Article:

Xu, Z., Zakharov, Y. orcid.org/0000-0002-2193-4334 and White, G. (2007) Antenna array optimisation using semidefinite programming for cellular communications from HAPs. Electronics Letters. pp. 67-69. ISSN: 0013-5194

https://doi.org/10.1049/el:20072947

Antenna Array Optimisation Using Semidefinite Programming for Cellular Communications from HAPs

Zhengyi Xu, Yuriy Zakharov and George White

Abstract

Array pattern optimisation based on semidefinite programming (SDP) is proposed to improve the coverage performance of cellular communications from High Altitude Platforms (HAPs). This optimisation, when applied to a linear vertical array of N omnidirectional antenna elements, allows a coverage performance better than that of an array of N narrowbeam aperture antennas forming hexagonal cells on the ground. In addition to the performance enhancement, the HAP antenna payload can be significantly reduced.

Introduction: High Altitude Platforms (HAPs) are considered as a future communication infrastructure, combining key advantages of both terrestrial and satellite systems [1]. Recently, there has been much interest in antenna design to improve the coverage performance from HAPs. HAP-based planar antenna arrays with predefined beam patterns have been proposed to form a cellular structure on the ground [2]. This does not require complicated real-time signal processing compared to adaptive beamforming. The use of one narrowbeam aperture antenna per hexagonal cell provides high coverage performance but bulky antenna payload [3]. Although a planar array of omnidirectional antenna elements could allow the same coverage performance, this would require significantly more elements than the number of aperture antennas [4]. The more the number of antenna elements, the more complicated is the antenna array RF circuit. Using vertical antenna arrays to form ring-shaped cells is proposed in [5]. The window method was shown in [5] to allow better coverage performance with a number of antenna elements slightly more than the number of the aperture antennas.

In this letter, we propose a beampattern optimisation based on semidefinite programming (SDP) [6] to further reduce the number of antenna elements. In particular, we show that this optimisation, when applied to a linear vertical array of omnidirectional antenna elements, allows a coverage performance better than that of an array of the same number of narrowbeam aperture antennas. Thus, our design shows significant advantage in both coverage performance and HAP antenna payload weight. Although SDP has recently been proposed for use in adaptive beamforming [7], [8], to our best knowledge, it has not been used for beampattern optimisation for cellular communications optimized with respect to a specific frequency reuse plan.

Beampattern optimisation: Fig.1 illustrates the communications scenario with a vertical linear antenna array mounted on a HAP at altitude H, providing coverage over a circular area of radius R. The coverage area is divided into M ring-shaped cells. Let $\Theta = [0, \theta_{max}]$ represent the interval of complementary elevation angles θ corresponding to the coverage area. The steering vector for an angle $\theta \in \Theta$ is given by

$$\mathbf{u}(\theta) = [1 \quad e^{-jk_0 d \cos\theta} \quad \cdots \quad e^{-jk_0(N-1)d\cos\theta}]^T, \tag{1}$$

where $j = \sqrt{-1}$, $k_0 = 2\pi/\lambda$ is the wave number, λ the wavelength, d the element spacing, and $[\cdot]^T$ denotes the transpose. The conventional beampattern optimisation aims to constrain the mainlobe to cover the corresponding cell whilst suppressing sidelobes; this results in low intercell interference. Applying window functions as antenna weights allows a good coverage performance with a relatively small number of antenna elements [5]. However, the coverage performance can be improved and the number of antenna elements can be reduced if a more sophisticated optimisation is used, in particular by taking into account a specific frequency reuse plan.

Suppose that the mth antenna array beam is steered to the mth cell. Let Θ_m be the set of angles θ corresponding to the mth cell and $\tilde{\Theta}_m$ be the set of angles θ corresponding to the other cells sharing the same channel of the frequency reuse plan as the mth cell. The beamformer weight vector $\mathbf{w}_m = [w_m^{(1)}, \cdots, w_m^{(N)}]^T$ providing a beampattern for covering the mth cell can be found by solving the following constrained optimisation problem

$$\mathbf{w}_m = \arg\min_{\mathbf{w}} \sum_{\theta_i \in \tilde{\Theta}_m} \left|\mathbf{w}^H \mathbf{u}(\theta_i)\right|^2 \text{ subject to } \left|\mathbf{w}^H \mathbf{u}(\theta_n)\right|^2 = 1 \;\; \forall\, \theta_n \in \Theta_m,$$
$$i = 1, \cdots, I_m \quad , \quad n = 1, \cdots, N_m, \tag{2}$$

where N is the number of array elements and $[\cdot]^H$ is the conjugate transpose. The constraints in (2) aim to generate a rectangular-shaped mainlobe in order to cover the desired cell. In (2), N_m angles θ_n within the set Θ_m result in N_m constraints. However, these constraints may not be solved by SDP if N_m is large, i.e., when small sample intervals for Θ_m are applied. Furthermore, it is difficult to determine the minimum sample interval for a given number of antenna elements and cellular configuration. Therefore, instead of solving (2), we solve the following optimisation problem:

$$\mathbf{w}_m = \arg\min_{\mathbf{w}} \sum_{\theta_i \in \tilde{\Theta}_m} \left|\mathbf{w}^H \mathbf{u}(\theta_i)\right|^2 \text{ subject to } \frac{1}{N_m} \sum_{\theta_n \in \Theta_m} \left|\mathbf{w}^H \mathbf{u}(\theta_n)\right|^2 = 1,$$
$$i = 1, \cdots, I_m \quad , \quad n = 1, \cdots, N_m. \tag{3}$$

Equation (3) sets one constraint to the sum of received power in the desired cell instead of N_m constraints in (2). Therefore, (3) can be simply solved by SDP using any sampling within Θ_m. The optimum sampling can be found by computer simulation to maximize Signal-to-Interference Ratio (SIR), as defined in the Numerical results section.

For a given sample interval, we optimize beampatterns only within the co-channel cells according to the frequency reuse plan instead of the whole coverage area. Thus, the SDP optimisation freedom is improved, since the the size of $\widetilde{\Theta}_m$ is reduced. Therefore, the larger the frequency reuse factor, the lower the sidelobes can be optimized in the cells that share the same channel as the cell of interest.

Numerical results: Assume that there are U cells that share the same channel, and users are randomly positioned with one user in each cell. The user in the mth cell of interest on the ground receives signals of powers $\{P_1, \cdots, P_U\}$ from all U beams of the HAP antenna. Downlink SIR for the user in the mth cell is given by [3]

$$\eta_m = \frac{P_m}{\sum_{u=1, u\neq m}^{U} P_u}. \tag{4}$$

Coverage performance is represented by the probability distribution function

$$C(\gamma) = Pr\{\eta_m > \gamma\}. \tag{5}$$

Fig.2 compares the coverage performance of four antenna configurations: 1) a set of narrowbeam aperture antennas forming hexagonal cells [3]; 2) a planar antenna array forming hexagonal cells [4]; 3) a vertical linear antenna array forming ring-shaped cells, using Hamming window as antenna element weights [5]; and 4) a vertical linear antenna array forming ring-shaped cells, with the proposed weight optimisation solved by SDP.

The respective communication scenario is defined as: 120 hexagonal cells for antenna configurations 1) and 2); 61 ring-shaped cells for antenna configurations 3) and 4); coverage radius R = 32.7 km; H = 20 km; 30 cells/channel. In Fig.2, it can be found that the planar antenna array requires almost 4 omnidirectional antenna elements per cell to provide similar 95% coverage performance as that of aperture antennas employed in [3]. Our proposed approach, however, can achieve about 3 dB better coverage performance using the same number of (omnidirectional, hence, physically simple) antenna elements as (narrowbeam, hence, physically large) aperture antennas, i.e., our optimisation shows significant coverage performance improvement while reducing antenna payload. This is desirable, particularly in regard to light, unmanned aeroplane HAPs (eg. NASA Pathfinder [9]). In [5], Hamming weights are applied for each vertical antenna array element and they provide 5 dB better coverage performance than aperture antennas or planar array. However, this requires 1.4 more antenna elements in order to support the same number of cells per channel. When compared with this result, our proposed beampattern optimisation improves coverage performance by 15 dB.

Conclusions: We have proposed an approach for antenna array optimisation that takes into account the frequency reuse plan and is based on semidefinite programming. This approach significantly improves the coverage performance of vertical linear antenna arrays providing ring-shaped coverage for cellular communications from HAPs. It is shown that for a fixed number of cells per frequency channel, a linear vertical antenna array achieves better coverage performance than narrowbeam aperture antennas or planar antenna arrays. This requires the same number of antenna elements as that of aperture antennas or approximately 4 times less number of elements than that of a planar array. The proposed approach could simplify antenna implementation and reduce HAP payload.

References

[1] D. Grace, N. E. Daly, T. C. Tozer, A. G. Burr, and D. J. Pearce, "Providing multimedia communications from High Altitude Platforms," *Int. J. Satellite Commun.*, , no. 19, pp. 559–580, Nov 2001.

[2] W. Li, X. Huang, and H. Leung, "Performance evaluation of digital beamforming strategies for satellite communications," *IEEE Trans. Aerospace and Electronic Systems.*, vol. 40, no. 1, pp. 12–25, Jan 2004.

[3] J. Thornton, D. Grace, M. H. Capstick, and T. C. Tozer, "Optimizing an array of antennas for cellular coverage from a High Altitude Platform," *IEEE Trans. Wireless Commun.*, vol. 2, no. 3, pp. 484–492, May 2003.

[4] Z. Xu, G. White, and Y. Zakharov, "Optimization of beampattern of high altitude platform antenna using conventional beamforming," *IEE Proc. Communications, to appear, Jan 2006.*

[5] Z. Xu, Y. Zakharov, and G. White, "Vertical antenna array and spectral reuse for ring-shaped cellular coverage from high altitude platform," *Loughborough Antennas and Propagation Conference (LAPC2006)*, pp. 209–212, April 2006.

[6] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Review.*, , no. 38(1), pp. 49–95, Mar 1996.

[7] M. Bengtsson and B. Ottersten, "Optimal downlink beamforming using semidefinite optimization," *in Proc. 37th Annual Allerton Conference on Communication, Control, and Computing.*, pp. 987–996, Sept. 1999.

[8] S.A. Vorobyov, A.B. Gershman, and Z.Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem," *IEEE Trans. Signal Processing.*, vol. 51, no. 2, pp. 313–324, Feb. 2003.

[9] S.D. Prince, S.N. Goward, and N.P. Hanan, "Estimation of global primary production using NOAA/NASA Pathfinder AVHRR land data set," *IGARSS 95.*, vol. 2, pp. 1000–1002, July 1995.

Authors' affiliations:

Zhengyi Xu, Yuriy Zakharov, and George White are with the Communication Research Group, Department of Electronics, University of York, York YO10 5DD, UK, (email: zx500, yz1, gpw100@ohm.york.ac.uk).

Figure captions

Fig.1 Ring-shaped cellular configuration for communications from HAPs.

Fig.2 Coverage performance of aperture antennas, a planar antenna array, and vertical linear antenna arrays.
□ 424-element planar antenna, 120 hexagonal cells, reuse factor 4 [4]
× 121-element aperture antenna, 120 hexagonal cells, reuse factor 4 [3]
◯ 121-element vertical antenna, 61 ring-shaped cells, reuse factor 2, beampattern optimization solved by SDP
◊ 171-element vertical antenna, 61 ring-shaped cells, reuse factor 2, Hamming window [5]
∗ 171-element vertical antenna, 61 ring-shaped cells, reuse factor 2, beampattern optimization solved by SDP

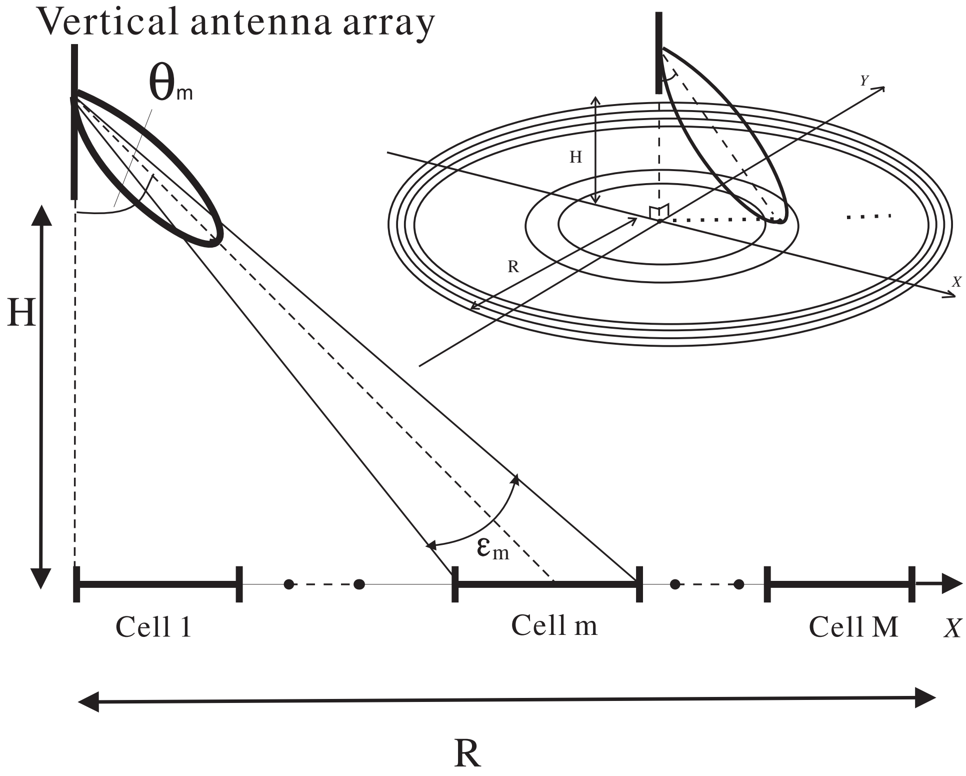
Vertical antenna array
θm
H
εm
Cell 1
Cell m
Cell M
X
R
H
R
Y
X

Fig. 1.

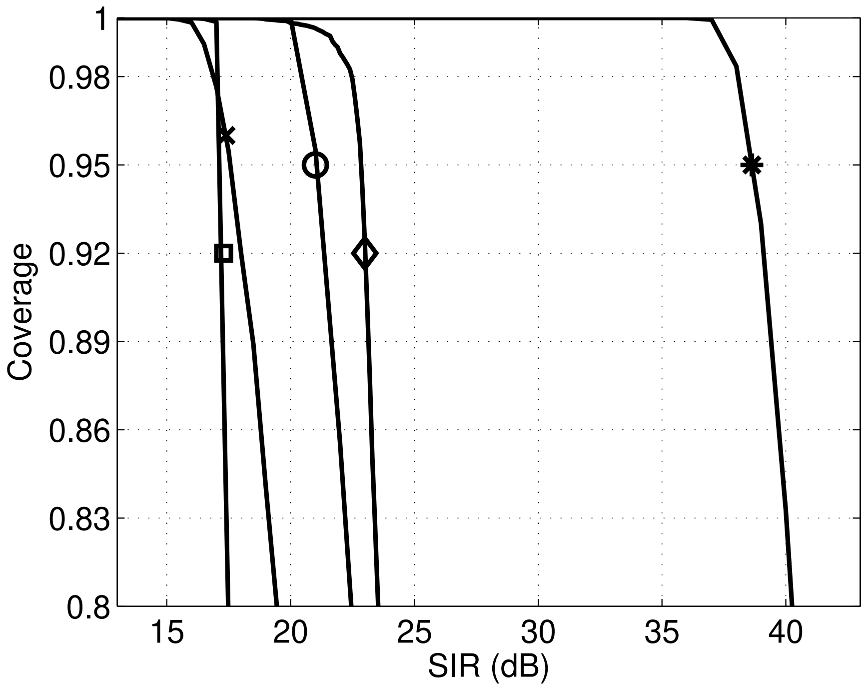
1
0.98
0.95
0.92
0.89
0.86
0.83
0.8
Coverage
15
20
25
30
35
40
SIR (dB)

Fig. 2.