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Constant Q and a fractal, stratified Earth

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ABSTRACT

Frequency dependent measurements of the quality factor Q typically show a constant behaviour for low frequencies and a positive power law dependence for higher frequencies. In particular, the constant Q pattern is usually explained using intrinsic attenuation models due to anelasticity with either a single or multiple superposed relaxation mechanisms – each with a particular resonance peak.

However, in this study, I show using wave localisation theory that a constant Q may also be due to apparent attenuation due to scattering losses. Namely, this phenomenon occurs if the Earth displays fractal characteristics. Moreover, if fractal characteristics exist over a limited range of scales only, even an absorption band can be created – in accordance with observations. This indicates that it may be very difficult to distinguish between intrinsic and scattering attenuation on the basis of frequency dependent measurements of the quality factor only.

Keywords: attenuation – fractals – inhomogeneous media – scattering – wave propagation

Frequency dependent measurements of the quality factor Q have been made since at least the beginning of the century [see e.g. Knopoff (1964) for some historical references]. Although Q is probably constant over a large frequency range in homogeneous materials, this is not the case in inhomogeneous media. Such frequency dependent estimates are required, since they enable us in principle to infer more about the existing attenuation mechanisms in the considered medium (Aki, 1980) or to invert for the lateral and vertical distribution of Q (Romanowicz, 1998). Both interpretations can then hopefully be related to geodynamical processes occurring in the Earth.

In the Earth, the obtained estimates typically show an approximately constant Q for periods between 10 s and 1 hour (0.28 10^{-4} to 0.1 Hz) and an increase with frequency for $f > 1$ Hz (Dziewonski, 1979; Sipkin and Jordan, 1979; Sato and Fehler, 1998). Namely, for S-waves and frequencies between 1 and 25 Hz, Q is proportional to f^α with α ranging between 0.6 and 0.8 in the average (Aki, 1980; Sato and Fehler, 1998). Low frequency estimates are usually obtained from normal mode and surface wave data, whereas higher frequencies are derived from body waves.

Combining the results of body wave data with the lower and constant Q measurements of normal mode and surface wave data, Aki (1980) conjectured the existence of a maximum in attenuation around 0.5 to 1 Hz. Figure 1 displays a sketch of the situation. Although the existence of the peak has not yet unambiguously been proven [see Sato and Fehler (1998) for the latest review], much research has been done to explain its conjectured position and magnitude. Notwithstanding the fact that its position can be explained using a single relaxation model, this can be caused by 2 completely different mechanisms. Firstly, it may be due to thermoelastic effects involving grain sizes or crack lengths of the order of 1 mm (Zener, 1948; Savage, 1966; Aki, 1980). Secondly, Aki (1980) proposed that apparent attenuation due to scattering losses may play a role. Subsequent research showed that in particular elastic scattering may cause the observed peak. Associated predictions indicate that heterogeneities with a typical scale-length of the order of 2 km may be involved (Sato and Fehler, 1998).

In this study, however, I focus on the constant Q behaviour which has had less attention lately. The phenomenon has always been explained by anelastic intrinsic attenuation using either a single or multiple superposed relaxation mechanisms. However, as I will show, a constant Q can also be caused by scattering in a fractal, stratified Earth. This is demonstrated using wave localisation theory (Shapiro and Zien, 1993; Van der Baan, 2001). This multiple scattering theory is used since it describes the exact attenuation and dispersion of a plane-wave traversing a 1-D medium. Although an acoustic version of the theory is employed, similar predictions can be made using the full elastic theory.

First, I review some explanations for the occurrence of constant Q. Then, I briefly present wave localisation theory and its predictions for a fractal, stratified Earth. Finally, I discuss some of its implications if Q measurements are to be interpreted in terms of scattering versus intrinsic attenuation mechanisms. I focus primarily on the physical implications of wave localisation theory, and in particular its predictions concerning scattering and apparent attenuation in fractal media in relation to constant Q behaviour. Van der Baan (2001) describes the theory in more detail and illustrates its correctness using numerical simulations. Some of these numerical simulations are used here for different purposes.

SOME EXPLANATIONS FOR CONSTANT Q

Constant Q behaviour is normally explained in 2 different ways. Either an anelastic mechanism is involved which attenuates waves independently of frequency or a multitude of relaxation mechanisms are required – each with a particular resonance peak. Their superposition is then assumed to yield a so-called absorption band resulting in a constant Q over a large frequency range.

Candidates for frequency independent intrinsic attenuation mechanisms include hysteresis in static stress-strain curves of rocks (Krasilnikov, 1963; McCall and Guyer, 1994), frictional sliding on dry surfaces of thin cracks (Walsh, 1966), and non-linear attenuation mechanisms (Knopoff and MacDonald, 1960). However, attributing constant Q behaviour to hysteresis raises the question which stress-strain model leads to a stress-strain cycle whose width is independent of frequency (Knopoff, 1964). On the other hand, hysteresis can be caused by cracks which open, close and slip during elastic loading (Sato and Fehler, 1998), thereby pointing again to the study of Walsh (1966).

The second explanation uses a collection of intrinsic attenuation mechanisms – each with an individual resonance (Debye) peak (Fig. 2.a). Since the total inverse quality factor Q_{tot}^{-1} is simply a summation over each individual inverse quality factor Q_i^{-1}, a large collection of attenuation mechanisms can in principle cause a constant absorption band as displayed in Fig. 2.b (Liu et al., 1976). These individual relaxation mechanisms can for instance be due to grain boundary sliding, the formation and movement of defects in crystal lattices and thermoelastic effects (Anderson, 1989).

However, it should be noted that a constant Q for all frequencies in combination with a non-zero effective phase velocity at zero frequency (i.e., in the long wavelength limit) is physically not possible. Namely, at zero frequency, the inverse quality factor should approach zero ($Q^{-1} = 0$), thereby rendering the medium effectively homogeneous, and the phase velocity should converge to the effective medium velocity. These conditions are required, since otherwise causality is violated and the phase velocity becomes unbounded (Futterman, 1962). This can, e.g., be shown using the Kramers-Krönig relations which relate attenuation and dispersion [see also Beltzer (1988)]. A constant Q alone is not in conflict with causality if the phase velocity is allowed to converge to zero in the long wavelength limit (Kjartansson, 1979). However, a zero phase velocity in the long wavelength limit conflicts with effective medium theories which state that it should converge to a finite constant.

Such a problem does not occur for the absorption band model of Liu et al. (1976). An additional advantage of their model is that it can deal with observations of Q being proportional to f^{α} for $f > 1$ Hz. It would simply indicate that the right edge, i.e. the corner frequency, starts at 1 Hz.

Unfortunately, however, the absorption band model is a phenomenological theory that does not give the faintest indication about which particular attenuation mechanisms are involved. To further complicate this problem, constant Q can also be caused by apparent attenuation due to scattering – as I will show using wave localisation theory.

WAVE LOCALISATION THEORY

Wave localisation theory originally comes from the quantum theory of disordered solids (Anderson, 1958). It implies that the energy of a wave has an exponential fall-off for large distances from its maximum. This causes the envelope of a pulse to decay exponentially for large propagation distances in random, 1-D media. Hence, the transmission coefficient T behaves as $|T| = \lim_{L\to\infty} \exp(-\gamma L)$ with γ the so-called Lyapunov exponent and L the thickness of the random medium. This happens for almost any frequency and realisation of the medium. The mathematical proof of the exponential decay follows from random matrix theory and in particular the work of Fürstenberg (1963), Oseledec (1968) and Virster (1979). For more background, the interested reader is referred to Van der Baan (2001).

In order to determine the apparent attenuation of a wave propagating in a fractal, stratified Earth, I consider a so-called matched-medium. That is, a random medium with thickness L is sandwiched between 2 homogeneous half-spaces with matching density ρ_0 and incompressibility κ_0^{-1}. The background density ρ_0 and incompressibility κ_0^{-1} are assumed to be constant and fluctuations occur in the incompressibility only. Thus, the medium is described by

$$\begin{aligned}
\kappa^{-1}(z) &= \begin{cases} \kappa_0^{-1} & z < 0,\ z > L \\ \kappa_0^{-1}[1+\sigma_\kappa(z)] & 0 \le z \le L \end{cases} \\
\rho(z) &= \rho_0 \qquad\qquad \text{everywhere}
\end{aligned} \tag{1}$$

with σ_κ the relative fluctuations of the incompressibility. For simplicity, I only treat the acoustic problem to prevent P-S and S-P conversions. In addition, the fluctuations are assumed to be stationary.

To determine the required Lyapunov exponent, Shapiro and Zien (1993) considered a plane-wave impinging from above on the inhomogeneous layer and used a second order perturbation of the resulting wave equation. In this way, they were able to show that, for vertical incidence, the Lyapunov exponent γ is given by

$$\gamma = \frac{1}{4}k_0^2 \int_0^\infty \mathrm{E}\left[\sigma_\kappa(z)\sigma_\kappa(z+\xi)\right]\cos(2k_0\xi)d\xi \tag{2}$$

with E[.] the autocorrelation function of the random layer. The variable k_0 equals to both the wavenumber in the background medium and the effective wavenumber in the random media, i.e. the wavenumber at zero frequency. The resulting expression for non-vertical incidence is only slightly more complicated.

IMPLICATIONS FOR A FRACTAL, STRATIFIED EARTH

To study the apparent attenuation in a fractal medium, I use the Von Kármán function given by

$$\mathrm{E}\left[\sigma_\kappa(z)\sigma_\kappa(z+\xi)\right] = \frac{2^{1-\nu}\left\langle\sigma_\kappa^2\right\rangle}{\Gamma(\nu)}\left(\frac{|\xi|}{a}\right)^\nu K_\nu\left(\frac{|\xi|}{a}\right), \tag{3}$$

where $K_\nu(.)$ represents the modified Bessel function of the third kind of order ν – also known as the MacDonald function. The constant a represents the typical scale-length of the heterogeneities, that is, of the fluctuations in the incompressibility.

For $\nu = 0$, this function has self-similar properties in the sense that it displays discontinuities on any scale. In addition, it has a constant variance for each octave interval of wavenumber for $k_0 a \gg 1$ (Frankel and Clayton, 1986). Moreover, its amplitude spectrum corresponds to a power law which is also indicative of fractal behaviour (Mandelbrot and Wallis, 1969).

Calculating integral (2) for the Von Kármán function (3) with $\nu = 0$ results in

$$\gamma_{VK} = \frac{\pi \langle \sigma_\kappa^2 \rangle}{4a\Gamma(0)} \frac{(k_0 a)^2}{(4(k_0 a)^2 + 1)^{1/2}}. \tag{4}$$

The reciprocal of the quality factor is obtained by means of

$$Q^{-1} = 2\gamma/k \approx 2\gamma/k_0, \tag{5}$$

yielding

$$Q_{VK}^{-1} \approx \frac{\pi \langle \sigma_\kappa^2 \rangle}{2\Gamma(0)} \frac{k_0 a}{(4(k_0 a)^2 + 1)^{1/2}}. \tag{6}$$

Wave localisation theory also predicts the exact dispersion of the phase velocity and therefore the exact frequency dependence of the wavenumber $k(f)$ (Van der Baan, 2001). However, for the present application, approximation (6) suffices since the phase velocity is a slowly monotonically rising function which also converges to a constant. Therefore, the overall trend of Q_{VK} is not changed.

The constant $\Gamma(0)$ is required to normalise the Von Kármán function for $\nu = 0$. However, in non-perfect fractals, i.e., in non-perfect realisations of the Von Kármán function, it is replaced by a finite constant.

Fig. 3 displays the behaviour of Q^{-1} for a large range of scales $k_0 a$. It can clearly be seen that Q^{-1} becomes effectively a constant for $k_0 a > 2$. Van der Baan (2001) showed that this phenomenon only happens for $\nu = 0$. Other functions, such as exponential and Gaussian functions, do not display such a behaviour either. In all these functions, a Debye peak occurs at $k_0 a \approx 1$. This indicates that, for such media, scattering is most efficient for wavelengths of the same order as the scale-length of the heterogeneities, i.e. due to Mie scattering. For fractal media, however, no such typical scale-length exists. Hence, any scale-length can be seen as a typical dimension and Mie scattering occurs at nearly all frequencies, thereby causing a constant Q.

The only exception occurs at zero frequency (long wavelength limit). In this case, the fractal medium becomes effectively homogeneous. Therefore, no scattering and thus attenuation occurs. Moreover, both the phase and group velocities converge to the expected effective medium and geometric optical velocities in respectively the low and high frequency limits (Van der Baan, 2001). Hence, causality is respected and the phase velocity is bounded, thereby preventing problems related to perfectly constant Q models (Futterman, 1962) as discussed before.

Van der Baan (2001) also demonstrated the correctness of the predicted behaviour of Q^{-1} for different autocorrelation functions using an acoustic version of the reflectivity code of Dietrich (1988). I use here some of these numerical simulations to show how effectively a constant absorption band can be created. In that study, a pulse propagated from the homogeneous halfspace below the inhomogeneous layer towards the surface. Its spectral content was compared to a reference wave traversing a homogeneous space and analysed using a wavelet transform. The quality factor was then obtained by means of a spectral ratio. The medium consisted of 4000 layers each 1 m thick with an average velocity of 4 km/s and a standard deviation of the fluctuations of the incompressibility σ_κ of 15 %. This corresponds to a standard deviation of 7.5 % of velocity. Sampling rate was 1 ms.

Fig. 4 displays the measured Q^{-1} at normal incidence for the model described above. The results of 4 different numerical simulations are shown in addition to the theoretical predictions. The overall behaviour of both the numerical simulations and the theoretical predictions agree well except for the highest frequencies. Namely, for these frequencies, Q^{-1} decays again. This is probably due to the fact that a discrete representation of the medium is used which cannot simulate a perfect fractal. On the other hand, it clearly shows that, in non-perfect fractals, a constant Q only occurs within a given frequency band and that Q^{-1} becomes proportional to $f^{-\alpha}$ for higher frequencies in accordance with observations.

DISCUSSION

Using different analysis methods, typical scale-lengths of heterogeneities have been established to exist in the Earth over a very large range [see, e.g., Wu and Aki (1988a) for an overview]. In this light, the existence of fractal distributions of heterogeneities within the Earth becomes more likely [see also Dolan et al. (1998)]. And, as I have demonstrated, a fractal stratified Earth may cause a constant Q because of apparent attenuation due to scattering. In addition, causality and a non-zero phase velocity at zero frequency are maintained. Moreover, as the numerical simulations have shown, in the case of non-perfect fractals, a constant Q only occurs over a certain frequency range, thereby effectively creating an absorption band.

Furthermore, if we assume that 1 Hz forms the corner frequency for S-waves ($v =$ 4 km/s), and that Mie scattering is most efficient ($k_0 a \approx 1$) then it is easily deduced that the heterogeneities of the lithosphere may have fractal characteristics for scales exceeding 1 km. In addition, wave localisation theory predicts a scattering Q of the order of 400 to 500 (Fig. 4) for vertical incidence and velocity fluctuations of 7.5 %, whereas measured Q values range from 50 to 500 (Fig. 1), thereby indicating that intrinsic attenuation may play a dominant role (if wave localisation is the appropriate scattering mechanism). On the other hand, scattering Q decreases with increasing angle of incidence towards 250 at 30° incidence, thereby reducing the role intrinsic attenuation plays (Van der Baan, 2001). Naturally, this does not take into account that the heterogeneities in the Earth are 3-D and elastic. However, the numerical simulations of Frankel and Clayton (1986) have shown that a constant Q also occurs for the same Von Kármán function in an elastic, 2-D medium.

More importantly, it also shows that it may be even more complicated to distinguish between intrinsic attenuation due to anelasticity and apparent attenuation due to scattering than anticipated before. Namely, not only can an absorption band be mimicked by scattering mechanisms, but Wennerberg and Frankel (1989) showed that even a single relaxation peak due to scattering attenuation is nearly indistinguishable from one created by intrinsic attenuation. This indicates that it may hardly be possible to distinguish between apparent and intrinsic attenuation by measuring Q only.

A possible way to circumvent this problem is to assume that either intrinsic attenuation or scattering is described by a particular physical mechanism with a known effect on for instance the energy density or Q. The non-explained part in the measurements is then solely due to the remaining mechanism.

Possible scattering mechanisms that can be invoked are wave localisation in finely layered media or radiative transfer of energy in 3-D structures. Approaches based on the latter mechanism describe the spatial (Wu, 1985) or spatial and temporal distribution (Zheng et al., 1991) of the energy density as a function of the contribution of scattering attenuation to total attenuation. Applications on real data can be found in Wu and Aki (1988b), Fehler et al. (1992) and Jin et al. (1994).

As an alternative, the phenomenological theory of Frankel and Wennerberg (1987) can be considered which does not rely on the existence of a particular physical scattering mechanism. Their energy flux model simply assumes that scattering involves a transfer of energy from the incident wave to the coda with conservation of total energy, whereas intrinsic attenuation reduces the total amount of energy. For weak-scattering, their model is sufficiently accurate (Zheng et al., 1991).

The drawback of applying such an approach lies, however, on the underlying implicit assumptions on which these mathematical theories are based. Namely, if their basic assumptions are violated, specific behaviour may be attributed to the wrong mechanism. In particular, these theories heavily rely on homogeneous and isotropic distributions of scatterers, and isotropic, pure-mode scattering. That is, scatterers are uniformly distributed in space, heterogeneities display identical horizontal and vertical scale-lengths, and no layering exists. In addition, scattering is angle independent, recorded energy consists entirely of either P or S-waves, and no mode conversion occurs. Some of these basic assumptions are presently being questioned (Margerin et al., 1998).

CONCLUSIONS

The observed constant Q pattern in the Earth for periods between 10 s and 1 hour has always been attributed to intrinsic attenuation mechanisms due to anelasticity. However, it has been demonstrated using wave localisation theory which takes multiple scattering into account that, in a fractal stratified Earth, a constant Q can also be caused by apparent attenuation due to scattering. Moreover, if the Earth displays fractal characteristics over a limited range of scales only then effectively an absorption band is created – as numerical simulations have shown. In this case, the inverse quality factor is only constant over a range of frequencies and tends to zero for other frequencies. This particular behaviour is in accordance with observations. This yields a further indication that it may be very difficult to distinguish between intrinsic and scattering attenuation using frequency dependent Q measurements only. In particular, in most cases, the common assumption that scattering and intrinsic effects can be separated by allowing for frequency dependent scattering only is not valid.

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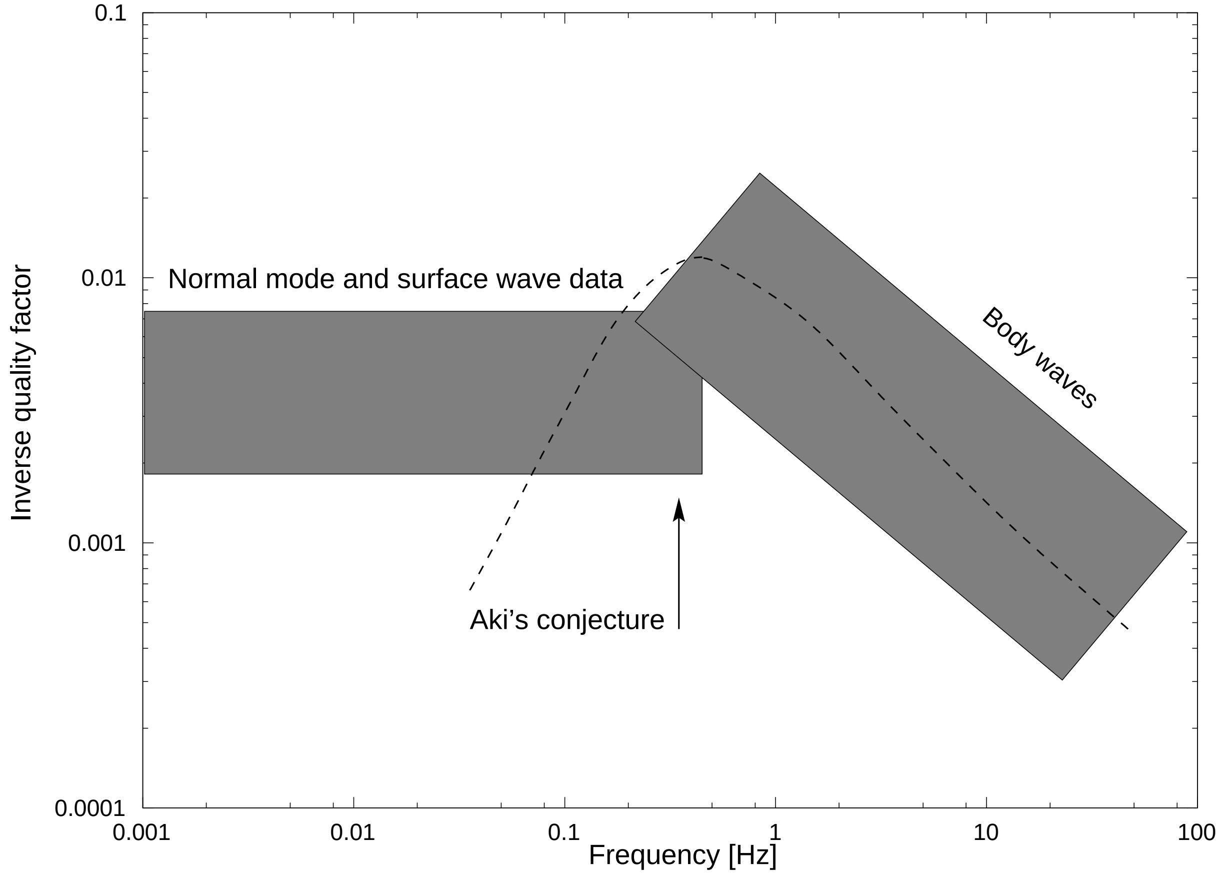
FIGURES

FIG. 1. Schematic sketch of frequency dependent Q measurements. The height of the boxes indicates roughly the uncertainty in measurements. The position of the resonance peak conjectured by Aki (1980) is also shown. After Sipkin and Jordan (1979), Aki (1980) and Sato and Fehler (1998).

FIG. 2. (a) A single relaxation mechanism with a Debye peak centred at a resonance frequency. (b) Several superposed mechanisms may form an absorption band. Adapted from Liu et al. (1976).

FIG. 3. Inverse quality factor as predicted for a fractal, stratified medium by wave localisation theory. Vertical scale is expressed in units of $\langle\sigma_\kappa^2\rangle/\Gamma(0)$.

FIG. 4. Numerical simulations for 4 different realisations of the Von Kármán function (broken lines) and the theoretical prediction (solid line). The factor $\Gamma(0)$ is replaced by the constant 13.0 to illustrate the general trend. After Van der Baan (2001).



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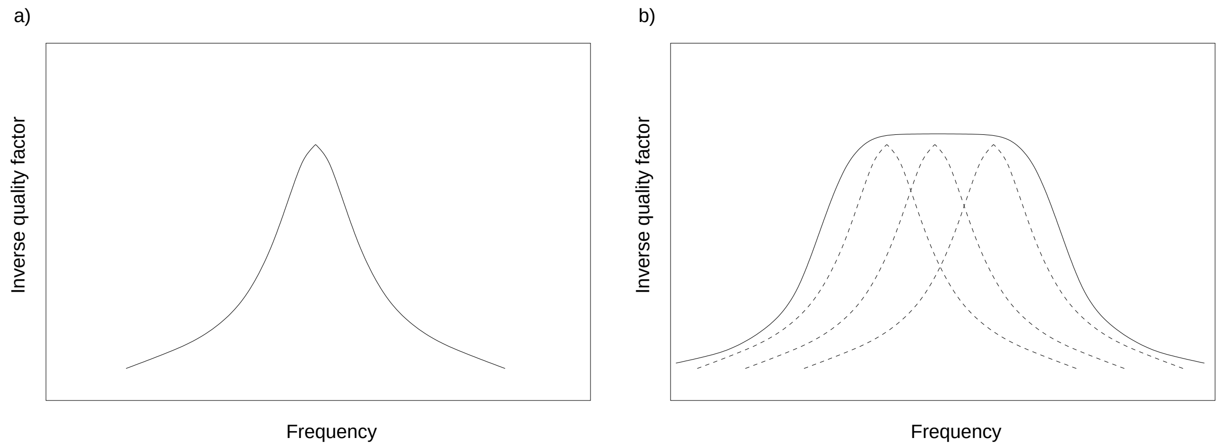

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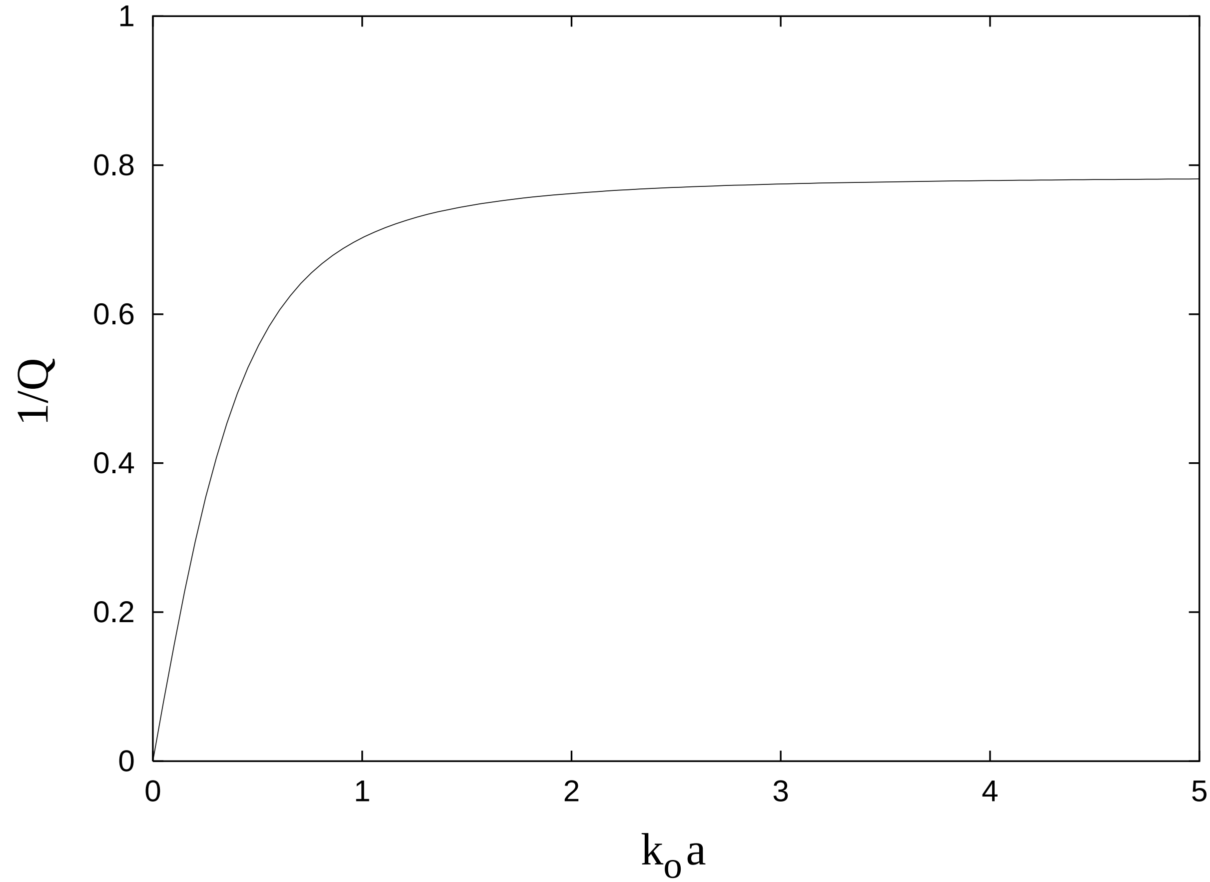

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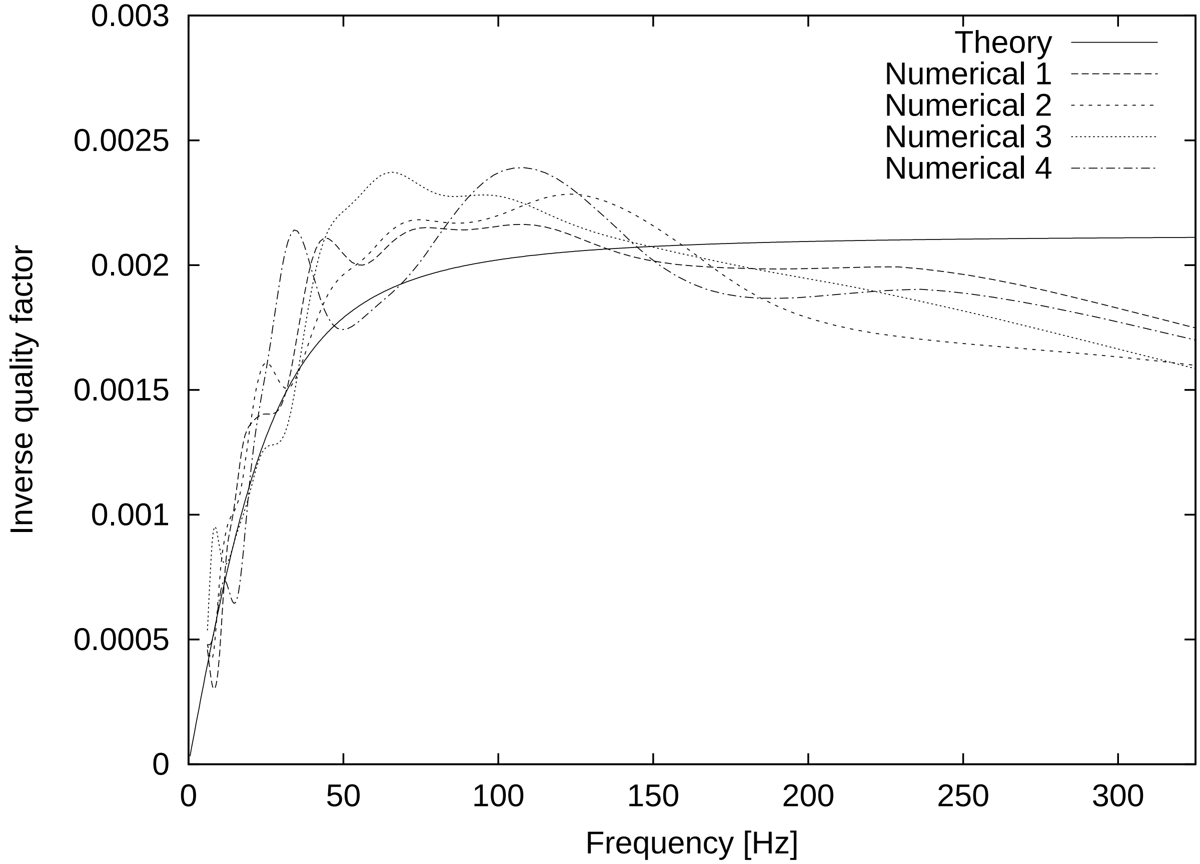

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