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Dynamic Tube Control for DC Microgrids

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***Abstract*— This paper proposes a dynamic tube control approach for DC Microgrids (MGs) connected to constant power loads (CPL) that guarantees boundedness of the system dynamics, satisfaction of the desired operational constraints and closed-loop stability. Contrary to many approaches in the literature, we consider an explicit model of the dynamics to investigate the geometric effect of the load demand perturbations on the behaviour of the closed loop system. Combined with the use of nominal dynamics, *i.e.* dynamics parametrized by a constant load demand, this allows us to formulate, for the first time, necessary conditions for the existence of a tube around a nominal solution that bounds all possible uncertain trajectories stemming from perturbations of the load demand. Furthermore, we show that the computation of the tube follows a fully decentralized approach and its size is dependent on the nominal dynamics, which we use in the regulation of the nominal solution to reduce the conservativeness of the controller. The effectiveness of the proposed control architecture is illustrated in a simulated scenario.**

***Index Terms*— microgrids, nonlinear control, robust control, networked systems**

I. Introduction

THE DC MG is a key element of the future smart power grid, exhibiting efficiency and reliability advantages over its AC counterpart [1]. One of the main challenges faced by DC MGs is the presence of constant power loads (CPLs) in the network. CPLs exhibit incremental negative impedance characteristics that reduce system damping and can introduce instabilities to the power system [2]. To address this, numerous studies investigate either small-signal methods [3] or variants of the droop control that introduce system robustness [4]. However, approximation methods only provide local stability guarantees, while most of the proposed studies omit explicit dynamical modelling.

In the context of MGs, system constraints take the form of current and voltage limits dictated by the physical limitations of the converters. It is also often desirable to adhere to predefined operational ranges that provide protection against large transients; in fact, this is especially important in the presence of CPLs, which induce high state transients during abrupt load fluctuations. Robust constrained control, which permits robustly stable operation within a constraint set, is an attractive solution to this problem. Specifically, the concept of robust tube-based MPC has gained increasing popularity as it allows to combine the advantages of both robust and optimal constrained control in a unified framework. In simple words, tube-based MPC refers to a collection of control schemes that utilize information on the system uncertainty to bound the system trajectories in a positive invariant set and drive this set to a reference subspace. A tube is then the sequence of sets containing every trajectory emanating from some common initial point. Substantial work has been devoted in the linear case, dating back from incorporating the minimum robust positive invariant set within the tube computation in [5], which evolved to dynamic tube in the case of parametric uncertainty in [6]. The linear approaches require the solution of the system dynamics to compute the constraint sets; this prohibits the direct extension to the nonlinear case, since a solution may not even exist. A nonlinear tube-based MPC was proposed in [7], where the computation of the tube was based on the global Lipschitz constant of the dynamics. In [8], feedback linearization was used to cancel the effect of the nonlinear terms, while in [9] a sliding-mode controller was used to bound the system trajectories and formulate a dynamic evolution of the tube. In [10] and [11], control contraction metrics were used to respectively minimize the effect of the disturbance on the system dynamics and to define an incremental Lyapunov function to parametrise the dynamic tube.

Considering the inherent problems of including CPLs in the network, a need for the extension of a dynamic tube-based control architecture arises naturally. To the authors' best knowledge, despite the potential advantages, such a control scheme is yet to be developed for the regulation of MGs. In contrast to other nonlinear dynamic tube-based approaches, we focus directly on the regulation of MGs with CPLs to reduce conservativeness and provide suitable tuning guidelines that achieve robustness to perturbations of the load demand, and overall system stability. Furthermore, we adopt an explicit system model and use this to characterize the geometric effect of the CPL on the system trajectories. In a previous work of the authors, a tube-based approach was proposed in [12], where the tube formulation was based on local input-to-state stability property of the dynamics, when the maximum effect of the disturbance was considered. This work was extended in [13], wherein a tube-based control is proposed considering a time-invariant size of the tube. In this paper, we significantly extend this work by incorporating the nominal dynamics, *i.e.* the dynamics parametrized by a nominal constant load, inside the tube computation. Thus, instead of considering the nominal state as an unknown bounded external input in the computation

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of the tube, we allow the tube boundary to evolve according to the distance of the nominal state from the boundary of the constraint set. In particular, we investigate the geometric effect of the load demand on the system dynamics to derive necessary conditions on the tuning parameters such that a uniform bound around the nominal dynamics exists and obtains the desired positive invariance properties. The derivation of the bounding functions follows a decoupling of the network dynamics to provide a fully decentralized computation of the dynamic tube that solely depends on locally available parameters. This dynamic tube allows us to parametrize the original constraint sets, such that the true dynamics are always contained within the desired operational range. We combine the above to introduce a framework for the control of DC MGs, that protects the converter components even during transients and achieve a desired network operation.

A. Notation

A MG can be seen as an undirected connected graph $\mathcal{G} = (\mathcal{M}, \mathcal{E})$ with set of nodes $\mathcal{M}$ representing a collection of power converters and local loads. The notation $\varepsilon = (i, j) \in \mathcal{E}$ denotes the edge connecting node i and j, where $\mathcal{E} \subseteq \mathcal{M} \times \mathcal{M}$ is the set of edges and (i, j) is an unordered pair. Then, $a \in \mathbb{R}^n$ denotes an n-dimensional vector and $[a] \in \mathbb{R}^{n\times n}$ is a diagonal matrix with $[a]_{ii} = a_i$. A continuous function $\alpha(\cdot)$ is said to belong to class $\mathcal{K}$ is it is strictly increasing and $\alpha(0) = 0$. A continuous function $\beta(\cdot)$ is said to be class $\mathcal{K}_\infty$ if it is class $\mathcal{K}$ and $\lim_{x\to\infty} \beta(x) = \infty$.

II. Problem Statement

A common representation of a meshed DC Microgrid is by a connected, undirected graph consisting of n number of nodes, where n is the number of local converters of the network. In this paper, we investigate the case, where each node $i \in \mathcal{M} := \{1, 2, \dots, n\}$ feeds a local CPL. Our aim is to introduce robustness of the network to perturbations of the load demand and guarantee restriction of the dynamics in a predefined operational range. The node voltage dynamics are formulated as,

$$C_i \frac{dv_i}{dt} = i_{in,i} - i_{o,i}, \tag{1}$$

where C_i is the capacitance of the output capacitor, $i_{in,i}$ is the control input current and $i_{o,i}$ is the output current flowing in the lines of the network and into the local CPL. The connections among the nodes of the graph can be represented by the weighted adjacency matrix $A(R) \in \mathbb{R}^{n\times n}$, where $a_{ij} = R_{ij}^{-1}$, with R_{ij}^{-1} the admittance of the line between nodes i and j, and $a_{ij} = 0$ if the edge (i, j) is not incident. It has been previously shown in the literature that cable inductance has no effect on the system stability [14]. Therefore, for simplicity, it is assumed in this study that the lines are purely resistive. The full topology of the network is represented by the Laplacian matrix $\mathcal{L} = [A(R)\mathbb{1}_n] - A(R)$. Therefore, the output network current can be modelled as

$$i_{o,i} = \frac{P_i}{v_i} + \mathcal{L}_{ii} v_i + \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} v_j, \tag{2}$$

where P_i represents the load demand and is assumed to be bounded in a compact set as $P_i \in \mathbb{P}_i \subset \mathbb{R}$. The control objective is to design and analyse a feedback control law for the current, $i_{in,i}$, at each node in the MG. The control laws should stabilize the system currents and voltages, despite unknown but bounded changes to the load, and moreover respect predefined operational limits. The latter are formulated as uncoupled state and input constraint sets denoted $\mathbb{X}_i$ and $\mathbb{C}_i$ respectively, where our aim is to guarantee that $v_i \in \mathbb{X}_i$ and $i_{in,i} \in \mathbb{C}_i$ at all times.

Assumption 1: It holds that $\mathbb{C}_i, \mathbb{X}_i \subset \mathbb{R}$ are compact and convex and $\{0\} \in \mathbb{C}_i$.

III. A Dynamic Tube-Based Voltage Control Law

A. Voltage Control Law

In this paper, we will decompose the dynamics into a nominal voltage and an error between the nominal and the uncertain state. We will exploit the form of the dynamics to show that the error is always contained in a positive invariant set provided that some conditions on the choice of the control parameters are satisfied. First, considering a constant nominal power demand $\bar{P}_i \in \mathbb{P}_i$, the network dynamics can be rewritten with respect to the deviation from the nominal load as

$$C_i \frac{dv_i}{dt} = i_{in,i} - \mathcal{L}_{ii} v_i - \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} v_j - \frac{\bar{P}_i}{v_i} - \frac{\delta P_i}{v_i}, \tag{3}$$

In order to satisfy the assumption $P_i \in \mathbb{P}$ we require $\delta P_i \in \mathbb{W}_i$ and $\{\bar{P}_i\} \oplus \mathbb{W}_i \subseteq \mathbb{P}_i$. Inspired by the tube-based approach to robust control, we propose and study a voltage control law of the form

$$i_{in,i} := -(K_{1,i} + K_{2,i}) e_i + u_i. \tag{4}$$

for $i \in \mathcal{M}$, where $K_{1,i}, K_{2,i} \in \mathbb{R}_{>0}$ are control gains. In addition,

$$e_i := v_i - z_i, \tag{5}$$

is the error between the voltage at node i and a corresponding nominal voltage, z_i, and u_i is a nominal control input. The pair (u_i, z_i) obey a nominal, i.e. disturbance free, version of the nodal voltage dynamics that can be written as

$$C_i \frac{dz_i}{dt} = u_i - \mathcal{L}_{ii} z_i - \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} z_j - \frac{\bar{P}_i}{z_i} \tag{6}$$

Therefore, the control law (4) is similar to a tube-based control law where the nominal input induces the system voltage to follow a nominal trajectory while the error feedback term mitigates the effect of uncertainties arising from the unknown fluctuations in the load being omitted in the determination of the nominal control input. Our aim is to study and analyse the use of this control law in the previously defined MG dynamics, and formulate conditions under which the currents and voltages satisfy the constraints and the voltage errors remain bounded. To expose the particular challenge, we substitute the control law (4) into (3) to yield,

$$C_i \frac{de_i}{dt} = -(\mathcal{L}_{ii} + K_{1,i} + K_{2,i}) e_i - \sum_{j\in\mathcal{N}_i} \mathcal{L}_{ij} e_j - \frac{\bar{P}_i}{e_i + z_i} + \frac{\bar{P}_i}{z_i} - \frac{\delta P_i}{e_i + z_i}, \tag{7}$$

B. Boundedness of the voltage within a robust positive invariant set

This section will introduce sufficient conditions on the controller parameters such that the error dynamics are bounded within a robust positive invariant RPI set. This section will initially provide a characterization of a candidate positive invariant error subspace by employing an energy-like function of the dynamics. Then, the behaviour of this function will be investigated in order to deduce sufficient lower bounds on both the control parameters and the nominal state trajectory, such that the desired closed-loop behaviour is guaranteed.

Observing the form of the dynamics in (7), it can be seen that the disturbance δP enters the system as a parametric disturbance where the magnitude of its effect also depends on the states e_i and z_i respectively. Now, for simplicity assume that $(K_{1,i} + K_{2,i}) = (K_{1,j} + K_{2,j})$ holds for all $(i, j) \in \mathcal{E}$. Then, consider the network error dynamics given by

$$C\frac{de}{dt} = -\left(\mathcal{L} + [K_{1,i}] + [K_{2,i}]\right)e - \left[\frac{\bar{P}_i}{e_i + z_i} + \frac{\bar{P}_i}{z_i} - \frac{\delta P_i}{e_i + z_i}\right] \tag{8}$$

where $C = \text{diag}\{C_1,\ C_2, \dots,\ C_n\}$, and a quadratic energy-like function $V(e) = \frac{1}{2}e^\top Ce$. Due to the properties of the Laplacian matrix, it holds that $\mathcal{L} + [K_{2,i}] \succ 0$ for any arbitrary small positive value of $K_{2,i}$. This yields a decoupled upper bound on the derivative of the energy-like function as

$$\frac{dV}{dt}(e) \leq \sum_{i\in\mathcal{M}} g_i(e_i, z_i), \tag{9}$$

where

$$g_i(e_i, z_i) = -K_{1,i}e_i^2 + \frac{\bar{P}_i}{z_i(e_i + z_i)}e_i^2 + w_i\frac{|e_i|}{e_i + z_i}, \tag{10}$$

and $w_i = \max|\delta P_i|$. Since the upper bound in (9) is a summation of individual functions at each node, the stability problem is decoupled and it suffices to investigate the properties of each scalar function $g_i(\cdot)$ to deduce positive invariance of the MG dynamics. Therefore, the problem of analysing stability and boundedness of the whole network dynamics is reduced to that of a single node problem. Next, we will first investigate the choice of control parameters such that (10) admits real roots. This will later be helpful to establish the existence of a closed RPI set around the origin.

Proposition 1: Consider the bound on the time derivative in (9), if the feedback gain and the nominal state satisfy respectively

$$K_{1,i} > \frac{\sqrt{(\bar{P}_i + 4w_i)^2 - 12w_i^2} - 2\sqrt{w_i(\bar{P}_i + w_i)}}{\bar{P}_i + 4w_i}$$

and

$$z_i \geq \beta_i$$

where

$$\beta_i = \sqrt{\frac{2\bar{P}_i + 4w_i + 4\sqrt{w_i(\bar{P}_i + w_i)}}{2K_{1,i}}}$$

then (10) admits two negative and one positive non-zero real roots.

Proof: For now we will assume that the nominal voltage is strictly greater than the error and thus $g_i(e_i, z_i)$ is continuous. However, we will later formulate conditions that guarantee this property. The non-zero roots of (10) are given by solving

$$K_{1,i}z_i|e_i|e_i + |e_i|(K_{1,i}z_i^2 - \bar{P}_i) - z_iw_i = 0. \tag{11}$$

We distinguish two cases: (a) $e_i > 0$ and (b) $e_i < 0$. For (a) the above yields

$$\Delta_1 = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 + 4K_{1,i}z_i^2w_i}}{2K_{1,i}z_i}$$

$$\Delta_2 = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) - \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 + 4K_{1,i}z_i^2w_i}}{2K_{1,i}z_i}.$$

Note that $(K_{1,i}z_i^2 - \bar{P}_i)^2 + 4K_{1,i}z_i^2w_i$ is always positive as it is a summation of positive terms. In order to show that $\Delta_1 > 0$, we require

$$\left|\sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 + 4K_{1,i}z_i^2w_i}\right| > \left|(K_{1,i}z_i^2 - \bar{P}_i)\right|.$$

This results in

$$4K_{1,i}z_i^2w_i > 0$$

which is indeed true and hence $\Delta_1 > 0$. Similarly, it can be shown that $\Delta_2 < 0$ which contradicts the assumption $e_i > 0$ and therefore is discarded as a root of the polynomial. Then, in the case of (b) we obtain

$$\Delta_3 = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2w_i}}{2K_{1,i}z_i}$$

$$\Delta_4 = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) - \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2w_i}}{2K_{1,i}z_i}.$$

Here, it is not necessarily true that the term under the square root is non-negative. In order to guarantee existence of real roots we impose the condition,

$$(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2w_i \geq 0,$$

or

$$K_{1,i}^2z_i^4 - z_i^2(2\bar{P}_iK_{1,i} + 4K_{1,i}w_i) + \bar{P}_i^2 \geq 0.$$

The largest root of the above exists and is given by,

$$\beta_i = \sqrt{\frac{2\bar{P}_i + 4w_i + 4\sqrt{w_i(\bar{P}_i + w_i)}}{2K_{1,i}}}$$

which results in the desired condition for the bound on the nominal state z_i. Consider the special case where $z_i = \beta_i$, then the requirement $e_i < 0$ yields

$$\frac{-K_{1,i}\beta_i^2 + \bar{P}_i}{2K_{1,i}z_i} < 0.$$

Solving the above with respect to $K_{1,i}$ and substituting with the explicit form of β_i results in the necessary condition

$$K_{1,i} > \frac{\sqrt{(\bar{P}_i + 4w_i)^2 - 12w_i^2} - 2\sqrt{w_i(\bar{P}_i + w_i)}}{\bar{P}_i + 4w_i}.$$

Note that the nominator of the above is always positive. Then, similarly to case (a) it can be shown that both roots Δ_3 and Δ_4 are negative and $\Delta_4 < \Delta_3$. ∎

The proof of Prop.1 reveals the existence of one negative and one positive root of $g_i(e_i, z_i)$ around the origin given by the functions $\alpha_1, \alpha_2\colon\ [\beta_i, \infty) \to \mathbb{R}$, where

$$\alpha_1(z_i) = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 + 4K_{1,i}z_i^2 w_i}}{2K_{1,i}z_i}, \tag{12a}$$

$$\alpha_2(z_i) = \frac{-(K_{1,i}z_i^2 - \bar{P}_i) + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2 w_i}}{2K_{1,i}z_i}. \tag{12b}$$

We are now able to define an explicit form of a candidate RPI set using (12a) and (12b), *i.e.*

$$\mathcal{S}(z_i) := \{e_i \in \mathbb{R}^n\colon\ \alpha_2(z_i) \leq e_i \leq \alpha_1(z_i)\}. \tag{13}$$

Therefore, the canditate RPI set is "dynamic" in that its size depends on z_i; thus, by regulating z_i one can effect the size of the tube and, equivalently, the extent to which the load uncertainty effects the voltage. In fact, the significance of Prop. 1 is that the tube width is a monotonically decreasing function of the nominal voltage; thus, the effects of load uncertainty can be reduced by raising nominal voltage levels. This is demonstrated by the following result, where we investigate the behaviour of the bounding functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, that later will also be useful in proving the desired positive invariant result.

Lemma 1: For all z_i that satisfy Proposition 1, the function $\alpha_1(z_i)$ (resp. $\alpha_2(z_i)$) is a strictly decreasing (resp. strictly increasing) function of z_i, which attains a maximum (resp. minimum) at $z_i = \beta_i$.

Proof: The derivative of (12a) is given by,

$$\frac{\partial \alpha_1(z_i)}{\partial z_i} = -\frac{\left(\bar{P}_i + K_{1,i}z_i^2\right)\left(-\gamma_i + \sqrt{\gamma_i^2 + 4K_{1,i}z_i^2 w_i}\right)}{2Kz^2\sqrt{\gamma_i^2 + 4K_{1,i}z_i^2 w_i}}. \tag{14}$$

where $\gamma_i = K_{1,i}z_i^2 - \bar{P}_i$. We have shown in the proof of Prop.1 that the second parenthesis of the nominator is always positive, hence we can conclude that $\frac{\partial \alpha_1(z_i)}{\partial z_i} < 0$ and $\alpha_1(z_i)$ is a strictly decreasing in its domain. Therefore, it immediately follows that it obtains a maximum at $z_i = \beta_i$. Similarly, we can show that $\alpha_2(\cdot)$ is strictly increasing and obtains a minimum value when $z_i = \beta_i$. ■

The next step is to address the continuity issue of the bounding function (10), *i.e.* the discontinuity at the point $e_i = -z_i$. The next result establishes that if $z_i \geq \beta_i > 0$, then $e_i < z_i$ is guaranteed; thus, continuity of (10) is established for $z_i \geq \beta_i$ and $e_i \in S(z_i)$.

Lemma 2: Consider the error (7) and nominal (6) dynamics respectively, if z_i satisfies Proposition 1, then the relation $z_i > e_i$ is guaranteed for all $e_i \in \mathcal{S}(z_i)$ and all $i \in \mathcal{M}$.

Proof: Using Lemma 1, we can formulate the necessary condition

$$z_i > \frac{K_{1,i}\beta_i^2 - \bar{P}_i}{2K_{1,i}\beta_i}.$$

Using Prop. 1, at worst case scenario we obtain $z_i = \beta_i$, which yields

$$K_{1,i}\beta_i^2 > -\bar{P}.$$

This is always true and therefore $z_i + e_i > 0$ holds for all $e_i \in \mathcal{S}_i(z_i)$ and $z_i \geq \beta_i$. ■

We are now ready to present the main result of this paper. The set $\mathcal{S}(z_i)$ is a RPI set for the error dynamics, when the previously established conditions on the choice of tuning parameters are met. This is shown in the following.

Theorem 1: The set $\mathcal{S}(z_i)$ is a robust positive invariant set for the error dynamics (7).

Proof: Using Prop. 1 and Lemma 2, we need to show that the part of (10) that does not include the disturbance is strictly negative. We will prove this by contradiction. Assume that for all $e_i \in \mathcal{S}(z_i)$ it holds that

$$-\left(K_{1,i} - \frac{\bar{P}_i}{z_i(e_i + z_i)}\right)e_i^2 > 0.$$

This implies that in $\mathcal{S}(z_i)$ we have

$$K_{1,i} - \frac{\bar{P}_i}{z_i(e_i + z_i)} < 0.$$

Note that in Lemma 2 we have guaranteed that $z_i(e_i + z_i) > 0$. Therefore,

$$e_i < \frac{\bar{P}_i - K_{1,i}z_i^2}{K_{1,i}z_i}.$$

Substituting for the bounding function (12b) the above imply

$$\frac{-(K_{1,i}z_i^2 - \bar{P}_i) + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2 w_i}}{2K_{1,i}z_i} < \frac{-\bar{P}_i - K_{1,i}z_i^2}{K_{1,i}z_i} \tag{15}$$

and thus

$$K_{1,i}z_i^2 - \bar{P}_i + \sqrt{(K_{1,i}z_i^2 - \bar{P}_i)^2 - 4K_{1,i}z_i^2 w_i} < 0.$$

However, it can be seen that $K_{1,i}z_i^2 - \bar{P}_i > 0$ by substituting $z_i = \beta_i$. This leads to a contradiction as the left hand side of the inequality is always positive and therefore the nominal term of (10) is negative for all $e_i \in \mathcal{S}(z_i)$. Furthermore, it is straightforward to show that the derivative of (10) does not vanish at the boundary points. Finally, using the fact that the third summand of (10) containing the disturbance is non-negative, we can conclude that $\mathcal{S}(z_i)$ is a robust positive invariant set for the error dynamics. This completes the proof. ■

The advantage of the proposed technique is that the respective bounding functions (12a) and (12b) on the local error dynamics depend solely on locally available information at the i^{th} node. In addition, the positive invariance property of the error dynamics can be used to formulate a type of Tube-MPC control scheme similar to [13], where the uncertainty-free nominal dynamics are regulated to desired equilibria, while satisfying a "tightened" version of the original constraint sets, *i.e.*

$$\mathbb{Z}_i(z_i) = \mathbb{X}_i \ominus \mathcal{S}_i(z_i), \tag{16a}$$

$$\mathbb{U}_i(z_i) = \mathbb{C}_i \ominus (-K_{1,i})\mathcal{S}_i(z_i), \tag{16b}$$

where $\mathbb{Z}(z)$, $\mathbb{U}(z)$ are the respective nominal constraint sets.

Remark 1: At this point, it is important to highlight the interaction between the feedback gain and the sizes of the voltage and current constraint sets. Higher values of the gain $K_{1,i}$ result in a "less tightened" state constraint set, which, however, inversely affects the size of the input current constraint set. This can be seen by the constraint set parametrisation in (16). Therefore, the choice of the gain $K_{1,i}$ needs to made according to the each individual case-study specifications to achieve the desired result and guarantee that the above constraint sets are non-empty. Furthermore, the condition $z_i \geq \beta_i$ needs to be incorporated inside the constraint set either in a direct manner, *i.e.* by intersecting the original constraint set, or indirectly by enforcing a larger positive lower bound on the nominal state. While the above may seem conservative, in practice the majority of the applications require a substantially higher voltage than the one resulting from the lower bound $z_i \geq \beta_i$. Ultimately, this also allows for the consideration of non-empty constraint sets, since the tube width is a monotonically decreasing function of the nominal voltage.

C. Closed-loop stability

In the previous section, we have shown boundedness of the error dynamics by considering the nominal state as an input. In the sequel, we show how to derive the stability of the decomposed dynamics. First, we assume the nominal dynamics can be driven to the desired reference point while satisfying the parametrised constraint sets from (16). This is formalised in the following assumption,

Assumption 2: There exist a control law $u_i(z_i)$, such that the solution of the nodal nominal dynamics (6) is driven to a desired, admissible equilibrium point $\bar{z}_i$, while satisfying the respective input and state constraint sets (16) at all times.

This assumption states that the proposed method can be combined with any admissible and stabilizing control law for (6). Initially, this may seem a strong assumption, however one needs to consider that the uncertainty of the dynamics is dealt within the proposed control method. Therefore, the nominal dynamics are uncertainty-free, hence there exist a plethora of different methods that satisfy Assumption 2, each with its own merits. For example, one may choose a linear feedback method to compensate the distance of the nominal voltage state from some reference value, which however would result in a relatively small Region of Attraction (RoA). On the other hand, it is possible to achieve larger RoA of the system equilibria by adopting more sophisticated methods, e.g. an MPC-based approach that computes a control action by considering the constraint requirements [13]. The following Theorem formalises the fact that adopting the proposed control method with any control law that satisfies Assumption 2 achieves the desired stability results of the cascaded dynamics.

Theorem 2: Let Assumptions 1 and 2 hold, then the decomposed dynamics given by (6),(7) admit asymptotically stable equilibrium point $(\bar{e}, \bar{z})$ in $\mathcal{S}(\bar{z}) \times \mathbb{Z}(\bar{z}) \subseteq \mathbb{X} \times \mathbb{C}$, where u_i is given by the control law postulated by Assumption 2.

Proof: Boundedness of the error dynamics follows from Theorem 1, by considering the nominal state trajectory $z(t)$ as an exogenous input. Using Assumption 2, the nominal dynamics admit asymptotically stable equilibrium points. Therefore, combining the above and applying [15, Theorem 10.3.1], we can conclude that the equilibrium pair $(\bar{e}, \bar{z})$ is asymptotically stable for the cascaded dynamics. ■

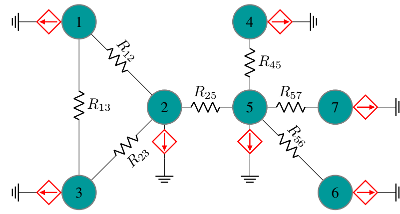

Fig. 1. Meshed MG with seven nodes locally connected to a CPL.

Time [ms]	$a_{1,7}(z_7) - a_{2,7}(z_7)$ [V]	time-invariant tube [V]
0.15	0.4164	0.4735
0.2	0.4048	0.4735
0.6	0.4033	0.4735

TABLE I
COMPARISON BETWEEN THE DYNAMIC AND TIME-INVARIANT TUBE WIDTH AT VARIOUS TIME INSTANTS FROM FIG 4.

IV. SIMULATIONS

In this section, we illustrate the theoretical properties of the proposed control scheme in a simulated scenario of a low-voltage network with topology depicted in Fig. 1. The network voltages are required to operate in the range 22 V $\leq v \leq$ 26 V. The input current magnitude of the converter is constrained as $|i_{in,i}| \leq 15$ A. The nominal load demand is chosen at 100 W, with maximum perturbations restricted in $|\delta P_i| \leq 20$ W. The tube upper and lower boundaries for each node are constructed according to Section III, with $K_{1,i} = 4$, $\forall i \in \mathcal{M}$. The respective lower bounds are computed according to Proposition 1 as $K_{i,lb} = 0.3786$ and $\beta_i = 7.72$ V. A MPC setup similar to [13] is used for the regulation of the nominal dynamics.

Reference changes occur at times $t = 0, 0.5, 1$ ms. Fig. 3 depicts convergence of the nominal trajectories to their respective reference points, while the true voltage is always contained within the dynamic tube. A clear demonstration of the controller properties can be seen in Fig. 2, depicting the state space of Node 1. Starting from initial state $x(t_0) = (v(t_0), i_{in}(t_0))$ the nominal trajectory defines the centre of the tube, with cross-sections shown by the rectangles, and the uncertain trajectory is contained within the Cartesian product of the constraint sets. Table I shows that the tube "shrinks" as the value of the nominal voltage increases, validating the conclusion drawn in Lemma 1 regarding the behaviour of the bounding functions with respect to the nominal voltage variations. In order to show the advantages of the proposed approach, a comparison between the time invariant tube approach from [13] and the dynamic tube are illustrated in Fig. 4 and Table I for the regulation of Node 7. Therein, it is seen that in the same operating conditions the dynamic tube avoids activation of the constraint set and results in a faster response. Despite the small differences in tube sizes, the time-invariant tube approach demonstrates longer convergence time.

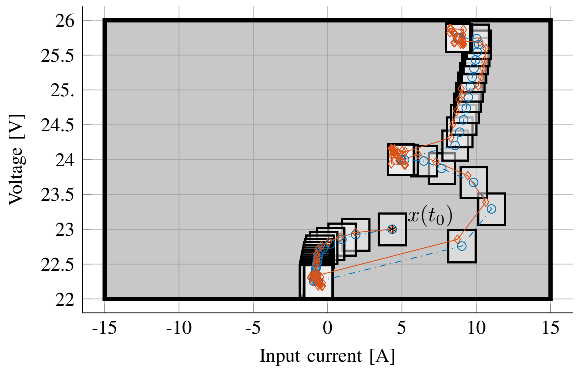

Fig. 2. State space of Node 1. The constraint set $\mathbb{Y} = \mathbb{X} \times \mathbb{C}$ is depicted as ▭ and the tube cross-sections with ▭. The nominal voltage defining the centre of the tube is depicted with (-·-·-) and the uncertain trajectory with (——).

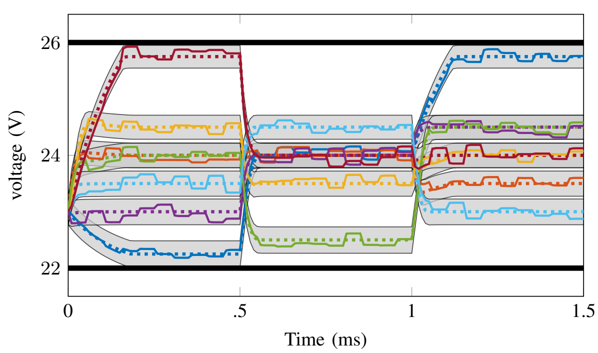

Fig. 3. Node uncertain and nominal voltages of Node 1 (——,······), Node 2 (——,······), Node 3 (——,······), Node 4 (——,······), Node 5 (——,······), Node 6 (——,······), and Node 7 (——,······). The black solid lines represent the upper and lower bound respectively of the nodal constraint set.

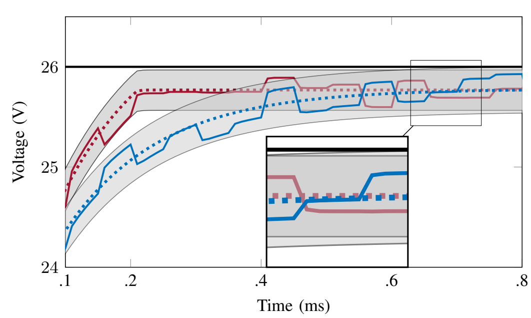

Fig. 4. Voltage comparison between the proposed approach (——) and the time-invariant tube (——) from [13].

V. Conclusions

In this study, we have proposed a dynamic tube-based control scheme for DC MGs with fluctuating load demand. We showed that the structure of the dynamics can be exploited to construct a tube that bounds all possible trajectories, stemming from different load profiles, around a nominal trajectory generated by considering a constant nominal load. We formulated conditions on the tuning parameters and the nominal trajectory such that existence of the tube is guaranteed. It was revealed that the size of the tube depends on the evolution of the nominal trajectory, which was used to reduce conservativeness arising from assuming a worst-case scenario for the nominal voltage. The results of this study were demonstrated in a simulated scenario of a meshed MG, where each node is locally connected to a CPL. Future approaches aim to include load forecasting techniques that estimate the magnitude of load demand perturbations δP_i and analyse the estimation effect on the overall system stability, as well as experimentally test the approach in medium-high voltage setups.

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