 1 if:

$$\tilde{F}^I < \frac{(\alpha - x)^2}{(1 + n_1)^2 x}. \quad (37)$$

The entrant's threshold operating cost if the incumbent did not provide route 1 would, from (35), be:

$$\tilde{F}^E = \frac{(\alpha - x)^2}{(1 + n_1^E)^2 x}. \quad (38)$$

The entrant would thus provide route 1 when the incumbent does not if:

$$\tilde{F}^E < \frac{(\alpha - x)^2}{(1 + n_1^E)^2 x}. \quad (39)$$

As $n_1^E < n_1$ we can see that (36) would always be smaller than (38). \square

As the entrant always provides route 1 when the incumbent does then we can concentrate on route 1 entrant's threshold operating cost as it determines when a complete network is provided, and we can compare this with the social planner's preferred provision of route 1.

Proposition 5. *Entry on route 1 can lead to the provision of a complete network that more closely matches the provision of the social planner than the monopolist's provision.*

Proof. Figures 4 and 5 depict simulations of (14), (22) and (38) indicating a non-empty parameter set under which entry on route 1 brings about the provision of a complete network, completing the proof. \square

Figure 4 is Figure 2 with the entrant's threshold operating cost added and this lies just beneath the social planner's operating cost so that the entrant would provide an incomplete network when the social planner would prefer a complete one. This means that with entry on route 1 we have the possibility that an incomplete network may be provided when a social planner would prefer a complete network - conversely a monopolist would provide route 1 for values that the social planner would not. However, the entrant's threshold operating cost is a closer representation of the social planner's preference than the monopolist's. Figure 4 looks at a higher level of α and shows that, again, for the most part, the entrant would provide a level of network provision that matches the social planner more closely than the monopolist, although this time at a level above the social planner and at low levels of β we have the possibility that the entrant would provide a complete network for values that the monopolist would not. This means that the network regulator could use entry on route 1 to provide a level of provision close to what the social planner would prefer, but would also have to carefully monitor the situation.

Route 1 is not the only viable place for the introduction of entrants and we now investigate what happens if competition is allowed on one of the radial routes - route 2 for convenience.

The general expression for the profit of the incumbent providing a complete network, using (1), is:

$$\pi^I = f_1(\alpha - x - f_1) + q_2^I(\beta - 1 - Q_3) + f_3(\beta - 1 - f_3) - F(2 + x), \quad (40)$$

where q_2^I is the demand supplied by the incumbent. If n_3^E is the total number of entrants then total output for route 3, Q_2 , can be defined by:

$$Q_2 = q_2^I + n_2^E q_2^{E_k}, \quad (41)$$

where $q_2^{E_k}$ is the demand supplied by each entrant. Profit for the k^{th} entrant becomes:

$$\pi^{E_k} = q_2^{E_k}(\beta - 1 - Q_2) - F. \quad (42)$$

Profit maximisation implies the first-order conditions (17a) and (17b) for $j = 2$ and given the Cournot assumption, on route 2 the equilibrium fare is:

$$f_2 = \frac{\beta - 1}{1 + n_2}, \quad (43)$$

where $n_2 = 1 + n_2^E$.

Calculating firm outputs then using this, (41) and (43) in (40) gives profits:

$$\pi^I = \frac{(1 + n_2)^2(\alpha - x)^2 + (n_2^2 + 2n_2 + 5)(\beta - 1)^2}{4(1 + n_2)^2} - f(2 + x), \quad (44a)$$

$$\pi^{E_k} = \frac{(\beta - 1)^2}{(1 + n_2)^2} - F. \quad (44b)$$

If, on the other hand, the incumbent chooses not to provide services on route 1, demands on routes 2 and 3 become interlinked by the diverted route 1 passengers. The profits for the incumbent and entrant, respectively, are:

$$\hat{\pi}^I = \frac{q_2^I}{2}(\alpha + \beta - 3 - f_2 - Q_2) + f_3(\alpha + \beta - 3 - f_2 - 2f_3) - 2F, \quad (45a)$$

$$\hat{\pi}^{E_k} = \frac{q_2^{E_k}}{2}(\alpha + \beta - 3 - f_2 - Q_2) - F. \quad (45b)$$

Maximising incumbent and entrant profits with respect to output on route 2, then substituting the values of q_3^I and Q_3 into (44a), before again maximising and solving gives the fare for route 2:

$$f_3 = \frac{n_2(6n_2 + 5)(\alpha + \beta - 3)}{4(4n_2^2 + 6n_2 + 3)}. \quad (46)$$

Calculating firm outputs, and using (14), (41) and (46) in (45) yields profits for the incumbent and entrant, respectively:

$$\hat{\pi}^I = \frac{(16n_2^4 + 96n_2^3 + 120n_2^2 + 64n_2 + 13)(\alpha + \beta - 3)^2}{8(4n_2^2 + 6n_2 + 3)^2} - 2F, \quad (47a)$$

$$\hat{\pi}^{E_k} = \frac{n_2(6n_2 + 5)^2(\alpha + \beta - 3)^2}{8(4n_2^2 + 6n_2 + 3)^2} - F. \quad (47b)$$

Proposition 6. *Entry on route 2 (or 3) can lead to the provision of a complete network that more closely matches the provision of the social planner than the monopolist's provision.*

Proof. Calculating the net gain to the incumbent from offering a complete network relative to an incomplete network by subtracting (44a) from (47a) and solve for F , gives:

$$\begin{aligned} \tilde{F}^{Iz} = & \frac{(1 + n_2)^2(\alpha - x)^2 + (n_2^2 + 2n_2 + 5)(\beta - 1)^2}{4x(1 + n_2)^2} \\ & - \frac{(16n_2^4 + 96n_2^3 + 120n_2^2 + 64n_2 + 13)(\alpha + \beta - 3)^2}{8x(4n_2^2 + 6n_2 + 3)^2}. \end{aligned} \quad (48)$$

Figures 6 and 7 depict simulations of (14), (22) and (48) indicating a non-empty parameter set under which entry on route 2 (or 3) increases profit for the incumbent by moving to a complete network from an incomplete network. \square

At first glance, it might seem that the incumbent would provide route 1 as a strategic tool against the entry on route 2. However, what actually happens is that the entry on route 2 causes a fall in the price that the incumbent can charge on route 1 and this reduces the range of values over which it provides a complete network. Figures 6 and 7 show this, but we can also see that with entry on route 2 there is the possibility that the incumbent will not provide route 1 when the social planner would prefer it to.

We may expect that route 1 is the preferable place to introduce entry as this is the route that completes the network; however this may not always be the case. Indeed, a

regulator could introduce entry on route 3 in preference to entry on route 1.

Proposition 7. *The welfare arising from complete network provision with entry on route 2 (or 3) can be larger than the welfare from complete network provision with entry on route 1, when all firms face the social planner's threshold operating cost.*

Proof. Using (17b) and (29) in (1) gives the following outputs:

$$Q_1^1 = \frac{n_1(\alpha - x)}{(1 + n_1)}, \quad (49a)$$

$$Q_2^1 = Q_3^1 = \frac{\alpha - 1}{2}. \quad (49b)$$

Welfare for each route can be calculated using:

$$W_i = f_i Q_i + \frac{1}{2}(c - f_i) Q_i - TF_i, \quad (50)$$

where c is the y-axis intercept on the demand curve and TF_i is the total constant operating cost associated with running the route. Substituting the relevant values for each route into (51) gives the welfare for each route:

$$W_1^1 = \frac{3n_1(\alpha - x)^2}{2(1 + n_1)^2} - n_1 F x, \quad (51a)$$

$$W_2^1 = W_3^1 = \frac{3(\alpha - 1)^2}{8} - F. \quad (51b)$$

Summing the elements of (52) together gives total welfare:

$$W_T^1 = \frac{3n_1(\alpha - x)^2}{2(1 + n_1)^2} + \frac{3(\beta - 1)^2}{8} - F(2 + n_1 F). \quad (52)$$

If the incumbent does not enter route 1 then welfare becomes:

$$W_T^1 = \frac{3n_1^E(\alpha - x)^2}{2(1 + n_1^E)^2} + \frac{3(\beta - 1)^2}{8} - F(2 + n_1^E F). \quad (53)$$

Now let us consider when there is entry on route 3. Using (17) and (44) in (1) gives the

following outputs:

$$Q_1^2 = \frac{(\alpha - x)}{2}, \quad (54a)$$

$$Q_2^2 = \frac{(\beta - 1)}{2}, \quad (54b)$$

$$Q_3^2 = \frac{n_2(\beta - 1)}{1 + n_2}. \quad (54c)$$

Substituting the relevant values for each route into (51) gives the welfare for each route:

$$W_1^2 = \frac{3(\alpha - x)^2}{8} - Fx, \quad (55a)$$

$$W_2^2 = \frac{3(\beta - 1)^2}{8} - F, \quad (55b)$$

$$W_3^2 = \frac{3n_3(\beta - 1)^2}{2(1 + n_3)^2} - nF. \quad (55c)$$

Summing the (56) together gives total welfare:

$$W_T^2 = \frac{3(\alpha - x)^2 + 3(\beta - 1)^2}{8} + \frac{3n_2(\beta - 1)^2}{2(1 + n_2)^2} - F(x + 1 + n_2). \quad (56)$$

If the incumbent does not enter route 1 then welfare becomes:

$$W_T^3 = \frac{3(\alpha - x)^2 + 3(\beta - 1)^2}{8} + \frac{3n_2^E(\beta - 1)^2}{2(1 + n_2^E)^2} - F(x + 1 + n_2^E). \quad (57)$$

Producing simulations of (51) and (56) using \tilde{F}^S as the operating cost gives Figure 8. □

6 Conclusions

We find a network operator may not always provide a full network that includes all the possible direct combinations of routes. The values at which a social planner and a network monopolist would provide a complete network are established to show that a monopolist will often provide a complete network when the social planner would prefer an incomplete network - suggesting that the monopolist ‘over-supplies’ the network, so despite the differences in our model we find results similar to those of Katz and

Shapiro (1985) and Lambertini and Orsini (2001). By considering a regulator that can allow entry on to one of the routes we find that a regulator, if they believe the socially preferable level of network interconnection is not being provided by an existing network monopolist, can ensure that a situation closer to the social planner's preference can be provided. More precisely, it can do this by allowing entry in one of two ways: onto the previously non-supplied route or where the incumbent is already operating.

A simple network model has been introduced and used to investigate network service provision. It can be shown that, when an operating cost is present, a network monopolist always offers a complete network when a social planner would. However, the monopolist will also tend to supply a complete network in situations that the social planner would prefer it not to. We consider two options open to the regulator that can reduce this 'over-provision'. Firstly, it can take direct action by allowing entry on route 1; this can cause the entrant to provide a level of complete network provision closer to the preferences of the social planner than the monopolist would. If allowing entry on route 1 is not plausible or desirable then the regulator can also ensure the provision of a complete network is closer to the social planner's preference by allowing entry on route 2 or 3. It is possible that route 3 entry that leads to a complete network may provide greater welfare than route 1 entry which result in a complete network.

Introducing entry is not without its disadvantages as it introduces the possibility that the provision of route 1 could drop to levels below those desired by the social planner. Route 1 entry could even lead to the provision of route 1 for a greater range than the monopolist. For both these reasons it is vital that the regulator continues to check that the appropriate conditions for the introduction of entry exist and continues to monitor the industry if it allows entry. The regulator should also carefully control the number of firms it allows into the market.

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Figure 1: A Simple Network

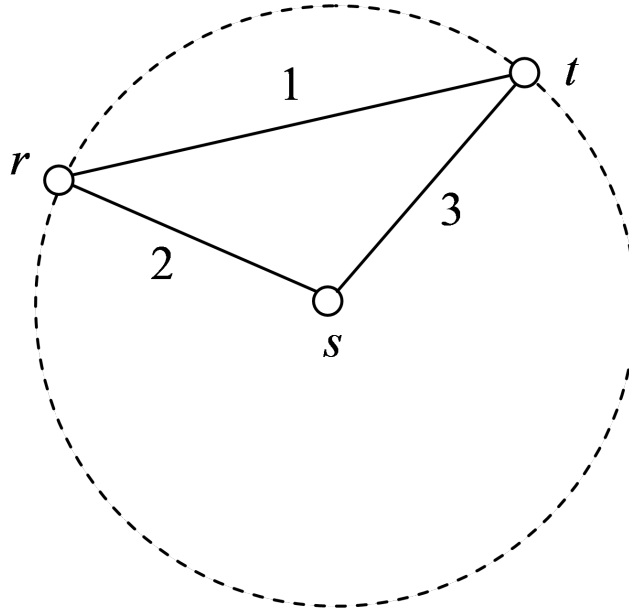


Figure 2: Network with $\alpha = 5$ and $x = 0.5$

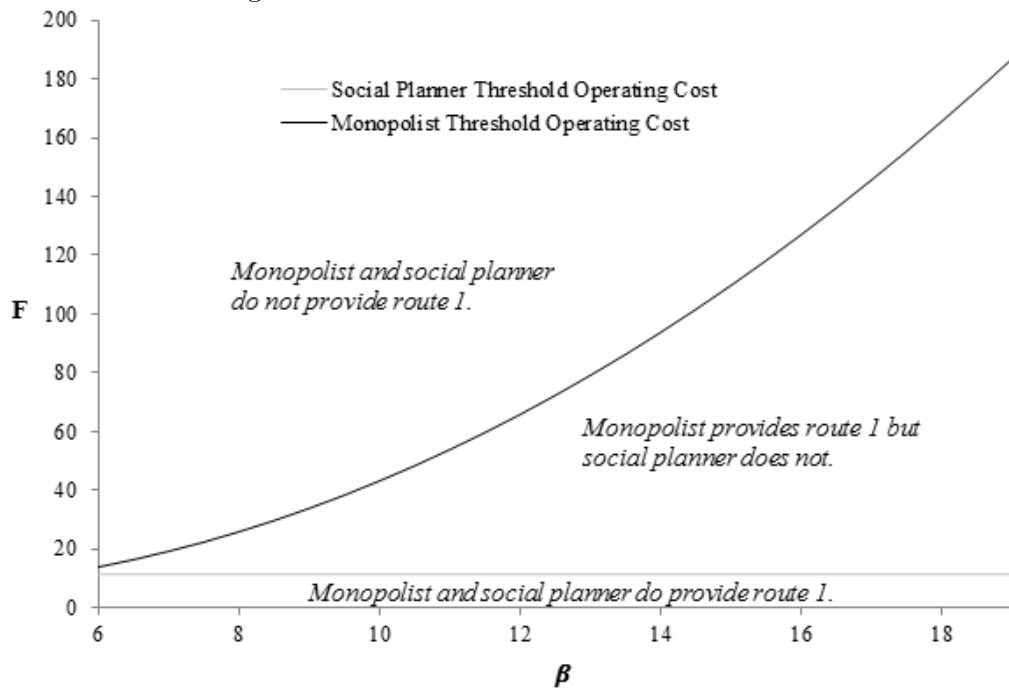


Figure 3: Network with $\alpha = \beta - 1 - x$ and $x = 0.5$

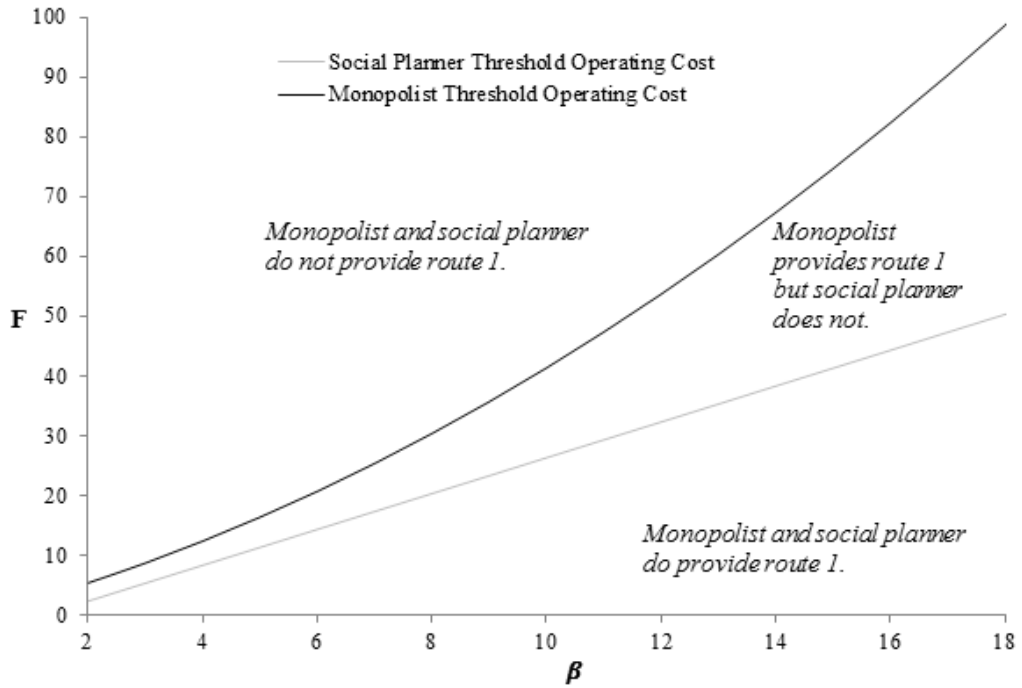


Figure 4: Network with $\alpha = 5$, $x = 0.5$ and $n = 1$

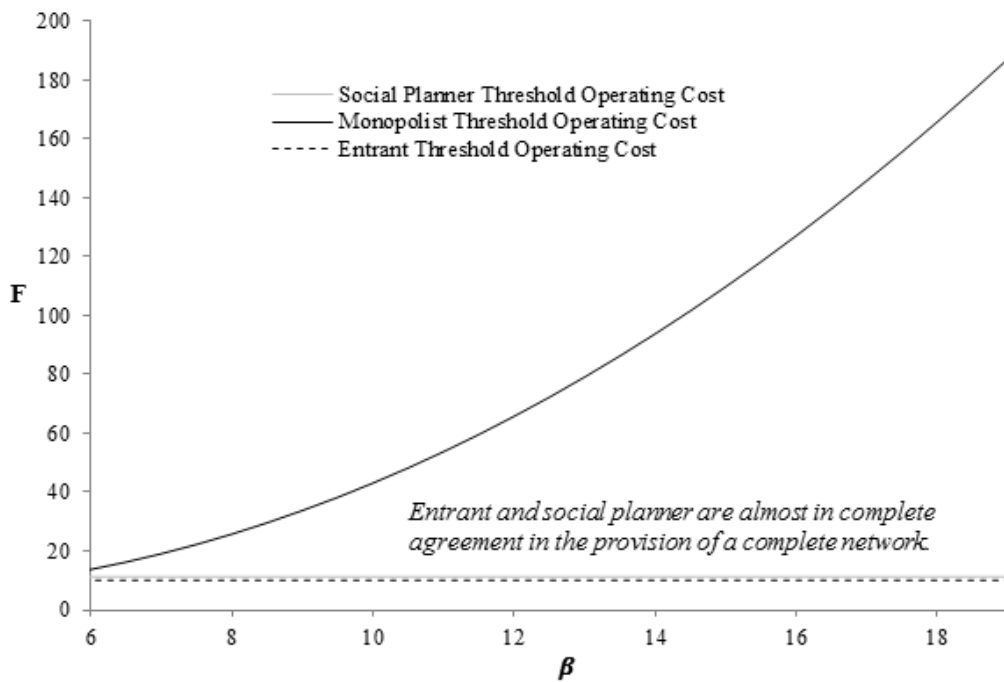


Figure 5: Network with $\alpha = 10$, $x = 0.1$ and $n = 1$

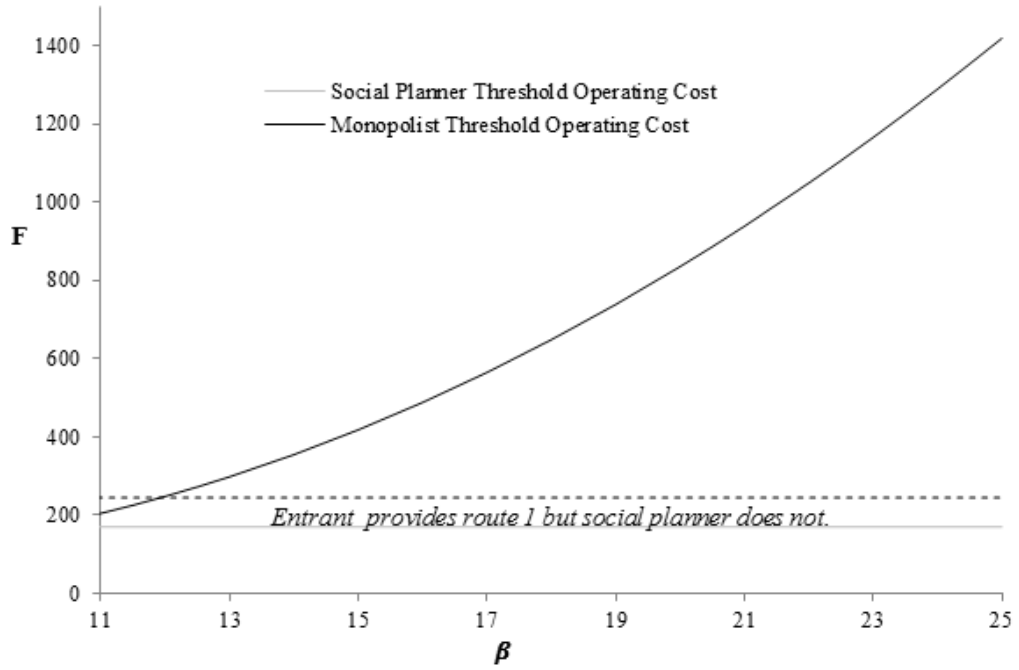


Figure 6: Network with $\alpha = 5$, $x = 0.1$ and $n = 1$

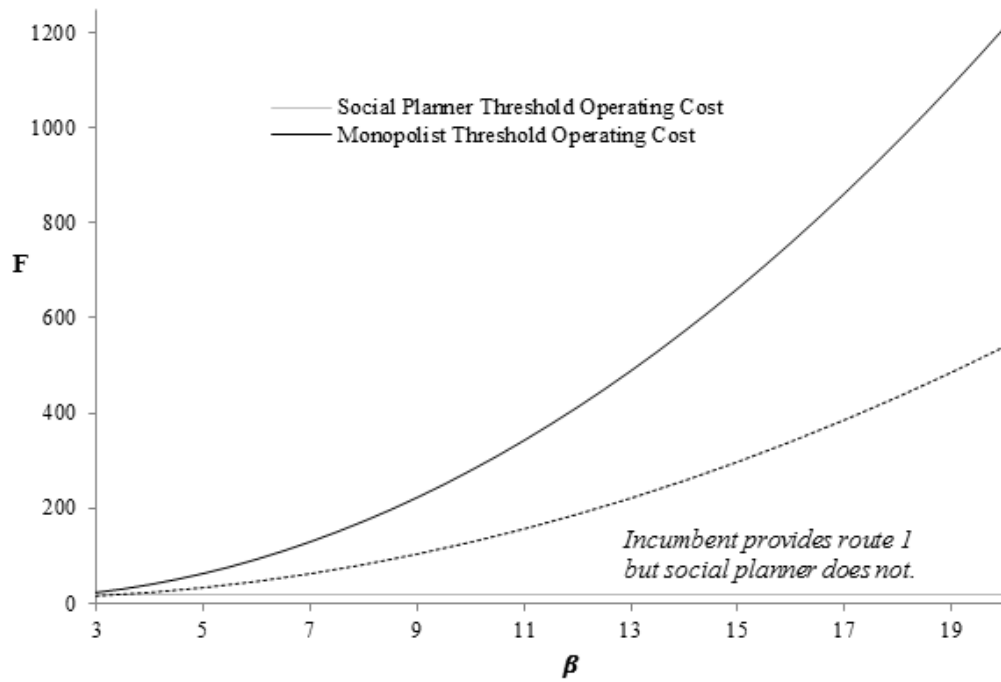


Figure 7: Network with $\alpha = 10$, $x = 0.5$ and $n = 1$

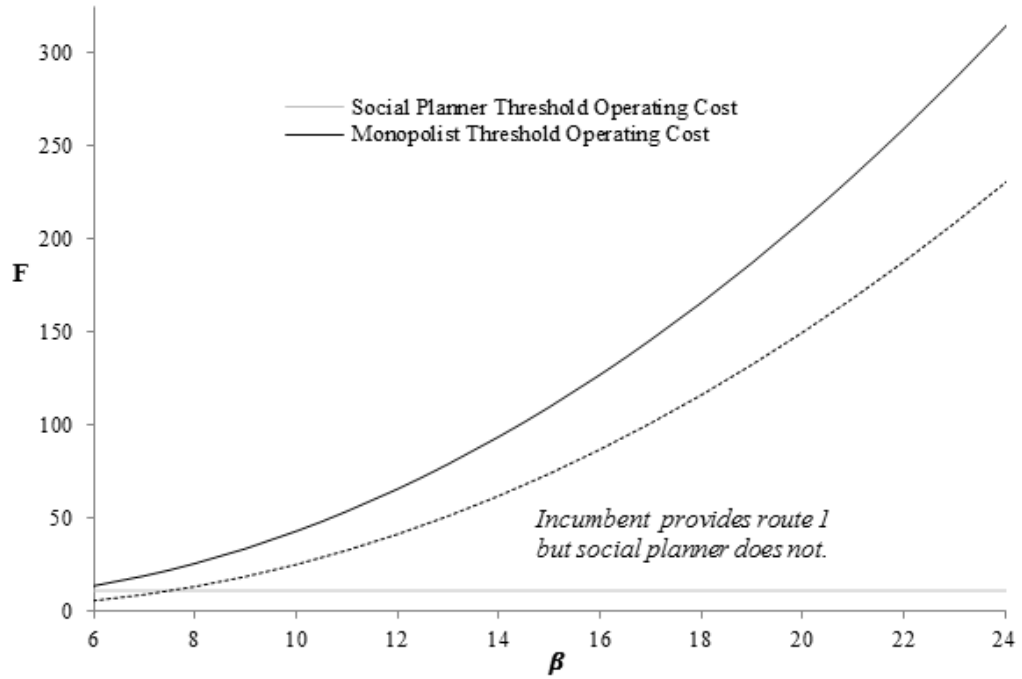
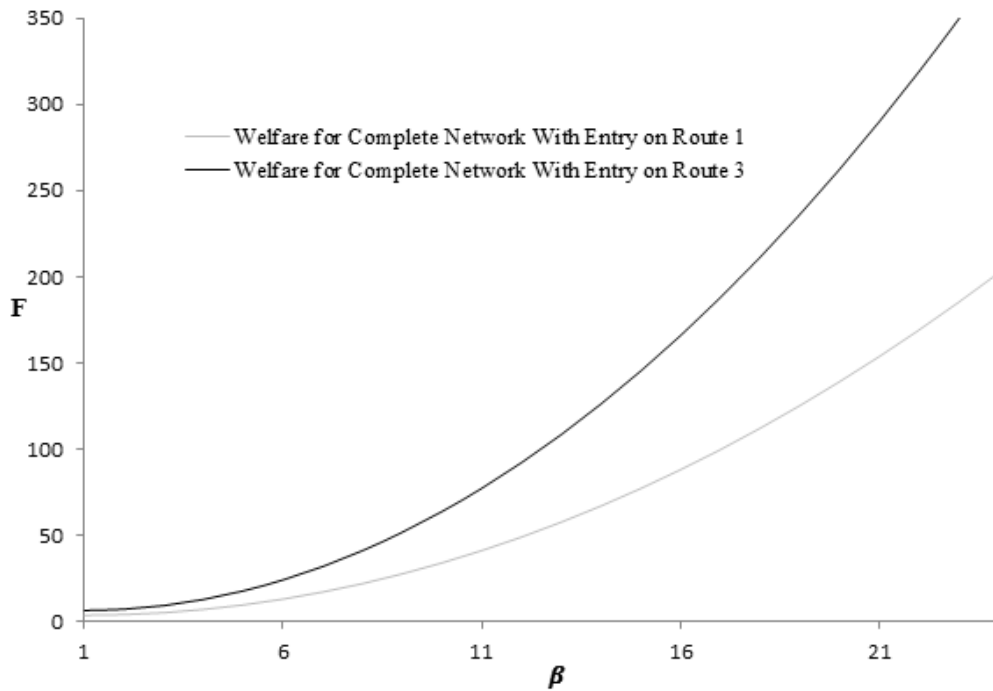


Figure 8: Network with where $n = 5$, $x = 1.6$, and $\alpha = 8$



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