Article:
Kemp-Benedict, E. (2017) Dynamic Stability of Post-Keynesian Pricing. The B.E. Journal of Theoretical Economics (BEJTE), 17 (2). 20160013. pp. 651-664. ISSN: 2194-6124

https://doi.org/10.1515/bejte-2016-0013

Title: Dynamic stability of post-Keynesian pricing

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Abstract

Conventional economic theory assumes a Walrasian pricing mechanism that is known to pose theoretical difficulties. Less well-known is that conventional price theory conflicts with empirical studies of price-setting in industrial firms. Post-Keynesian theory, which assumes mark-up pricing on normal costs and infrequent price changes, is consistent with observation, and we show in this paper that post-Keynesian pricing, unlike conventional pricing, features stable dynamics. We focus on the short run, because post-Keynesian theory posits complex and historically-contingent long-term price dynamics. Specifically, we show that under very general conditions, prices converge to a unique equilibrium price vector.

Keywords: general equilibrium; prices; post-Keynesian; Perron-Frobenius theorem; Sraffian

JEL: E3: Prices, Business Fluctuations, and Cycles; C67: Input–Output Models; B5: Current Heterodox Approaches

1. Introduction

The conventional view of price determination in economics remains that of Walrasian general equilibrium, in which prices are determined by the intersection of downward-sloping demand curves and upward-sloping supply curves in markets where the law of one price prevails. Careful work on this model established beyond doubt that the Walrasian dynamic leads to instability, rather than stability (Kirman 1989; Saari 1995), but these results have had little to no impact on economic practice. Instead, a "representative consumer" is postulated, who both draws an income and owns the means of production – but it is precisely this sort of individual-like aggregate that theory says cannot be constructed (Kirman 1992).

Adding to the theoretical problem with the conventional view of price determination is an empirical one: firms do not set prices as convention says they do. From the pioneering work of Hall and Hitch (1939) through a major study on price-setting in firms (Blinder 1994; 1998) to more recent surveys carried out by central banks, the dominant strategy used by industrial firms is found to be mark-up pricing on costs, with negligible attention to changes in demand. Moreover, prices are adjusted rarely; Blinder (1994; 1998) reported that the median firm adjusted prices annually, a finding that is consistent with more recent surveys. As detailed in Coutts and Norman (2013), econometric evidence is no kinder to the conventional pricing model. While these results are unsettling to those working with conventional theory, they are fully consistent with post-Keynesian pricing theory (Downward and Lee 2001; Coutts and Norman 2013). In this theory, prices are set as a markup on normal costs and are adjusted infrequently. This behavior is thought by post-Keynesian theorists to explain the comparative (although not absolute) stability of prices.

Following Coutts and Norman (2013), we refer to the combination of mark-up pricing and infrequent adjustment as a "pure" post-Keynesian pricing dynamic. The dynamic follows from the assumptions that firms seek profits in order to grow, participate in oligopolistic markets, and cannot know how consumers and competitors will respond to their price changes. Post-Keynesian models represent dynamics in actual historical time (Robinson 1980; Setterfield 1995) rather than the logical time of Arrow and Debreu (Arrow and Debreu 1954; Debreu 1959); the importance of historical time for Walrasian dynamics was recognized by Fisher (1983). Full post-Keynesian pricing theory is more nuanced and complex than the pure version, as it postulates that prices for raw materials are determined in competitive markets, assumes irreversible processes in historical time, admits a variety of pricing strategies, provides a role for institutions, and allows for some price adjustment due to changing consumer demand. Also, post-Keynesian pricing theory, like other post-Keynesian theory, allows for both upward (micro-to-macro) and downward (macro-to-micro) causation (King 2012).

In this paper we show that under very general conditions, a discrete-time formulation of pure post-Keynesian pricing converges on a unique, but historically contingent, set of prices. Moreover, the result holds even though firms possess only local information: their own costs at the current time. This result stands in sharp contrast to neoclassical general equilibrium theory, in which prices are not guaranteed to converge, even to a local equilibrium under restrictive conditions, unless agents have access to implausibly broad information (Fisher 1983; Kirman 1989; Saari 1995). The results presented in this paper confirm the intuition that the central features of pure post-Keynesian pricing – cost-determined prices and gradual, not instantaneous, adjustment – are consistent with a stable process of price adjustment.

The model presented below applies in the short run, during which we assume no change in pricing strategy or technique. We assume that firms wish to maintain profit margins at levels determined through historical processes, the details of which are not relevant in the short run. We consider adjustment when current prices are away from their equilibrium values. In the first time period, which might reasonably be assumed to be a week, month, or quarter, firms are assumed to maintain their prices. As their costs change, firms may adjust their prices in subsequent time periods to close the gap between realized and desired profit margins. Those price changes then propagate through the economy, because one firm's price is another firm's cost (Steedman 1992). As costs of intermediates change, firms make further adjustments in a sequence that either converges towards or diverges away from a set of prices consistent with desired profit margins. Convergence indicates a stable price system, while divergence – which must eventually be checked by some additional mechanism – indicates instability. We show that convergence obtains under very general conditions.

The proofs in the paper use linear algebra, in contrast to the differential geometry, topology and game theory of neoclassical general equilibrium theory. Reference is made to the Perron-Frobenius theorem (e.g., Sternberg 2010, chap. 9), which applies to non-negative matrices. Two parts of the theorem are particularly relevant. The first is that a non-negative irreducible matrix that is not all zero possesses a positive eigenvalue that is larger in magnitude than all other eigenvalues. The second is that if the elements of one non-negative matrix are greater than or equal to those of another (non-identical) non-negative matrix, then the maximum eigenvalue of the first matrix is strictly greater than that of the second matrix.

The techniques and results in this paper are formally similar to those used in Sraffian analysis, but we allow profit margins to differ between firms. The approach is similar to that used for neoclassical dynamic input-output models, although we focus on short-term adjustment rather than long-term growth and development, and introduce different price-setting behavior.

2. Relationship to existing literature

The post-Keynesian theory pursued in this paper stands in sharp contrast to the mainstream neo-Walrasian theory. It uses the formal apparatus of Sraffian (sometimes called neo-Ricardian) analysis, but the motivating assumptions would not be shared by all Sraffian authors. Both neo-Walrasian and Sraffian analyses tend toward formalism, starting with a parsimonious set of premises and deriving general results. In contrast, in the post-Keynesian tradition, mathematics is seen as a means to an end, and the economic system is allowed to be "messy". Nevertheless, some rules apply: one entity's money income is another entity's money expenditure; businesses and households cannot run at a loss indefinitely; investors expect to receive an income out of profits; and human physiological needs set a lower bound to household consumption. In this section we summarize the neo-Walrasian and Sraffian approaches and contrast them with the post-Keynesian theory presented in this paper. The presentations are necessarily simplified, but we believe they capture the spirit of the neo-Walrasian and Sraffian programs.

Neo-Walrasian theory

We take the classic paper by Arrow and Debreu (1954) to define the essential neo-Walrasian position. The Arrow-Debreu economy contains firms, commodities, and "consumption units", which includes both households and institutional final consumers. Each firm has a choice amongst a set of production schedules, with positive entries reflecting outputs and negative entries reflecting inputs. Arrow and Debreu then assume: AD1) non-increasing returns to scale for each firm, AD2) the impossibility of output (in the aggregate) without some input, and AD3) a net input of labor, in that labor is a necessary input to production, but cannot be an output of production. They also allow for forward markets, in that commodities are differentiated by time and location, as well as by their physical properties.

Each consumption unit has a choice of consumption schedules, with positive entries reflecting consumption and negative entries reflecting provision of labor. Only physically feasible consumption schedules are allowed: a household must consume enough to meet its physiological needs, and no individual can work more than 24 hours in a day. Consumption units are presumed to start with an existing bundle of commodities and a contractual claim on the profits of firms. In this model, AD4) all profits are distributed, so firms do not retain any profits. Arrow and Debreu then introduce two crucial behavioral rules: AD5) firms choose the production schedule that maximizes their profits given a set of prevailing prices; AD6) consumption units choose the consumption bundle that maximizes subjective utility, subject to a budget constraint, where the budget includes distributed profits. Furthermore, utility is assumed to satisfy: AD7) continuity over the set of consumption schedules; AD8) no saturation, in that there is no consumption bundle the consumer would prefer over all others, regardless of income; AD9) convex indifference surfaces.

With these assumptions, Arrow and Debreu proceed to show, using game theoretic arguments, that there exists a competitive equilibrium in which prices for all commodities are non-negative. If a price is zero, then it constitutes a "free good" whose supply is in excess of demand.

From the perspective of post-Keynesian pricing, Arrow-Debreu theory rests on several problematic assumptions. First, assumption AD1 posits non-declining returns to scale. There is evidence that constant returns are typical: Simon and Bonini (1958) argued that under approximately constant returns to scale, the distribution of firm sizes should follow a Pareto distribution. The data available at the time supported a Pareto distribution of firm sizes, and a more recent analysis confirmed it (Axtell 2001). There is also more direct statistical evidence supporting a hypothesis of constant returns, at least for the typical industry (Burnside 1996). Yet, firms are heterogeneous, and although the typical firm may experience constant returns, some may also experience increasing returns (Basu and Fernald 1997). Lambrecht (2004) points out that mergers do not make sense with declining or even constant returns, so the fact that mergers do take place is *prima facie* evidence of increasing returns for at least some firms. We conclude that the empirical evidence for assumption AD1 is too weak to require it in a theory of price determination. Second, firms do retain profits, which are an important source of funds for investment (Lintner 1956; Brav et al. 2005), so assumption AD4 cannot be maintained. Third, while most people are employed by small firms, which may substantially be price takers, the firm size distribution has a long tail (Axtell 2001), so as a contribution to output, large firms predominate. Firms are moreover highly constrained in what they can produce and sell by their existing capital, expertise, and customer base, so major changes in the production schedule are infrequent and "lumpy" (Sakellaris 2004). Furthermore, many firms adopt policies to maximize their chance of survival through expanding market share, even at the expense of immediate profits (Chamberlain and Gordon 1989).[1] Together, these observations mean that assumption AD5, that firms flexibly choose their production schedule to maximize profits, must be abandoned. Fourth, while consumers do indeed respond to prices, whether through a subjective utility function or other mechanism, post-Keynesian pricing theory (Coutts and Norman 2013) argues that oligopolistic firms set their prices based on cost, and largely ignore the price response of demand, so assumptions AD6-AD9 are irrelevant; they are also problematic on their own merits (Lavoie 1994; Ackerman 1997). This leaves us with AD2) the necessity of some input to production and AD3) a net input of labor. We accept both of these assumptions. The first is inescapable on physical grounds (Glucina and Mayumi 2010), and while the second is not impossible, we are very far from such an economy at present, so we assume that all firms employ some workers.

Setting the problematic assumptions aside, Arrow and Debreu's substantive contribution was also relatively modest. They showed only that at least one competitive equilibrium exists. There may be many such equilibria, and they need not be stable. Much of the subsequent literature on stability sought general forms for excess demand functions, or the difference between demand and supply, for a given set of prices. At equilibrium, either the excess demand for each commodity is zero, or its price is zero. Away from equilibrium, prices rise if excess demand is positive, and fall otherwise (while remaining above zero). The stability of the price system depends on the response of the excess demand function to a change in prices. Unfortunately for

[1] In the "shareholder value era", firms were increasingly oriented toward maximizing market value. This can partly be achieved by increasing profits – for example, by cutting payroll – but also by any action that improves perceptions of the firm's prospects – such as an acquisition – or increases the value of its existing stock – such as buy-backs. After some notable bubbles and crashes, and facing the possibility of secular stagnation, it is not obvious that this is, in fact, the best way to run a company.

the theory, the work of Sonnenschein (1972), Mantel (1974) and Debreu (1974) (collectively resulting in the so-called SMD theorem), reinforced by the work of Kirman and Koch (1986), definitively closed this route to demonstrating stability. Briefly, the SMD theorem says that an excess demand function having nearly any form can be derived from a population with well-behaved individual utility functions. That is, nothing general can be said about excess demand functions on the basis of individual rational choice, and therefore no general conclusions can be drawn regarding the stability of the price system.

Fisher (1983) and Saari (1985) followed a different approach. Rather than identifying equilibria and then asking if they were locally stable, Fisher and Saari asked whether a procedure exists for reaching an equilibrium from an arbitrary starting point. Fisher (1983) sought an economically realistic process of adjustment, and was able to draw only very limited conclusions. Reflecting on his work later (Fisher 2011), he offered little hope, writing, "The search for stability at great levels of generality is probably a hopeless one." Asking whether any process, even one that was not economically realistic, could reach an equilibrium, Saari (1985) drew similarly disappointing conclusions, arguing that if only a finite amount of information is available at each step in the procedure, then it was not guaranteed to succeed.

In conclusion, the neo-Walrasian program, despite making unrealistically strong assumptions about how people and firms behave, was nonetheless unable to explain the general stability of the price system. Given the very general framework in which these authors worked, the reader might wonder how a post-Keynesian price mechanism could reach an equilibrium. The answer is that, contrary to Walras, prices are not assumed to clear markets; rather, they are set to recover costs, pay investors, and provide the funds for further investment. The iron law of supply and demand becomes manifest over time, not at each instant – if firms cannot sell their goods at a sufficiently high price, then they go out of business (or never get started).

Sraffian theory

Formally, the theory presented in this paper is Sraffian, but the underlying assumptions sometimes diverge from those found in the Sraffian literature. While we do not wish to make too much of the differences, given the considerable diversity within the Sraffian tradition (Aspromourgos 2004), we focus on some crucial ones.

The classic source on Sraffa's theory is his book, *Production of Commodities by Means of Commodities* (Sraffa 1960). However, we do not refer to it, for three reasons. First, the subtitle of Sraffa's book, "Prelude to a Critique of Economic Theory", reflected his project of critiquing marginalist theories of value, in particular the belief that there is a well-defined "quantity of capital" whose marginal product gives the price of capital (see also Sraffa 1962). Sraffa's capital critique is not the focus of this paper. Second, Sraffa resisted the advice of the mathematicians whom he consulted (Sraffa 1960, vii) and adopted non-conventional notation, which complicates any attempt to relate his work to others. Third, Sraffa inspired a variety of active research programs (Aspromourgos 2004), so more recent books and papers are a more relevant guide. In this section we refer to the book by Abraham-Frois and Berrebi (1997), which includes an exposition of core Sraffian concepts, and to a paper by Boggio (1992) on price stability.

First, we introduce some notation. Sraffian theory focuses on the input-output relationships in a production economy. We therefore introduce the physical inter-industry matrix $\mathbf{A} = [A_{ij}]$, where the entries have the following interpretation,

$$A_{ij} = \frac{\text{quantity of input from sector } i}{\text{quantity of output of sector } j}. \tag{1}$$

For example, if electricity is used to produce steel, then the corresponding element in the inter-industry matrix would have units of (kWh of electricity)/(tonne of steel). We adopt the convention that all vectors are column vectors. Given a price vector $\mathbf{p}$, intermediate costs for sector i per unit of output can be computed as

$$\text{unit intermediate costs for sector } i = \sum_{j=1}^{n} p_j A_{ji}, \tag{2}$$

where n is the number of sectors. Note that the price vector is multiplied by the transpose of the inter-industry matrix, so the subscripts on $\mathbf{A}$ are reversed. Direct unit costs u_{0i}, which include both labor and raw materials costs, are added to unit intermediate costs to get total costs. Firm (and sector average) prices must be sufficient to cover the total,

$$p_i \geq \sum_{j=1}^{n} p_j A_{ji} + u_{0i}. \tag{3}$$

The ratio of price to cost is the profit margin μ_i, so by definition,

$$p_i \equiv \mu_i \left(\sum_{j=1}^{n} p_j A_{ji} + u_{0i} \right), \quad \mu_j \geq 1. \tag{4}$$

Using a prime to denote a transpose, and a tilde to denote a diagonal matrix constructed out of a vector by placing the vector elements along the diagonal, we can write this equation in matrix form as

$$\mathbf{p} = \tilde{\boldsymbol{\mu}} \cdot \left(\mathbf{A}' \cdot \mathbf{p} + \mathbf{u}_0 \right). \tag{5}$$

To clarify the relationship to terminology found elsewhere in the literature, the profit margin is related to the profit mark-up m_i by

$$\mu_i = 1 + m_i. \tag{6}$$

In the Sraffian literature, m_i is referred to as the "rate of profit". However, in the post-Keynesian literature, the profit rate is the ratio of profits to the value of the capital stock, so we avoid the Sraffian terminology.

Sraffian authors assume that all firms apply the same profit margin μ unless they are absolutely constrained,[2] so in the Sraffian literature equation (5) simplifies to

$$\mathbf{p} = \mu \left(\mathbf{A}' \cdot \mathbf{p} + \mathbf{u}_0 \right). \tag{7}$$

The solution to this system is the set of production prices $\mathbf{p}^*$,

$$\mathbf{p}^* = \left(\mathbf{1} - \mu \mathbf{A}' \right)^{-1} \cdot \mu \mathbf{u}_0, \tag{8}$$

where $\mathbf{1}$ is the identity matrix. Suppose that there is an initial price vector $\mathbf{p}^{(0)}$, which differs from the production prices. Suppose further that firms always apply the profit margin μ, regardless of costs, so the price vector $\mathbf{p}^{(1)}$ in the next time period is given by equation (7),

$$\mathbf{p}^{(1)} = \mu \left(\mathbf{A}' \cdot \mathbf{p}^{(0)} + \mathbf{u}_0 \right). \tag{9}$$

More generally, at time t,

$$\mathbf{p}^{(t)} = \mu \left(\mathbf{A}' \cdot \mathbf{p}^{(t-1)} + \mathbf{u}_0 \right). \tag{10}$$

Iterating this equation, it is straightforward to show that

[2] For example, by a "blocking sector": see Abraham-Frois and Berrebi (1997, 74–75).

$$\mathbf{p}^{(t)} = \left(\mu\mathbf{A}'\right)^t \mathbf{p}^{(0)} + \left[\sum_{s=0}^{t-1}\left(\mu\mathbf{A}'\right)^s\right]\mu\mathbf{u}_0. \tag{11}$$

This sequence converges to the production prices in equation (8) as long as

$$\mu\rho\left(\mathbf{A}\right) < 1, \tag{12}$$

where $\rho(\mathbf{A})$ is the spectral radius of matrix **A**, and we have used the fact that the spectrum of a matrix is equal to the spectrum of its transpose. As Boggio (1992, 266) points out, stability of this "full cost" pricing dynamic is rather easy to demonstrate, as long as the inequality in (12) holds. Xu and Yan (2011) found maximum eigenvalues for the inter-industry matrix for Australia, China, Japan, the UK, and the US to lie between 0.4 and 0.7, with corresponding maximum profit margins between 1.4 and 2.5. As typical sector markups are around 1.2 (Oliveira Martins, Scarpetta, and Pilat 1996), this appears reasonable.

From a post-Keynesian perspective this result is promising, but unsatisfying. Empirically, firms and sectors exhibit a wide range of profit margins, because their market power, cost structure, and capital intensity differ widely. While a typical sectoral average at the 3-4 digit ISIC level is around 1.2, they can be much smaller, barely above 1.0, and can rise above 2.0 (Oliveira Martins, Scarpetta, and Pilat 1996). We therefore cannot maintain the Sraffian assumption of a consistent profit margin across firms or sectors. Also, as noted in the introduction, firms do not typically update their prices continuously. Rather, they adopt pricing periods that can be as long as a year. In this paper we extend the standard Sraffian result to the more complex setting of sector-specific profit margins and partial price adjustment.

A further distinction is on the concept of long-period production prices. In this paper we take a position that some Sraffians share, but many would reject. In the system above, the production prices **p*** provide a "center of gravitation" for the price system. If actual prices differ from production prices, then they will move toward them over time. However, while we accept the notion of equilibrium production prices as the unobservable solution to the price system, there is nothing truly "long-run" about them, because direct unit costs, firm pricing strategies and purchasing decisions change continually, while firms – even entire sectors – can appear and disappear. We therefore distinguish between the stability of production prices and the stability of the process of price adjustment when actual prices differ from production prices. We focus on the stability of the adjustment process, while noting that production prices move continually, driven by forces beyond the price system itself.

Before turning to the arguments specific this paper, we discuss the Sraffian concepts of basic and non-basic sectors (Abraham-Frois and Berrebi 1997, chap. 3). A basic sector produces goods that are used, directly or indirectly, by every other basic sector; the classic example of a non-basic good is a luxury good. That is, given a source sector i and a consuming sector j, if both are basic then there is a sequence of non-zero entries in the inter-industry matrix (perhaps a sequence with only one element), $A_{ik_1}, A_{k_1k_2}, \dots, A_{k_nj}$, by which the product of sector i becomes an input into sector j. With this definition, the part of the inter-industry matrix that constitutes the basic sectors is seen to be irreducible. This is important for the application of the Perron-Frobenius theorem, which applies to irreducible and non-negative matrices.

If the inter-industry matrix includes non-basic sectors, then it is possible to construct a permutation of rows and columns such that (Abraham-Frois and Berrebi 1997, 60–63)

$$\mathbf{A} \to \mathbf{A}_{\text{perm}} = \begin{pmatrix} \mathbf{A}^{\alpha} & \mathbf{A}^{\alpha\beta} \\ 0 & \mathbf{A}^{\beta} \end{pmatrix}, \tag{13}$$

where $\mathbf{A}^{\alpha}$ is a square irreducible matrix, $\mathbf{A}^{\beta}$ is a square matrix, and $\mathbf{A}^{\alpha\beta}$ is rectangular. What this notation indicates is that, like luxury goods, the products of the β sectors are not used in the α sectors. However, the α sector goods may be used by the β sectors. To take a concrete example, a jewelry maker may use solder, which is also used in manufacturing, and the jewelry may be bought by a boutique for sale, but the manufacturing sector does not buy jewelry from either the jewelry maker or the boutique as an intermediate input to production.

Production prices are given by equation (5), which becomes

$$\begin{pmatrix} \mathbf{p}^{\alpha} \\ \mathbf{p}^{\beta} \end{pmatrix} = \begin{pmatrix} \tilde{\boldsymbol{\mu}}^{\alpha} \\ \tilde{\boldsymbol{\mu}}^{\beta} \end{pmatrix} \cdot \left[\begin{pmatrix} \mathbf{A}^{\alpha\prime} & 0 \\ \mathbf{A}^{\alpha\beta\prime} & \mathbf{A}^{\beta\prime} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{p}^{\alpha} \\ \mathbf{p}^{\beta} \end{pmatrix} + \begin{pmatrix} \mathbf{u}_0^{\alpha} \\ \mathbf{u}_0^{\beta} \end{pmatrix} \right]. \tag{14}$$

The system for the irreducible set of basic sectors α is self-contained,

$$\mathbf{p}^{\alpha} = \tilde{\boldsymbol{\mu}}^{\alpha} \cdot \left(\mathbf{A}^{\alpha\prime} \cdot \mathbf{p}^{\alpha} + \mathbf{u}_0^{\alpha} \right). \tag{15}$$

The system for the set of non-basic sectors β includes purchases from the α sector,

$$\mathbf{p}^{\beta} = \tilde{\boldsymbol{\mu}}^{\beta} \cdot \left(\mathbf{A}^{\beta\prime} \cdot \mathbf{p}^{\beta} + \mathbf{A}^{\alpha\beta\prime} \cdot \mathbf{p}^{\alpha} + \mathbf{u}_0^{\beta} \right). \tag{16}$$

Prices from the basic goods sector, $\mathbf{p}^{\alpha}$, appear as direct unit costs in the pricing decisions of the non-basic sectors.

The theory presented in this paper applies to the set of basic sectors, because these are the sectors where profit margins are constrained by the system of reciprocal inter-industry exchange. Profit margins in non-basic sectors may be constrained by consumer and competitor behavior – for example, a corner grocery store or a filling station may face steep competition. However, the flexibility allows luxury goods manufacturers to set very high profit margins, if their clients are prepared to pay the price.

3. A model for post-Keynesian pricing

We can now state the goals of this paper with reference to the post-Keynesian, neo-Walrasian and Sraffian literature. Following the Sraffian literature, we seek a process by which prices adjust toward a (typically changing) set of production prices in discrete time. In keeping with post-Keynesian pricing theory, and in contrast to the neo-Walrasian literature (and Boggio 1985; 1992), we assume that producers do not adjust prices in response to changes in demand. Taking note of Saari's (1985) contribution, we seek an adjustment process that uses only local information, while with Fisher (1983) we seek an economically realistic process that takes place in historical time. We limit analysis to the set of basic sectors, because these are the sectors in which profit margins are constrained by the structure of inter-industry exchange. We depart from the Sraffian literature by assuming profit margins to vary by sector and allow firms to depart from their target profit margins when their costs are changing.

We start by reproducing the price equation (5), which determines equilibrium production prices

$$\mathbf{p} = \tilde{\boldsymbol{\mu}} \cdot \left(\mathbf{A}' \cdot \mathbf{p} + \mathbf{u}_0 \right). \tag{17}$$

We note that prices in this system are always strictly positive, because profit margins are greater than one, the inter-industry matrix is non-negative, and all sectors have at least some labor input, so direct unit costs $\mathbf{u}_0$ are strictly positive. If there is less demand for the good at the manufacturer's price than the amount produced, then inventories build up and firms cut back on production. In the extreme case, firms may be forced to close and either discard inventory or sell it at a discount, but they do not give away their products for free.

Price-setting behavior

Given the inter-industry matrix, sector profit margins, and costs, the solution to equation (17) is the set of production prices **p***,

$$\mathbf{p}^{*} = \left(\mathbf{1} - \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}'\right)^{-1} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{u}_0. \tag{18}$$

We now describe an adjustment process that applies when the initial set of prices, $\mathbf{p}^{(0)}$, is not equal to the equilibrium production price indices.

Given an initial vector of prices $\mathbf{p}^{(0)}$, firms either leave their price at its current value, or fully or partially adjust it to the price that yields them their desired profit margin. As a diverse set of firms do this across a sector, the net result is a set of sectoral next-period prices

$$p_i^{(1)} = \left(1 - \beta_i\right) p_i^{(0)} + \beta_i \mu_i \left(\sum_{j=1}^{n} p_j^{(0)} A_{ji} + u_{i0} \right), \quad 0 < \beta_i \leq 1. \tag{19}$$

This adjustment process uses only local information: the costs facing each firm in a sector and their own price from the previous period. In matrix notation, this can be written

$$\begin{aligned} \mathbf{p}^{(1)} &= \left(\mathbf{1} - \tilde{\boldsymbol{\beta}}\right) \cdot \mathbf{p}^{(0)} + \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \left(\mathbf{A}' \cdot \mathbf{p}^{(0)} + \mathbf{u}_0\right) \\ &= \left[\left(\mathbf{1} - \tilde{\boldsymbol{\beta}}\right) + \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}'\right] \cdot \mathbf{p}^{(0)} + \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{u}_0. \end{aligned} \tag{20}$$

Defining for convenience matrices **M** and **N**,

$$\mathbf{M} = \left(\mathbf{1} - \tilde{\boldsymbol{\beta}}\right) + \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}', \quad \mathbf{N} = \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}}, \tag{21}$$

the price-setting equation can be written

$$\mathbf{p}^{(1)} = \mathbf{M} \cdot \mathbf{p}^{(0)} + \mathbf{N} \cdot \mathbf{u}_0. \tag{22}$$

At a general time, *t*,

$$\mathbf{p}^{(t)} = \mathbf{M} \cdot \mathbf{p}^{(t-1)} + \mathbf{N} \cdot \mathbf{u}_0. \tag{23}$$

The solution to this system is

$$\mathbf{p}^{(t)} = \mathbf{M}^{t} \cdot \mathbf{p}^{(0)} + \left(\sum_{s=0}^{t-1} \mathbf{M}^{s} \right) \cdot \mathbf{N} \cdot \mathbf{u}_0. \tag{24}$$

This will converge as long as

$$\rho(\mathbf{M}) < 1. \tag{25}$$

If this does hold, then in the limit of large t, the first term in equation (24) becomes negligibly small, while the second term can be replaced by the limit of the series,

$$\lim_{t\to\infty} \mathbf{p}^{(t)} = (\mathbf{1} - \mathbf{M})^{-1} \cdot \mathbf{N} \cdot \mathbf{u}_0. \tag{26}$$

Substituting for **M** and **N** from (21), we find that prices converge to production prices,

$$\lim_{t\to\infty} \mathbf{p}^{(t)} = \left(\tilde{\boldsymbol{\beta}} - \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}'\right)^{-1} \cdot \tilde{\boldsymbol{\beta}} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{u}_0 = \left(\mathbf{1} - \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}'\right)^{-1} \cdot \tilde{\boldsymbol{\mu}} \cdot \mathbf{u}_0 = \mathbf{p}^*. \tag{27}$$

We must therefore determine under what conditions the convergence criterion (25) holds.

4. Convergence condition for post-Keynesian pricing

For convenience, we define the product of the profit margins and the transpose of the inter-industry matrix as a new matrix, **Z**,

$$\mathbf{Z} \equiv \tilde{\boldsymbol{\mu}} \cdot \mathbf{A}'. \tag{28}$$

Note that **Z** is a non-negative matrix, because both the inter-industry matrix **A** and the profit margins are non-negative. Writing the **M** matrix in terms of **Z**, we have

$$\mathbf{M} = \left(\mathbf{1} - \tilde{\boldsymbol{\beta}}\right) + \tilde{\boldsymbol{\beta}} \cdot \mathbf{Z}. \tag{29}$$

M is also a non-negative matrix, because from equation (19) the β_i are assumed to all be less than or equal to one.

We note that from the price equation (17) and the definition of **Z** in equation (28), production prices are given by

$$\mathbf{p}^* = \left(\mathbf{Z} \cdot \mathbf{p}^* + \tilde{\boldsymbol{\mu}} \cdot \mathbf{u}_0\right). \tag{30}$$

It may be that no such set of production prices exists. However, we show that if it does exist then condition (25) is necessarily satisfied. We first write this equation in terms of elements,

$$p_i^* = \left(\sum_{j=1}^{n} Z_{ij} p_j^* + \mu_i u_{i0} \right). \tag{31}$$

Dividing through by p_i^*, which we can do, because prices are strictly positive, gives

$$1=\sum_{j=1}^{n}\frac{1}{p_i^*}Z_{ij}p_j^*+\mu_i\frac{u_{i0}}{p_i^*}. \tag{32}$$

Because we assume some labor costs, the final term in this equation is strictly positive, and we have

$$\sum_{j=1}^{n}\frac{1}{p_i^*}Z_{ij}p_j^*<1. \tag{33}$$

Now, if we define a matrix $\underline{\mathbf{M}}$, with elements

$$\underline{M}_{ij}=\frac{1}{p_i^*}M_{ij}p_j^*=1-\beta_i+\beta_i\frac{1}{p_i^*}Z_{ij}p_j^*, \tag{34}$$

then the spectrum of $\underline{\mathbf{M}}$ is identical to that of $\mathbf{M}$, because, if a vector $\mathbf{m}$, with elements m_i, is an eigenvector of $\mathbf{M}$ with eigenvalue m, then by multiplying through it is possible to show that a vector $\underline{\mathbf{M}}$ with elements m_i/p_i^* is an eigenvector of $\underline{\mathbf{M}}$, also with eigenvalue m:

$$\sum_{j=1}^{n}\underline{M}_{ij}\underline{m}_j=\sum_{j=1}^{n}\frac{1}{p_i^*}M_{ij}p_j^*\frac{m_j}{p_j^*}=\frac{1}{p_i^*}\sum_{j=1}^{n}M_{ij}m_j=m\frac{1}{p_i^*}m_i=m\underline{m}_i. \tag{35}$$

We therefore have

$$\rho(\underline{\mathbf{M}})<1\Rightarrow\rho(\mathbf{M})<1. \tag{36}$$

A basic result for spectral radii is that they are bounded above by any consistent norm of the matrix,

$$\rho(\underline{\mathbf{M}})\leq\|\underline{\mathbf{M}}\|. \tag{37}$$

In particular, the infinity norm, which is given by the maximum row sum of the absolute value of the matrix elements, provides an upper-bound. Because the $\mathbf{M}$ matrix is non-negative, we have

$$\|\underline{\mathbf{M}}\|_\infty=\max_i\sum_{j=1}^{n}\underline{M}_{ij}=\max_i\left(1-\beta_i+\beta_i\sum_{j=1}^{n}\frac{1}{p_i^*}Z_{ij}p_j^*\right). \tag{38}$$

From (19), each of the β_i is strictly greater than zero and less than or equal to one. Also, from (33), the sum is strictly less than one. We therefore have

$$\sum_{j=1}^{n} \frac{1}{p_i^*} Z_{ij} p_j^* \le 1 - \beta_i + \beta_i \sum_{j=1}^{n} \frac{1}{p_i^*} Z_{ij} p_j^* < 1. \quad (39)$$

We conclude

$$\rho(\underline{\mathbf{M}}) \le \|\underline{\mathbf{M}}\|_\infty < 1. \quad (40)$$

Comparing this to (36), we find that the stability condition (25) is satisfied.

We have shown that if a set of equilibrium production prices exists, then the price-setting process will converge toward it. Instabilities arise in the adjustment process when profit margins are set such that no equilibrium prices exist. In that case, the price evolution (24) still holds, but it leads to a price spiral; there is cost-push inflation due to excessively high profit margins. This is not a typical source of inflation, and for good reason. If any firm were to set its profit margin so high as to rapidly push other firms' costs up, then someone could under-cut them. Even without experiencing the macroeconomic consequences of excessive profit margins, oligopolistic firms are constrained by their desire to restrict entry by rival firms. Indeed, in price wars, firms undercut their rivals, not price above them, by temporarily operating at a loss in order to gain market share.

We note that the result also follows if a more stringent, and more easily checked, condition holds, that

$$\rho(\mathbf{Z}) \le \mu_{\max} \rho(\mathbf{A}) < 1. \quad (41)$$

As an empirical matter, it is plausible to assume this normally does hold. Average sector-level profit margins in OECD countries at the 3-4 digit ISIC level are around 1.20 (Oliveira Martins, Scarpetta, and Pilat 1996; Wu 2009), so the product of a typical (not maximum) profit margin of 1.20 and a typical value for $\rho(\mathbf{A})$ of 0.55 gives a value of 0.66, which is less than one. Assuming the same value of 0.55 for $\rho(\mathbf{A})$, the sufficient condition (41) puts a bound on profit margins of 1.82. This bound is satisfied in all but four of the more than 700 entries in the data provided by Oliveira Martins et al. (1996). The exceptions are: Office & computing machinery in Finland, 1980-1992 (1.92); Radio, TV and Communications Equipment in Australia, 1970-1992 (2.02); and Tobacco products in France (3.12 from 1970-1992 and 3.17 from 1980-1992). The existence of these high values need not imply that the price systems were not viable. Tobacco products are mainly for consumption, so they are among the non-basic sectors. Also, if markups are high enough for (41) to be violated, the necessary condition (25) may still hold. Moreover, both conditions may fail to hold in real economies from time to time, leading to temporarily unstable prices. However, in the case of Australia even the sufficient condition may not have been violated: the average value for $\rho(\mathbf{A})$ in Australia in the 1968-1989 period was 0.48 (Xu and Yan 2011, Table 6), and multiplying that by 2.02 gives 0.97, which is still less than one (Xu and Yan do not report a value for Finland). In fact, all products of maximum profit margins from Oliveira Martins et al. (1996) and average maximum eigenvalues of the technical matrix from Xu and Yan (2011) are less than one. We conclude that although the condition (41) is stricter than it needs to be, it is reasonable to assume that it holds in practice.

5. Discussion

The central result of this paper confirms the intuition of post-Keynesian price theorists that two features of the theory – mark-up pricing and infrequent adjustment – are consistent with a stable price system. The stability condition is satisfied whenever a set of equilibrium production prices exists. This seems to be the case empirically, and it is plausible to assume that profit margins high enough to destabilize prices will be rapidly removed through competition. Dynamic stability of the price adjustment process is therefore expected to hold under very general conditions.

Stability of the price adjustment mechanism does not mean that prices themselves are stable, because the equilibrium “production prices” shift continually. Production prices change whenever labor or raw material costs change; firm pricing policy changes; firms enter or exit from markets; or technological change affects firm cost structure. From the mathematical result in this paper and the expected competitive behavior of firms, actual prices follow the moving production prices with a locally stable dynamic, so any instability is due to those other causes – for example, a wage-price spiral or inflation following an oil shock. This is where the speed of adjustment, as reflected in the values β_i, adds some stability to economic life. The speed of adjustment does not affect the convergence of prices toward production prices, but a slower speed of adjustment does mean that rapid fluctuations in production prices are not transmitted through the price system. We note that higher profit margins and a greater intensity of intermediate consumption as reflected in the inter-industry matrix, also damps rapid fluctuations.

The pricing behavior in this paper focused on the irreducible core of the economy – the set of basic sectors. The raw materials brought into production are often traded on exchanges, while firms typically buy forward contracts. The most productive resources are exploited first, so marginal productivity tends to decline. Markets for raw materials thus look more like an Arrow-Debreu economy than they do a post-Keynesian manufacturing sector. A full post-Keynesian pricing model would include a distinct pricing mechanism for raw materials. Also, consumers interact with the non-basic sector, where non-price competition, brand recognition, and customer loyalty affect purchasing behavior. Long-lived consumer goods are traded in markets with distinct characteristics, including the housing and automobile markets. A great deal is known about price setting in markets for consumer goods, and a full post-Keynesian pricing theory would include these sectors as well. Adding both raw material and consumption goods sectors would raise the issue of a price response to changing demand, because while manufacturing firms tend to operate in oligopolistic markets, prices of raw materials and retail prices are responsive to demand. A post-Keynesian pricing theory with consumers should take into account the drawbacks of the mainstream model of consumer behavior. Post-Keynesian consumer theory views consumers as meeting a hierarchy of needs (Lavoie 1994) and a common core of needs can help to resolve the indeterminacy of neo-Walrasian pricing theory (Kemp-Benedict 2013). The manufacturing sector also responds to demand, but over longer times as firms seek to expand market share. The processes of establishing, maintaining, and expanding market share are complex, and the industrial organization literature offers a fruitful source of information.

Some aspects of post-Keynesian pricing theory call for an open-system analysis, because the influences on production prices are not confined to the price system itself. Business fads can affect pricing policy and cost structure; technological change and firm entry are driven by external processes of discovery and innovation; technology, geology, climate, and geopolitics can influence raw materials costs; firms can exit a market because of the death or departure of a leader; and workers have more or less influence over the wage setting process. We emphasize that this does not mean that economists must remain silent on these issues. Rather, they should construct models that allow for external influences due to social and institutional change.

Evolutionary economics is a promising source of insight into price dynamics in the longer run. Gintis (2006; 2007) presents an evolutionary agent-based model populated by neoclassical firms and workers. The model was found to always converge on a unique set of prices, although the mechanism is not fully understood. In the model, firms either survive or fail based on their pricing strategy; successful pricing strategies are emulated, and "mutations" simulate innovation. Although Gintis' model unrealistically excludes durable capital investment (Bilancini and Petri 2008) and is inconsistent with the empirical studies of price determination by firms discussed in the introduction to this paper, it is intriguing because it results in mark-up pricing after starting with a neoclassical foundation. A post-Keynesian evolutionary model might start from a different set of assumptions. A firm could attract investors only if it had a reasonable expectation of making a profit and some prospects for growth. Firms would seek to grow by reinvesting profits, and become resilient by establishing lines of credit and creating cash buffers. Consumers would learn how to use products, while firms would learn from consumers. Wages, salaries, and other compensation would be set and emulated across firms, while both compensation and profit margins would be influenced by the level of demand. These dynamics and others would drive production prices, but, as this paper has shown, we expect that with each change the underlying post-Keynesian price-setting process, combined with inter-firm competition, would lead firm prices to track production prices.

An alternative approach is to apply the neo-Kaldorian concept of macroeconomic "regimes" (e.g., Setterfield 2002; Araujo and Trigg 2015). In such models the determinants of production prices are endogenized, but with parameters that differ between regimes. Thus, in contrast to the stochastic agent-based evolutionary models, neo-Kaldorian models are deterministic, but with dynamics that are conditional on external factors. Those factors are captured in model parameters whose values distinguish between regimes.

6. Conclusion

Conventional economic theory proceeds as though a modern economy can be modeled as a utility-maximizing representative consumer who owns the profit-maximizing representative firm. Yet work within this tradition has established, with increasing strictness, that the assumption is theoretically invalid (Kirman 1989; 1992; Saari 1995). Furthermore, findings from empirical studies of price determination by firms are inconsistent with conventional economic theory. In contrast, those findings are fully consistent with post-Keynesian pricing theory (Lavoie 2001; Downward and Lee 2001; Coutts and Norman 2013). This paper demonstrates that "pure" post-Keynesian pricing theory, which assumes mark-up pricing and infrequent price changes, leads to stable price dynamics under very general conditions. It thus shows that the post-Keynesian price mechanism, while being consistent with empirical studies, has good theoretical properties as well.

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