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THE TRANSLOG COSTS FUNCTION APPLIED TO EUROPEAN RAILWAYS

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CONTENTS

ABSTRACT

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This paper expands the exploratory analysis undertaken by Vigoroux-Steck (1990), who used statistical cost techniques, in conjunction with UIC published data, to estimate cost functions for 13 Western European railway operations. Four improvements have been made to Steck's work. Firstly, the data has been up-dated from 1987 to 1990 and re-indexed to incorporate the most recent information on international prices. Secondly, consistency is introduced into the Returns to Scale and Density measures by including all relevant estimated parameter values. Thirdly, we constrain the regression model to insure linear homogeneity of degree one in factor prices. Fourthly, we attempt to reduce statistical problems by redefining variables around the sample mean (although there is further work to be undertaken in this respect).

Three further amendments were undertaken with limited success. Attempts to introduce a more complex treatment of technological change led to implausible returns to scale. Attempts to re-define the returns to density measure by using a length of line variable rather than a density (train kilometres divided by length of line) variable also led to some implausible results. A re-definition of Steck's returns to density measure gave more plausible results.

Despite these statistical problems, some common results do come through. In particular, it appears that Western Europe's largest railways exhibit decreasing returns to scale and increasing returns to density. This does suggest that some European railways are operating beyond the point of maximum efficient scale and re-organisation into smaller units may be sensible. Our findings are less robust on how small these units should be. Railways with less than 3,000 km of route may be below the minimum efficient scale. Another important finding is that some Western Europe railways do exhibit diseconomies of density most notably those of Switzerland and the Netherlands. Proposed expansion of infrastructure in these countries may be sensible. Lastly, in terms of an index of managerial efficiency, we find that the railway of Sweden is a consistently high performer and those of Austria and Belgium poor performers.

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THE TRANSCENDENTAL LOGARITHMIC (TRANSLOG) COST FUNCTION APPLIED TO EUROPEAN RAILWAYS

1. INTRODUCTION

This paper revises and expands on exploratory work by Vigoroux Steck (1990) who used statistical cost analysis techniques using railway statistics for 13 countries taken from the publication International Railway Statistics (UIC) for the period 1979 - 1987. The countries included, with principal operators bracketed, are Great Britain (BR), Switzerland (CFF), Republic of Ireland (CIE), Germany (DB), Denmark (DSB), Italy (FS), Netherlands (NS), Norway (NSB), Austria (OBB), Sweden (SJ), Belgium (SNCB), France (SNCF), and Finland (VR). For uniformity we retain the operating names of these European railways as used by Steck, despite the name changes made by a number of the organisations and the fragmentation of the Swedish rail operation since the commencement of the period under observation.

Our revisions are as follows. Firstly, we extend the database to 1990. Secondly, we introduce the constrained regression model to ensure linear homogeneity in input prices. Thirdly, we focus on the redefinition of variables around the sample mean and attempt to increase the representation of technological change measures in the translog specification adopted by Steck.

The aim of this paper is to reconsider the conclusions drawn regarding economies of scale and density in the light of the additional statistics and where appropriate introduce refinements to the estimation technique and models. We believe such work is particularly opportune given that many countries are considering alternative organisational arrangements. In particular, in Britain a recent White Paper (Cm 2012, 1992) proposes a radical new structure for BR's rail operations which may involve around 20 franchised passenger rail operators and four privatised freight operators. Such fragmentation suggests that economies of density and, particularly economies of scale are not significant in the rail industry. This study aims to provide new empirical evidence on this issue to supplement previous evidence collated by Nash and Preston (1992). The translog model is unable to examine economies of scope as it can not handle cases where one output is zero. However, this issue has recently been examined, using the same data base with a quadratic cost function, by Jara-Diaz and Munizaga (1992). Our work concentrates on the translog models 25 and 26 developed by Steck and reproduced in the Appendix. Variables are denoted as they were in Steck's thesis with his definitions reproduced in the Appendix.

1.1 PREVIOUS WORK

Using pooled data Steck's preferred model form to estimate European railway cost functions using the flexible transcendental logarithmic (Translog) cost function, associated with Christensen et al, 1973, and taking the following specification:

$$\ln RTC = a_0 + \sum_{i=1}^{n} a_i \ln Y_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} \ln Y_i \ln Y_i + \sum_{j=1}^{m} b_j \ln w_j + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} b_{ij} \ln w_i \ln w_j + \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} \ln Y_i \ln w_j + \sum_{k=1}^{p} d_k D_k + \varepsilon$$

where $i = 1, ..., n$ is the number of outputs, $j = 1, ..., m$ is the number of input prices,

$d = k, ..., p$ is the number of firm specific dummy variables, *RTC* is the total operating cost, Y_i and w_j the output and factor prices respectively, D_k the dummy for each railway operator and a_i, a'_{ij}, b_j, b'_{ij}, c_{ij}, d_k as parameters.

Output measures were total train kilometres (*TKT*) and passenger train kilometres as a percentage of the total (*%TKP*) thus reducing multicollinearity. Models 25 and 26 (reproduced in the Appendix) are translog cost functions that differ by the inclusion of a basic technological change variable, the year of operation (*YEAR*), in the latter. A traffic
density variable (*DEN*), defined as total train kilometres per route mile is also included in each model.

Each model has a negative first order price coefficient for the materials and services variable and the main output variable *TKT* has a negatively signed coefficient in model 25. A number of variables are insignificant at the 10% level (14 out of 40 for model 25 and 11 out of 41 for model 26). In particular a large number of cross product terms are significant. Our major intention however is not to tamper with the specification of the model but to work within its restrictions to assess the change in operating characteristics of the principal European railways.

Steck calculated elasticities of real operating cost (*RTC*) with respect to the output measure total train kilometres (*TKT*) and traffic density (*DEN*). We denote these elasticities η_T and η_D respectively. Returns to scale (*RTS*) are then calculated as $^1/\eta_T$ and are increasing where *RTS* is greater than one. Returns to density are calculated as $1-\eta_D$ and are increasing where *RTD* is greater than one. The following partial derivatives were used for model 25 after excluding variables with insignificant parameters based on a 10% rejection criteria:

$$\eta_T = \partial \, ln \, RTC / \partial \, ln \, TKT = -6.225 + 0.842 \, ln \, TKT + 0.095 \, ln \, WV - 0.189 \, ln \, \%TKP - 0.779 \, ln \, DEN$$

$$\eta_D = \partial \, ln \, RTC / \partial \, ln \, DEN = 5.778 + 0.784 \, ln \, DEN - 0.779 \, ln \, TKT + 0.427 \, ln \, \%TKP$$

The figures in table 1 (based around Steck table 5.1) were derived by evaluating these derivatives for the final year of the study period, 1987. Steck noted the decreasing returns to scale (ie. increases in level of operation lead to higher increases to total costs and hence higher average costs) and the increasing returns to traffic density (ie. increases in the use of the network lead to lower increases to total costs and hence lower average costs) for the larger railway operators (BR, DB, FS, SJ, SNCF).

The additional bracketed figures are values based on partial derivatives including insignificant variables. Returns to scale increase and to density decrease for all networks in this case, but not so as to change the conclusion for any of the larger operators.

Model 26 including the technological change variable *YEAR*, albeit the most simplistic in structure, results in partial derivatives as follows:

Table 1:Model 25: Returns to scale and density in 1987

Railway	Elasticity of *RTC* with respect to		*RTS*	*RTD*
	TKT	*DEN*		

BR	2.30	(2.01)	-1.87	(-1.55)	0.43	(0.50)	2.87	(2.55)
CFF	0.87	(0.58)	-0.55	(-0.22)	1.15	(1.72)	1.55	(1.22)
CIE	0.46	(0.14)	-0.27	(+0.13)	2.19	(7.06)	1.27	(0.87)
DB	2.72	(2.43)	-2.34	(-1.99)	0.37	(0.41)	3.34	(2.99)
DSB	0.65	(0.36)	-0.37	(-0.05)	1.54	(2.80)	1.37	(1.05)
FS	2.31	(2.04)	-1.87	(-1.61)	0.43	(0.49)	2.87	(2.61)
NS	0.73	(0.44)	-0.44	(-0.11)	1.37	(2.26)	1.44	(1.11)
NSB	1.06	(0.77)	-0.88	(-0.53)	0.94	(1.30)	1.88	(1.53)
OBB	1.47	(1.19)	-1.15	(-0.85)	0.68	(0.84)	2.15	(1.85)
SJ	1.92	(1.64)	-1.69	(-1.38)	0.52	(0.61)	2.69	(2.38)
SNCB	0.99	(0.70)	-0.69	(-0.35)	1.01	(1.43)	1.69	(1.35)
SNCF	2.92	(2.64)	-2.55	(-2.24)	0.34	(0.38)	3.55	(3.24)
VR	1.30	(1.02)	-1.21	(-0.88)	0.77	(0.98)	2.21	(1.88)

NB. The value of the elasticity of *RTC* with respect to *TKT* for OBB railway was incorrectly typed in the original thesis.

$$\partial\, ln\, RTC / \partial\, ln\, TKT = 0.575 + 0.222\, ln\, TKT - 0.168\, ln\, WM + 0.076\, ln\, WV - 0.095\, ln\, \%TKP - 0.252\, ln\, DEN$$

$$\partial\, ln\, RTC / \partial\, ln\, DEN = -0.517 + 0.584\, ln\, DEN + 0.164\, ln\, WM - 0.252\, ln\, TKT + 0.405\, ln\, \%TKP$$

In each elasticity equation the intercept is insignificant, as is the logged variable *%TKP* in the first of these. The criteria for the retention and rejection of variables was not outlined in Steck's paper and other insignificant variables have been omitted. Table 2A (based around Steck Table 5.2) gives the elasticity values and returns to scale and density for 1987 operating data based on model 26.

Due to the inclusion of the insignificant variables mentioned, the addition of the few remaining insignificant variables to the elasticity equations make negligible difference as the bracketed values show.

Table 2A Model 26: Returns to scale and density in 1987

Railway	Elasticity of *RTC* with respect to *TKT*		DEN		*RTS*		*RTD*	
BR	1.17	(1.17)	-0.45	(-0.44)	0.86	(0.86)	1.45	1.44
CFF	0.74	(0.74)	+0.12	(+0.13)	1.35	(1.36)	0.88	0.87
CIE	0.66	(0.66)	-0.30	(-0.30)	1.51	(1.51)	1.30	1.30
DB	1.29	(1.29)	-0.72	(-0.72)	0.78	(0.78)	1.72	1.72
DSB	0.69	(0.69)	+0.01	(+0.02)	1.45	(1.46)	0.99	0.98
FS	1.21	(1.21)	-0.56	(-0.54)	0.83	(0.83)	1.56	1.54
NS	0.69	(0.69)	+0.20	(+0.21)	1.46	(1.46)	0.80	0.79
NSB	0.87	(0.87)	-0.55	(-0.54)	1.15	(1.15)	1.55	1.54
OBB	1.04	(1.04)	-0.44	(-0.43)	0.96	(0.96)	1.44	1.43
SJ	1.13	(1.13)	-0.83	(-0.83)	0.88	(0.89)	1.83	1.83
SNCB	0.81	(0.81)	-0.07	(-0.07)	1.23	(1.23)	1.07	1.07
SNCF	1.39	(1.39)	-0.96	(-0.95)	0.72	(0.72)	1.96	1.95
VR	0.97	(0.96)	-0.79	(-0.78)	1.04	(1.04)	1.79	1.78

NB. The sign of the elasticity of *RTC* with respect to *DEN* for OBB railway was incorrectly typed in the original thesis.

Steck noted the larger networks show similar patterns as that brought out by model 25 ie. decreasing returns to scale and increasing returns to traffic density; and that increasing returns to scale were also evident for smaller operators CFF, CIE, DSB, NS and SNCB. The values suggest the smaller networks exhibit increasing returns to traffic density except for NS, and to a lesser extent CFF, which appear to have no spare capacity on their network that could be utilised further.

However, comparison should be undertaken cautiously as model 25 elasticity equations contain only statistically significant variables and applying this criteria to Model 26 changes the picture drastically with all operators exhibiting increasing returns to scale. Interestingly on applying this criteria the larger networks, excluding BR and FS, continue to show increasing returns to traffic density. VR also exhibits economies of density. BR and FS, together with NSB and OBB, exhibit approximate constant returns to traffic density whilst the remaining operators CFF, CIE, DSB, NS and SNCB suggest decreasing returns to traffic density (see Table 2B).

Table 2B: Model 26: Returns to scale and density in 1987 based on elasticity equations containing variables significant at the 10% level

Railway	*RTS*	*RTD*
BR	1.75	0.93
CFF	7.19	0.37
CIE	19.07	0.78
DB	1.49	1.20
DSB	10.36	0.48
FS	1.64	1.04
NS	9.81	0.29
NSB	3.89	1.03
OBB	2.36	0.92
SJ	1.97	1.32
SNCB	4.75	0.55
SNCF	1.30	1.44
VR	2.96	1.27

The overall conclusion drawn by Steck was the preference for model 26 due primarily to its inclusion of some form of technological progress but also because the elasticities suggested are consistent with those of a time series analysis he performed on the same data. However, his findings are extremely sensitive to the treatment of statistically insignificant parameter values. Table 2B, with respect to *RTS*, suggests that simply ignoring the insignificant parameter values will lead to implausible results (eg the very high values for CIE and DSB). The correct procedure would be to re-estimate the model with the insignificant variables omitted but then the model would no longer be translog in nature.

1.2 THE MULTIPRODUCT COST FUNCTION

Using the data on which Steck based his work, Jara-Diaz and Munizaga (1992) have analysed operating costs using an estimated multiproduct cost function, C(W,Y,X). Given the level of fixed inputsX , input prices (W) are varied to generate a four dimensional flow vector (Y) containing passenger, passenger kilometre, tonne and tonne-kilometre output measures. A geographically based density variable (R), defined as the ratio of track length to area of country operated over, was included in the best model to capture network shape and coverage, very much different to that included by Steck which depended on the total train kilometres being operated.

The form of the chosen cost function was quadratic which is defined for zero levels of output and thus suited to the calculation of economies of scope. Cross product terms of interest were retained and after analysis only combinations of one fixed factor were included. The best model contained 26 estimated parameters and was considered good fitting, robust and without heteroskedastic problems. The model suggested marginal costs increase with freight transported and decrease with passenger numbers. The marginal cost of freight with respect to density was found to increase, and that of passengers to fall.

In conclusion, Jara-Diaz and Munizaga found each operator exhibiting scale returns with the exception of OBB who appeared to be operating at constant returns to scale. The scale of the returns was not related to network size. Economies of scope were implied by the figures suggesting advantages were gained by each operator running both freight and passenger services. The only exception to this was SNCF and DB whose values were statistically insignificant and whose operating costs would be unlikely to vary from a disaggregate operation.

2. UPDATING THE ANALYSIS

2.1 UPDATING THE DATABASE

The initial requirement of extending the database to include the three additional years of data met with a slight complication in that we were unable to reproduce the implicit price index used by Steck. Based on gross domestic product (GDP) figures abstracted from the OECD National Accounts 1969-1987 publication, the relevant figures in the revised 1960-1990 version differed marginally and necessitated the re-indexing of Steck's work to provide a consistent base for comparative purposes. The resulting re-indexed model 25 and 26 are shown in the Appendix as model 25(I) and 26(I) respectively. Tables 3 and 4 are re-indexed versions of tables 1 and 2, the difference in values being small as was the difference in indices.

The re-indexing actually causes an additional model 25 parameter to become significant at the 10% level in the second of the two elasticity equations ($\partial lnRTC/\partial lnDEN$) yet the resulting effect on the elasticity values is only to increase them marginally with no change to their interpretation. Overall, the conclusions to be drawn are as before.

Inclusion of 1988, 1989 and 1990 data results in models 25(IU) and 26(IU) as shown in the Appendix, which follow the translog specification of models 25 and 26 from Steck's work.

In model 25(IU) the coefficients of all first order and second order variables have maintained the same sign compared with model 25(I), and although a number of variables have increased in statistical insignificance a greater number have shown the opposite trend. The first order price coefficients of labour and energy are positive, though only the latter significant, and that of materials and services negative albeit insignificant. The magnitude and statistical significance of the labour price variable has fallen. One would expect each of these coefficients to be positive to confirm short run variable costs to be an increasing function of factor prices.

Where significant the coefficients of cross-product variables have retained the same sign as in model 25(I), yet in the majority of cases a loss of significance has occurred. The intercept coefficient has increased in magnitude and become significant.

Table 3:Model 25(I): Returns to scale and density in 1987

Railway	Elasticity of *RTC* with respect to *TKT*	DEN	*RTS*	*RTD*
BR	2.34	-2.16	0.43	3.16
CFF	0.93	-0.89	1.08	1.89
CIE	0.47	-0.58	2.13	1.58
DB	2.76	-2.62	0.36	3.62
DSB	0.69	-0.70	1.45	1.70
FS	2.35	-2.15	0.43	3.15
NS	0.78	-0.77	1.28	1.77
NSB	1.06	-1.17	0.94	2.17
OBB	1.51	-1.46	0.66	2.46
SJ	1.93	-1.98	0.52	2.98
SNCB	1.04	-1.02	0.96	2.02
SNCF	2.94	-2.81	0.34	3.81
VR	1.30	-1.51	0.77	2.51

Table 4:Model 26(I): Returns to scale and density in 1987

Railway	Elasticity of *RTC* with respect to *TKT*	DEN	*RTS*	*RTD*
BR	1.18	-0.43	0.85	1.43
CFF	0.75	+0.13	1.34	0.87
CIE	0.66	-0.27	1.53	1.27
DB	1.30	-0.69	0.77	1.69
DSB	0.69	+0.03	1.45	0.97
FS	1.22	-0.53	0.82	1.53
NS	0.69	+0.20	1.45	0.80
NSB	0.87	-0.56	1.15	1.56
OBB	1.04	-0.42	0.96	1.42
SJ	1.13	-0.82	0.89	1.82
SNCB	0.81	-0.05	1.23	1.05
SNCF	1.39	-0.93	0.72	1.93
VR	0.95	-0.79	1.05	1.79

The first order coefficient of the main output variable, total train kilometres (*TKT*) is highly significant yet negative in sign whilst the secondary output measures, passenger train kilometres as a percentage of the total (*%TKP*) and the density measure (*DEN*) are positive and significant as would be expected. Coefficients of all second order variables are positive except that of the labour input price.

The coefficients for the dummy variables are summarised in table 5 for the translog models excluding and including the technological change variable. Adjustments have been made because of the logged nature of the dependent variable in the regressions and so as to include Finnish railways (VR), which was used as a base. The 1987 values are taken from the re-indexed models 25(I) and 26(I).

Table 5: Operator comparisons

	25(I):1987	25(IU):1990	26(I):1987	26(IU):1990
BR	1.01*	0.56	1.40	0.95*
CFF	2.29	2.61	1.48	1.52
CIE	0.90*	1.27	0.90*	1.13*
DB	0.61	0.22	1.46	0.79*
DSB	1.43	1.73	1.19	1.27
FS	1.21*	0.74	1.60	1.17*
NS	1.68	2.03	1.07*	1.17
NSB	0.93	1.13	0.94	1.02*
OBB	2.27	2.34	1.97	1.99
SJ	0.63	0.50	0.79	0.67
SNCB	2.34	2.61	1.75	1.80
SNCF	0.30	0.10	0.99*	0.51
VR	1.00	1.00	1.00	1.00

Note: * = the corresponding dummy variables were insignificant at the 10% level.

These dummy variables are very basic yet allow a simple comparison between railway operators, ceteris paribus, ie. from 1987 values the Austrian railway operation (OBB) appears at least twice as expensive to operate as the Finnish equivalent (VR). Interpretation of these figures should be made with some reservation because of the basic nature. The values for the same operator in the same year, but with different technological change assumptions in the translog specification (ie. the difference in models 25 and 26), suggest different conclusions in some cases. However a comparison of 1987 and 1990 values does show that the larger network operators, namely BR, DB, FS, SJ and SNCF, have increased their efficiency.

Model 26(IU) is the updated version of the Steck translog model including the technological change variable *YEAR* and it should be compared to model 26(I) the re-indexed version of Steck's original model 26. The intercept coefficient remains negative in sign and insignificant in the model with only two of the three first order price coefficients being positive. As with model 25(IU) these are the coefficients of the labour and energy variables and both are highly significant. The materials and services input price variable is both negative and significant.

The coefficient of the *TKT* and *DEN* variables are negative yet insignificant and of the *%TKP* variable, positive and significant. The sign of the *%TKP* is intuitively correct with greater costs associated with a greater reliance on passenger services.

The technological change coefficient is negative, implying reduced costs with advancing years, and is highly significant. All second order and cross product variables show little change from model 26(I) with marginal movements in statistical significance and the only sign change from that of an insignificant variable.

Applying the significance criteria to model 25(IU) as applied to model 25, such that all insignificant coefficients at the 10% level are dropped, the partial derivatives are:

$$\partial \ln RTC / \partial \ln TKT = -9.349 + 1.228 \ln TKT + 0.062 \ln WV - 1.153 \ln DEN$$

$$\partial \ln RTC / \partial DEN = 6.890 + 1.030 \ln DEN - 1.153 \ln TKT + 0.264 \ln WM$$

These produce the returns to scale and density shown in table 6.

Including variables with insignificant parameter values produces the bracketed figures shown. The larger railway networks continue to perform at levels of decreasing returns to scale and increasing returns to traffic density. Indeed a comparison with table 3 show the returns to scale and density are more pronounced for all of these operators (BR, DB, FS, SJ, SNCF). Due to changes in significance at the margin, the above partial derivatives differ slightly from Steck's yet returns based on his original equations suggest the same conclusion. The bracketed returns to scale values indicate a degree of stability in the BR, DB and FS cases with SJ and SNCF a little more inconclusive. As with the 1987 returns to density values, all railway operators are subject to increasing returns.

Table 6:Model 25(IU): Returns to scale and density

Railway operator	*RTS*		*RTD*	
BR	0.34	(0.36)	5.71	(3.18)
CFF	1.24	(1.43)	3.79	(1.28)
CIE	4.98	(9.32)	3.09	(0.49)
DB	0.29	(0.30)	6.26	(3.84)
DSB	2.12	(2.74)	3.44	(0.89)
FS	0.35	(0.36)	5.64	(3.13)
NS	1.40	(0.61)	3.72	(1.20)
NSB	0.95	(0.96)	3.96	(1.46)
OBB	0.63	(1.50)	4.44	(2.05)
SJ	0.44	(2.18)	5.09	(2.80)
SNCB	1.04	(0.86)	3.92	(1.47)
SNCF	0.27	(3.65)	6.47	(4.07)
VR	0.68	(1.38)	4.35	(2.12)

Table 7 summarises the returns to scale and density brought out by model 26(IU). Steck's criteria has been applied with the following partial derivatives resulting:

$$\partial \ln RTC / \partial \ln TKT = -0.593 + 0.424 \ln TKT - 0.204 \ln WM + 0.059 \ln WV - 0.025 \ln \%TKP$$

$$\partial \ln RTC / \partial \ln DEN = -1.351 + 0.660 \ln DEN + 0.435 \ln WM - 0.441 \ln TKT + 0.438 \ln \%TKP$$

In these derivatives both the intercept coefficients are insignificant as are the material and services input price variables. This was also the case with Steck's derivatives. The bracketed figures are values brought out by including all insignificant variables in the partial derivative equations.

Table 7:Model 26(IU): Returns to scale and density

Railway operator	*RTS*		*RTD*	
BR	0.63	(0.64)	1.80	(1.73)
CFF	1.24	(1.27)	0.94	(0.88)
CIE	1.58	(1.58)	1.03	(0.99)
DB	0.55	(0.56)	2.19	(2.16)
DSB	1.36	(1.38)	0.96	(0.91)
FS	0.62	(0.63)	1.93	(1.82)
NS	1.26	(1.29)	0.91	(0.87)
NSB	1.09	(1.09)	1.39	(1.36)
OBB	0.82	(0.83)	1.61	(1.53)
SJ	0.69	(0.70)	2.13	(2.07)
SNCB	1.09	(1.10)	1.20	(1.17)
SNCF	0.52	(0.52)	2.43	(2.36)
VR	0.84	(0.85)	1.96	(1.93)

The values of Table 7 again imply the larger networks exhibit diseconomies of scale and, in a similar manner to those of Table 6, the values suggest a more pronounced conclusion than those three years previously (see Table 4). All remaining networks, with the exception of CIE, have also moved in a similar manner with VR and OBB now exhibiting diseconomies when in 1987 the figures suggested constant returns to scale.

The returns to density figures have, with the exception of CIE again, risen across the board. NS and CFF continue to exhibit diseconomies of density, though to a lesser extent than in 1987, whilst the CIE value suggests constant returns to density. All other networks are operating at increasing returns to traffic density.

2.2 INTRODUCING CONSTRAINTS

We applied the seemingly unrelated regression estimation procedure first adopted by Zellner by utilising the SYSLIN SUR procedure in the SAS computer package, and by introducing RESTRICT statements we applied a number of constraints on the model. These constraints ensure linear homogeneity in input prices by restricting the estimated parameter values. Denoting each parameter by its associated variable they were as follows.

$$ln\,\mathrm{WE} + ln\,WM + ln\,\mathrm{WV} = 1$$

$$(ln\,\mathrm{WE})^2 + ln\,\mathrm{WE} \,.\, ln\,WM + ln\,\mathrm{WE} \,.\, ln\,\mathrm{WV} = 0$$

$$(ln\,WM)^2 + ln\,WM \,.\, ln\,\mathrm{WE} + ln\,WM \,.\, ln\,\mathrm{WV} = 0$$

$$(ln\,\mathrm{WV})^2 + ln\,\mathrm{WV} \,.\, ln\,WM + ln\,\mathrm{WV} \,.\, ln\,\mathrm{WE} = 0$$

$\ln TKT \cdot \ln WM + \ln TKT \cdot \ln \mathrm{WE} + \ln TKT \cdot \ln \mathrm{WV} = 0$

$\ln \%TKP \cdot \ln WM + \ln \%TKP \cdot \ln \mathrm{WE} + \ln \%TKP \cdot \ln \mathrm{WV} = 0$

$\ln DEN \cdot \ln WM + \ln DEN \cdot \ln \mathrm{WE} + \ln DEN \cdot \ln \mathrm{WV} = 0$

The symmetry condition $\ln WM \cdot \ln \mathrm{WE} = \ln \mathrm{WE} \cdot \ln WM$ was applied throughout for all such cross products to ensure the translog cost function is continuously twice differentiable.

The translog models with and without the technological variable are shown as models 25(IUC) and 26(IUC) in the Appendix and should be compared directly to models 25(IU) and 26(IU), the unconstrained versions. A number of coefficient sign changes have taken place in both models, yet of the first order variables only *DEN* in model 26(IUC) is one of these, becoming positive in sign yet remaining insignificant. Table 8 summarises the returns to scale and density values from the two models. The bracketed values are those borne out from the partial derivatives retaining insignificant variables whilst the unbracketed values only include variables statistically significant at the 10% level.

For model 25(IUC), all operators who exhibited diseconomies of scale or economies of scale in model 25(IU) continue to do so, but with all values moving towards one (ie. constant returns) from above and below.

Table 8: Constrained returns to scale and density

Railway Operator	Model 25(IUC)				Model 26 (IUC)			
	RTS		*RTD*		*RTS*		*RTD*	
BR	0.38	(0.38)	3.06	(2.77)	0.45	(0.67)	2.77	(1.59)
CFF	1.17	(1.19)	1.24	(0.96)	0.63	(1.17)	1.76	(0.59)
CIE	3.46	(3.46)	1.45	(1.20)	0.62	(1.13)	2.78	(1.66)
DB	0.32	(0.33)	3.60	(3.37)	0.42	(0.60)	3.10	(2.00)
DSB	1.65	(1.68)	1.23	(0.96)	0.64	(1.16)	2.09	(0.93)
FS	0.39	(0.39)	3.14	(2.83)	0.44	(0.65)	2.98	(1.76)
NS	1.20	(1.22)	1.17	(0.90)	0.64	(1.17)	1.72	(0.56)
NSB	0.97	(0.97)	2.09	(1.85)	0.56	(0.94)	2.93	(1.81)
OBB	0.68	(0.68)	2.10	(1.84)	0.52	(0.84)	2.50	(1.36)
SJ	0.49	(0.49)	3.03	(2.79)	0.46	(0.71)	3.33	(2.21)
SNCB	1.00	(1.01)	1.55	(1.31)	0.59	(1.03)	2.12	(1.02)
SNCF	0.30	(0.30)	3.99	(3.72)	0.40	(0.56)	3.51	(2.37)
VR	0.72	(0.73)	2.54	(2.32)	0.51	(0.83)	3.27	(2.18)

On restricting the partial derivative equation to significant variables only, all operators continue to exhibit increasing returns to density, although all values are lower than the corresponding figures from table 6. Comparing the bracketed values of tables 6 and 8 shows the application of the constraint has caused the returns to density value to fall for all operators except CIE, DSB, NSB and VR, which have increased, and SJ which has remained unchanged. The implications are economies of density for all networks bar NS (diseconomies) CFF and DSB (constant returns to traffic density).

The model 26(IUC) unbracketed figures in table 8 have no direct comparison with the unconstrained model 26(IU) and it is the bracketed figures that can be compared with those of table 7. Again values move towards constant returns to scale (*RTS=1*) from each end of the range although movement is more marginal than that of the model 25(IUC) and 25(IU) comparison. Conclusions from the returns to density values are on the whole the same as those in the unconstrained model.

The dummy variables from these two constrained models are shown in table 9 with increases for every operator except CIE in model 26(IUC), compared with values from the unconstrained models (Table 5).

Table 9: Operator comparisons

Operator	Model 25(IUC)	Model 26 (IUC)
BR	0.70*	1.19*
CFF	2.89	1.70
CIE	1.12*	1.13*
DB	0.33	1.03*
DSB	2.36	1.68
FS	1.07*	1.42
NS	2.39	1.36
NSB	1.26	1.13
OBB	2.64	2.23
SJ	0.59	0.74
SNCB	3.10	2.12
SNCF	0.16	0.70*
VR	1.00	1.00

Note: * = the corresponding dummy variables were insignificant at the 10% level.

In conclusion, the application of the constraints makes little different to the specification of the model except for a slight tendency for movement towards constant returns. Consequently returns to scale and density do not vary significantly from those borne out in the unconstrained regression models.

2.3 FURTHER MODIFICATIONS

Further amendment to Steck's work were undertaken in line with the literature. Firstly, an attempt was made to better model technological change by introducing the second order and cross product terms for the time trend variable *YEAR*. This was done for two models, Model 27, a single output variable model in which variable *% TKP* was dropped in order to regain degrees of freedom, whilst Model 28 contains the two output measures
(*TKT* and *% TKP*) used in the rest of this work. The model results are given in the Appendix with the most noticeable feature being the increase in size of the intercept. Table 10 shows that the two models give plausible results in terms of operator comparisons (they are similar to the results of models 26 (IUC)). However, Table 11 shows that, with these models, the *RTS* measures, as defined looses its meaning, despite the inclusion of all parameter values, whether statistically significant or not. The

reasons for this are not clear. By contrast, the *RTD* measures for model 27 are plausible and for model 28 could be provided constant returns is re-defined as zero with increasing returns above and decreasing returns below. Again the cause of this requires further investigation. A possible explanation is that the inclusion of second order and cross product terms leads to an unrealistic expression for the change in costs with respect to time, which dominates the effects of changes in TKT or DEN on costs.

Table 10: Operator comparisons

Operator	Model 27	Model 28
BR	1.53	1.75
CFF	2.16	1.97
CIE	0.87*	0.95*
DB	1.27*	1.58*
DSB	1.65	1.82
FS	1.92	2.01
NS	1.27	1.73
NSB	1.02	1.07*
OBB	2.46	2.46
SJ	0.82	0.85
SNCB	2.48	2.48
SNCF	0.81*	1.03*

Note: * = Insignificantly different from VR

Table 11: Model 27 and 28 Returns to scale and density

	RTS		*RTD*	
Operator	Model 27	Model 28	Model 27	Model 28
BR	1.37	0.56	1.68	0.67
CFF	-24.39	0.90	0.72	-0.38
CIE	-4.83	0.89	1.50	0.69
DB	1.04	0.52	2.03	0.99
DSB	-7.41	0.89	0.95	-0.02
FS	1.38	0.55	1.72	0.82
NS	-16.67	0.89	0.71	-0.39
NSB	10.10	0.75	1.81	0.88
OBB	3.61	0.70	1.22	0.30
SJ	1.83	0.60	2.14	1.23
SNCB	21.28	0.81	1.05	0.01
SNCF	0.94	0.49	2.33	1.39
VR	3.68	0.68	2.11	1.20

Our work so far leads us to believe that model 26 IUC is our most plausible model. However, we were aware that models of this type are affected by heteroscedasticity. Visual inspection of scatterplots (see, for example, Figure 1) did not suggest this was a problem, although there was clearly a grouping of the data into four clusters, broadly corresponding to large, medium, medium - small and small railways. However, when a Park Glesjer test was performed (by regressing the absolute residuals against the independent variables) it was found that 21 of 41 parameter were significant at the 5% level.

The traditional way to deal with this problem with the translog function is to re-define all variables around a point of expansion, usually the sample mean. This procedure was adopted in model 26 (IUCM) which is given in the Appendix. Although 13 of the 41 parameter values in this model were statistically insignificant at the 10% level, this was a slight improvement on 26 (IUC) where 14 parameter values were insignificant. The key indicators are given in Table 12. It should be noted that the estimation of *RTS* and *RTD* was revised to take into account the variable re-definition as follows. Suppose we have the simple model:

$$\frac{\ln RTC_i}{(\sum_{i=1}^{n} \ln RTC_i)/n} = \alpha + \beta \frac{\ln TKT_i}{(\sum_{i=1}^{n} \ln TKT_i)/n} + \gamma \frac{\ln DEN_i}{(\sum_{i=1}^{n} \ln DEN_i)/n}$$

Then:

$$\eta_s = \frac{\partial \ln RTC_i}{\partial \ln TKT_i} = \beta \frac{((\sum_{i=1}^{n} \ln TKT_i)/n - \ln TKT_i/n)}{[(\sum_{i=1}^{n} \ln TKT_i)/n]^2} . (\sum_{i=1}^{n} \ln RTC_i)/n$$

and

$$\eta_D = \frac{\partial \ln RTC_i}{\partial \ln DEN_i} = \gamma \frac{((\sum_{i=1}^{n} \ln DEN_i)/n - \ln DEN_i/n)}{[(\sum_{i=1}^{n} \ln TKT_i)/n]^2} . (\sum_{i=1}^{n} \ln RTC_i)/n$$

where n = Number of observations (260).

From Table 12 we have re-confirmation of the finding that the largest railways have decreasing returns to size, with SNCF, DB, FS, SJ and BR most afflicted, in that order, by the problem. Some of the smaller railways exhibit increasing returns to scale, most notably NS, CFF and DSB, although other exhibit constant returns, particularly CIE.

Figure 1: Scatterpoint of actual values against predicted values

Table 12: Model 26 (IUCM): Key indicators

Operator	RTS	RTD	Operators' Comparison[1]
BR	0.76	1.25	0.99
CFF	1.35	0.16	1.00
CIE	1.00	1.12	0.94*
DB	0.68	1.57	0.97*
DSB	1.25	0.51	1.00
FS	0.71	1.39	1.01
NS	1.42	0.15	0.98
NSB	0.91	1.68	0.94
OBB	0.92	0.72	1.05
SJ	0.73	2.22	0.89
SNCB	1.15	0.55	1.04
SNCF	0.61	2.14	0.91*
VR	0.81	2.34	1.00

Note: [1] Mean dummy variable for VR would be 20/260 = 0.076; Exp (0.076) = 1.080. All values standardised around this value for consistency with earlier tables.
* Insignificantly different from zero.

By contrast, we have confirmation of the finding that the largest railways exhibit economies of density, SNCF, DB, FS and BR, as do the railway systems of the sparsely populated Nordic countries, VR, SJ and NSB. Some of more heavily trafficked railways have diseconomies of density, most notably NS, CFF and, less markedly, DSB and SNCF. The railway most closely exhibiting constant returns to density is CIE, although this may reflect averaging the densely used Dublin suburban line with the lightly trafficked rest of the network. This finding of only mildly increasing returns to density contrasts with the findings of McGeehan (1988).

In terms of operator comparisons, all other things being equal, Table 12 shows that our redefining of the variables has led to a considerable narrowing of differences. Indeed compared to our chosen bench mark rail system (VR), only SJ and NSB have markedly significant lower costs (by 11% and 6% respectively) and only OBB and SNCB have markedly higher costs (by 5% and 4% respectively).

One further amendment to Steck's work was undertaken. The measure of returns to density, as defined, is a long run one in that in order to increase density, given constant total train-km, track length must be reduced. A more common, short run, measure of density examines the changes in costs as a result of changes in total train kms, given constant track length (see, for example, Caves et al., 1985). As a result, we re-ran translog model 26 (IUCM) by replacing the *DEN* varible by *LL*. The results are given as model 29, Appendix M. We had previously rejected using *LL* on the basis that it was highly correlated with *TKT* but by expanding around the point of means, we have reduced this objection. However 14 out of 41 parameters are insignificant at the 10% level, including the *LL* first order term and four out of five cross-product terms. Nonetheless, it was decided to investigate the cost elasticities and returns to scale and density. Two cost elasticities were estimated:

$$\eta_1 = \frac{\partial \ln \mathrm{RTC}}{\partial \ln \mathrm{TKT}} \text{ and } \eta_2 = \frac{\partial \ln \mathrm{RTC}}{\partial \ln \mathrm{LL}}$$

with Returns to Density (*RTD*) estimated as $1/\eta_1$ and Returns to Scale estimated as $1/(\eta_1 + \eta_2)$. The results are given in Table 13. It can be seen that this model replicates our finding of decreasing returns to scale for the largest railway companies. For returns to density, our findings highlight the diseconomies of density for the NS and CFF rail systems but also indicate that certain rail systems (SNCF, NSB and, to a lesser extent, CIE) exhibit very pronounced economies of density. For VR and SJ, it might be argued that the cost elasticity with respect to *TKT* given constant *LL* (η_1) is of the wrong (negative sign) but is indicative of marked economies of density. The operators' comparison results are almost identical to those in Table 12.

Table 13: Model 29 Key indicators

Operator	η_1	η_2	*RTS*	*RTD*	Operators' Comparison
BR	0.66	0.68	0.74	1.50	0.99
CFF	1.25	-0.49	1.30	0.80	1.00
CIE	0.28	0.74	0.97	3.57	0.94*
DB	0.46	1.04	0.66	2.17	0.97*
DSB	0.93	-0.10	1.20	1.08	0.99
FS	0.47	0.96	0.70	2.11	1.00
NS	1.29	-0.55	1.36	0.75	0.98
NSB	0.16	0.97	0.88	6.19	0.93
OBB	0.75	0.37	0.88	1.33	1.04
SJ	-0.06	1.47	0.71	-15.02	0.89
SNCB	0.96	-0.07	1.12	1.04	1.04
SNCF	0.08	1.59	0.60	12.11	0.91*
VR	-0.17	1.44	0.79	-5.62	1.00

Note: Compiled as per Table 12

The results in Table 13 highlight two features. Firstly, it is clear that *LL* is only a crude measure of network capacity, particularly for Scandinavian railways, with their high proportion of single tracks. The use of the *DEN* variable had disguised this fact. It would be preferable to make use of track-km rather than route-km but this information was not available for some observations in our data set (SNCF 1972-78). Secondly, Caves et al. (op cit.) would argue, using the analysis of Mundlak (1978), that traditional measures of *RTD* are biased but, assuming the cross-section element of our data set dominates the time series element, estimates of *RTS* are not biased. They suggest a two-stage estimation procedure involving expansion around the (non-transformed) sample mean and firm mean for each observation. The econometric implications of our approach using the transformed sample means requires further investigation.

An alternative amendment to Steck's work would be to re-define returns to density *(RTD)* as $1/(\eta_T + \eta_D)$ rather than $1/\eta_D$. This can be interpreted as examining change in costs with respect to train km whilst holding route km constant and is consistent with the returns to scale *(RTS)* measure $1/\eta_T$. In Table 14 we revise the *RTD* measure for model 26 IUC (Table 8) and model 26 IUCM (Table 12) and Steck's original model 26 (Table 2A).

Table 14: Amended return to density measures

	Model 26 Original	Amended	Model 26 UIC Original	Amended	Model 26 IUCM Original	Amended
BR	1.44	1.37	1.59	1.11	1.25	0.93
CFF	0.87	1.15	0.59	0.79	0.16	0.63
CIE	1.30	2.78	1.66	4.54	1.12	1.14
DB	1.72	1.75	2.00	1.49	1.57	1.11
DSB	0.98	1.41	0.93	1.08	0.51	0.78
FS	1.54	1.49	1.76	1.30	1.39	0.98
NS	0.79	1.11	0.56	0.78	0.15	0.65
NSB	1.54	3.00	1.81	4.00	1.68	2.38
OBB	1.43	1.63	1.36	1.20	0.72	0.73
SJ	1.83	3.33	2.21	5.00	2.22	6.66
SNCB	1.07	1.35	1.02	1.05	0.55	0.76
SNCF	1.95	2.27	2.37	2.38	2.14	2.00
VR	1.78	5.55	2.18	50.00	2.34	-9.09

Compared to Steck's original model 26 results, CIE and the Nordic railways exhibit much greater returns to density, whilst CFF and NS no longer exhibit decreasing returns. However in subsequent amendments to model 26, CFF and NS do exhibit decreasing returns to density (model 26 IUC), joined in model 26 IUCM by DSB, OBB and SNCB, whilst BR and FS exhibit constant or slightly decreasing returns.

3. CONCLUSIONS

We have improved on Steck's earlier work in four ways. Firstly, we have updated the data to 1990. Secondly, we introduce consistency into the Returns to Scale and Returns to Density measures by including all relevant estimated parameter values, whether statistically significant or not. Thirdly, we constrain the regression model to ensure linear homogeneity in input prices. Fourthly, we reduce the statistical problems that are associated with pooled data by redefining variables around the sample mean. All but the second improvement led to important changes in model results. In this work we have seen that the translog model is not robust, a large proportion of parameter values are insignificant and in certain situations our measures of returns to density and, particularly, scale loose their meaning, most notably when time cross-effects are introduced in an attempt to better model technological change.

However, some common results do come through. Table 15 compares, in broad terms, the results of our two preferred models. In terms of returns to scale the most consistent finding is that the largest railways (with over 10,000 km of route) exhibit decreasing returns to size, suggesting they are beyond the point of maximum efficient scale. Our findings on constant returns are less consistent, railways ranging from CIE (1,944 route kms) to VR (5,867 route kms) have broadly constant returns, although model 26 (IUCM) does indicate that some railways with less than 3,000 km have increasing returns (CFF, DSB, NS). If 3,000 route km was the minimum efficient scale, this would suggest that BR should be broken up into 5 or 6 operators rather than 25 or 26.

Table 15: Comparison of model results

	Network size (1990 length of line) -km	*RTS*		*RTD*		Managerial Efficiency	
		26 (IUC)	26 (IUCM)	26 (IUC)	26 (IUCM)	26 (IUC)	26 (IUCM)
BR	16584	D	D	C	C	M	H
CFF	2978	C	I	D	D	L	M
CIE	1944	C	C	I	C	M	M
DB	26949	D	D	I	C	M	M
DSB	2344	C	I	C	D	L	M
FS	16066	D	D	I	C	L	L
NS	2798	C	I	D	D	L	H
NSB	4044	C	C	I	I	L	H
OBB	5624	C	C	C	D	L	L
SJ	10801	D	D	I	I	H	H
SNCB	3479	C	C	C	D	L	L
SNCF	34070	D	D	I	I	M	M
VR	5867	C	D	I	I	M	M

Notes: I = Increasing returns (> 1.2) H = Higher efficiency than VR
C = Constant returns (0.8 - 1.2) M = Same efficiency to VR
D = Decreasing returns (< 0.8) L = Lower efficiency than VR.
RTD measure amended in light of Table 14

In terms of returns to density, four railways exhibit increasing returns; SNCF, SJ, VR and NSB. CFF and NS consistently exhibit decreasing returns, reflecting Switzerland's and the Netherlands' roles as railway cross roads of Europe. If the operator dummy variable is interpretated as a measure of managerial efficiency, only SJ consistently performs better than VR, whilst FS, OBB and SNCB consistently perform worse. This finding concurs with other studies (Jackson, 1992, Table IV).

Overall, we conclude that, despite some statistical problems, the translog cost function has allowed us some useful insights into the cost characteristics of western European railways.

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APPENDIX

CONTENTS

A. VARIABLE DEFINITIONS

Variables applicable to this paper are the following:

a) Composition and resources of the railway system

LL	=	length of lines at the end of the year (km)
SF	=	average railway staff strength (road and shipping services staff are not included).

b) Output measures

TKP	=	passenger train-kilometres (thousands) for all types of traction
TKT	=	total train-kilometres (thousands) for all types of traction.

c) Financial results (specific costs in millions)

NCS	=	salaries of active staff
NCE	=	energy consumption
NCV	=	various, materials, services rendered by third parties

and hence

NTC	=	NCS + NCE + NCV

where NTC is the total (nominal) value of operating costs. The specific costs of pensions, social charges, taxes and depreciation vary between countries and are excluded from the data set for this reason.

Conversion to real costs involved the use of an implicit price level (IPL) based on gross domestic product (GDP) for each country (see OECD, 1960-1990, National Accounts). The value of IPL for 1990 is 100.

To overcome the currency problem we adopt the procedure used by Steck and convert all costs to pounds sterling by use of purchasing power parities (PPP). These values are preferable to exchange rates for our purpose since they overcome cost of living differentials.

The respective values of the purchasing power parities (PPP) for 1990, in national currency units per US dollar, are:

BR	=	Great Britain (0.609)
CFF	=	Switzerland (2.19)
CIE	=	Ireland (0.688)
DB	=	West Germany (2.08)
DSB	=	Denmark (9.41)
FS	=	Italy (1415)
NS	=	Netherlands (2.16)
NSB	=	Norway (9.81)
OBB	=	Austria (14.0)
SJ	=	Sweden (9.35)

SNCB	=	Belgium (39.3)
SNCF	=	France (6.59)
VR	=	Finland (6.40)

Denoting the PPP value of BR for 1990 as PPP_{BR}, and in general the PPP for rail operator i as PPP_i, the real operating cost for rail operator i is calculated as follows:

$RTC = 100(NTC/IPL)\ (PPP_{BR}/PPP_i)$ (millions of pounds sterling).

Input prices are defined as total expenditures of labour, energy consumption and materials and services divided by total labour and total train-kilometres. Hence, the

price of labour, *WM*, is the ratio between the cost of real salaries and staff number (SF), expressed in pounds per number of staff

price of energy per train-kilometre, WE is the ratio between real energy cost and total train-kilometres (*TKT*), expressed in thousands of pounds per train-kilometre.

price of materials and services per train-kilometre, WV, is the ratio between real various cost and total train-kilometre (*TKT*), expressed in thousands of pounds per train-kilometre.

The traffic density variable *DEN*, is defined as total train-kilometres (*TKT*) per route mile (LL).

The output measures are *TKP* and *TKT* and Multicollinearity problems are avoided by using the variable *%TKP*, the ratio of *TKP* to *TKT*. Technological change is introduced by the inclusion of the time trend variable *YEAR*, the year of operation.

Dummy variables, namely DBR, DCFF, DCIE, DDB, DDSB, DFS, DNS, DNSB, DOBB, DSJ, DSNCB and DSNCF, were defined for each railway.

B. TRANSLOG MODEL 25:
RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*DEN*,dummies)

$$\ln \mathrm{RTC} =$$

$$\underset{(0.5)}{5.37} + \underset{(2.7)}{5.208} \ln\mathrm{WM} + \underset{(3.8)}{2.069} \ln\mathrm{WE} - \underset{(-1.6)}{0.761} \ln\mathrm{WV} - \underset{(-6.6)}{6.225} \ln\mathrm{TKT}$$

$$+ \underset{(4.3)}{12.963} \ln\%\mathrm{TKP} + \underset{(4.9)}{5.778} \ln\mathrm{DEN} - \underset{(-2.9)}{0.277} (\ln\mathrm{WM})^2 + \underset{(4.2)}{0.049} (\ln\mathrm{WE})^2$$

$$+ \underset{(3.1)}{0.030} (\ln\mathrm{WV})^2 + \underset{(9.4)}{0.421} (\ln\mathrm{TKT})^2 + \underset{(0.2)}{0.070} (\ln\%\mathrm{TKP})^2 + \underset{(4.6)}{0.392} (\ln\mathrm{DEN})^2$$

$$- \underset{(-4.0)}{0.217} \ln\mathrm{WM} \bullet \ln\mathrm{WE} + \underset{(0.1)}{0.003} \ln\mathrm{WM} \bullet \ln\mathrm{WV} - \underset{(-0.8)}{0.032} \ln\mathrm{WM} \bullet \ln\mathrm{TKT}$$

$$- \underset{(-4.7)}{1.323} \ln\mathrm{WM} \bullet \ln\%\mathrm{TKP} + \underset{(0.4)}{0.041} \ln\mathrm{WM} \bullet \ln\mathrm{DEN} + \underset{(1.5)}{0.024} \ln\mathrm{WE} \bullet \ln\mathrm{WV}$$

$$- \underset{(-1.0)}{0.013} \ln\mathrm{WE} \bullet \ln\mathrm{TKT} - \underset{(-0.3)}{0.037} \ln\mathrm{WE} \bullet \ln\%\mathrm{TKP} + \underset{(1.6)}{0.066} \ln\mathrm{WE} \bullet \ln\mathrm{DEN}$$

$$+ \underset{(7.4)}{0.095} \ln\mathrm{WV} \bullet \ln\mathrm{TKT} + \underset{(1.7)}{0.170} \ln\mathrm{WV} \bullet \ln\%\mathrm{TKP} - \underset{(-0.1)}{0.002} \ln\mathrm{WV} \bullet \ln\mathrm{DEN}$$

$$- \underset{(-2.1)}{0.189} \ln\mathrm{TKT} \bullet \ln\%\mathrm{TKP} - \underset{(-9.5)}{0.779} \ln\mathrm{TKT} \bullet \ln\mathrm{DEN} + \underset{(2.1)}{0.427} \ln\%\mathrm{TKP} \bullet \ln\mathrm{DEN}$$

$$+ \underset{(0.4)}{0.06}\mathrm{DBR} + \underset{(11.2)}{0.83}\mathrm{DCFF} - \underset{(-0.9)}{0.12}\mathrm{DCIE} - \underset{(-2.1)}{0.45}\mathrm{DDB} + \underset{(4.7)}{0.35}\mathrm{DDSB} + \underset{(1.6)}{0.22}\mathrm{DFS}$$

$$+ \underset{(6.8)}{0.53}\mathrm{DNS} - \underset{(-0.6)}{0.03}\mathrm{DNSB} + \underset{(17.4)}{0.83}\mathrm{DOBB} - \underset{(-6.1)}{0.45}\mathrm{DSJ} + \underset{(17.0)}{0.85}\mathrm{DSNCB} - \underset{(-5.2)}{1.20}\mathrm{DSNCF}$$

$$R^2 = 0.9994 \qquad F = 9737.8$$

NB. The parameter value for the variable pair *ln* WV . *ln DEN* was incorrectly typed in the original thesis.

C. TRANSLOG MODEL 26:

RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*DEN*,*YEAR*,dummies)

$$
\begin{aligned}
\ln \mathrm{RTC} = {} & \\
& -\underset{(-1.2)}{11.05} + \underset{(3.7)}{5.930}\ln \mathrm{WM} + \underset{(6.1)}{2.726}\ln \mathrm{WE} - \underset{(-3.1)}{1.192}\ln \mathrm{WV} + \underset{(0.5)}{0.575}\ln \mathrm{TKT} \\
& + \underset{(4.4)}{10.940}\ln \%\mathrm{TKP} - \underset{(-0.4)}{0.517}\ln \mathrm{DEN} - \underset{(-9.4)}{0.009}\mathrm{YEAR} \\
& - \underset{(-3.2)}{0.247}(\ln \mathrm{WM})^2 + \underset{(7.0)}{0.070}(\ln \mathrm{WE})^2 \\
& + \underset{(6.5)}{0.054}(\ln \mathrm{WV})^2 + \underset{(2.2)}{0.111}(\ln \mathrm{TKT})^2 - \underset{(-0.8)}{0.241}(\ln \%\mathrm{TKP})^2 + \underset{(4.1)}{0.292}(\ln \mathrm{DEN})^2 \\
& - \underset{(-5.9)}{0.261}\ln \mathrm{WM} \bullet \mathrm{Ln WE} + \underset{(1.7)}{0.067}\ln \mathrm{WM} \bullet \ln \mathrm{WV} - \underset{(-4.7)}{0.168}\ln \mathrm{WM} \bullet \ln \mathrm{TKT} \\
& - \underset{(-5.2)}{1.195}\ln \mathrm{WM} \bullet \ln \%\mathrm{TKP} + \underset{(2.0)}{0.164}\ln \mathrm{WM} \bullet \ln \mathrm{DEN} - \underset{(-2.9)}{0.044}\ln \mathrm{WE} \bullet \ln \mathrm{WV} \\
& + \underset{(0.2)}{0.002}\ln \mathrm{WE} \bullet \ln \mathrm{TKT} + \underset{(1.7)}{0.191}\ln \mathrm{WE} \bullet \ln \%\mathrm{TKP} - \underset{(-0.3)}{0.012}\ln \mathrm{WE} \bullet \ln \mathrm{DEN} \\
& + \underset{(7.0)}{0.076}\ln \mathrm{WV} \bullet \ln \mathrm{TKT} + \underset{(2.3)}{0.186}\ln \mathrm{WV} \bullet \ln \%\mathrm{TKP} - \underset{(-0.1)}{0.002}\ln \mathrm{WV} \bullet \ln \mathrm{DEN} \\
& - \underset{(-1.3)}{0.095}\ln \mathrm{TKT} \bullet \ln \%\mathrm{TKP} - \underset{(-2.9)}{0.252}\ln \mathrm{TKT} \bullet \ln \mathrm{DEN} + \underset{(2.5)}{0.405}\ln \%\mathrm{TKP} \bullet \ln \mathrm{DEN} \\
& + \underset{(2.7)}{0.35}\mathrm{DBR} + \underset{(4.9)}{0.38}\mathrm{DCFF} - \underset{(-1.1)}{0.12}\mathrm{DCIE} + \underset{(2.0)}{0.40}\mathrm{DDB} + \underset{(2.4)}{0.15}\mathrm{DDSB} + \underset{(4.3)}{0.48}\mathrm{DFS} \\
& + \underset{(0.8)}{0.06}\mathrm{DNS} - \underset{(-1.7)}{0.06}\mathrm{DNSB} + \underset{(16.5)}{0.69}\mathrm{DQBB} - \underset{(-3.6)}{0.24}\mathrm{DSJ} + \underset{(10.5)}{0.55}\mathrm{DSNCB} + \underset{(0.0)}{0.00}\mathrm{DSNCF}
\end{aligned}
$$

$$R^2 = 0.9996 \quad F = 14117.9$$

NB. The sign of the parameter for variable DDB above was incorrectly typed in the original thesis.

D. TRANSLOG MODEL 25(I):
RTC = f(*WM*,WE,WV, *TKT*, *%TKP*, *DEN*, dummies)

$$\ln RTC =$$

$$
\begin{aligned}
&\underset{(0.4)}{3.88} + \underset{(2.9)}{5.627} \ln WM + \underset{(3.5)}{1.798} \ln WE - \underset{(-2.1)}{0.949} \ln WV - \underset{(-6.5)}{6.242} \ln TKT \\
&+ \underset{(4.6)}{13.115} \ln \%TKP + \underset{(4.7)}{5.360} \ln DEN - \underset{(-3.2)}{0.305} (\ln WM)^2 + \underset{(4.4)}{0.053} (\ln WE)^2 \\
&+ \underset{(3.1)}{0.029} (\ln WV)^2 + \underset{(9.3)}{0.420} (\ln TKT)^2 - \underset{(-0.1)}{0.039} (\ln \%TKP)^2 + \underset{(4.2)}{0.368} (\ln DEN)^2 \\
&- \underset{(-3.7)}{0.193} \ln WM \bullet \ln WE + \underset{(0.4)}{0.019} \ln WM \bullet \ln WV - \underset{(-0.8)}{0.032} \ln WM \bullet \ln TKT \\
&- \underset{(-5.0)}{1.366} \ln WM \bullet \ln \%TKP + \underset{(0.8)}{0.076} \ln WM \bullet \ln DEN + \underset{(1.8)}{0.030} \ln WE \bullet \ln WV \\
&- \underset{(-0.7)}{0.009} \ln WE \bullet \ln TKT - \underset{(-0.4)}{0.048} \ln WE \bullet \ln \%TKP + \underset{(1.7)}{0.071} \ln WE \bullet \ln DEN \\
&+ \underset{(7.8)}{0.098} \ln WV \bullet \ln TKT + \underset{(1.8)}{0.179} \ln WV \bullet \ln \%TKP - \underset{(-0.2)}{0.005} \ln WV \bullet \ln DEN \\
&- \underset{(-2.1)}{0.180} \ln TKT \bullet \ln \%TKP - \underset{(-9.3)}{0.756} \ln TKT \bullet \ln DEN + \underset{(2.4)}{0.480} \ln \%TKP \bullet \ln DEN \\
&+ \underset{(0.1)}{0.01} DBR + \underset{(11.5)}{0.83} DCFF - \underset{(-0.9)}{0.11} DCIE - \underset{(-2.3)}{0.49} DDB + \underset{(4.8)}{0.36} DDSB + \underset{(1.4)}{0.19} DFS \\
&+ \underset{(6.6)}{0.52} DNS - \underset{(-1.7)}{0.07} DNSB + \underset{(17.6)}{0.82} DOBB - \underset{(-6.1)}{0.46} DSJ + \underset{(17.0)}{0.85} DSNCB - \underset{(-5.2)}{1.22} DSNCF
\end{aligned}
$$

$$R^2 = 0.9995 \qquad F = 10439.3$$

E. TRANSLOG MODEL 26(I):
RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*DEN*,*YEAR*,dummies)

$$\ln RTC =$$

$$-\underset{(-0.9)}{8.36} + \underset{(3.4)}{5.506}\ln WM + \underset{(5.6)}{2.487}\ln WE - \underset{(-3.5)}{1.357}\ln WV + \underset{(0.3)}{0.376}\ln TKT$$

$$+\underset{(4.7)}{11.445}\ln \%TKP - \underset{(-0.7)}{0.811}\ln DEN - \underset{(-8.9)}{0.008}YEAR$$

$$-\underset{(-3.0)}{0.235}(\ln WM)^2 + \underset{(7.0)}{0.071}(\ln WE)^2$$

$$+\underset{(6.4)}{0.052}(\ln WV)^2 + \underset{(2.2)}{0.114}(\ln TKT)^2 - \underset{(-1.1)}{0.326}(\ln \%TKP)^2 + \underset{(3.9)}{0.284}(\ln DEN)^2$$

$$-\underset{(-5.5)}{0.243}\ln WM \bullet \ln WE + \underset{(2.0)}{0.082}\ln WM \bullet \ln WV - \underset{(-4.3)}{0.156}\ln WM \bullet \ln TKT$$

$$-\underset{(-5.6)}{1.259}\ln WM \bullet \ln \%TKP + \underset{(2.4)}{0.193}\ln WM \bullet \ln DEN - \underset{(-2.4)}{0.038}\ln WE \bullet \ln WV$$

$$+\underset{(0.3)}{0.003}\ln WE \bullet \ln TKT + \underset{(1.4)}{0.153}\ln WE \bullet \ln \%TKP + \underset{(0.1)}{0.004}\ln WE \bullet \ln DEN$$

$$+\underset{(7.4)}{0.080}\ln WV \bullet \ln TKT + \underset{(2.5)}{0.204}\ln WV \bullet \ln \%TKP - \underset{(-0.3)}{0.009}\ln WV \bullet \ln DEN$$

$$-\underset{(-1.4)}{0.099}\ln TKT \bullet \ln \%TKP - \underset{(-2.8)}{0.245}\ln TKT \bullet \ln DEN + \underset{(2.6)}{0.434}\ln \%TKP \bullet \ln DEN$$

$$+\underset{(2.4)}{0.34}DBR + \underset{(5.0)}{0.39}DCFF - \underset{(-0.9)}{0.10}DCIE - \underset{(1.8)}{0.38}DDB + \underset{(2.7)}{0.17}DDSB + \underset{(4.1)}{0.47}DFS$$

$$+\underset{(0.9)}{0.07}DNS - \underset{(-1.7)}{0.06}DNSB + \underset{(16.4)}{0.68}DOBB - \underset{(-3.5)}{0.23}DSJ + \underset{(10.5)}{0.56}DSNCB - \underset{(-0.1)}{0.01}DSNCF$$

$$R^2 = 0.9996 \qquad F = 14538.0$$

F. TRANSLOG MODEL 25(IU):
RTC = f(*WM*,WE,WV, *TKT*, *%TKP*, *DEN*, dummies)

$$\ln RTC =$$

$$
\begin{aligned}
&\underset{(3.0)}{36.81} + \underset{(0.8)}{1.668}\ln WM + \underset{(4.3)}{2.619}\ln WE - \underset{(-0.7)}{0.369}\ln WV - \underset{(-7.9)}{9.349}\ln TKT \\
&+ \underset{(4.1)}{14.751}\ln \%TKP + \underset{(5.0)}{6.890}\ln DEN - \underset{(-1.4)}{0.138}(\ln WM)^2 + \underset{(5.5)}{0.084}(\ln WE)^2 \\
&+ \underset{(3.4)}{0.041}(\ln WV)^2 + \underset{(10.8)}{0.614}(\ln TKT)^2 + \underset{(0.5)}{0.202}(\ln \%TKP)^2 + \underset{(5.0)}{0.515}(\ln DEN)^2 \\
&- \underset{(-4.4)}{0.268}\ln WM \bullet \ln WE - \underset{(-0.4)}{0.022}\ln WM \bullet \ln WV - \underset{(-0.2)}{0.011}\ln WM \bullet \ln TKT \\
&- \underset{(-4.8)}{1.634}\ln WM \bullet \ln \%TKP + \underset{(2.3)}{0.264}\ln WM \bullet \ln DEN + \underset{(0.7)}{0.014}\ln WE \bullet \ln WV \\
&+ \underset{(0.8)}{0.012}\ln WE \bullet \ln TKT + \underset{(0.9)}{0.142}\ln WE \bullet \ln \%TKP - \underset{(-0.1)}{0.004}\ln WE \bullet \ln DEN \\
&+ \underset{(3.9)}{0.062}\ln WV \bullet \ln TKT + \underset{(1.2)}{0.144}\ln WV \bullet \ln \%TKP + \underset{(1.4)}{0.053}\ln WV \bullet \ln DEN \\
&- \underset{(-0.4)}{0.046}\ln TKT \bullet \ln \%TKP - \underset{(-11.5)}{1.153}\ln TKT \bullet \ln DEN + \underset{(1.6)}{0.415}\ln \%TKP \bullet \ln DEN \\
&- \underset{(-3.0)}{0.58}DBR + \underset{(11.7)}{0.96}DCFF + \underset{(1.7)}{0.24}DCIE - \underset{(-5.6)}{1.52}DDB + \underset{(6.0)}{0.55}DDSB - \underset{(-1.8)}{0.30}DFS \\
&+ \underset{(8.1)}{0.71}DNS + \underset{(2.6)}{0.12}DNSB + \underset{(13.7)}{0.85}DOBB - \underset{(-8.1)}{0.70}DSJ + \underset{(14.8)}{0.96}DSNCB - \underset{(-8.2)}{2.35}DSNCF
\end{aligned}
$$

$$R^2 = 0.9989 \qquad F = 6248.5$$

G. TRANSLOG MODEL 26(IU):
RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*DEN*,*YEAR*,dummies)

$$\ln RTC =$$

$$-\underset{(-0.6)}{6.49} + \underset{(3.8)}{6.820}\ln WM + \underset{(5.6)}{2.791}\ln WE - \underset{(-3.2)}{1.462}\ln WV - \underset{(-0.5)}{0.593}\ln TKT$$

$$+ \underset{(4.6)}{13.662}\ln \%TKP - \underset{(-1.0)}{1.351}\ln DEN - \underset{(-10.4)}{0.009}\,YEAR$$

$$- \underset{(-3.8)}{0.314}(\ln WM)^2 + \underset{(7.6)}{0.095}(\ln WE)^2$$

$$+ \underset{(5.7)}{0.057}(\ln WV)^2 + \underset{(3.5)}{0.212}(\ln TKT)^2 - \underset{(-0.6)}{0.234}(\ln \%TKP)^2 + \underset{(3.8)}{0.330}(\ln DEN)^2$$

$$- \underset{(-5.3)}{0.267}\ln WM \bullet \ln WE + \underset{(2.2)}{0.102}\ln WM \bullet \ln WV - \underset{(-4.6)}{0.204}\ln WM \bullet \ln TKT$$

$$- \underset{(-5.6)}{1.547}\ln WM \bullet \ln \%TKP + \underset{(4.6)}{0.435}\ln WM \bullet \ln DEN - \underset{(-3.4)}{0.061}\ln WE \bullet \ln WV$$

$$+ \underset{(1.6)}{0.019}\ln WE \bullet \ln TKT + \underset{(2.3)}{0.307}\ln WE \bullet \ln \%TKP - \underset{(-1.0)}{0.040}\ln WE \bullet \ln DEN$$

$$+ \underset{(4.6)}{0.059}\ln WV \bullet \ln TKT + \underset{(1.7)}{0.171}\ln WV \bullet \ln \%TKP + \underset{(0.8)}{0.025}\ln WV \bullet \ln DEN$$

$$- \underset{(-0.3)}{0.025}\ln TKT \bullet \ln \%TKP - \underset{(-4.1)}{0.441}\ln TKT \bullet \ln DEN + \underset{(2.1)}{0.438}\ln \%TKP \bullet \ln DEN$$

$$- \underset{(-0.3)}{0.05}DBR + \underset{(4.9)}{0.42}DCFF - \underset{(-1.0)}{0.12}DCIE - \underset{(-0.9)}{0.23}DDB + \underset{(3.0)}{0.24}DDSB + \underset{(1.1)}{0.16}DFS$$

$$+ \underset{(1.8)}{0.16}DNS + \underset{(0.4)}{0.02}DNSB + \underset{(12.9)}{0.69}DOBB - \underset{(-5.3)}{0.40}DSJ + \underset{(9.1)}{0.59}DSNCB - \underset{(-2.3)}{0.67}DSNCF$$

$$R^2 = 0.9993 \qquad F = 9068.7$$

H. TRANSLOG MODEL 25(IUC):
RTC = f(*WM*,WE,WV, *TKT*, *%TKP*, *DEN*,dummies)

$$\ln RTC =$$

$$\underset{(4.1)}{23.75} + \underset{(6.0)}{1.260} \ln WM + \underset{(0.1)}{0.014} \ln WE - \underset{(-1.2)}{0.274} \ln WV - \underset{(-5.9)}{7.093} \ln TKT$$

$$+ \underset{(1.4)}{2.673} \ln \%TKP + \underset{(6.0)}{6.167} \ln DEN - \underset{(-7.0)}{0.032} (\ln WM)^2 + \underset{(3.4)}{0.055} (\ln WE)^2$$

$$+ \underset{(4.7)}{0.063} (\ln WV)^2 + \underset{(8.1)}{0.517} (\ln TKT)^2 - \underset{(-1.6)}{0.749} (\ln \%TKP)^2 + \underset{(6.0)}{0.667} (\ln DEN)^2$$

$$+ \underset{(1.6)}{0.020} \ln WM \bullet \ln WE + \underset{(1.1)}{0.012} \ln WM \bullet \ln WV - \underset{(-2.3)}{0.054} \ln WM \bullet \ln TKT$$

$$- \underset{(-4.0)}{0.731} \ln WM \bullet \ln \%TKP + \underset{(0.4)}{0.024} \ln WM \bullet \ln DEN - \underset{(-8.0)}{0.075} \ln WE \bullet \ln WV$$

$$+ \underset{(1.1)}{0.019} \ln WE \bullet \ln TKT + \underset{(1.6)}{0.273} \ln WE \bullet \ln \%TKP - \underset{(-0.8)}{0.044} \ln WE \bullet \ln DEN$$

$$+ \underset{(2.1)}{0.036} \ln WV \bullet \ln TKT + \underset{(3.5)}{0.459} \ln WV \bullet \ln \%TKP + \underset{(0.5)}{0.020} \ln WV \bullet \ln DEN$$

$$+ \underset{(2.4)}{0.246} \ln TKT \bullet \ln \%TKP - \underset{(-8.9)}{0.970} \ln TKT \bullet \ln DEN + \underset{(0.2)}{0.067} \ln \%TKP \bullet \ln DEN$$

$$- \underset{(-1.5)}{0.35} DBR + \underset{(11.7)}{1.06} DCFF + \underset{(0.7)}{0.11} DCIE - \underset{(-3.6)}{1.11} DDB + \underset{(9.7)}{0.86} DDSB - \underset{(-0.4)}{0.07} DFS$$

$$+ \underset{(10.7)}{0.87} DNS + \underset{(4.8)}{0.23} DNSB + \underset{(16.5)}{0.97} DQBB - \underset{(-5.9)}{0.53} DSJ + \underset{(18.3)}{1.13} DSNCB - \underset{(-5.6)}{1.81} DSNCF$$

$$R^2 = 0.9985 \qquad F = 5453.7$$

I. TRANSLOG MODEL 26(IUC):
RTC = f(*WM*,WE,WV, *TKT*, *%TKP*, *DEN*, *YEAR*, dummies)

$$\ln \mathrm{RTC} =$$

$$\underset{(2.9)}{14.02} + \underset{(7.0)}{1.208} \ln \mathrm{WM} + \underset{(1.9)}{0.298} \ln \mathrm{WE} - \underset{(-2.7)}{0.506} \ln \mathrm{WV} - \underset{(-0.6)}{0.692} \ln \mathrm{TKT}$$

$$+ \underset{(2.2)}{3.429} \ln \%\mathrm{TKP} + \underset{(0.6)}{0.578} \ln \mathrm{DEN} - \underset{(-10.2)}{0.009} \mathrm{YEAR}$$

$$- \underset{(-1.7)}{0.008} (\ln \mathrm{WM})^2 + \underset{(6.2)}{0.083} (\ln \mathrm{WE})^2$$

$$+ \underset{(6.5)}{0.072} (\ln \mathrm{WV})^2 + \underset{(2.6)}{0.166} (\ln \mathrm{TKT})^2 - \underset{(-2.1)}{0.829} (\ln \%\mathrm{TKP})^2 + \underset{(5.9)}{0.555} (\ln \mathrm{DEN})^2$$

$$- \underset{(-0.1)}{0.001} \ln \mathrm{WM} \bullet \ln \mathrm{WE} + \underset{(1.0)}{0.009} \ln \mathrm{WM} \bullet \ln \mathrm{WV} - \underset{(-4.1)}{0.081} \ln \mathrm{WM} \bullet \ln \mathrm{TKT}$$

$$- \underset{(-4.2)}{0.636} \ln \mathrm{WM} \bullet \ln \%\mathrm{TKP} + \underset{(1.1)}{0.055} \ln \mathrm{WM} \bullet \ln \mathrm{DEN} - \underset{(-10.5)}{0.082} \ln \mathrm{WE} \bullet \ln \mathrm{WV}$$

$$+ \underset{(1.6)}{0.023} \ln \mathrm{WE} \bullet \ln \mathrm{TKT} + \underset{(1.7)}{0.237} \ln \mathrm{WE} \bullet \ln \%\mathrm{TKP} - \underset{(-1.6)}{0.070} \ln \mathrm{WE} \bullet \ln \mathrm{DEN}$$

$$+ \underset{(4.0)}{0.058} \ln \mathrm{WV} \bullet \ln \mathrm{TKT} + \underset{(3.6)}{0.400} \ln \mathrm{WV} \bullet \ln \%\mathrm{TKP} + \underset{(0.4)}{0.015} \ln \mathrm{WV} \bullet \ln \mathrm{DEN}$$

$$+ \underset{(1.4)}{0.120} \ln \mathrm{TKT} \bullet \ln \%\mathrm{TKP} - \underset{(-4.0)}{0.417} \ln \mathrm{TKT} \bullet \ln \mathrm{DEN} + \underset{(0.4)}{0.091} \ln \%\mathrm{TKP} \bullet \ln \mathrm{DEN}$$

$$+ \underset{(0.9)}{0.17} \mathrm{DBR} + \underset{(5.8)}{0.53} \mathrm{DCFF} + \underset{(0.9)}{0.12} \mathrm{DCIE} + \underset{(0.1)}{0.03} \mathrm{DDB} + \underset{(6.5)}{0.52} \mathrm{DDSB} + \underset{(2.2)}{0.35} \mathrm{DFS}$$

$$+ \underset{(3.5)}{0.31} \mathrm{DNS} + \underset{(2.7)}{0.12} \mathrm{DNSB} + \underset{(15.5)}{0.80} \mathrm{DQBB} - \underset{(-3.9)}{0.30} \mathrm{DSJ} + \underset{(11.9)}{0.75} \mathrm{DSNCB} - \underset{(-1.2)}{0.36} \mathrm{DSNCF}$$

$$R^2 = 0.9990 \qquad F = 7687.3$$

J. TRANSLOG MODEL 27:

RTC = f(*WM*,WE,WV,*TKT*,*YEAR*,*DEN*,dummies)

K. TRANSLOG MODEL 28:
RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*YEAR*,*DEN*,dummies)

$$\ln \mathrm{RTC} =$$

$$\underset{(-7.4)}{-2638.1} + \underset{(2.1)}{7.725}\ln \mathrm{WM} + \underset{(0.5)}{1.474}\ln \mathrm{WE} - \underset{(-3.6)}{8.200}\ln \mathrm{WV} + \underset{(2.5)}{3.207}\ln \mathrm{TKT}$$

$$- \underset{(-0.6)}{7.765}\ln \%\mathrm{TKP} + \underset{(7.3)}{2.624}\mathrm{YEAR} - \underset{(-1.1)}{5.606}\ln \mathrm{DEN} - \underset{(-3.3)}{0.13}(\ln \mathrm{WM})^2 + \underset{(4.5)}{0.057}(\ln \mathrm{WE})^2$$

$$+ \underset{(4.2)}{0.050}(\ln \mathrm{WV})^2 + \underset{(2.5)}{0.173}(\ln \mathrm{TKT})^2 - \underset{(-2.4)}{0.965}(\ln \%\mathrm{TKP})^2 - \underset{(-7.3)}{0.001}(\mathrm{YEAR})^2 + \underset{(6.7)}{0.588}(\ln \mathrm{DEN})^2$$

$$- \underset{(0.3)}{0.003}\ln \mathrm{WM} \bullet \ln \mathrm{WE} + \underset{(1.3)}{0.010}\ln \mathrm{WM} \bullet \ln \mathrm{WV} - \underset{(-3.1)}{0.054}\ln \mathrm{WM} \bullet \ln \mathrm{TKT}$$

$$- \underset{(-2.0)}{0.287}\ln \mathrm{WM} \bullet \ln \%\mathrm{TKP} - \underset{(-1.8)}{0.003}\ln \mathrm{WM} \bullet \mathrm{YEAR} - \underset{(-7.0)}{0.059}\ln \mathrm{WE} \bullet \ln \mathrm{WV}$$

$$- \underset{(-0.9)}{0.039}\ln \mathrm{WM} \bullet \ln \mathrm{DEN} - \underset{(-0.5)}{0.018}\ln \mathrm{WE} \bullet \ln \mathrm{DEN} + \underset{(1.7)}{0.058}\ln \mathrm{WV} \bullet \ln \mathrm{DEN}$$

$$- \underset{(-3.6)}{0.433}\ln \mathrm{TKT} \bullet \ln \mathrm{DEN} + \underset{(1.4)}{0.004}\mathrm{YEAR} \bullet \ln \mathrm{DEN} - \underset{(-0.8)}{0.177}\ln \%\mathrm{TKP} \bullet \ln \mathrm{DEN}$$

$$+ \underset{(1.0)}{0.014}\ln \mathrm{WE} \bullet \ln \mathrm{TKT} - \underset{(-0.5)}{0.061}\ln \mathrm{WE} \bullet \ln \%\mathrm{TKP} - \underset{(-0.5)}{0.001}\ln \mathrm{WE} \bullet \mathrm{YEAR}$$

$$+ \underset{(3.0)}{0.40}\ln \mathrm{WV} \bullet \ln \mathrm{TKT} + \underset{(3.5)}{0.349}\ln \mathrm{WV} \bullet \ln \%\mathrm{TKP} + \underset{(3.5)}{0.004}\ln \mathrm{WV} \bullet \mathrm{YEAR}$$

$$+ \underset{(2.2)}{0.180}\ln \mathrm{TKT} \bullet \ln \%\mathrm{TKP} - \underset{(-2.4)}{0.002}\ln \mathrm{TKT} \bullet \mathrm{YEAR} + \underset{(0.6)}{0.004}\ln \%\mathrm{TKP} \bullet \mathrm{YEAR}$$

$$+ \underset{(2.5)}{0.56}\mathrm{DBR} + \underset{(7.2)}{0.68}\mathrm{DCFF} - \underset{(-0.3)}{0.05}\mathrm{DCIE} + \underset{(1.4)}{0.46}\mathrm{DDB} + \underset{(7.8)}{0.60}\mathrm{DDSB} + \underset{(3.7)}{0.70}\mathrm{DFS}$$

$$+ \underset{(5.3)}{0.55}\mathrm{DNS} + \underset{(1.5)}{0.07}\mathrm{DNSB} + \underset{(18.5)}{0.90}\mathrm{DOBB} - \underset{(1.7)}{0.16}\mathrm{DSJ} + \underset{(13.4)}{0.91}\mathrm{DSNCB} + \underset{(0.1)}{0.03}\mathrm{DSNCF}$$

$$R^2 = 0.9994 \qquad F = 8685.7$$

L. TRANSLOG MODEL 26 (IUCM)

RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*DEN*,*YEAR*,dummies).

$$\ln RTC =$$

$$\underset{(3.2)}{2.53} + \underset{(35.75)}{1.043}\ln WM + \underset{(0.8)}{0.008}\ln WE - \underset{(-2.0)}{0.051}\ln WV - \underset{(-0.5)}{1.190}\ln TKT$$

$$+ \underset{(1.8)}{0.082}\ln \%TKP + \underset{(1.0)}{0.417}\ln DEN - \underset{(-11.6)}{3.12}\ln YEAR$$

$$+ \underset{(7.4)}{0.031}(\ln WM)^2 + \underset{(4.6)}{0.005}(\ln WE)^2$$

$$+ \underset{(6.2)}{0.013}(\ln WV)^2 + \underset{(1.9)}{2.670}(\ln TKT)^2 - \underset{(-2.6)}{0.025}(\ln TKP)^2 + \underset{(6.6)}{0.779}(\ln DEN)^2$$

$$- \underset{(-6.0)}{0.011}\ln WM \bullet \ln WE - \underset{(-6.6)}{0.020}\ln WM \bullet \ln WV - \underset{(-2.9)}{0.092}\ln WM \bullet \ln TKT$$

$$+ \underset{(2.4)}{0.020}\ln WM \bullet \ln \%TKP - \underset{(-0.1)}{0.003}\ln WM \bullet \ln DEN + \underset{(4.6)}{0.006}\ln WE \bullet \ln WV$$

$$+ \underset{(0.2)}{0.002}\ln WE \bullet \ln TKT - \underset{(-1.4)}{0.006}\ln WE \bullet \ln \%TKP - \underset{(-1.3)}{0.015}\ln WE \bullet \ln WV$$

$$+ \underset{(0.2)}{0.089}\ln WV \bullet \ln TKT - \underset{(-2.4)}{0.013}\ln WV \bullet \ln \%TKP + \underset{(-1.3)}{0.018}\ln WV \bullet \ln DEN$$

$$- \underset{(-0.2)}{0.014}\ln TKT \bullet \ln \%TKP - \underset{(-2.2)}{2.191}\ln TKT \bullet \ln DEN - \underset{(-0.4)}{0.014}\ln \%TKP \bullet \ln DEN$$

$$+ \underset{(2.5)}{0.073}BR + \underset{(5.5)}{0.081}CFF + \underset{(0.8)}{0.016}CIE + \underset{(1.1)}{0.051}DB + \underset{(5.9)}{0.076}DSB + \underset{(3.4)}{0.084}FS$$

$$+ \underset{(4.1)}{0.061}NS + \underset{(2.1)}{0.014}NSB + \underset{(15.0)}{0.126}OBB - \underset{(-3.3)}{0.040}SJ + \underset{(11.4)}{0.118}SNCB - \underset{(-0.3)}{0.02}SNCF$$

$$R^2 = 0.9989 \qquad F = 6424.7$$

M. TRANSLOG MODEL 29:
RTC = f(*WM*,WE,WV,*TKT*,*%TKP*,*LL*,*YEAR*,dummies)

$$\ln \text{RTC} =$$

$$\underset{(3.2)}{2.53} + \underset{(35.8)}{1.043} \ln \text{WM} + \underset{(1.9)}{0.007} \ln \text{WE} - \underset{(-2.0)}{0.051} \ln \text{WV} - \underset{(0.5)}{0.557} \ln \text{TKT}$$

$$+ \underset{(1.8)}{0.083} \ln \%\text{TKP} - \underset{(-1.0)}{1.329} \ln \text{LL} - \underset{(-11.6)}{3.119} \text{YEAR}$$

$$+ \underset{(7.4)}{0.031} (\ln \text{WM})^2 + \underset{(4.6)}{0.005} (\ln \text{WE})^2$$

$$+ \underset{(6.2)}{0.014} (\ln \text{WV})^2 + \underset{(7.2)}{6.692} (\ln \text{TKT})^2 - \underset{(-2.6)}{0.025} (\ln \%\text{TKP})^2 + \underset{(6.6)}{7.638} (\ln \text{LL})^2$$

$$- \underset{(-6.0)}{0.011} \ln \text{WM} \bullet \ln \text{WE} - \underset{(-6.6)}{0.012} \ln \text{WM} \bullet \ln \text{WV} - \underset{(-1.3)}{0.104} \ln \text{WM} \bullet \ln \text{TKT}$$

$$- \underset{(2.4)}{0.020} \ln \text{WM} \bullet \ln \%\text{TKP} + \underset{(0.1)}{0.009} \ln \text{WM} \bullet \ln \text{LL} - \underset{(4.6)}{0.006} \ln \text{WE} \bullet \ln \text{WV}$$

$$+ \underset{(-1.6)}{0.061} \ln \text{WE} \bullet \ln \text{TKT} + \underset{(-1.4)}{0.006} \ln \text{WE} \bullet \ln \%\text{TKP} + \underset{(1.3)}{0.048} \ln \text{WE} \bullet \ln \text{LL}$$

$$+ \underset{(2.9)}{0.164} \ln \text{WV} \bullet \ln \text{TKT} - \underset{(-2.4)}{0.014} \ln \text{WV} \bullet \ln \%\text{TKP} - \underset{(1.2)}{0.057} \ln \text{WV} \bullet \ln \text{DEN}$$

$$+ \underset{(-0.6)}{0.077} \ln \text{TKT} \bullet \ln \%\text{TKP} - \underset{(-5.7)}{13.071} \ln \text{TKT} \bullet \ln \text{LL} + \underset{(0.4)}{0.049} \ln \%\text{TKP} \bullet \ln \text{LL}$$

$$+ \underset{(2.5)}{0.07} \text{DBR} + \underset{(5.5)}{0.08} \text{DCFF} + \underset{(0.8)}{0.02} \text{DCIE} + \underset{(1.1)}{0.05} \text{DDB} + \underset{(5.9)}{0.07} \text{DDSB} + \underset{(3.4)}{0.08} \text{DFS}$$

$$+ \underset{(4.1)}{0.06} \text{DNS} + \underset{(2.1)}{0.01} \text{DNSB} + \underset{(15.0)}{0.13} \text{DOBB} - \underset{(-3.2)}{0.04} \text{DSJ} + \underset{(11.4)}{0.12} \text{DSNCB} - \underset{(-0.3)}{0.01} \text{DSNCF}$$

$$R^2 = 0.9989 \qquad F = 6424.7$$