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ECONOMETRIC MODELLING OF COMPETITION BETWEEN TRAIN TICKET TYPES

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1. INTRODUCTION

The railways in Britain have a long history of using price discrimination backed up with product differentiation to significantly increase revenue over what might be obtained in an undifferentiated market. Whilst not as sophisticated as the yield management systems widely used in the airline industry, rail ticketing strategies are continually evolving, with new products emerging, unsuccessful products discontinued and gradual refinement of others.

In recent years, there has been increased interest in modelling competition between different ticket types. The re-organisation of the railway industry in Great Britain has provided a greater commercial incentive to operators to price differentiate in order to maximise the revenue from their franchises. The policy of moderation of competition has allowed limited on-track competition, largely based around overlapping franchises but also with service extensions and new entrants, and this has stimulated product development and hence interest in ticket choice. Partly in response to the greater commercialisation of the railway industry, particularly where here is a degree of market power, the regulatory bodies have taken a greater interest in the range of tickets offered and their associated prices, travel restrictions and availability (SRA, 2003).

This paper reports on research which was conducted as part of an update to the Passenger Demand Forecasting Handbook (PDFH), which contains a forecasting framework and recommended demand parameters that are widely used in the railway industry in Great Britain (ATOC, 2002), and as part of a project to provide the Strategic Rail Authority with evidence on cross elasticities between ticket types for use in its review of how it regulates rail fares.

2. BACKGROUND

2.1 Ticket Structure

Historically the railways in Britain charged fares according to the distance travelled with a 50% mark-up for first class travel compared with standard. Indeed, the Railway Act, 1844 (commonly known as the "Parliamentary Trains Act") deriving from Gladstone's Committee of Inquiry into railway policy, specified that railway companies had to provide a minimum service: one train each day each way, travelling at not less than 12 miles per hour and stopping at every station, charging no more than 1d. per mile for third class passengers.

As the railways developed, so the pricing structure evolved, with lower-priced

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excursion tickets as the leisure market grew and discounts for regular travellers by the use of season tickets as commuting increased. Nevertheless, it was still the case in about 1980 that fares were based on mileage, just that the mileage rate varied according to the ticket type (first class, second class, cheap day return, weekend return, economy return). Since then, we have seen the development of an increasingly market based fare structure where prices are based on what the market will bear with the objective of increasing revenue, encouraging off-peak travel and discouraging peak travel. The distinctions on any one route rely primarily on segmentations according to peak or off-peak travel, with off-peak tickets invalid at certain times of day. Across routes, distinctions are apparent according to the size of the available capacity relative to the (potential) demand. Thus, while an unrestricted standard class return from Manchester to London is priced at £175 for a (one-way) distance of 186 miles, the equivalent ticket for the 189 mile Leeds to London journey is £132. For off-peak travel, though, the prices are £50 and £63.30 respectively.

As well as the traditional walk-on fares, we have seen massive growth in advance purchase tickets based on the yield management techniques widely used by airlines. Some operators have sought to promote these ahead of walk-on categories which, apart from the full fare tickets, have seen increasing restrictions on when they can be used. This has led to a greater range of tickets being offered. New low prices are available for those able to plan ahead and not requiring flexibility to change travel plans but at the expense of increased real prices for users requiring the walk-on facility. Thus from Leeds to London, those booking 7 days ahead can purchase a return for £25 or £36 (sold by quota so that the ticket becomes unavailable when the quota for a particular train has been reached); and from Manchester to London, returns are available (subject to quota) at £22, £31 and £42 booking 14, 7 and 3 days ahead respectively. Groups of 4 travelling together can obtain even lower prices; £72 from Manchester to London, £54 from Leeds to London and £48 in both cases if travel is via the competing Midland Mainline service. In all there are 33 quoted fares from Leeds to London and 28 from Manchester to London. This is before one considers the discounts open to those aged 16-24, those aged over 60 and family groups, all of which are available on the purchase of a railcard.

2.2 Rail Ticket Sales Models

Since the 1970's, rail ticket sales data in Great Britain has provided a means by which the properties of rail travel behaviour can be examined. These aggregate direct demand models have been developed to examine the impact on rail demand of factors external to the rail industry, such as the levels of income, employment, car ownership and competition from other modes, as well as the impacts of fare and service quality which are directly under the control of train companies. Fares are central determinants of the revenue earned by train companies and this, along with the traditionally strong interest in Britain in pursuing price differentiation and market segmentation, has resulted in a considerable amount of empirical analysis in this area (Hughes, 1980; Oldfield and Tyler, 1981; Stark, 1981; Rail Operational Research 1982, 1989, 1995a, 1995b, 1995c; Glaister, 1983; Owen and Phillips, 1987; Phillips, 1987; Wardman, 1997a, 1997b; AEAT, 1999). A recent meta-analysis of British evidence (Wardman and Shires, 2003) uncovered 99 fare elasticity values for suburban rail travel and 456 values for inter-urban travel, and by far the largest source of this evidence is studies which have analysed ticket sales data.

These models can be split into those which pooled data across ticket types and those which aimed to improve upon this approach by estimating separate models for different ticket types. There are a number of drawbacks of pooling data across different ticket types. These are:

- The usual means of representing fare in these models has been to use average revenue per trip. This can lead to misleading and in some instances even wrong sign estimates of the market elasticity.
- Estimating a single model fails to provide important insights into different levels of price sensitivity by ticket type and the degree of substitutability which are essential to effective price discrimination
- A single model constrains all other variables, such as those relating to service quality and to external factors, to be the same across the different market segments which the various tickets cater for. This is unlikely to be empirically justified.
- Estimating a single model to pooled data reduces the amount of data and could result in less variation in fare than otherwise. These will result in less precise coefficient estimates.

Some ticket sales based studies have estimated separate models by ticket type. In such cases, it is necessary to specify appropriate cross elasticity terms but most have not done so (Owen and Phillips, 1987; Phillips, 1987; Wardman, 1997b; Ahmed, 1998; CEBR, 1998; Steer Davies Gleave, 1999). However, attempts were made in Britain in the 1990's to estimate elasticities specific to particular categories of ticket and the cross-elasticities between them.

Rail Operational Research (1995a) examined competition between Travelcard season tickets, which also allow travel on bus and underground in London, and point-to-point seasons. This covered trips in the South East to London between 1987 and 1994. The key findings are reported in Table 1. The cross-elasticities of point-to-point seasons with respect to Travelcard fare vary considerably, and by more than would be expected according to variation in market share, but there was no evidence of a cross-elasticity effect on Travelcard demand.

Table 1: Analysis of Travelcard and Point-to-Point Sales

Ticket Type	Distance	Own Elasticity	Cross Elasticity
Travelcard	Within Boundary	-0.5	-
	Across Boundary	-0.8	-
Point-to-Point	Within Boundary	-0.3	2.00
	Across Boundary	-0.5	0.15

AEAT (1999) conducted a major investigation into fare elasticities based on the analysis of annual ticket sales data covering the period 1991 to 1999 for over 5000 flows. The main purpose was to estimate fare elasticities disaggregated by ticket type and the relevant cross-elasticities. The four ticket types considered were: first class; standard full fare, where there are no travel restrictions; standard reduced fare, where travel is not permitted in the peak; and Apex, where advance purchase is required and travel restrictions apply. It was assumed that passengers would not be prepared to trade-down by more than two categories or to trade-up by more than one category.

The elasticity estimates along with 95% confidence intervals are presented in Table 2. There are two immediately apparent features of the results. Firstly, two of the eight estimated cross-elasticities are wrong sign. Secondly, the precision with which the cross elasticities are estimated is not generally high.

Table 2: Inter-Urban Ticket Type Models

Ticket Type	Own Elasticity	Cross Elasticities			
		First	Full	Reduced	Apex
First non Season	-0.6 ±0.08	-	0.06 ±0.03	0	0
Standard Class Full	-0.7 ±0.05	-0.01 ±0.01	-	-0.08 ±0.03	0
Standard Class Reduced	-0.4 ±0.03	0.01 ±0.01	0.03 ±0.01	-	0.05 ±0.01
Standard Class APEX	-0.3 ±0.12	0	0.02 ±0.03	0.33 ±0.08	-

Note: Given the form of model estimated, the own elasticities are conditional

A simpler model was also estimated, distinguishing only between what were termed premium fares, covering first class and standard class full fares, and discount fares, incorporating reduced and Apex fares. The models are reported in Table 3 and fared little better, with one cross-elasticity wrong sign despite being statistically significant.

Table 3: Simplified Inter-Urban Ticket Type Models

Ticket Type	Own Elasticity	Cross Elasticity
Premium	-1.1 ±0.04	-0.08 ±0.02
Discount	-0.6 ±0.05	0.18 ±0.03

With regard to the estimation of cross-elasticities AEAT (1999) concluded that, "Estimation of this effect is fraught with problems, even with our large sample size". There are two principal difficulties encountered by ticket sales models which endeavour to estimate separate models by ticket type:

- There is strong correlation between the fares of different ticket types given the common practice of applying across the board annual fare revisions. This means that the fare coefficients will be estimated less precisely than otherwise, and the model will have problems discerning their separate effects. The positive correlation between fares will result in negative correlation amongst the coefficient estimates. Thus an own elasticity (in absolute) will be too large (small) and a cross elasticity too large (small).

- Cross elasticities tend to be somewhat smaller than own elasticities and thus are inherently more difficult to estimate precisely.

To the extent that there are common effects across ticket types, the estimation of separate models for each ticket type, as opposed to a joint estimation procedure, is inefficient and will lead to less precise coefficient estimates. Moreover, the estimation of separate models fails to exploit demand variation in one model that contributes to an understanding of demand variation in another.

2.3 Why Aggregate Models?

Disaggregate choice models estimated to the decisions individuals make are particularly well suited to the analysis of competition between tickets, covering travel restrictions and aspects of service quality differentiation as well as different fares. Where there are currently a range of tickets with different features, revealed preference (RP) methods are suitable. These can be enhanced with stated preference (SP) exercises, particularly where new tickets are under investigation. There have been a number of British studies that have conducted disaggregate analysis of choices between tickets (Steer Davies Gleave, 1993; Accent and HCG, 1996; Rail Operational Research, 1996; Whelan et al., 1997; Wardman and Murphy, 1999).

Although disaggregate approaches have a number of attractions and the sophisticated market can be expected to support the development of robust models, the analysis here is based on aggregate methods which examine variations in the volume of rail demand between specified stations. There are a number of reasons why we have adopted an aggregate approach:

- A large amount of reliable ticket sales data is available at relatively low cost. At the least, this complements analysis based on more disaggregate means.

- There is an opportunity to use more sophisticated analytical methods to overcome the difficulties faced in previous studies.

- Aggregate data more readily supports the analysis of external factors, and thus it is possible to simultaneously estimate separate GDP and other elasticities for the market segments covered by the different ticket types

- The resulting models fit relatively easily into the demand forecasting procedures contained in the PDFH.

- Variations in fares not only stimulate switching between ticket types but can lead to significant variations in the total volume of rail travel. This variation is automatically included within aggregate models.

- Disaggregate ticket choice models can be complex to develop, given the wide range of tickets available and travellers' limited information about many of them. Application of the models for forecasting need to address the serious misperceptions that will exist regarding the range of tickets and their characteristics whilst appropriate forecasting methods have heavy data requirements.

Another paper at this conference examines the impact of changes in the fares of different tickets at a more detailed level (Whelan and Johnson, 2003).

3. MODELLING APPROACH

As we have seen, previous studies which have examined competition between ticket types based on ticket sales data have not met with a great deal of success. Some models failed to specify cross elasticities whilst those that did often found them to be either wrong sign or statistically insignificant. Our conclusion as to the cause of these problems is the high degree of correlation between the fares of different tickets. This compounds the already difficult task of estimating what are relatively small effects.

A way forward is to harness relationships from economic theory, a procedure which has often been neglected in the empirical analysis of travel demand data. Two relationships are of particular interest here. The Slutsky symmetry equation expresses the relationship between two income compensated cross fare elasticities as:

$$f_{ji} = f_{ij} \frac{V_i F_i}{V_j F_j} \tag{1}$$

V_i is the volume of sales of ticket i and F_i is the fare of ticket i, whereupon the cross-elasticity of demand for ticket j with respect to the fare of ticket i is equal to the product of the cross-elasticity of demand for ticket i with respect to the fare of ticket j and the relative revenue of the two tickets. The relationship relies upon the assumptions of neo-classical micro-economic theory and, since it applies at the individual level, we must assume that aggregate behaviour reflects that of the typical individual. However, an additional relationship exists between cross and own elasticities (Dodgson, 1986) that is true by definition:

$$f_{ij} = -f_{jj} \frac{V_j}{V_i} \delta_{ji} \tag{2}$$

The cross-elasticity of ticket type i with respect to the price of ticket type j can be deduced from the own price elasticity of demand for ticket j (f_{jj}), the relative share of the two tickets (V_j/V_i) and what is termed the diversion factor (δ_{ji}) which denotes the proportion of those who divert from ticket j to ticket i when ticket j becomes unacceptable to them.

On London routes, four generic ticket types cover the vast majority of non-season ticket sales relevant to the business and leisure markets. These are:

- First class tickets, which allow travel on this premium product at anytime and can be bought at anytime;
- Standard class full fare tickets, where travel is allowed at anytime and the tickets can be bought at anytime;
- Standard reduced fare tickets, which place restrictions on the time of travel but which can be bought at anytime;
- Standard class Apex tickets, which allow travel only on specified trains and which must be purchased in advance. Their availability is quota controlled.

We can specify a system of demand equations to cover these generic tickets. Adopting the standard constant elasticity form and ignoring terms other than a size factor (a) for each we have:

$$V_1 = a_1 P_1^{f_{11}} P_F^{f_{1F}} P_R^{f_{1R}} g_1(V_A) \tag{3.1}$$

$$V_F = a_F P_1^{f_{F1}} P_F^{f_{FF}} P_R^{f_{FR}} g_F(V_A) \tag{3.2}$$

$$V_R = a_R P_1^{f_{R1}} P_F^{f_{RF}} P_R^{f_{RR}} g_R(V_A) \tag{3.3}$$

$$V_A = a_A Q \tag{3.4}$$

The demand for first (1), full (F) and reduced (R) ticket types are a function of their own fare and the fares of the other tickets except Apex. The assumption within this framework is that the demand for Apex is supply constrained, that is, more tickets than the quota (Q) available could be sold at the price charged. An Apex fare elasticity therefore has no meaning. As far as the other tickets are concerned, it makes sense to relate the demand for them not to the Apex fare but to its availability.

Apex is a quota controlled ticket, and the quotas are set per train rather than at the level of the i-j flow which is the unit at which the analysis is undertaken. Even if train operating companies had historical data on Apex quotas and were prepared to supply such information, it would be of little use to us since we have no way of knowing how the quotas related to specific i-j movements.

However, a good proxy for the competition offered by Apex is the actual sales of these tickets, and this is available alongside the sales of other tickets. Hence the volume of Apex sales (V_A) enters the demand functions above. We return to the specification of this term below.

Given that fares often move in parallel, the own elasticities (f_{11}, f_{FF} and f_{RR}) will not provide an accurate reflection of the demand change after a common proportionate change in all fares because there will also be second order effects due to switching between ticket types. In such cases, we can derive the conditional elasticity (C) for a particular ticket as the sum its own elasticity and the cross elasticities with respect to the fares of other tickets.

The overall elasticity (η_{OC}) when all fares are changed in the same proportion is the sum of all the conditional elasticities each weighted by their volume share:

$$\eta_O = C_1 \frac{V_1}{V} + C_F \frac{V_F}{V} + C_R \frac{V_R}{V} \tag{4}$$

An appropriate estimation procedure can be used to exploit all the data within a single estimation and ensure that the theoretical relationships between the parameters of equations 3.1, 3.2 and 3.3 are satisfied. Given Q is not known, the analysis is restricted to the demand for first, full and reduced tickets.

For any ticket k, the volume of rail demand between stations i and j in time period t is here specified as:

$$V_{ijkt} = \mu_{ijkt} P_{ijkt}^{f_{kk}} P_{ijlt}^{f_{kl}} P_{ijmt}^{f_{km}} GT_{ijt}^{\alpha} G_{it}^{\beta} O_{it}^{\gamma} T_{ijt}^{\kappa} C_{ijt}^{\lambda} e^{\theta H_{it}} \tag{5}$$

P represents the fare of ticket k and of two competing tickets (l and m) with own elasticity f_{kk} and cross elasticities f_{kl} and f_{km}.

Generalised journey time (GT) is the standard measure of timetable related service quality used in the railway industry in Britain. It contains station-to-station journey time, service headway and interchange and is expressed as an equivalent amount of journey time.

Since GT varies across ticket types as a result of, for example, different time of travel restrictions, it is therefore possible, in principle, to estimate GT cross elasticities between ticket types. However, the limited variation in GT on London based flows means that it is a challenge to estimate a significant and plausible own GT elasticity let alone a cross elasticity and thus we therefore did not proceed with such a specification.

G and O are measures of per capita GDP and population relevant to the origin, T and C represent car time and cost, and H is the proportion of households in the origin with a car. The parameters α, β, γ, κ and λ are all elasticities. H enters in this rather than constant elasticity form since the parameter θ was obtained from analysis of National Travel Survey data and used this functional form, albeit with H taking only the values of 0 or 1, to estimate the effect of whether a household had a car on the number of rail trips made. The elasticity with respect to car ownership is θH_{it}.

μ_{ijkt} is a size effect, representing the generating potential of origin stations not accounted for by income and population and the attracting potential of destination stations. To avoid specifying variables to represent effects which are not of interest here, we can instead examine changes in demand between two time periods (1 and 2) and assume that μ_{ijkt} is effectively constant. The model then takes the form:

$$\frac{V_{ijk2}}{V_{ijk1}}=\left(\frac{P_{ijk2}}{P_{ijk1}}\right)^{f_{kk}}\left(\frac{P_{ijl2}}{P_{ijl1}}\right)^{f_{kl}}\left(\frac{P_{ijm2}}{P_{ijm1}}\right)^{f_{km}}\left(\frac{GT_{ij2}}{GT_{ij1}}\right)^{\alpha}\left(\frac{G_{i2}}{G_{i1}}\right)^{\beta}\left(\frac{O_{i2}}{O_{i1}}\right)^{\gamma}$$
$$\left(\frac{T_{ij2}}{T_{ij1}}\right)^{\kappa}\left(\frac{C_{ij2}}{C_{ij1}}\right)^{\lambda} e^{\theta(H_{i2}-H_{i1})+\tau(V_{ijA2}-V_{ijA1})/V_{ijA1}} \tag{6}$$

In equations 3.1, 3.2 and 3.3 above, allowance was made for the effect of Apex sales (V_A) on the demand for each ticket. The most realistic approach would be to allow changes in V_A to have an additive effect, whereupon the change in V_A would reduce the demand for a ticket k (V_K) by some proportion of V_A (π_K). This could be achieved within the above multiplicative model by instead specifying the term containing V_A as:

$$\frac{V_{ijk2}/(V_{ijk2}+\pi_k V_{ijA2})}{V_{ijk1}/(V_{ijk1}+\pi_k V_{ijA1})} \tag{7}$$

The shortcoming with this approach is that the elasticities can vary quite considerably. Each elasticity in equation 6 would, for each ticket type, be multiplied by:

$$\frac{V_{ijkt}+\pi_k V_{ijAt}}{V_{ijkt}} \tag{8}$$

Whilst we have no objections in principle to variations in elasticities, it would be a step-change in the procedures involved in PDFH which might not prove acceptable, whilst empirical studies which test for elasticity variation within ticket sales models struggle to obtain convincing and statistically significant effects let alone the potentially large variation implied by this formulation.

In order to leave the elasticities unaffected, we could have entered V_A in constant elasticity form. However, this would be unrealistic since a given proportionate change in V_A will have a larger impact on V_k when V_A is large relative to V_k than when it is small. Alternatively, V_A could enter in exponential form. The final term would then be:

$$e^{\tau(V_{ijA2}-V_{ijA1})} \tag{9}$$

This would not impact on the elasticities. It implies that a given absolute difference V_A will have the same proportionate impact on V_k, but this will not generally be the case. We specified V_A as in equation 6 which is the same as in equation 9 but deflated by the actual volume of Apex trips in the based period since this gave the best fit to the data.

It is not possible to estimate sensible parameters for all the external factors simultaneously, since they are so highly correlated. The solution is to isolate the effects of O, T, C and H by constraining γ, κ, λ and θ to equal the best estimates available, and to freely estimate the GDP elasticity (β) which is then conditional upon the constraints used.

The base year (1) is taken to be1998 since the recording of sales is likely to be more reliable for the most recent year. However, the results vary little with other selections of before and after years. A logarithmic transformation of equation 6 is taken prior to parameter estimation. The variance of the error term (ε_{ij}) is:

$$Var(\varepsilon_{ij}) = \frac{1}{V_{ij1}^2} Var(V_{ij1}) + \frac{1}{V_{ij2}^2} Var(V_{ij2}) - \frac{2Cov(V_{ij1}V_{ij2})}{V_{ij1}V_{ij2}} \tag{10}$$

Weighted least squares is used to correct for the variation in the error variance across observations.

4. RESULTS

On the key commuter routes in the South East, there is competition between different season tickets and between season and daily tickets but data on season tickets was not available to us. The data which was available related to the annual volume of trips by first, full, reduced and apex tickets for 5010 flows for the period 1990 to 1998. On some Non London inter-urban routes, there is little effective competition between ticket types since first class is not offered and reduced fare tickets are valid on all trains whilst on other routes the competition only really emerges towards the latter part of the time period.

The analysis is therefore conducted on flows over 100 miles to and from London, where there has long been a relatively sophisticated ticketing structure. It is based on annual data for the period 1993 to 1998 since it is only from 1993 that Apex tickets have been available. In addition, 1994 was removed given the serious industrial action in that year.

The number of routes in our data are 441 between 100 and 200 miles and 269 over 200 miles. Given that there are five years of data, and hence four ratios of demand changes, we have a maximum of 1764 and 1076 observations on the two sets of flows respectively. However, there are some cases where the changes in demand seemed on a first, full or reduced ticket appeared to be too large. Years were removed from the analysis where year-on-year demand more than doubled or halved. This reduced the data sets to 1608 and 900 observations but we feel it removes the worst excesses of data inaccuracy.

Across the years in question, the correlation between the first and full fares, first and reduced fares and full and reduced fares were 0.89, 0.87 and 0.82 respectively for the 100-200 mile flows and 0.42, 0.50 and 0.34 for the over 200 mile flows. The fare variation, in absolute, across the 5 years was 8.7%, 6.6% and 6.7% for first, full and reduced tickets on the 100-200 mile flows and 15.2%, 14.5% and 9.1% on the over 200 mile flows. Hence there has been an appreciable amount of fare variation.

Table 4 lists the parameter constraints given the inability to freely estimate these parameters within equation 6 due to multicollinearity. These figures are largely based upon the recommendations for business and leisure travel contained in the Railway Industry Forecasting Framework (Steer Davies Gleave, 1999), weighted for a 40:60 split between the two journey purposes.

Table 4: Constrained Parameter Values

Purpose	Share	Population γ	Car Time κ	Car Cost λ	Car θ
Business	40%	1.0	0.30	0.10	0.00
Leisure	60%	1.0	0.30	0.30	-0.94
Overall		1.0	0.30	0.22	-0.56

Table 5 lists the diversion factors used in this work. Enquiries were made with all the relevant train operating companies about whether they had information on diversion factors between ticket types. No such evidence was available and thus the figures in Table 5 are based on the collective judgments of members of the study team involved in updating the PDFH. Thus we are assuming that when a fare variation becomes sufficiently large to cause a behavioural switch, 40% of current first class travellers will switch to full fare tickets.

Table 5: Assumed Diversion Factors

	First	Full	Reduced	Apex	Not Rail
First to:	-	40%	20%	0%	40%
Full to:	20%	-	50%	5%	25%
Reduced to:	0%	40%	-	30%	30%

The zero diversion from reduced to first means that the cross elasticity between first and reduced is zero and hence no cross elasticity term (f_{1R}) was specified in the demand system. Although the diversion between first and reduced is not zero, the volume of reduced travel far exceeds that by first class such that the cross elasticity would be very low. As a result, no cross elasticity was specified between reduced and first class (f_{R1}). In any event, the symmetry condition would require it to be zero.

The technique of iterative seemingly unrelated regression has been used to estimate the system of equations with the parameter restrictions imposed. There are two Slutsky symmetry restrictions, between f_{1F} and f_{F1} and between f_{FR} and f_{RF}, and four restrictions based on the diversions factors between f_{F1} and f_{11}, f_{1F} and f_{FF}, f_{RF} and f_{FF}, and f_{FR} and f_{RR}. These restrictions were imposed at the mean revenue shares for equation 1 and the mean volumes in equation 2 across routes and years.

Table 6 reports separate models for 100-200 miles (Models 1a-1e) and for over 200 miles (Models 2a-2e). All the freely estimated parameters are reported. If the variables which are highly correlated with GDP and whose parameters are constrained are removed from the equation, the GDP elasticity increases but the estimated fare elasticities hardly differ. The reported GDP elasticity estimates are conditional upon the constrained elasticity values of Table 4.

Models 1a and 2a are estimated as a system of equations but no constraints are imposed. The results are poor in almost all respects. Some cross elasticities are wrong sign, others that are correct sign are far from statistically significant, whilst the pattern of own elasticities is not entirely convincing and indeed the conditional elasticity for full fare in Model 2a is wrong sign.

Models 1b and 2b impose the symmetry condition of equation 1, whereupon the relevant pairs of cross elasticities have the same t ratios. The imposition of this condition does not improve matters greatly and indeed two of the four cross-elasticities in Model 1b are wrong sign whilst in Model 2b two cross elasticities are far from significant and the conditional elasticity for full fare tickets remains positive.

Models 1c and 2c ignore the symmetry conditions but do impose the relationship between cross and own elasticities using diversion factors as specified in equation 2. The jointly estimated parameters have the same t ratios. Model 1c now has all its cross elasticities as right sign and significant, although as a consequence the conditional elasticity for full fare tickets is very low. Model 2c can hardly be regarded an improvement over 2b since there are still two insignificant cross elasticities and the conditional elasticity for full fare tickets remains positive.

Both the symmetry and the diversion factors constraints are imposed simultaneously in Models 1d and 2d. Given that the constraints uniquely determine the relativities, all that remains for the estimation process is to determine the scale of the coefficients. The joint estimation returns the same t ratio for all the fare coefficients and this is far higher than those generally obtained in the other models. In addition, all the elasticities are correct sign and their order of magnitude seems reasonable.

As would be expected, imposition of the constraints reduces the goodness of fit, although only by a little, but offsetting this is the much greater degree of confidence that can be placed in the estimated parameters.

Table 6: Estimated Models

	First	Full	Reduced	Cond	GDP	GT
100-200 miles						
1a: No Constraints						
First	-0.83 (15.1)	0.03 (0.3)	-	-0.80	2.05 (36.1)	-0.49 (3.7)
Full	0.45 (14.1)	-1.64 (19.5)	-0.16 (4.0)	-1.35	2.88 (53.4)	-0.49 (3.7)
Reduced	-	-0.53 (10.9)	-1.53 (54.9)	-2.06	0.75 (19.6)	-2.13 (20.6)
Adjusted R^2 = 0.868						
1b: Slutsky Symmetry						
First	-0.89 (18.1)	0.36 (15.0)	-	-0.53	1.98 (39.6)	-0.39 (3.0)
Full	0.44 (15.0)	-1.66 (22.1)	-0.29 (8.5)	-1.51	2.82 (53.8)	-0.39 (3.0)
Reduced	-	-0.19 (8.5)	-1.50 (53.8)	-1.69	0.79 (21.8)	-2.24 (21.8)
Adjusted R^2 = 0.863						
1c: Diversion Factors						
First	-0.78 (23.5)	0.26 (14.1)	-	-0.52	1.86 (36.5)	-0.90 (6.7)
Full	0.39 (23.5)	-1.08 (14.1)	0.51 (49.1)	-0.18	3.06 (58.1)	-0.90 (6.7)
Reduced	-	0.21 (14.1)	-1.45 (49.1)	-1.24	0.83 (20.4)	-2.59 (23.6)
Adjusted R^2 = 0.832						
1d: Both						
First	-0.90 (53.7)	0.36 (53.7)	-	-0.54	1.84 (36.5)	-0.89 (6.7)
Full	0.45 (53.7)	-1.51 (53.7)	0.47 (53.7)	-0.59	3.16 (63.9)	-0.89 (6.7)
Reduced	-	0.30 (53.7)	-1.35 (53.7)	-1.05	0.88 (21.8)	-2.46 (23.7)
Adjusted R^2 = 0.828						
1e: Overall						
Overall (excl Apex)		-0.96 (26.0)			1.80 (48.9)	-2.17 (25.2)
Adjusted R^2 = 0.733						
Overall (incl Apex)		-1.08 (33.5)			1.62 (45.5)	-2.30 (22.8)
Adjusted R^2 = 0.711						
Over 200 miles						
2a: No Constraints						
First	-0.56 (16.0)	0.18 (3.4)	-	-0.38	2.61 (40.9)	n.s.
Full	0.83 (17.1)	-0.07 (0.9)	-0.02 (0.3)	0.74	0.60 (7.7)	n.s.
Reduced	-	0.04 (0.7)	-1.92 (23.8)	-1.88	1.45 (7.3)	n.s.
Adjusted R^2 =0.783						
2b: Slutsky Symmetry						
First	-0.59 (17.1)	0.37 (16.9)	-	-0.22	2.44 (56.7)	n.s.
Full	0.74 (16.9)	0.01 (0.1)	0.04 (0.7)	0.69	0.54 (7.1)	n.s.
Reduced	-	0.01 (0.7)	-1.95 (24.3)	-1.94	1.48 (8.2)	n.s.
Adjusted R^2 = 0.783						
2c: Diversion Factors						
First	-0.73 (22.5)	0.00 (0.2)	-	-0.73	2.79 (71.6)	n.s.
Full	0.18 (22.5)	-0.01 (0.2)	0.40 (23.4)	0.57	0.61 (7.8)	n.s.
Reduced	-	0.00 (0.2)	-1.80 (23.4)	-1.80	1.74 (10.2)	n.s.
Adjusted R^2 = 0.774						
2d: Both						
First	-0.56 (27.3)	0.08 (27.3)	-	-0.48	2.68 (73.8)	n.s.
Full	0.14 (27.3)	-1.00 (27.3)	0.27 (27.3)	-0.59	1.29 (18.9)	n.s.
Reduced	-	0.08 (27.3)	-1.23 (27.3)	-1.15	2.50 (19.6)	n.s.
Adjusted R^2 = 0.773						
2e: Overall						
Overall (excl Apex)		-1.05 (6.1)			2.85 (34.3)	n.s.
Adjusted R^2 = 0.489						
Overall (incl Apex)		-1.44 (25.8)			2.04 (38.7)	n.s.
Adjusted R^2 = 0.422						

No significant effects upon first or full sales were obtained from V_A. For the 100-200 miles flows, the estimated coefficient (τ) was -0.058 (9.9). This implies a 0.5% reduction in the volume of reduced ticket sales after a 10% increase in the volume of Apex tickets. The corresponding figure for the over 200 miles flows was -0.141 (4.9) which implies that a 10% increase in Apex sales will lead to a 1.5% fall in reduced ticket sales.

There is little variation in the GDP elasticities across the different models for the 100-200 mile flows, although more is apparent for the over 200 mile flows. For the shorter distances, it is the first and full markets which grew fastest, and these are dominated by business travel. However, our feeling is that tighter restrictions on the use of reduced fare tickets, which we did not have the data to model, will have contributed to the high GDP elasticity for full tickets. It may also have impacted on the estimated fare elasticities. For the longer distance flows, where there are fewer restrictions on the use of reduced tickets, sales of reduced tickets grew by more than full fare tickets, although again the GDP elasticity for first class is high.

The conditional elasticities are sensible in relation to each other. The first class market is dominated by business travel and high income leisure travel and thus the fare elasticity is expected to be lower than for full fare tickets which does have a large share of business travel but by generally less senior employees. Reduced tickets are largely used by leisure travellers and hence the elasticity is expected to be relatively high.

We cannot compare the cross-elasticities against other evidence since the other aggregate models that have been developed were unsatisfactory in this respect whilst the disaggregate ticket choice models that have been estimated do not report such figures. In any event, the latter cross elasticities would be strongly dependent upon the both the fare levels and market shares making comparison difficult. We can, however, compare the conditional elasticities estimated here with what can be interpreted to be approximately conditional elasticities derived from ticket sales models which split by ticket type but did not specify cross-elasticity terms.

Owen and Phillips (1987) analysed demand on 20 long distance London based flows. The median first and standard class fare elasticity estimates were -1.0 and -1.17. Ahmed (1998) reports fare elasticities on London routes for first, full and reduced tickets of -0.71 (±0.22), -0.81 (±0.12) and -0.83 (±0.12) respectively. Wardman and Dunkerley (1999) analysed ticket sales data for Great Western services and estimated elasticities for first class fares of -0.14 on the top 25 revenue earning flows and -0.73 on other London flows. The corresponding figures for standard class, which combines full and reduced tickets, were -0.59 and -0.96. AEAT (1999) examined a large data set of flows and estimated models split by ticket type and distance. Results relevant for this study are presented in Table 7.

Table 7: Elasticities by Ticket Type

Distance	First non Season	Standard Full	Standard Reduced
75-149 miles	-0.5±0.11	-0.5±0.08	-0.7±0.05
150-249 miles	-0.3±0.12	-0.3±0.08	-1.1±0.12
250 miles and over	-0.6±0.14	-0.3±0.11	-1.5±0.22

Source: AEAT (1999) Table 1.

Finally, in a recent meta-analysis covering 902 fare elasticities from British studies, of which 456 were inter-urban rail travel obtained from 57 studies, Wardman and Shires (2003) estimated a model to explain variations in the elasticities. The model predicts non-conditional and conditional fare elasticities on London based flows of 150 miles of -0.73 and -0.68 for first, -0.82 and -0.66 for full, and -1.23 and -0.93 for reduced.

The conditional elasticities estimated here are therefore broadly consistent with other empirical evidence.

The final models (1e and 2e) estimate overall models using the average fare across the different ticket types. Models were estimated to trips including and excluding those using Apex tickets, As expected, the elasticities are higher when the price sensitive Apex market is included. These models cannot explain trips as well as when there are separate equations by ticket type, and this is to be expected. However, it turns out that the elasticities estimated to the volume of trips excluding the Apex fares closely resembles what would be an average across the elasticities estimated by ticket type.

In summary, we have demonstrated that the use of economic theory in the form of parameter constraints allows the estimation of models for separate ticket types with plausible own and cross elasticities. This procedure required that both the Slutsky symmetry and diversion factor based constraints were used. In the absence of these constraints, the results were most unsatisfactory and reminiscent of the findings of previous research in this area.

Whilst the reported models represent a step forward in this area, there are a number of limitations:

- Equations 1 and 2 indicate that the cross-elasticities will vary widely according to market conditions, whereas we have estimated only a single set of cross-elasticities. The diversion factors can be expected to vary, as the strength of competition between tickets varies, and the revenue and volume shares differ over years and routes. The constraints were based on the mean values of the revenue shares and relative volumes due to software limitations.

- Whilst more variation could be permitted by estimating separate sets of equations, or separate parameters within a single set of equations, for different train operating companies or conditions, this would reduce the precision of the parameter estimates and in any event would not be practical for smaller companies.

- The approach cannot model new tickets.
- In its current form, the approach does not allow for changes in the travel restrictions associated with reduced tickets. Whilst in principle the approach could be amended to include terms which represent changes to travel restrictions, collecting the information would be a major task. Moreover, in many cases these are relatively small effects which would be difficult to detect.

5. A COMPLEMENTARY APPROACH

More recent work has extended the research to address some of the above limitations. This included the development of a more flexible procedure for estimating cross-elasticities and the extension to a broader set of flows.

We can deduce the own and cross elasticities of a system of demand equations from the relationships between own and cross elasticities of equation 2 and knowledge of the conditional elasticities (Toner et al., 2001). The conditional elasticity for a ticket i (C_i) is the sum of own and the relevant cross elasticities. To illustrate a two ticket system, we have:

$$f_{ii} + f_{ij} = C_i \qquad (11.1)$$

$$f_{ji} + f_{jj} = C_j \qquad (11.2)$$

and:

$$f_{ij} = -f_{jj} \frac{S_j}{S_i} \delta_{ji} \qquad (12.1)$$

$$f_{ji} = -f_{ii} \frac{S_i}{S_j} \delta_{ij} \qquad (12.2)$$

These equations can be solved simultaneously to yield:

$$f_{ii} = \frac{C_i + C_j \frac{s_j}{s_i} \delta_{ji}}{1 - \delta_{ij}\delta_{ji}} \qquad (13.1) \qquad\qquad f_{ij} = -f_{jj} \frac{s_j}{s_i} \delta_{ji} \qquad (13.2)$$

$$f_{ji} = -f_{ii} \frac{s_i}{s_j} \delta_{ij} \qquad (13.3) \qquad\qquad f_{jj} = \frac{C_j + C_i \frac{s_i}{s_j} \delta_{ij}}{1 - \delta_{ij}\delta_{ji}} \qquad (13.4)$$

The system readily extends to any number of ticket types. Given that the volume of sales of each ticket are known, own and cross elasticities tailored to specific situations can be estimated. Further situation specific results are obtained if information on how diversion factors vary is available.

As part of research for the Strategic Rail Authority to support its review of fare regulation, we extended the empirical study in order to:

- Disaggregate the own and cross elasticities which had been estimated for inter-urban London based flows by the five train operating companies to allow for the specific circumstances of each;
- Estimate own and cross elasticities for train operating companies not covered in our empirical study;
- Include new tickets in the system in anticipation of how the market might evolve;
- Conduct market research to improve understanding of diversion factors.

We report below the results estimated for a specific train operating company operating long distance London based services, for comparison with our empirical results, and for a train operating company providing services on the dense suburban network centred around London in the South East of England.

Table 8 contains the diversion factors and conditional elasticities used. The former were obtained from market research conducted on a variety of routes in January 2003. The latter were taken from the Passenger Demand Forecasting Handbook (ATOC, 2002), assuming that a ticket specific elasticity is a conditional elasticity. The point-point tickets cover travel between rail stations only, whereas the Travelcard tickets additionally include travel on public transport within London.

Table 8: Diversion Factors and Conditional Elasticities

South East to London						
	Point-Point Season	Travelcard Season	Full Day	Point-Point Reduced	Travelcard Reduced	Cond Elas
Point-Point Season to:	-	0.67	0.08	0.03	0.00	-0.3
Travelcard Season to:	0.72	-	0.08	0.03	0.00	-0.3
Full Day to:	0.03	0.03	-	0.05	0.15	-0.3
Point-Point Reduced to:	0.02	0.02	0.25	-	0.55	-1.0
Travelcard Reduced to:	0.00	0.00	0.05	0.70	-	-1.0
To London 100-200 miles						
	First	Full	Reduced	Apex	Cond Elas	
First to:	-	0.45	0.03	0.00	-0.5	
Full to:	0.12	-	0.35	0.13	-0.6	
Reduced to:	0.00	0.25	-	0.40	-1.1	
Apex to:	0.00	0.08	0.40	-	0.0	

The deduced elasticities are reported in Table 9. Since product specific elasticities are outside our realms of experience, it is difficult to say whether the own elasticities are reasonable or not: they could well be somewhat higher than the sort of elasticity figures we are accustomed to in these markets. However, we can comment upon the cross elasticity terms.

As expected, there are high cross-elasticities between the travelcard and point-point tickets which serve as close substitutes. The degree of competition in the off-peak market is particularly marked. Nor is it surprising that there is generally little interaction between the season tickets and the off-peak tickets. With regard to the full fare daily ticket, there is a strong interaction with the point-point reduced ticket. Thus they are prepared to switch departure times but would seem to be a different market to Travelcard users. As for those using a reduced ticket with Travelcard, the strong interaction with full day tickets could be explained by lesser price sensitivity, which underlies their purchase of the Travelcard add-on, and thus when they switch they are more prepared to pay for the premium full fare product.

Table 9: Deduced Own and Cross Elasticities

South East to London						
	Point-Point Season	Travelcard Season	Full Day	Point-Point Reduced	Travelcard Reduced	Conditional Elasticity
Point-Point Season	-1.26	0.86	0.04	0.06	0.00	-0.3
Travelcard Season	1.04	-1.47	0.05	0.08	0.00	-0.3
Full Day	0.15	0.14	-1.99	1.21	0.19	-0.3
Point-Point Reduced	0.03	0.03	0.05	-2.40	1.80	-1.0
Travelcard Reduced	0.00	0.00	0.39	3.45	-4.84	-1.0

To London 100-200 miles					
	First	Full	Reduced	Apex	Conditional Elasticity
First	-0.86	0.35	0.00	0.00	-0.5
Full	0.25	-1.88	0.89	0.13	-0.6
Reduced	0.01	0.33	-1.77	0.33	-1.1
Apex	-	-	-	-	0

The London elasticities differ from those empirically estimated. This is to be expected given the different diversion factors used and that the relative shares relate to a specific route in 2001/2 rather than an average across the period 1993 and 1998 for flows across many different routes. Nonetheless, the results do have similarities with those empirically estimated. For example, the full fare own elasticity is high, reflecting the strong competition it faces, and there is virtually no interaction between first and reduced. The impacts of variations in full fare on first and reduced demand are also similar between the two methods.

In principle, the approach can be extended to include new tickets, but this requires robust estimates of the likely shares of the new tickets and their conditional elasticities. The former is simplified if the new tickets exist on other train operating companies whilst it might be possible to deduce a conditional elasticity from the elasticities of adjacent tickets.

The Apex fare is allowed to impact on the demand for the other tickets but does not have its own elasticity because it is quota controlled. There is no problem having more 'columns' than 'rows' since the additional cross elasticities are simply deduced from equation 2.

The procedure above attaches particular importance to the availability of conditional elasticity estimates. In some cases, only one elasticity is known: the overall market elasticity with respect to average revenue. In this case, and for two tickets, we have:

$$\frac{dV}{dP}\frac{P}{V}=b \quad where\, V=V_1+V_2 \quad and\, P=\frac{P_1V_1+P_2V_2}{V_1+V_2}$$

This can be decomposed into:

$$C_1\frac{V_1}{V_1+V_2}+C_2\frac{V_2}{V_1+V_2}=b$$

that is, the volume weighted-sum of the conditional elasticities equals the overall market elasticity. So given b and the volumes, specifying one conditional elasticity immediately generates the other. A way forward in practice could be to use complementary techniques, such as disaggregate choice models based on RP or SP data to determine the relativities of C_1 and C_2, along with ticket sales models to estimate b.

6. CONCLUSIONS

The research reported here has enhanced aggregate rail demand models based on ticket sales data by estimating a system of equations representing different tickets where explicit allowance is made for the degree of competition between them. This has harnessed relationships apparent within economic theory, which turned out to be critical to the estimation of robust models with reasonable parameters. This contrasts with previous unsuccessful studies in this area. The results have been included within the recommendations contained in the Passenger Demand Forecasting Handbook (ATOC, 2002) which is widely used in the railway industry in Great Britain.

We have then proceeded to illustrate a method which can be used to provide a much greater level of detail than is possible using the econometric approach. This allows the own and cross elasticities within a system of equations representing the demand for different tickets to be deduced from evidence on conditional elasticities and diversion factors between tickets. This has enabled own and cross elasticity estimates to be obtained for a much wider range of situations and has contributed to the Strategic Rail Authority's review of fare regulation.

We do not claim that this is the final word on this subject; indeed it represents only the beginning of what will hopefully be more detailed analysis of competition between tickets and operators. It is hoped that the new procedure outlined here will have a role to play in enabling generalisation of the recommendations contained in PDFH and a more flexible approach to forecasting.

The two methods have a degree of complementarity about them. The empirical method requires the Slutsky symmetry condition, which relies upon the assumption of conventional economic theory, because it contributes to the estimation of reasonable conditional elasticities. On the other hand, the deductive method does not use the Slutsky method but is reliant upon sensible conditional elasticities. Whilst the deductive method provides much more flexibility than the econometric method, it is the latter which provides the conditional elasticities and also a range of other elasticity evidence.

Further work should involve the inclusion of variables representing travel restrictions in both approaches, a better understanding of variations in diversion factors, more variation in the conditional and own elasticities according to the fare charged and market share, and the exploration of possible links with disaggregate means of analysing ticket choice.

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