Article:
Zhang, K. and Gubbins, D. (2000) Is the geodynamo process intrinsically unstable? Geophysical Journal International, 140 (1). F1-F4. ISSN: 0956-540X

https://doi.org/10.1046/j.1365-246x.2000.00024.x

Geophys. J. Int. (2000) **140,** F1–F4

FAST-TRACK PAPER

Is the geodynamo process intrinsically unstable?

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Accepted 1999 September 8. Received 1999 July 14; in original form 1999 March 2

SUMMARY

Recent palaeomagnetic studies suggest that excursions of the geomagnetic field, during which the intensity drops suddenly by a factor of 5–10 and the local direction changes dramatically, are more common than previously expected. The 'normal' state of the geomagnetic field, dominated by an axial dipole, seems to be interrupted every 30–100 kyr; it may not therefore be as stable as we thought. We have investigated a possible mechanism for the instability of the geodynamo by calculating the critical Rayleigh number (R_c) for the onset of convection in a rotating spherical shell permeated by an imposed magnetic field with both toroidal and poloidal components. We have found R_c to be a very sensitive function of the poloidal field at the very small Ekman number pertaining to the core. The magnetic Reynolds number, and therefore the dynamo action, is equally sensitive to the applied field because of its dependence on the difference between the Rayleigh number and its critical value. This explains why numerical dynamo simulations at small Ekman number fail when similar magneto-convection calculations succeed: the fluctuating magnetic field of the dynamo leads to rapid swings in convection strength that cannot be resolved numerically. The geo-dynamo may be unstable for the same reason, with the strength of convection varying wildly in response to the inevitable small changes in magnetic field. Frequent geo-magnetic excursions may therefore be a manifestation of the instability arising from the core's very weak viscosity and the controlling effects of the Earth's rotation.

Key words: geodynamo, magnetoconvection.

1 INTRODUCTION

Recent studies suggest that the Earth's magnetic field has fallen dramatically in magnitude and changed direction repeatedly since the last reversal 700 kyr ago (Langereis *et al.* 1997; Lund *et al.* 1998). These important results paint a rather different picture of the long-term behaviour of the field from the conventional one of a steady dipole reversing at random intervals: instead, the field appears to spend up to 20 per cent of its time in a weak, non-dipole state (Lund *et al.* 1998). One of us (Gubbins 1999) has suggested that this is evidence of a rapid natural timescale (500 yr) in the outer core, and that the magnetic field is usually prevented from reversing completely by the longer diffusion time of the inner core (2–5 kyr). This raises a number of important but difficult questions for geodynamo theory. How can the geomagnetic field change so rapidly and dramatically? Can slight variations of the geomagnetic field affect the dynamics of core convection significantly? If so, is the geodynamo process intrinsically unstable?

Of course, an ideal way to answer the above questions is to simulate the geodynamo directly (Glatzmaier & Roberts 1995; Glatzmaier & Roberts 1996; Kuang & Bloxham 1997; Jones *et al.* 1995). However, it is impossible to simulate the strong effects of rotation in the Earth because it produces very small-scale solutions that vary rapidly with time (Zhang & Jones 1997). In this paper we argue, on the basis of results from magnetoconvection studies, that large swings in the geomagnetic field result from extreme sensitivity of core convection to changes in the poloidal geomagnetic field. Furthermore, this behaviour cannot be simulated by the present generation of geodynamo models, and may be the root cause of apparent numerical instabilities reported by some authors (e.g. Walker *et al.* 1998).

Core magnetohydrodynamics (MHD) is subtle because of the competing effects of rotation and geomagnetic fields. There are six major forces: Coriolis, $\mathbf{F}_c$, buoyancy, $\mathbf{F}_b$, magnetic (Lorentz), $\mathbf{F}_l$, inertial, $\mathbf{F}_i$, viscous, $\mathbf{F}_v$, and pressure, $\mathbf{F}_p$. These must be in balance at any instant of time:

$$\mathbf{F}_c+\mathbf{F}_b+\mathbf{F}_l+\mathbf{F}_i+\mathbf{F}_v+\mathbf{F}_p=0\,. \tag{1}$$

In dimensionless form, with length measured by the core radius and time by the magnetic diffusion time, the ratio of

Coriolis to viscous forces is given by the Ekman number, E, which is extremely small [$E=O(10^{-15})$]; the buoyancy force is measured by the Rayleigh number, R_E; the Lorentz force by the Elsasser number, Λ; and the inertial forces by the Rossby number, R_o, which is also extremely small [$R_o=O(10^{-7})$]. Note that our Rayleigh number, R_E, is the same as that used by Roberts (1968) and others in the problem of convection and different by a factor of E from the so-called modified Rayleigh number, $R_{\mathrm{mod}}=ER_E$. The pressure force, $\mathbf{F}_\mathrm{p}$, is passive in the Boussinesq approximation.

The small Ekman number E causes an intriguingly subtle balance among the six forces in eq. (1) that must be satisfied by a dynamic dynamo all the time. In non-magnetic convection a force balance is struck between buoyancy, pressure and viscous forces. This leads to very small length scales $O(E^{1/3})$ and very large critical Rayleigh numbers, the smallest value of R_E for which convection occurs, of order $O(E^{-4/3})$ (Roberts 1968; Busse 1970). This makes numerical simulations difficult but possible at low Ekman number ($E\leq 10^{-5}$, Sun *et al.* 1993). In magnetoconvection with an externally imposed magnetic field the primary force balance is magnetostrophic—between Coriolis, buoyancy, pressure and Lorentz forces—provided the magnetic field is strong enough [Elsasser number $\Lambda=O(1)$]. The viscous force is not required in the leading force balance; the solution can be large scale and therefore presents few numerical difficulties. Why then is the dynamo calculation so much more difficult at low E, when the only difference is that the field is self-generated rather than being imposed?

2 THE MAGNETOCONVECTION MODEL

We consider a spherical shell of electrically conducting Boussinesq fluid with constant thermal diffusivity κ, magnetic diffusivity λ, thermal expansion coefficient α and kinematic viscosity ν in which convection is maintained by a uniform distribution of heat sources. The fluid is confined in a spherical shell of inner radius r_i and outer radius r_o, with $r_\mathrm{i}/r_\mathrm{o}=0.4$. The whole system rotates with a constant angular velocity $\boldsymbol{\Omega}$. We assume that the inner and outer bounding spherical surfaces are stress-free and impenetrable, since it is well known that the choice of velocity boundary condition does not affect the leading-order convection solution. Perfectly magnetic insulating boundaries are assumed at both the inner and outer bounding surfaces of the shell.

In the problem of magnetoconvection, we impose a large-scale magnetic field upon the spherical shell of electrically conducting fluid. Our imposed axisymmetric magnetic field contains both toroidal and poloidal parts with dipole symmetry:

$$\mathbf{B}=B_0(\epsilon\mathbf{B}_\mathrm{P}+\mathbf{B}_\mathrm{T}), \tag{2}$$

scaled so that $|\mathbf{B}_\mathrm{P}|_{\max}=1$ and $|\mathbf{B}_\mathrm{T}|_{\max}=1$. We use the same functional form of $\mathbf{B}_\mathrm{T}$ and $\mathbf{B}_\mathrm{P}$ as in a previous study of magnetic field instability (Zhang & Fearn 1994; Zhang & Fearn 1995). Let us look at the form of the poloidal magnetic field $\mathbf{B}_\mathrm{P}$ as an example. Any mean poloidal field can be represented as a linear combination of functions $H_{ln}(\theta, r)$ that are solutions of

$$(\beta_{ln}^2+\nabla^2)H_{ln}(\theta, r)=0\,, \tag{3}$$

where β_{ln} are to be determined and spherical polar coordinates (r, θ, ϕ) are used. The boundary conditions are to match the potential fields in the exterior of the shell that satisfy

$$\nabla^2 H_{ln}(\theta, r)=0 \tag{4}$$

for $r>r_\mathrm{o}$ or $r<r_\mathrm{i}$, which yields

$$H_{ln}(\theta, r)=P_l(\cos\theta)[\,j_l(r\beta_{ln})n_{l-1}(r_\mathrm{o}\beta_{ln})-j_{l-1}(r_\mathrm{o}\beta_{ln})n_l(r\beta_{ln})]\,, \tag{5}$$

where $P_l(\cos\theta)$ is the Legendre function, and $j_l(r\beta_{ln})$ and $n_l(r\beta_{ln})$ are the spherical Bessel functions of the first and second kinds. The parameter β_{ln} is then determined by the equation

$$j_l(r_\mathrm{i}\beta_{ln})n_{l-1}(r_\mathrm{o}\beta_{ln})-j_{l-1}(r_\mathrm{o}\beta_{ln})n_l(r_\mathrm{i}\beta_{ln})=0\,, \tag{6}$$

where $0<\beta_{l1}<\beta_{l2}<\beta_{l3}\ldots$. Here n in β_{ln} reflects the complexity of the poloidal field in the radial direction. We choose the poloidal field with the largest scale with $l=1$ and $n=1$,

$$\mathbf{B}_\mathrm{P}=\left(-r\nabla^2 H_{11}+\frac{1}{r}\frac{\partial}{\partial r}r^2\frac{\partial H_{11}}{\partial r}\right)\hat{r}+\frac{1}{r}\frac{\partial}{\partial\theta}\frac{\partial(rH_{11})}{\partial r}\,\hat{\theta}\,. \tag{7}$$

Our toroidal field $\mathbf{B}_\mathrm{T}$ is chosen in a very similar way.

The magnetoconvection problem is characterized by five independent dimensionless parameters: the Rayleigh number, R_E, the Ekman number, E, the Prandtl number, Pr, the Roberts number, q, and the Elsasser number, Λ, all with the usual definitions. Pr and q represent material properties of an electrically conducting fluid.

The primary objective of our calculation is to show that the MHD convection system is very sensitive to small variations of the poloidal field at small E, which can lead to rapid swings in convection strength that cannot be resolved numerically and which may lead to instabilities of a geodynamo model. To achieve this objective, we have neglected the inertial term $\partial\mathbf{u}/\partial t+\mathbf{u}\cdot\nabla\mathbf{u}$ in the equation of motion by taking the large Pr limit (see also Glatzmaier & Roberts 1995). This can be justified on the basis that convection relevant to dynamo action is on a much longer timescale than the period of rotation. We take the Roberts number $q=1$ and fix the Elsasser number at $\Lambda=10$. This value of Λ ensures we are in the strong-field regime, in which length scales are large and the main force balance is magnetostrophic. It is also typical of the values obtained from large-scale geodynamo calculations, which have $\Lambda=O(10)$ based on the average field (Sarson *et al.* 1998).

We fix E, Pr, q and Λ and solve the equations of motion, heat and induction simultaneously for many different values of the Rayleigh number R_E, to determine the smallest R_E (which is referred to as R_c) at which convection can take place. Repeating the calculations for different values of ϵ, the strength of the poloidal field, gives the variation of R_c with ϵ.

Our simulation cannot reach the value of the Ekman number for the core, which is about 10^{-15}. However, we can reach the asymptotic region for small E. Table 1 gives results for $\epsilon=0$. It shows the solution approaching a limit with $R_\mathrm{c}\approx 12E^{-1}$ and drift rate $C\approx 8.5$ as $E\to 0$. The scaling of R_c arises from the necessity of buoyancy to remain in the force balance.

We have simulated 30 solutions at small E by increasing ϵ gradually from zero. The results are shown in Fig. 1 for the most unstable linear mode, which is $m=1$, except very close to $\epsilon=0$, when R_c for $m=2$ becomes comparable. Increasing ϵ slightly from zero to 0.07 reduces the critical Rayleigh number R_c from $R_\mathrm{c}\approx 12E^{-1}$ to $R_\mathrm{c}\approx 1$. The range of R_c is larger for smaller E: at $E=10^{-15}$ it ranges from 1 to 10^{16}, a huge effect

Table 1. Results of linear magnetoconvection calculation for $\epsilon=0$, $m=1$, showing approach to an asymptotic limit as $E \to 0$. C is the dimensionless oscillation frequency of the solution (drift rate).

E	R_c	ER_c	C
1.0×10^{-4}	9.68×10^{4}	9.69	7.02
5.0×10^{-5}	2.12×10^{5}	10.1	7.61
1.0×10^{-5}	1.18×10^{6}	11.9	8.12
5.0×10^{-6}	2.42×10^{6}	12.1	8.25
1.0×10^{-6}	12.3×10^{6}	12.3	8.32

for such a small change in the field. The corresponding drift rate changes from positive (eastwards) to negative (westwards). We therefore expect the amplitude and pattern of convection, and hence the size of R_m, also to change dramatically in response to small variations in the poloidal field.

3 IMPLICATIONS FOR NUMERICAL SIMULATIONS

Now consider the more complicated scenario when the magnetic field is self-generated by dynamo action. Differential rotation can produce a large $\mathbf{B}_T$ with dipole symmetry, and a poloidal field will arise from the action of radial motion on the toroidal field, or from small-scale flows, with timescales of centuries.

Negative values of R_c in Fig. 1 correspond to magnetic instabilities, which draw their energy from the imposed magnetic field rather than the applied heat sources. Such a solution could not be maintained indefinitely if the field were generated from dynamo action, although it could occur temporarily as part of a time-dependent solution. The part of the curve with $R_c \geq 0$ is therefore the most relevant for dynamo calculations.

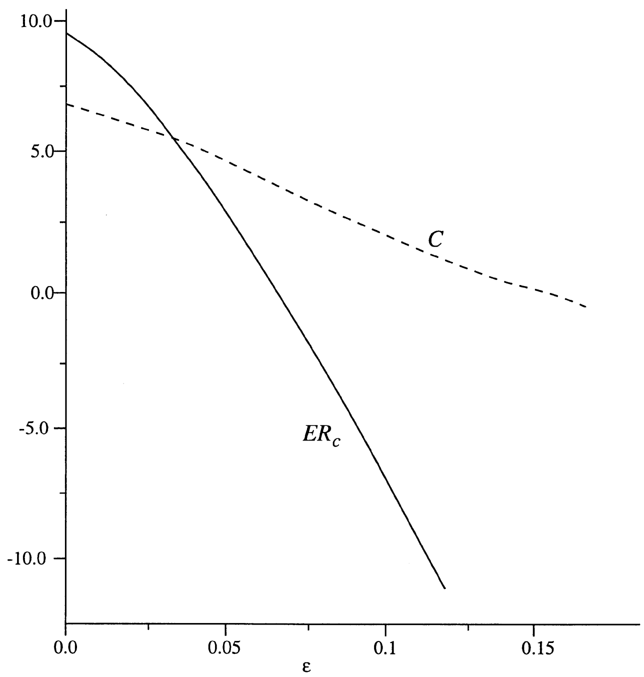

Figure 1. The scaled critical Rayleigh number, $E^{-1}R_c$, and the corresponding drift rate, C, are plotted against ϵ, the poloidal field strength. $E=10^{-4}$ here, but Table 1 shows this is in the asymptotically small Ekman number regime.

When the Ekman number is small, R_c can be wildly and sensitively dependent on the strength and form of the magnetic field. It can change from $R_c=O(10^{20})$ when the magnetic field is weak (Busse 1970; Zhang 1992) to $R_c \approx 1$ in the magnetostrophic regime. We show here that R_c is extremely sensitive to small variations in the magnetic field, particularly the poloidal field. It follows that R_m can swing rapidly through a wide range of values because of its dependence on (R_E-R_c). It is difficult to anticipate the existence of a quasi-steady geodynamo if the whole system is so sensitive to small variations of the field and R_m varies so wildly. The results suggest that the dynamic geodynamo is intrinsically unstable and is characterized by a strong time dependence.

These results also provide a clue as to why numerical integrations of an Earth-like dynamo model characterized by small Ekman number (rather than hyperdiffusivity with large effective Ekman number at small wavelength) prove to be formidably difficult (Walker *et al.* 1998), while no such difficulties arise in the corresponding non-magnetic problem (Sun *et al.* 1993). Although non-magnetic convection may be highly chaotic, the driving force that determines the average amplitude of convection, measured by (R_E-R_c), is fixed and time-independent. In the geodynamo problem, because the Lorentz force enters eq. (1) together with a small Ekman number, the dynamic balance becomes highly variable even though R_E is fixed. This inevitably leads to rapid variations and collapses of the magnetic field, and the many numerical problems that arise in computer simulations.

This discussion is based on linear simulations of the magnetoconvection, but the real problem is non-linear. However, we believe the dynamic behaviour discussed in this paper would be manifested in the non-linear problem because all the key elements of the dynamic force balance (1) in the fluid core at small Ekman number have been captured. We imagine a non-linear solution in which the applied field varies with time. The linear calculations reported here will not reflect this time dependence, which changes the nature of the stability analysis. However, there is no reason to expect the strong dependence of R_c on $\mathbf{B}_p$ to change. We shall investigate the effects of a time-dependent field in a future study.

We also imagine the field to be self-generated through a dynamo mechanism rather than imposed, which is much more difficult to investigate or quantify. Dynamo action occurs through non-linear interaction of the convection with the magnetic field via the term $\nabla\times(\mathbf{v}\times\mathbf{B})$ in the induction equation, which may stabilize or destabilizethe system. A further study of the dependence of the generated field on the flow is underway using kinematic dynamo theory.

4 IMPLICATIONS FOR THE EARTH'S MAGNETIC FIELD

This study was inspired by recent palaeomagnetic results, which suggest the geomagnetic field is rather unstable and undergoes collapses in strength and large changes in direction after a few tens of thousands of years. The interval of weak, non-dipolar field lasts only 2–5 kyr before the field grows once more to its typical modern strength and dipole-dominated character.

Current geodynamo simulations do not show such dramatic behaviour. We attribute the extra stability to the larger effective viscosity in the numerical calculations, necessitated by the limited temporal (and spatial) resolution offered by even the largest computers available. High viscosity applied to small-wavelength convection will prevent it from reaching the high R_c regime in Fig. 1, thus limiting the swings in convective and dynamo power.

We therefore envisage a true geodynamo operating mainly in magnetostrophic balance with occasional collapses into a high-R_c regime. What happens then is a matter for speculation at the moment because the flow would be small scale, rapidly time-varying and beyond present numerical resolution. Observations show clearly that geodynamo action continues and the large-scale, magnetostrophic state is quickly re-established. The inner core may play a stabilizing role by giving the poloidal field a longer timescale based on electrical diffusion rather than fluid advection (Hollerbach & Jones 1993; Gubbins 1999).

Our intention has been to isolate the effect of a poloidal magnetic field on magnetoconvection in a rapidly rotating spherical system and to show it can have a dramatic effect on the convection. We have therefore excluded other possible effects such as those of differential rotation, which may be important in understanding the exchange of angular momentum between core and mantle (Jault *et al.* 1988), and non-linear stability in the magnetostrophic approximation (e.g. McLean & Fearn 1999). The effects of differential rotation are expected to be of secondary importance in eq. (1) simply because the inertial term $\mathbf{u}\cdot\nabla\mathbf{u}$ can be neglected to leading order on the long timescale that is relevant to dynamo action.

The difficult theoretical question now posed is not why the geodynamo is so unstable, but why the large-scale magnetostrophic state is as stable as it is, persisting for tens of thousands of years or about one magnetic diffusion time.

ACKNOWLEDGMENTS

This work was supported by PPARC grant GR/L22973 (KZ) and by NERC grant GR3/9741 (DG).

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