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# BLOCK SYNCHRONISATION FOR JOINT CHANNEL AND DC-OFFSET ESTIMATION USING DATA-DEPENDENT SUPERIMPOSED TRAINING

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## ABSTRACT

In this paper, we propose a new (single-step) block synchronisation algorithm for joint channel and DC-offset estimation for data-dependent superimposed training (DDST). While a (two-step) block synchronisation algorithm for DDST has previously been proposed in [5], due to interference from the information-bearing data it performed sub-optimally, resulting in channel estimates with unknown delays. These delay ambiguities (also present in the equaliser) were then estimated in [5] in a non-practical manner. In this paper we avoid the need for estimation of this delay ambiguity by exploiting the special structure of the channel output's cyclic mean vector. The result is a BER performance superior to the DDST synchronisation algorithm first published in [5].

## 1. INTRODUCTION

In communication systems the channel estimation problem is often solved by the inclusion of a training sequence. An alternative method is the superimposed training (ST) scheme, where a periodic training sequence is added to the data sequence at the expense of a small data power loss ([1]-[5]). In [4] the interference between data and the training is effectively removed by adding an additional data-dependent sequence at the transmitter thus giving the data dependent ST (DDST). A reduction in SNR at the transmitter is offset by an improved channel estimation at the receiver.

Since we are using first-order statistics and as the direct conversion receivers suffer from the DC-offset problem [2, 3], the DC-offset has to be included in the mathematical model of DDST. Since DDST is a block transmission scheme it is also important that the position within the received sequence that corresponds to the start of a transmitted block, referred to as block synchronisation (BS), is known at the receiver. BS for DDST was first studied in [5], but required  $P \geq 2M + 1$ , where  $P$  is the training sequence period and  $M$  is the number of channel taps. In [5] a 2-step algorithm for block synchronisation based on projection operators was used. In the

first step the training sequence synchronisation (TSS) was attempted and based on this TSS the final BS was obtained. But as we will show later, the TSS algorithm presented in [5] suffered from the interference of the data and so resulted in poor BS. The method in [5] is outperformed by the new algorithm presented here.

**Objectives and Contributions:** In this paper, we propose a single step algorithm (as opposed to the two step TSS and then BS of [5]) to remove the effects of the information-data “noise” on the BS process. This new method of BS gives a much better performance than the existing method for DDST BS in [5], both in terms of MSE of channel estimates and also the BER.

**Notation:** Superscript ‘ $T$ ’ denotes the transpose operator. For matrix  $\mathbf{A}$ , define  $\mathbf{A}^{[L]c}$  and  $\mathbf{A}_{[L]c}$  to correspond respectively to its first and last  $L$  columns. Furthermore,  $\mathbf{A}^{[L]r}$  and  $\mathbf{A}_{[L]r}$  respectively denote its first and last  $L$  rows.  $\mathbf{1}_{P \times Q}$  and  $\mathbf{0}_{P \times Q}$  correspond respectively to  $P \times Q$  matrices of ones and zeros.  $\mathbf{I}_P$  is the  $P \times P$  identity matrix. Finally,  $\|\cdot\|$  represents the Euclidean norm of a vector.

## 2. SYSTEM MODEL

Consider a cyclic-prefixed DDST based block transmission system over a frequency-selective channel. To avoid interblock interference, we assume the length ( $Z$ ) of the cyclic prefix to be equal to the length ( $M$ ) of the channel. So the channel's input signal is a succession of data blocks, each block being  $N$ -samples long and separated by a cyclic prefix. At the receiver, the received data block (under perfect BS and after removing the cyclic prefix) has the following form [4, 5]:

$$x(k) = \sum_{l=0}^{M-1} h(l)b(k-l) + \sum_{l=0}^{M-1} h(l)c(k-l) + \sum_{l=0}^{M-1} h(l)e(k-l) + n(k) + d \quad (1)$$

where  $k = 0, 1, \dots, N - 1$ ;  $b(k)$  is the information bearing sequence;  $h(k)$  is the channel impulse response;  $e(k)$  is a periodic (period  $P$ ) data-dependent sequence [4] given by  $e(k) = -\frac{1}{N_P} \sum_{i=0}^{N_P-1} b(iP + k)$ ,  $k = 0, 1, \dots, P - 1$  with

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$N_P = \frac{N}{P}$ ;  $n(k)$  is additive, white, Gaussian noise; and  $d$  represents an unknown DC-offset term due to using first-order statistics [2, 3]. It is assumed that all terms in (1) are complex; that  $b(k)$  and  $n(k)$  are from independent, identically distributed (i.i.d), random, zero-mean processes, with powers  $\sigma_b^2$  and  $\sigma_c^2$  respectively; and that the channel is of known order  $M - 1$ , i.e.  $h(0) \neq 0$  and  $h(M - 1) \neq 0$ . Furthermore,  $c(k)$  is the periodic superimposed training sequence (period  $P \geq M$ ) with power  $\sigma_c^2 = \frac{1}{P} \sum_{k=0}^{P-1} |c(k)|^2$ .

Now as in [2], we can write

$$\hat{y}(j) = \frac{1}{N_P} \sum_{i=0}^{N_P-1} x(iP + j) \quad (2)$$

with  $j = 0, 1, \dots, P - 1$ , and where  $\hat{y}(j)$  is an estimate of the periodic (period  $P$ ) true cyclic mean  $y(j) \equiv E\{x(iP + j)\}$ . So from (1) and (2)

$$\begin{aligned} \hat{y}(j) = & \sum_{l=0}^{M-1} h(l) \tilde{b}(j-l) + \sum_{l=0}^{M-1} h(l) c(j-l) + \\ & + \sum_{l=0}^{M-1} h(l) e(j-l) + \tilde{n}(j) + d \end{aligned} \quad (3)$$

where  $\tilde{b}(k) = \frac{1}{N_P} \sum_{i=0}^{N_P-1} b(iP + k)$  with  $k = 1 - P, 2 - P, \dots, P - 1$  and  $\tilde{n}(j) = \frac{1}{N_P} \sum_{i=0}^{N_P-1} n(iP + j)$  with  $j = 0, 1, \dots, P - 1$ . So (3) can now be written as

$$\hat{\mathbf{y}} = (\mathbf{C}^{[M]c} + \tilde{\mathbf{B}}^{[M]c} + \mathbf{E}^{[M]c}) \mathbf{h} + d \mathbf{1}_{P \times 1} + \tilde{\mathbf{n}} \quad (4)$$

where  $\mathbf{h} = [h(0) \dots h(M - 1)]^T$  and  $\hat{\mathbf{y}} = [\hat{y}(0) \dots \hat{y}(P - 1)]^T$ , with a similar expression for  $\tilde{\mathbf{n}}$ . Note that  $\mathbf{C}$  is a  $P \times P$  circulant matrix with first column  $[c(0) c(1) \dots c(P - 1)]^T$ ;  $\mathbf{E}$  is also circulant with first column  $[e(0) e(1) \dots e(P - 1)]^T$  and  $\tilde{\mathbf{B}} = \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2$ , where  $\tilde{\mathbf{B}}_1$  is  $P \times P$  circulant with first column  $[\tilde{b}(0) \tilde{b}(1) \dots \tilde{b}(P - 1)]^T$  and  $\tilde{\mathbf{B}}_2$  is  $P \times P$  upper triangular Toeplitz and  $[\frac{\tilde{b}(-k) - \tilde{b}(N-k)}{N_P}]$  are the elements of the  $k$ -th ( $k = 1, 2, \dots, P - 1$ ) upper diagonal. Due to the use of a cyclic prefix, we have  $\tilde{\mathbf{B}}_2 = \mathbf{0}$ . Now it is not difficult to see that if we choose  $e(k) = -\tilde{b}(k)_P$ , with  $k = 0, 1, \dots, N - 1$  and  $(\cdot)_P$  implying arithmetic modulo  $P$ , then  $\mathbf{E} = -\tilde{\mathbf{B}}_1$  and so (4) becomes (removing the interference of data upon the cyclic mean estimates)

$$\hat{\mathbf{y}} = \mathbf{C}^{[M]c} \mathbf{h} + d \mathbf{1}_{P \times 1} + \tilde{\mathbf{n}} \quad (5)$$

which will be used later.

### 3. PROPOSED BLOCK SYNCHRONISATION ALGORITHM

To begin with we start by re-formulating the TSS/BS method of [5]. When there is no block synchronisation between the

transmitter and the receiver, (1) can be re-written as

$$\begin{aligned} x(k) = & \sum_{l=0}^{M-1} h(l) b(k - \tau_s - l) + \sum_{l=0}^{M-1} h(l) c(k - \tau_s - l) + \\ & + \sum_{l=0}^{M-1} h(l) e(k - \tau_s - l) + n(k) + d \end{aligned} \quad (6)$$

where  $k = 0, 1, \dots, N - 1$ . The effect of  $\tau_s$  in our set-up represents the synchronisation offset between the transmitter and the receiver. Both the transmitted and received blocks are  $N$ -samples long. For a proper operation of DDST  $\tau_s$  must be determined modulo- $N$ -i.e., block synchronisation must be achieved. So the problem now becomes to establish block synchronisation i.e., derive the estimate  $\hat{\tau}_s$  for DDST and then estimate the channel  $\{h(m)\}_{m=0}^{M-1}$  from  $\{x(k)\}_{k=0}^{N-1}$ . Now define the cyclic means (obtained by shifting the  $N$ -point window of (6) one sample to the right for each value of  $k$ ) as

$$\hat{y}_k(j) = \frac{1}{N_P} \sum_{i=0}^{N_P-1} x(iP + j + k) \quad (7)$$

where  $j = 0, 1, \dots, P - 1$  and  $k = 0, 1, \dots, N - 1$ . Note that in [5], the cyclic prefix was assumed to be equal to the length of the training sequence  $P$ , so the  $c(k)$  term in (6) is still periodic, even when there is no BS. Using this fact, and irrespective of whether we have BS or not, after some manipulations we can express  $\{\hat{y}_0(j)\}_{j=0}^{P-1}$  in (7) as

$$\mathbf{P}_0 \hat{\mathbf{y}}_0 = \mathbf{P}_0 (\mathbf{C}^{[M]c} + \tilde{\mathbf{B}}'^{[M]c} + \mathbf{E}'^{[M]c}) \mathbf{h} + d \mathbf{1}_{P \times 1} + \tilde{\mathbf{n}} \quad (8)$$

where the permutation matrix  $\mathbf{P}_0$  is a circulant matrix. Note that  $\tilde{\mathbf{B}}'$  and  $\mathbf{E}'$  are formed similarly to  $\tilde{\mathbf{B}}$  and  $\mathbf{E}$  in (4), except that we now use  $b(k - \tau_s)$  and  $e(k - \tau_s)$  from (6). In [5] the columns of the matrix  $\mathbf{C}$  (previously defined) span two orthogonal subspaces defined by the columns of  $\mathbf{C}^{[M]c}$  and  $\mathbf{C}_{[P-M]c}$ , and this orthogonality is used to obtain the TSS. However, in (8), the term  $(\tilde{\mathbf{B}}'^{[M]c} + \mathbf{E}'^{[M]c})$  will prevent the TSS from being obtained unless we can set  $\mathbf{E}'^{[M]c} = -\tilde{\mathbf{B}}'^{[M]c}$ , and this can only occur if there is perfect BS, which we do not have! Hence the TSS in [5] suffers from interference from data (i.e.,  $\tilde{\mathbf{B}}'^{[M]c}$ ) and this results in errors in the estimated TSS. Due to the fact that [5] uses the TSS to later achieve the BS in the next step so the BS also suffers from errors.

Now in order to achieve BS and DC-offset estimation in this paper we propose to rewrite (5) as

$$\hat{\mathbf{y}} = \mathbf{C} \left\{ [\mathbf{h}^T \mathbf{0}_{1 \times (P-M)}]^T + m \mathbf{1}_{P \times 1} \right\} + \tilde{\mathbf{n}} \quad (9)$$

where  $m = d/P\bar{c}$  and  $\bar{c} = \frac{1}{P} \sum_{k=0}^{P-1} c(k)$ , and reformulate again as

$$\mathbf{C}^{-1} \hat{\mathbf{y}} = [\mathbf{h}^T \mathbf{0}_{1 \times (P-M)}]^T + \hat{\mathbf{m}} \quad (10)$$

where  $\hat{\mathbf{m}} \approx m\mathbf{1}_{P \times 1}$ . It can be observed that the last  $P - M$  elements of the RHS of (10) are all equal. Now we will show that this special structure can be used not only to estimate the scaled DC-offset but also used for BS. Now in order to achieve BS (which will now automatically include TSS) in a single step we set out to search for this special structure from the received sequence  $x(k)$ . But, in order to ensure that the last  $P - M$  equal elements in (10) do not naturally occur in  $\mathbf{h}$ , we let  $P - M > M$  so that  $P \geq 2M + 1$ . Therefore in order to achieve the BS we propose to exploit the special structure of the vector in the RHS of (10), which has its last  $P - M$  elements of equal magnitude. So we compute the cyclic mean for each value of  $k$  in (7), where  $k = 0, 1, \dots, N - 1$ , and search for the special structure (just mentioned) for each value of  $k$ . Hence BS is achieved (in practice) using

$$\hat{\tau}_s = \arg \min_{0 \leq k \leq N-1} \|\mathbf{V}(\mathbf{C}^{-1}\hat{\mathbf{y}}_k)_{[P-M]_r}\| \quad (11)$$

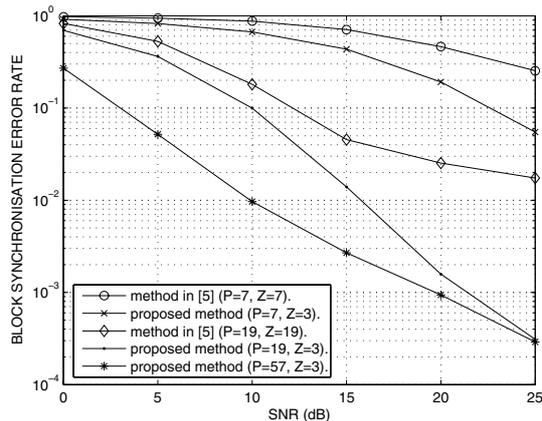
where  $\mathbf{V} := \mathbf{I}_{(P-M)} - \frac{1}{P-M}\mathbf{1}_{(P-M) \times (P-M)}$  acting on a vector produces the same vector with its mean removed from each element, and so this helps (11) identify when the last  $(P - M)$  elements of  $\mathbf{C}^{-1}\hat{\mathbf{y}}_k$  are nearly all equal, and so achieves BS. As we will see later in the simulations, there has to be a compromise on the size of the training sequence as a smaller  $P$  will result in poor block synchronisation while a larger  $P$  will result in more data distortion at the transmitter and hence a poor final BER performance. Also the length of the cyclic prefix  $Z$  in our proposed method can be  $Z \geq M$  whereas the method in [5] must use the larger value  $Z = P$ .

Once the BS is achieved we can show that the channel estimates can be obtained with

$$\hat{\mathbf{h}} = \left( (\mathbf{C}^{-1})^{[M]_r} - \frac{1}{P-M}\mathbf{1}_{M \times (P-M)}(\mathbf{C}^{-1})_{[P-M]_r} \right) \hat{\mathbf{y}}_{\hat{\tau}_s} \quad (12)$$

where we have used the last  $P - M$  elements of (10) to estimate the scaled DC-offset and then used it along with first  $M$  elements of (10) to obtain the channel estimates. Note that  $\hat{\mathbf{y}}_{\hat{\tau}_s}$  comes from (7) and (11).

**Remark 1:** As the first step in the method presented in [5] suffers from interference from data, so we have an error in estimation of the BS offset. This error has the effect of delays/advances in estimated channel impulse response and so this delay ambiguity also causes equaliser output to be similarly delayed. Therefore [5] resorted to estimating this (equalisation) delay by comparing the equalised sequence, shifted by different amounts, with the transmitted symbols  $\{b(k)\}$  in order to compute the BER. So the delay providing the smallest BER is the actual true equalisation delay, and this can now be compensated for and is also used to compute the correct MSE of channel estimates. The problem is, of course, that in practice we cannot use the transmitted symbols to calculate this equalisation delay, via BER. In this paper (in all simulations for all methods) we avoid compensating for the equalisation delay which cannot be obtained in practical applications.



**Fig. 1.** Block synchronisation error rate (see (13)) using the proposed method. The results using the method in [5] are also included for comparison.

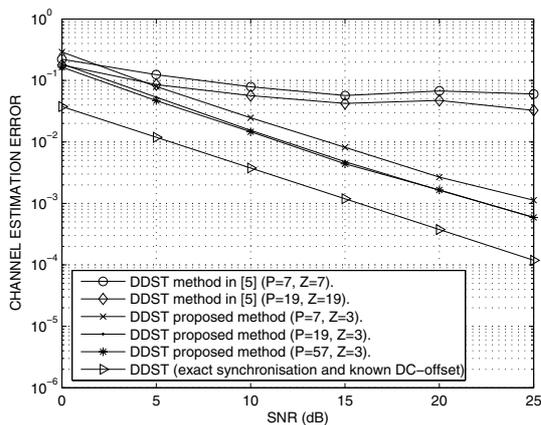
#### 4. SIMULATION RESULTS

We will now proceed with some simulations to test our new synchronisation algorithm. The channel  $h(k)$  was three tap complex Rayleigh fading: both real and imaginary parts of channel taps follow a normal distribution, rescaled to achieve unit mean energy channel ( $E[\sum_{m=0}^{M-1} |h(m)|^2] = 1$ ). The data was a BPSK sequence, to which a training sequence fulfilling  $\mathbf{C}\mathbf{C}^H = P\sigma_c^2\mathbf{I}$  as in [2] was added before transmission. The training to information power ratio ( $\text{TIR} = \frac{\sigma_c^2}{\sigma_b^2}$ ) was set to 0.2,  $N = 399$ , DC-offset  $d = \sqrt{0.1}$  and a linear MMSE equaliser of length  $Q = 11$  taps was used throughout as in [5]. The MMSE equaliser operates using its optimum delay—i.e., for a given delay  $\alpha$  an MMSE equaliser ( $\mathbf{w}_\alpha$ ) was computed, and  $\alpha_{opt} = \arg \min_\alpha \left\{ \sum_{k=-\infty}^{\infty} |\delta(k-\alpha) - (\hat{\mathbf{h}} * \mathbf{w}_\alpha)(k)|^2 \right\}$ , was used. A cyclic prefix of length  $Z = 3$  is added at the beginning of each block, whereas for the method in [5] it is larger value  $Z = P$ . Training sequences with different periods i.e.,  $P = 7, 19, 57$  are used to show the dependence of the BS algorithm on the size of the training sequence period. In each Monte Carlo run, a random synchronisation offset ( $\tau_s$ ) between 0 and  $N + P - 1$  was introduced between the transmitter and the receiver, so that we could be at any sample index within the first block at the receiver. The BS error rate ( $\gamma$ ) is defined as

$$\gamma = \frac{\text{No. of errors in estimating the BS offset}}{\text{Total no. of Monte Carlo runs}} \quad (13)$$

where an error is said to occur when  $\hat{\tau}_s \neq \tau_s$ . So Figure 1 shows the BS error rate. It can be seen that the proposed method for synchronisation completely outperforms the only existing DDST block synchronisation scheme [5]. Now Figure 2 shows the MSE of the channel estimates. We can ob-

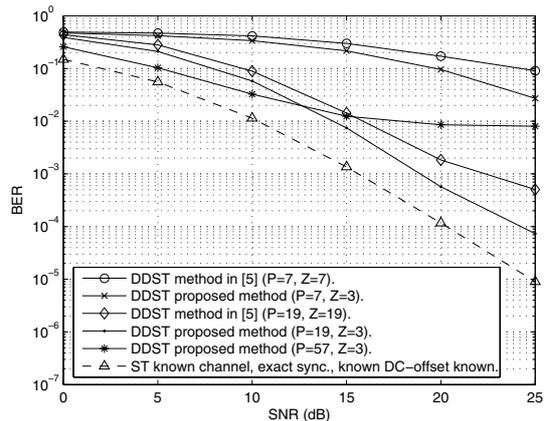
serve that as the BS algorithm improves for  $P = 19$  and  $P = 57$ , the MSE of channel estimates also improves and converges at high SNR whereas the method in [5] gives poor performance. Finally, Figure 3 shows the BER: in step 1 (not shown), we derive the estimate  $\hat{b}(k)$  and then get the estimate  $\hat{e}(k)$  in the next iteration to improve the BER as in [4]. This result is shown in the Figure 3. It is clear that for low  $P$  we have poor BS, poor channel estimation and this results in poor BER. As we increase  $P$ , both BS and channel estimation improve as does BER. But for very large  $P$  (i.e.,  $P = 57$ ), the improved BS and improved channel estimation does not result in improved BER (especially at high SNR). This is because with large  $P$  we have zeroed (via  $e(k)$ )  $P$  DFT bins of the data  $b(k)$  in (1), and so increase data distortion at the transmitter. So we increase  $P$  to a point where the effect of the data distortion (on BER) outweighs the improvement (on BER) gained by improved BS and channel estimation. In conclusion, we can see that the proposed method of BS completely outperforms the existing conventional DDST BS scheme and requires a cyclic prefix of length only  $Z = M$  whereas the method in [5] requires a larger value of  $Z = P$ .



**Fig. 2.** MSE of channel estimates using the proposed method. The results using the method in [5], together with the result assuming known DC-offset and perfect BS for DDST, are also included for comparison.

## 5. CONCLUSION

In this paper, we have presented a new (single-step) block synchronisation algorithm (BS) for the joint channel and DC-offset estimation for data-dependent superimposed training (DDST). No a-priori BS was assumed and a DC-offset could be present at the receiver output. The proposed method of BS is based on the particular structure of the channel output's cyclic mean vector. We have shown that the proposed method of BS is dependent on the training sequence period ( $P$ ) and the SNR. Since the proposed method does not suffer due to



**Fig. 3.** BER using the proposed method. The results using the method in [5], together with the result assuming known DC-offset and perfect synchronisation for ST when the channel is completely known, are also included for comparison.

the interference by information-bearing data as in [5], it does not require any knowledge of the equalisation delay as in [5] (which anyway cannot be obtained in practical applications). The simulations show that the proposed method completely outperforms the existing DDST-based method of [5] and requires a smaller cyclic prefix of length  $Z = M$  as apposed to  $Z = P$  in [5]. Finally we get a performance close to the ideal case where we have exact synchronisation and known DC-offset.

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