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Article

Analysis of Topological Properties and Robustness of Urban Public Transport Networks

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Abstract: With the acceleration of urbanization, public transport networks are an important part of urban transport systems, and their robustness is critical for city operation. The objective of this study is to analyze the topological properties and robustness of an urban public transport network (UPTN) with a view to enhancing the sustainability of urbanization. In order to present the topological structure of the UPTN, the L-Space complex network modeling method is used to construct a model. Topological characteristics of the network are calculated. Based on single evaluation indices of station significance, a comprehensive evaluation index is proposed as the basis for selecting critical stations. The UPTN cascading failure model is established. Using the proportion of the maximum connected subgraph as the evaluation index, the robustness of the UPTN is analyzed using different station significance indices and deliberate attack strategies. The public transport network of Xuzhou city is selected for instance analysis. The results show that the UPTN in Xuzhou city has small-world effects and scale-free characteristics. Although the network has poor connectivity, it is a convenient means to travel for residents with many independent communities. The network's dynamic robustness is demonstrably inferior to its static robustness due to the prevalence of cascading failure phenomena. Specifically, the failure of important stations has a wider impact on the network performance. Improving their load capacity and distributing the routes via them will help bolster the network resistance against contingencies. This study provides a scientific basis and strategic recommendations for urban planners and public transport managers to achieve a more sustainable public transport system.

Keywords: urban public transport network (UPTN); complex network; topological property; cascading failure; robustness

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1. Introduction

With the advantages of convenience, efficiency, and popularity, urban public transport has become a cornerstone of urban infrastructure, and plays a crucial role in the sustainable development of cities [1]. It provides an efficient and economical way of traveling and reduces reliance of residents on private vehicles, thereby effectively alleviating traffic congestion. The provision of convenient services by urban public transport encourages residents to opt for more environmentally friendly modes of traveling. Meanwhile, the advance of new-energy buses continues to enhance the positive influence of the urban public transportation system on the environment [2]. Its benefits include reducing urban pollution, improving air quality, and decreasing greenhouse gas emissions. Furthermore, the rational use of urban space and the optimization of urban planning are promoted by public transport. By providing accessibility to residents from different social and economic backgrounds, all people can enjoy the opportunities offered by cities, like employment. This contributes to the sustainable growth and gradual prosperity in urban economies.

With the global focus on the Sustainable Development Goals (SDGs), the role of the urban public transport network (UPTN) is becoming more prominent. It is not only relevant to the daily lives of urban residents and the long-term prosperity of cities, but is also a key factor in achieving environmentally friendly, resource-efficient, and inclusive economic growth. Therefore, this study focuses on the systematic analysis of the topological properties and robustness of the UPTN to provide guidance for the sustainable development of urban planning and traffic management.

The topological properties of the UPTN manifest its structural characteristics. A comprehensive grasp of the network topology will facilitate the planning and optimization of the public transport system, thereby improving resource allocation and resident experience. In this paper, the L-Space complex network modeling method is adopted to construct the UPTN model. The analyses of the distribution of each topological characteristic verify whether the UPTN has small-world effects and scale-free characteristics, and detect whether the UPTN possesses great connectivity and convenience.

The robustness of the UPTN has an implication for the damage resilience of the public transport system. Excellent robustness will promote the sustainable development of the public transport system and reduce its maintenance costs and economic losses due to accidental events. Using the ratio of the maximum connected subgraph of the network as the evaluation index, the study contrasts and analyzes the static and dynamic robustness of the UPTN, with and without the introduction of the cascading failure model.

However, related research and analyses of UPTNs mainly focuse on first-tier cities, and few scholars have carried out in-depth discussions on UPTNs in second-tier and third-tier cities. At present, second-tier and third-tier cities are undergoing a phase of rapid urbanization, accompanied by population growth and economic expansion. A well-planned UPTN can have a positive impact on social resource allocation and balanced region development, thus enhancing urban competitiveness and talent attraction. Moreover, the economic and cultural development level of these cities is more representative and universal. This study employs Xuzhou UPTN as a case to gain a comprehensive understanding of its topological properties and robustness, thus providing a common reference value and guiding significance for the sustainable development of public transport in many second-tier and third-tier cities.

2. Literature Review

2.1. Significance of UPTN

Nowadays, more and more studies are being carried out in the field of transport, especially of the public transport system, due to its great significance and functions. YU et al. [3] pointed out that the vigorous development of transport infrastructure underlies flourishing economic growth, embodied in the adjustment of industrial structure, acceleration of the urbanization process, and realization of common wealth. Hu et al. [4] argued that transport convenience is advantageous for the high-level service industry. It could accelerate the dissemination of production elements and improve the accessibility of the market, leading to the spread of high-level services.

Given the importance of the public transport system, it is necessary to understand and analyze its properties, and maintain and safeguard its proper operation. In order to realize this, the introduction of urban road traffic examinations [5] and road traffic safety evaluation methods [6] may be considered.

2.2. Topological Properties of UPTN

Small-world effects [7] and scale-free characteristics [8] in complex networks were proposed, which initiated the popularity of applying complex networks in all kinds of large-scale integrated systems. At present, complex network theory has become a powerful tool for constructing and analyzing UPTNs. More and more scholars have begun to study the topological structure of UPTNs using complex network modeling methods, mainly including the L-Space method, the P-Space method, and the C-Space method, as shown in Table 1.

Table 1. Complex network modeling methods.

Modeling Methods	Constructing Form	Features	Literatures
L-Space method	Nodes denote bus stations, and edges denote that two adjacent stations have direct routes.	The method intuitively reflects the topology of the network and explicitly indicates the location of the bus stations in bus routes and the public transport network.	Literature [9,10]
P-Space method	Nodes denote bus stations, and edges denote that two stations are in the same route.	The method reflects the interchange situation of bus routes and the interchange experience of passengers.	Literature [11,12]
C-Space method	Nodes denote bus routes, and edges denote that two routes can realize interchange in this station.	The method reflects the interconnectivity of bus routes.	Literature [13]

When it comes to analyzing the topological properties, most scholars adopt the L-Space method. Zheng et al. [9] and Luo et al. [10] applied the L-Space method to Beijing UPTN and concluded that it belonged to a scale-free network. Meanwhile, other methods could assist with the analysis. Zhang et al. [11] adopted the P-Space method in studying the community structure and transfer accessibility of Beijing UPTN. Zhang et al. [12] proved that Shanghai UPTN had small-world effects with the P-Space method. Wang et al. [13] applied the C-Space method to analyze Chengdu UPTN.

2.3. Robustness of UPTN

With the deepening of research, scholars focus on the robustness of transport networks. The first step in analyzing robustness is to screen out the critical stations in the network. Scholars have preferred to select stations based on the network topological properties, as shown in Table 2.

Table 2. Network topological properties.

Topological Properties	Definition	Meanings	Literatures
Degree	The number of direct bus routes at a bus station.	Degree reflects the connectivity of a bus station.	Literature [14]
Clustering coefficient	The interconnectivity between neighbouring stations of a bus station.	Clustering coefficient reflects the closeness of a bus station.	Literature [15]
Betweenness centrality	The frequency of a bus station acting as shortest path mediator.	Betweenness centrality reflects the ability to control traffic of a bus station.	Literature [16,17]
Path length	The length of a path between two bus stations.	Path length reflects the travel efficiency of passengers.	Literature [18]

Liu et al. [14] constructed the 'E-value' index with degree to select critical stations of Beijing UPTN. Wang et al. [15] analyzed the significance of cities in China's air transport network. Yang et al. [16] selected key stations of Shanghai urban rail transit network on the basis of betweenness centrality. Wang et al. [17] used betweenness centrality in selecting key stations in the city road network. Piraveenan et al. [18] proposed transportation centrality based on path length to identify critical sites in Delhi and Seoul road networks.

When analyzing the robustness of the network, the cascading failure model [19] has often been introduced into the study. Scholars improved this model for experiments, as shown in Table 3.

Table 3. Improvements in the cascading failure model.

Literatures	Improvements	Conclusions	Limitations
Literature [20]	Improved the load capacity model with the consideration of the non-linear relationship between load and capacity.	The capacity parameter had a certain influence on the robustness of the logistics network, and a reasonable increase in the capacity parameter could enhance the robustness of the logistics network.	Fewer redistribution strategies are used to fully assess network robustness.
Literature [21]	Proposed 'M2 model', 'M3 model', and 'M4 model' based on 'M1 model' according to the idea of degree distribution and average distribution.	The new model more accurately reflected the redistribution of actual passenger flows, especially 'M4 model' performing best.	The lack of a specific basis for the selection of the critical stations might result in deliberate attacks that may not be accurately targeted.
Literature [22]	Seleted maximum connectivity component and global efficiency as the network robustness evaluation index.	The maximum connectivity component outperformed global efficiency in assessing the network robustness.	The selection of key stations was determined by a single topological property that failed to provide a comprehensive measure of station importance.
Literature [23]	Considered the cascading failure phenomenon of nodes and edges in the network.	A smaller coupling strength could increase the network's resistance to cascading failures.	A full assessment of network robustness might not be possible using load redistribution based on passenger traffic alone.

Wang et al. [20] improved the load capacity model to simulate cascading failure attacks against the postal logistics network in Northwest China. Sonnam et al. [21] proposed new load redistribution strategies, the 'M2 model', 'M3 model', and 'M4 model', based on the 'M1 model' and adopted them to simulate the cascading failure process on the Beijing bus–subway coupled network. Wang et al. [22] selected the maximum connectivity component of the network and the global efficiency as the evaluation metrics to analyze the static robustness of the railway freight network in China. Yang et al. [23] stimulated both the node and edge cascading failure processes on Qingdao UPTN to analyze its robustness under extreme weather conditions. In addition, Muller-Hannemann et al. [24] started to use machine learning to approximate the robustness of the public transport system.

3. Methodology

3.1. UPTN Topological Structure Modeling

A UPTN is mainly composed of bus stations and bus routes. There are three main methods for modeling its topological structure: the L-Space method, the P-Space method, and the C-Space method. Since the L-Space method can intuitively reflect the topology of the urban public transport, and explicitly indicate the location of bus stations in bus routes and the public transport network, the L-Space method is used in this paper to construct the UPTN model.

The directed weighted UPTN is constructed using bus stations as nodes, bus route connections as directed edges, and the number of bus routes as the weight of the edges. It is defined as follows:

$$G = (V, E) \tag{1}$$

where $V = \{v_i | i = 1, 2, \ldots, N\}$ denotes the set of nodes, v_i is the i-th station, N is the total number of nodes with $N = |V|$; $E = \{e_{ij} | i, j = 1, 2, \ldots, N\}$ denotes the set of edges, e_{ij} is the connection of bus routes from v_i to v_j, M is the total number of edges with $M = |E|$. The adjacency matrix $\mathbf{A} = (a_{ij})$ and the weight matrix $\mathbf{W} = (w_{ij})$ can represent the connections of bus stations. If v_i can reach v_j, then $a_{ij} = 1$ and w_{ij} is equal to the weight of e_{ij} (i.e., the number of bus routes of e_{ij}); otherwise, $a_{ij} = 0$ and $w_{ij} = 0$.

3.2. UPTN Topological Characteristics

3.2.1. Degree of Bus Stations

The degree of a bus station is the number of bus route connections at a bus station, reflecting the connectivity of that bus station. It is defined as follows:

$$k_i = k_i^{\text{out}} + k_i^{\text{in}} = \sum_{j=1}^{N} a_{ij} + \sum_{j=1}^{N} a_{ji}, i = 1, 2, \ldots, N \tag{2}$$

where k_i, k_i^{out}, and k_i^{in} respectively denote degree, outcoming degree, and incoming degree of the station v_i. The average degree of the UPTN is the average of all degrees of bus stations in the network. It is defined as follows:

$$k = \frac{1}{N} \sum_{i=1}^{N} k_i \tag{3}$$

where k denotes the average degree of the UPTN.

3.2.2. Strength of Bus Stations

The strength of a bus station is the sum of the weight of bus route connections at a bus station, which expresses the activity of that bus station. It is defined as follows:

$$s_i = s_i^{\text{out}} + s_i^{\text{in}} = \sum_{j=1}^{N} w_{ij} + \sum_{j=1}^{N} w_{ji}, i = 1, 2, \ldots, N \tag{4}$$

where s_i, s_i^{out}, and s_i^{in} respectively denote strength, outcoming strength, and incoming strength of the station v_i. The average strength of the UPTN is the average of all strengths of bus stations in the network. It is defined as follows:

$$s = \frac{1}{N} \sum_{j=1}^{N} s_i \tag{5}$$

where s denotes the average strength of the UPTN.

3.2.3. Pressure of Bus Stations

The pressure of a bus station is the ratio of the strength to the degree of a bus station, reflecting the traffic pressure carried by the bus station. It is defined as follows:

$$p_i = \frac{s_i}{k_i}, i = 1, 2, \ldots, N \tag{6}$$

where p_i denotes the pressure of the station v_i. If $k_i = 0$, then $p_i = 0$. The average pressure of the UPTN is the average of all pressures of bus stations in the network. It is defined as follows:

$$p = \frac{1}{N} \sum_{i=1}^{N} p_i \tag{7}$$

where p denotes the average pressure of the UPTN.

3.2.4. Clustering Coefficient of Bus Stations

The clustering coefficient of a bus station is the probability that neighbouring stations are connected to each other, i.e., the ratio of the number of actual bus route connections to the number of maximum possible bus route connections between neighbouring stations, reflecting the closeness of that bus station to its neighbouring stations. It is defined as follows:

$$c_i = \frac{2|E_i|}{|V_i|(|V_i - 1|)}, i = 1, 2, \ldots, N \tag{8}$$

where c_i denotes the clustering coefficient of the station v_i; V_i denotes the set of neighbouring stations of v_i; E_i denotes the set of bus route connections between neighbouring stations of v_i. The average clustering coefficient of the UPTN is the average of all clustering coefficients of bus stations in the network. It is defined as follows:

$$c = \frac{1}{N}\sum_{i=1}^{N} c_i \tag{9}$$

where c denotes the average clustering coefficient of the UPTN.

3.2.5. Average Shortest Path Length of Bus Stations

The average shortest path length is the average of the sum of the shortest path length from a bus station to all other bus stations, which can reflect the convenience of that bus station to reach other bus stations. It is defined as follows:

$$d_i = \frac{1}{N-1}\sum_{j=1, j\neq i}^{N} d_{ij}, i = 1, 2, \ldots, N \tag{10}$$

where d_i denotes the average shortest path length of the station v_i; d_{ij} denotes the shortest path length from v_i to v_j. The average shortest path length of the UPTN is the average of all average shortest path lengths of bus stations in the network. It is defined as follows:

$$d = \frac{1}{N}\sum_{i=1}^{N} d_i \tag{11}$$

where d denotes the average shortest path length of the UPTN.

In addition, the network diameter, another index, is the maximum average shortest path length of bus stations in the network. It shows the maximum number of bus stations that a passenger may pass through in a single trip. It is defined as follows:

$$D = d_{\max} \tag{12}$$

where D denotes the network diameter of the UPTN; $d_{\max}$ denotes the maximum average shortest path length of bus stations.

3.2.6. Betweenness Centrality of Bus Stations

The betweenness centrality of a bus station is the probability that a bus station acts as a mediator of the shortest paths between other stations, i.e., the ratio of the number of shortest paths passing through a bus station to the number of shortest paths in the network, showing the ability of the bus station to control transport. It is defined as follows:

$$b_i = \sum_{p=1, q=1, p\neq q}^{N} \frac{\sigma_{pq}(i)}{\sigma_{pq}}, i = 1, 2, \ldots, N \tag{13}$$

where b_i denotes the betweenness centrality of the station v_i; σ_{pq} denotes the number of shortest paths from v_p to v_q; $\sigma_{pq}(i)$ denotes the number of shortest paths from v_p to v_q via v_i. The average betweenness centrality of the UPTN is the average of all betweenness centralities of bus stations in the network. It is defined as follows:

$$b = \frac{1}{N}\sum_{i=1}^{N} b_i \tag{14}$$

where b denotes the average betweenness centrality of the UPTN.

3.3. UPTN Station Significance Evaluation

3.3.1. Single Evaluation Index

Network topological characteristics can visually reflect a certain property of a bus station. In this paper, the degree, strength, pressure, clustering coefficient, and betweenness centrality of bus station are selected as single evaluation indices for the significance of the bus station, and can respectively embody the level of connectivity, activity, carrying, tightness, and influence of the bus station.

3.3.2. Comprehensive Evaluation Index

Since a single evaluation index cannot comprehensively measure the significance of a bus station in the network, a combination of single evaluation indices is necessary.

The degree can reflect the cross-linking degree of a bus station in bus routes; the strength can show the heavy degree of the transport task at a bus station; the clustering coefficient can reflect the interchange convenience of a bus station in the local area; and the betweenness centrality can embody the pivotal function of a bus station in the network. Hence, the importance degree, a comprehensive evaluation index for the significance of the bus station, is proposed as a comprehensive based on the indices above. It is defined as follows:

$$I_i = \frac{k_i}{k_{\max}} + \frac{s_i}{s_{\max}} + \frac{c_i}{c_{\max}} + \frac{b_i}{b_{\max}}, i = 1, 2, \ldots, N \tag{15}$$

where I_i denotes the importance degree of the station v_i; $k_{\max}$, $s_{\max}$, $c_{\max}$, and $b_{\max}$ respectively denote the maximum degree, strength, clustering coefficient, and betweenness centrality of bus station.

3.4. UPTN Cascading Failure Model

When a bus station in the UPTN fails due to contingencies, it may impact neighbouring stations and affect the whole network in turn. This phenomenon is referred to as the cascading failure phenomenon. As shown in Figure 1, when a certain station breaks down, the traffic load on it will be distributed to its neighbouring stations. If the load on the neighbouring station exceeds its maximum load capacity, a new failed station will be generated and conduct another cascading load distribution.

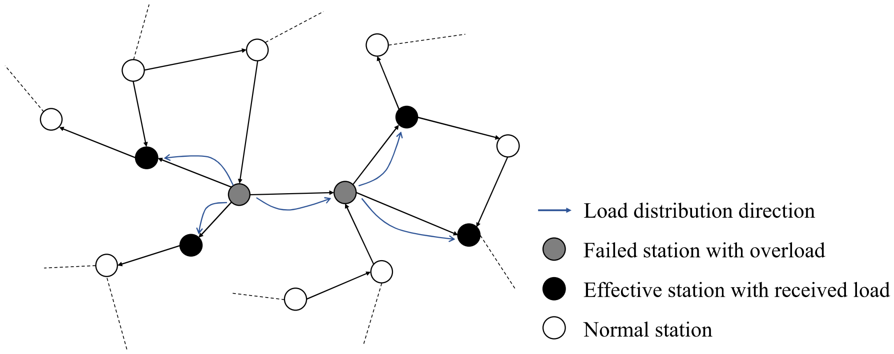


Figure 1. Schematic diagram of cascading failure phenomenon.

3.4.1. Initial Load and Maximum Load Capacity of Bus Stations

According to the cascading failure model in complex networks [19], the maximum load capacity is proportional to the initial load as follows:

$$C_i = (1 + \alpha)L_i^0, i = 1, 2, \ldots, N \tag{16}$$

where L_i^0 denotes the initial load of the station v_i; C_i denotes the maximum load capacity of v_i; and $\alpha > 0$ is the capacity parameter. Since the pressure of a bus station can largely reflect its load situation, it is chosen as the initial load of the bus station, as follows:

$$L_i^0 = p_i, i = 1, 2, \ldots, N \tag{17}$$

3.4.2. Load Redistribution Strategy of Bus Stations

If a bus station fails with overload, its load needs to be distributed to neighbouring stations according to a certain strategy. Assume that the redistributed load of the neighbouring station is proportional to the load of the failed station, as follows:

$$\Delta L_j = \beta_j L_i, v_j \in V_i \tag{18}$$

where L_i denotes the load of the failed station v_i; ΔL_j denotes the redistributed load of the neighbouring station v_j; β_j denotes the redistributed proportion of v_j. Depending on the load redistribution strategy, the formula of β_j differs as follows:

$$\beta_j = \begin{cases} 1/|V_i|, & \text{average distribution} \\ L_j^0 \Big/ \sum\limits_{v_j \in V_i} L_j^0, & \text{initial load distribution} \\ k_j \Big/ \sum\limits_{v_j \in V_i} k_j, & \text{degree distributuion} \\ s_j \Big/ \sum\limits_{v_j \in V_i} s_j, & \text{strength distribution} \\ c_j \Big/ \sum\limits_{v_j \in V_i} c_j, & \text{clustering coefficient distribution} \\ b_j \Big/ \sum\limits_{v_j \in V_i} b_j, & \text{betweenness centrality distribution} \\ I_j \Big/ \sum\limits_{v_j \in V_i} I_j, & \text{importance degree distribution} \end{cases} \tag{19}$$

where β_j depends on the number of v_i's neighbouring stations under the average distribution strategy; the total initial load of v_i's neighbouring stations under the initial load distribution strategy; the total degree of v_i's neighbouring stations under the degree distribution strategy; the total strength of v_i's neighbouring stations under the strength distribution strategy; the total clustering coefficient of v_i's neighbouring stations under the clustering coefficient distribution strategy; the total importance centrality of v_i's neighbouring stations under the clustering coefficient distribution strategy; and the total importance degree of v_i's neighbouring stations under the importance degree distribution strategy.

3.4.3. Cascading Failure Process of Bus Station

According to the load redistribution model in Equation (18), the load of a bus station at a certain moment is the sum of its load at the previous moment and its redistributed load, as follows:

$$L_i(t) = L_i(t-1) + \Delta L_i, i = 1, 2, \ldots, N \tag{20}$$

where $L_i(t)$ denotes the load of the station v_i at the t-th moment.

Due to the cascading load distribution, the load of a bus station changes in real time. After each update of the load of the bus station, it is necessary to determine whether it exceeds the maximum load capacity. If it exceeds, another cascading load redistribution is conducted for the station until all distributed stations meet their load capacity; otherwise, no cascading load redistribution is performed. This is shown as follows:

$$T_i = \begin{cases} 0, \ L_i \leq C_i \\ 1, \ L_i > C_i \end{cases}, i = 1, 2, \ldots, N \tag{21}$$

where T_i denotes the failure of the station v_i. If $T_i = 0$, then v_i functions; otherwise, v_i fails.

3.5. UPTN Robustness Evaluation

The ratio of the maximum connected subgraph is one of the important metrics used to measure the network connectivity. It indicates the ratio of the number of nodes in the maximum connected subgraph to the number of nodes in the original network after removing several nodes from the network. A directed network has two kinds, which are the ratio of the maximum strongly connected subgraph and the ratio of the maximum weakly connected subgraph. Both of these are selected as evaluation indices for the robustness of the UPTN. They are defined as follows:

$$S_s = \frac{N_s}{N} \tag{22}$$

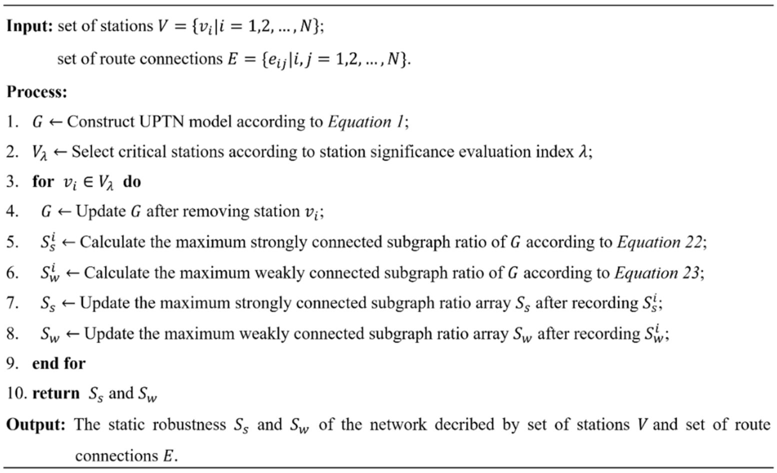
$$S_w = \frac{N_w}{N} \tag{23}$$

where S_s and S_w respectively denote the ratio of the maximum strongly connected subgraph and the ratio of the maximum weakly connected subgraph of the UTPN; N_s and N_w respectively denote the number of nodes of the maximum strongly connected subgraph and the number of nodes of the maximum weakly connected subgraph.

3.6. UPTN Robustness Simulation Algorithm

3.6.1. Static Robustness Simulation Algorithm

The static robustness of the UPTN is analyzed by focusing on its connectivity performance in the face of deliberate attacks without considering the cascading failure model. In order to assess the resilience of the network to immediate failures, the change in the ratio of the maximum connected subgraph is observed as critical stations are progressively removed from UPTN. The simulation algorithm is shown in Figure 2.



```
Input: set of stations $V = \{v_i | i = 1,2, \ldots, N\}$;
       set of route connections $E = \{e_{ij} | i, j = 1,2, \ldots, N\}$.
Process:
$G$ ← Construct UPTN model according to Equation 1;
$V_\lambda$ ← Select critical stations according to station significance evaluation index $\lambda$;
for $v_i \in V_\lambda$ do
    $G$ ← Update $G$ after removing station $v_i$;
    $S_s^i$ ← Calculate the maximum strongly connected subgraph ratio of $G$ according to Equation 22;
    $S_w^i$ ← Calculate the maximum weakly connected subgraph ratio of $G$ according to Equation 23;
    $S_s$ ← Update the maximum strongly connected subgraph ratio array $S_s$ after recording $S_s^i$;
    $S_w$ ← Update the maximum weakly connected subgraph ratio array $S_w$ after recording $S_w^i$;
end for
return $S_s$ and $S_w$
Output: The static robustness $S_s$ and $S_w$ of the network decribed by set of stations $V$ and set of route
        connections $E$.
```

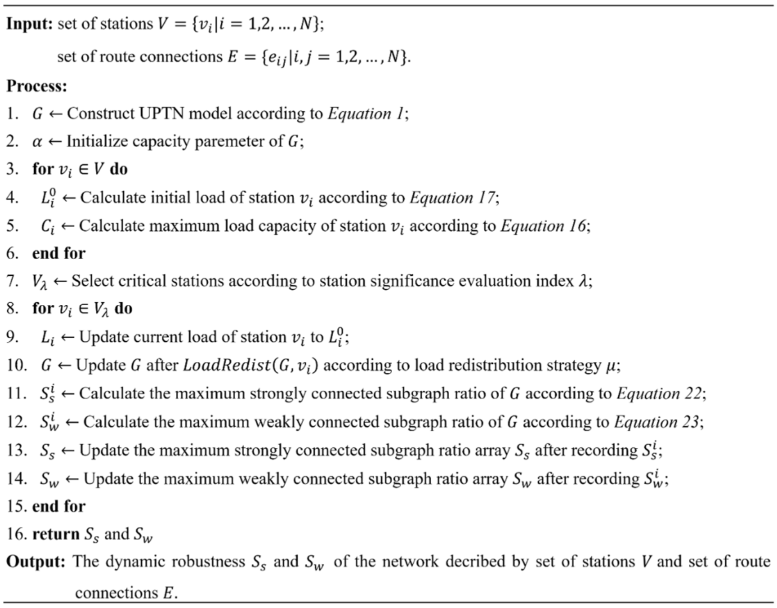
Figure 2. Algorithm of static robustness analysis simulation.

The detailed steps are as follows:

1. The UPTN, a directed weighted network, is constructed by abstracting bus stations as network nodes, bus route connections as network edges, and the number of bus routes as the weights of edges.
2. Critical stations are removed from the network sequentially and marked as failed. Then the network status is updated.
3. The ratio of the maximum connected subgraph is calculated in order to evaluate the network robustness.

3.6.2. Dynamic Robustness Simulation Algorithm

The dynamic robustness of the UPTN focuses on its connectivity change under the cascading failure phenomenon. The removal of critical stations results in the distribution of their load to neighbouring stations. Similarly, newly generated overloaded stations are redistributed with the load until no further station failiures occur. By simulating the cascading failure process, the algorithm evaluates the stability and reliability of the network against the propagation of complex failures during the long-term operation. This is shown in Figure 3, and the load redistribution simulation algorithm is shown in Figure 4.



```
Input: set of stations $V = \{v_i | i = 1,2,\dots,N\}$;
       set of route connections $E = \{e_{ij} | i,j = 1,2,\dots,N\}$.
Process:
$G$ ← Construct UPTN model according to Equation 1;
$\alpha$ ← Initialize capacity paremeter of $G$;
for $v_i \in V$ do
    $L_i^0$ ← Calculate initial load of station $v_i$ according to Equation 17;
    $C_i$ ← Calculate maximum load capacity of station $v_i$ according to Equation 16;
end for
$V_\lambda$ ← Select critical stations according to station significance evaluation index $\lambda$;
for $v_i \in V_\lambda$ do
    $L_i$ ← Update current load of station $v_i$ to $L_i^0$;
    $G$ ← Update $G$ after $LoadRedist(G, v_i)$ according to load redistribution strategy $\mu$;
    $S_s^i$ ← Calculate the maximum strongly connected subgraph ratio of $G$ according to Equation 22;
    $S_w^i$ ← Calculate the maximum weakly connected subgraph ratio of $G$ according to Equation 23;
    $S_s$ ← Update the maximum strongly connected subgraph ratio array $S_s$ after recording $S_s^i$;
    $S_w$ ← Update the maximum weakly connected subgraph ratio array $S_w$ after recording $S_w^i$;
end for
return $S_s$ and $S_w$
Output: The dynamic robustness $S_s$ and $S_w$ of the network decribed by set of stations $V$ and set of route
        connections $E$.
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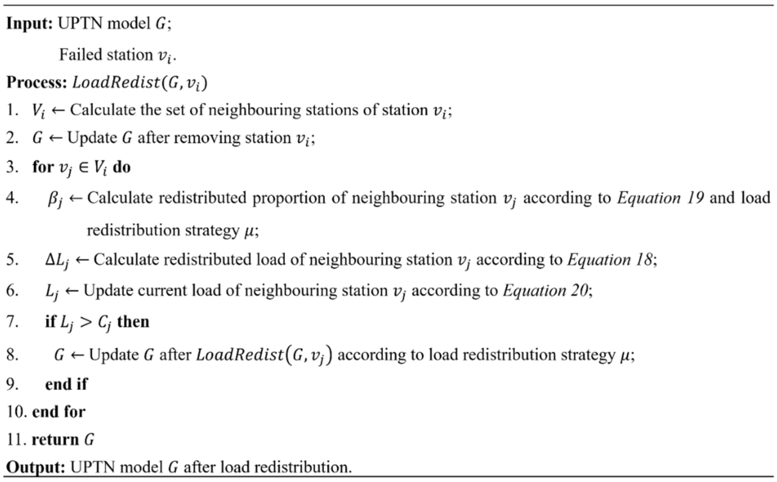
Figure 3. Algorithm of dynamic robustness analysis stimulation.



```
Input: UPTN model $G$;
       Failed station $v_i$.
Process: $LoadRedist(G, v_i)$
$V_i$ ← Calculate the set of neighbouring stations of station $v_i$;
$G$ ← Update $G$ after removing station $v_i$;
for $v_j \in V_i$ do
    $\beta_j$ ← Calculate redistributed proportion of neighbouring station $v_j$ according to Equation 19 and load
        redistribution strategy $\mu$;
    $\Delta L_j$ ← Calculate redistributed load of neighbouring station $v_j$ according to Equation 18;
    $L_j$ ← Update current load of neighbouring station $v_j$ according to Equation 20;
    if $L_j > C_j$ then
        $G$ ← Update $G$ after $LoadRedist(G, v_j)$ according to load redistribution strategy $\mu$;
    end if
end for
return $G$
Output: UPTN model $G$ after load redistribution.
```

Figure 4. Algorithm of load redistribution stimulation.

The detailed steps are as follows:

1. The UPTN, a directed weighted network, is constructed by abstracting bus stations as network nodes, bus route connections as network edges, and the number of bus routes as the weights of edges.
2. The capacity parameter is initialized. The initial load and maximum load capacity of bus station are calculated.
3. Critical stations are selected according to significance evaluation indices, removed from the network, and marked as failed. Then the network status is updated.
4. Load redistribution is conducted on the failed stations. Their loads are distributed to neighbouring stations based on load redistribution strategies.
5. The efficacy of redistributed stations is evaluated. If the load of a station exceeds its maximum load capacity, mark it as failed, and then repeat Step 4; otherwise, proceed to Step 6.
6. The ratio of the maximum connected subgraph is calculated in order to evaluate the network robustness.

4. Experiments and Results

4.1. Xuzhou UPTN Topological Structure Modeling

This paper uses ‘8684 Bus (v15.3.43)’ as the data source. Bus routes are divided into upward routes and downward routes according to the actual operation. The bus operation data of Xuzhou city (five districts, three cities, and two counties) as of March 2024 are counted, with 428 routes and 2516 stations. Xuzhou UPTN is constructed using the L-Space method as shown in Figure 5.

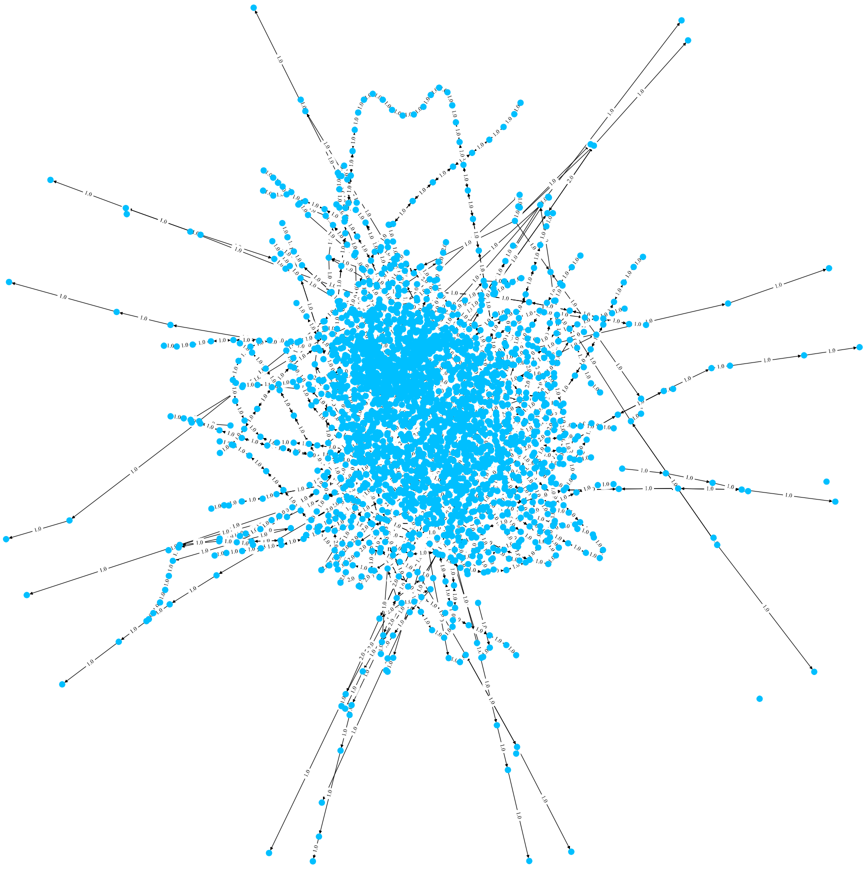


Figure 5. Topological structure of Xuzhou UPTN. Blue dots denote bus stations; lines denote bus route connections; numbers denote weights of bus route connections.

The aforementioned model is proposed based on the following assumptions.

1. Considering the possible human or technical errors in the operational data, it is assumed that the selected data sets are verified and calibrated;
2. Considering that the data at different time points will reveal different network properties, it is assumed that the data at the selected time points can reflect the general operation of the bus network;
3. Considering that the weight of each route connection in actual operation will be affected by vehicle frequency, passenger flow, road conditions, etc., it is assumed that the weight of each route connection in this experiment only depends on the number of bus routes on it;
4. Considering that the actual network robustness is affected by many factors, including traffic flow, passenger behavior, and environmental changes, it is assumed that the robustness in this experiment depends only on the network topological structure.

4.2. Analysis of Xuzhou UPTN Topological Properties

'Python (v3.12.4)' and third-library 'Networkx (v3.2.1)' were used to program and calculate the basic topological characteristic. The results are shown in Table 4. There are 2516 stations and 6443 route connections in Xuzhou UPTN. The average degree, average clustering coefficient, and average betweenness centrality of the network are all low, indicating that the network has poor connectivity and tightness.

Table 4. Basic topological characteristics of Xuzhou UPTN.

Topological Characteristic	Calculated Value
Number of stations	2516
Number of route connections	6443
Average degree	5.12
Average strength	8.36
Average pressure	1.48
Average clustering coefficient	0.0586
Average betweenness centrality	0.0052
Average shortest path length	14.3
Network diameter	49

4.2.1. Degree Distribution of Bus Station

The degree distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 6. The 20 stations with the highest degree are shown in Table 5 (the stations with the same name are the same stations in the upward route and downward route). Almost 80% of stations have a degree of 4 or less, reflecting the uneven distribution of neighbouring stations in the network. Only a few stations have a key connectivity role. The average degree of the network is 5.12, i.e., each station is adjacent to about 5 stations, indicating that the network has a low connectivity.

The degree distribution of Xuzhou UPTN is fitted. The result is shown in Figure 7. The fitting function is $P(k) = 44.8k^{-3.31}$, which means that the degree probability distribution of Xuzhou UPTN obeys the power law distribution. In the network, the vast majority of stations have a low degree, while a small minority of them have a high degree, indicating that it satisfies scale-free characteristic [8].

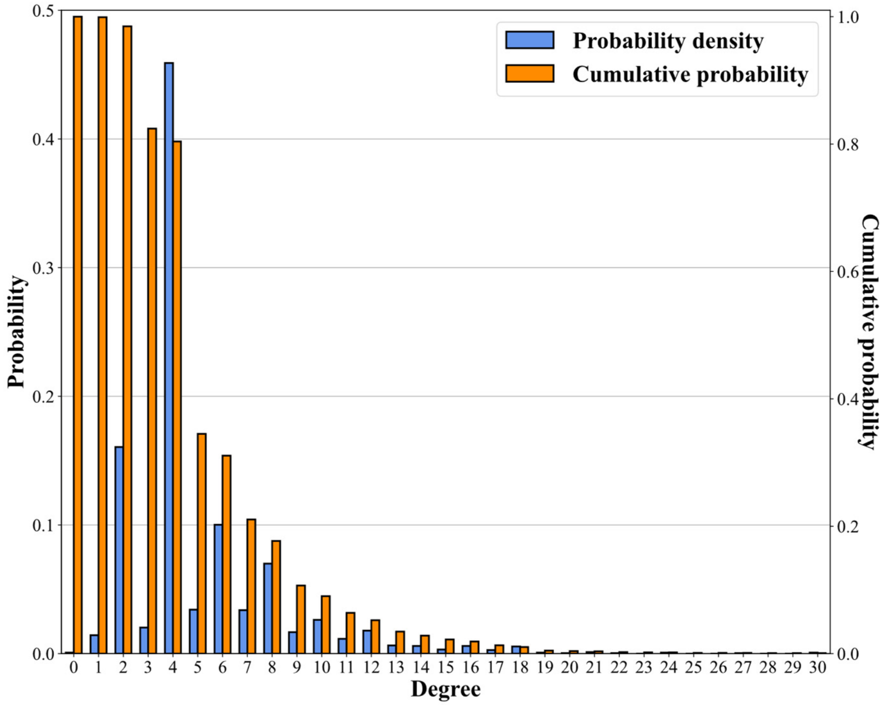


Figure 6. Degree distribution and its cumulative distribution of Xuzhou UPTN.

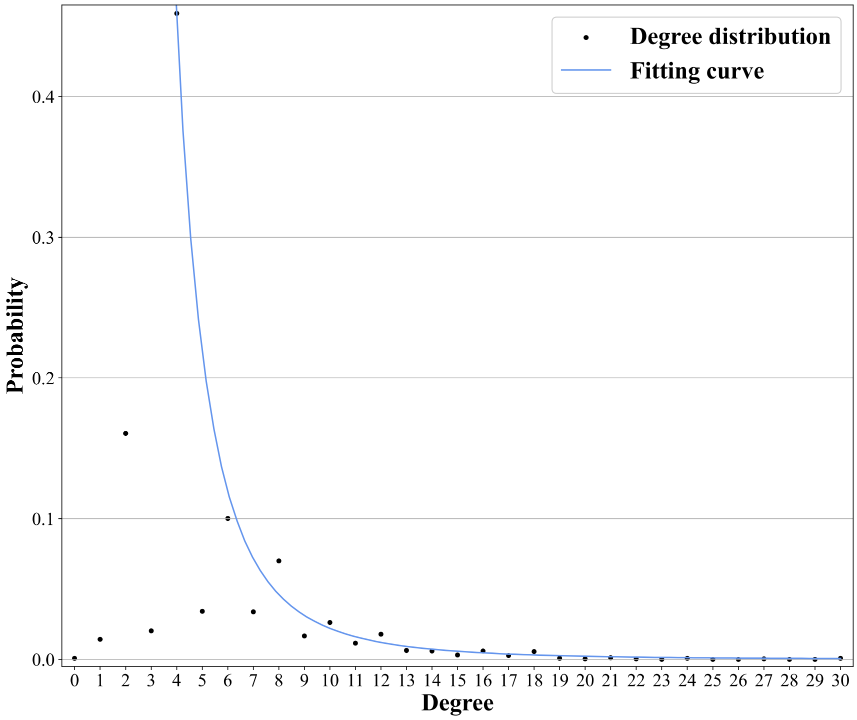


Figure 7. Fitting result for degree distribution of Xuzhou UPTN.

Table 5. Partial stations with a high degree in Xuzhou UPTN.

No.	Station Name	Degree	No.	Station Name	Degree
1	North Crossing	30	11	Xuzhou Station (Chaoyang)	19
2	Huaihai Cultural and Exposition Park (Food City)	30	12	Maternal and Child Health Hospital	19
3	Xuanwu Market	27	13	Bazi Street	18
4	Xuzhou Station (Chaoyang)	24	14	Dongdianzi	18
5	Mazhuang	24	15	No.97 Hospital	18
6	Cigarette Factory	22	16	Central General Store Building	18
7	First Hospital of Xuzhou city	21	17	Enhua Edifice	18
8	Cultural Club (Wanhongqiao Electronic Market)	21	18	South Democracy Road	18
9	Xuanwu Market	21	19	Memorial Archway	18
10	Duanzhuang	20	20	Sidao Street	18

4.2.2. Strength Distribution of Bus Stations

The strength distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 8. The 20 stations with the highest strength are shown in Table 6. Over 80% of stations have a strength of 4 or less, while nearly 10% of them have a strength more than 20. The extremely unbalanced strength distribution of the network tends to cause network congestion. The average strength of the network is 8.36, i.e., about 7 routes pass through each station, indicating a large amount of traffic.

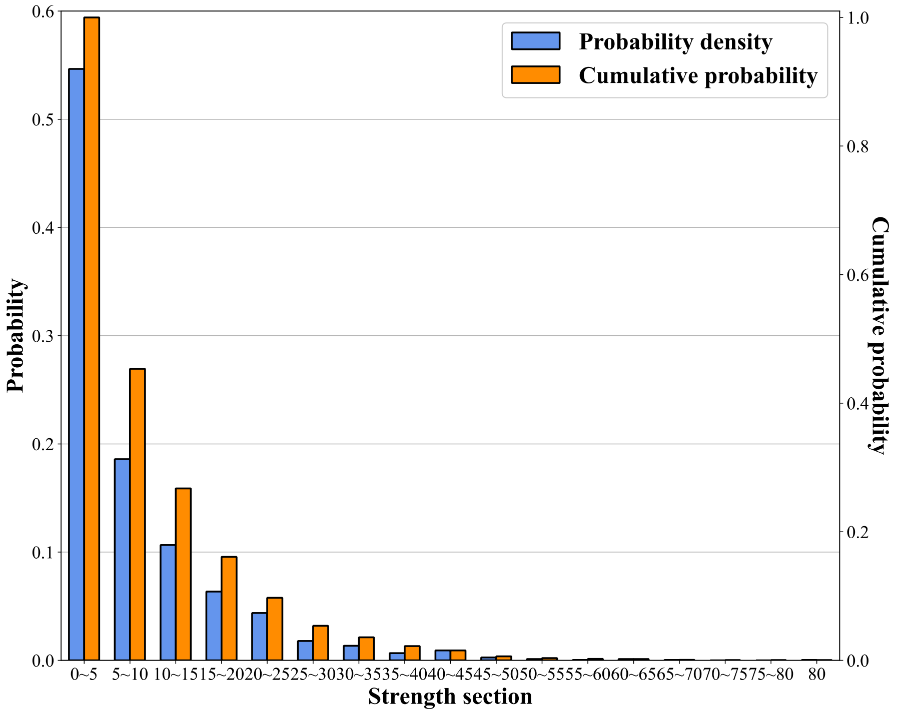


Figure 8. Strength distribution and its cumulative distribution of Xuzhou UPTN.

Table 6. Partial stations with a high strength of Xuzhou UPTN.

No.	Station Name	Strength	No.	Station Name	Strength
1	North Crossing	80	11	Bus Repair Factory	48
2	Sidao Street	66	12	Xiyuan Community	48
3	Zhongshan Restaurant	64	13	Cigarette Factory	48
4	Hospital of Traditional Chinese Medicine	64	14	East Gate of Han Street	48
5	Huaihai Cultural and Exposition Park (Food City)	64	15	North Gate of Han Street	48
6	Memorial Tower	56	16	No.4 Hospital of Xuzhou city	46
7	Xuanwu Market	53	17	China University of Mining and Technology	44
8	No.97 Hospital	52	18	Pengzu Avenue	44
9	South Bus Station	50	19	Tianqiao East	44
10	Xuzhou Station (Chaoyang)	49	20	Zhaishan Market	44

4.2.3. Pressure Distribution of Bus Stations

The pressure distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 9. The 20 stations with the highest pressure are shown in Table 7. Most stations have a pressure between 1.0 and 1.5, while a few have a pressure of 5 or more. This indicates that most of the transport pressure of the network is concentrated in a small number of stations, which may lead to the station collapse due to overloaded traffic.

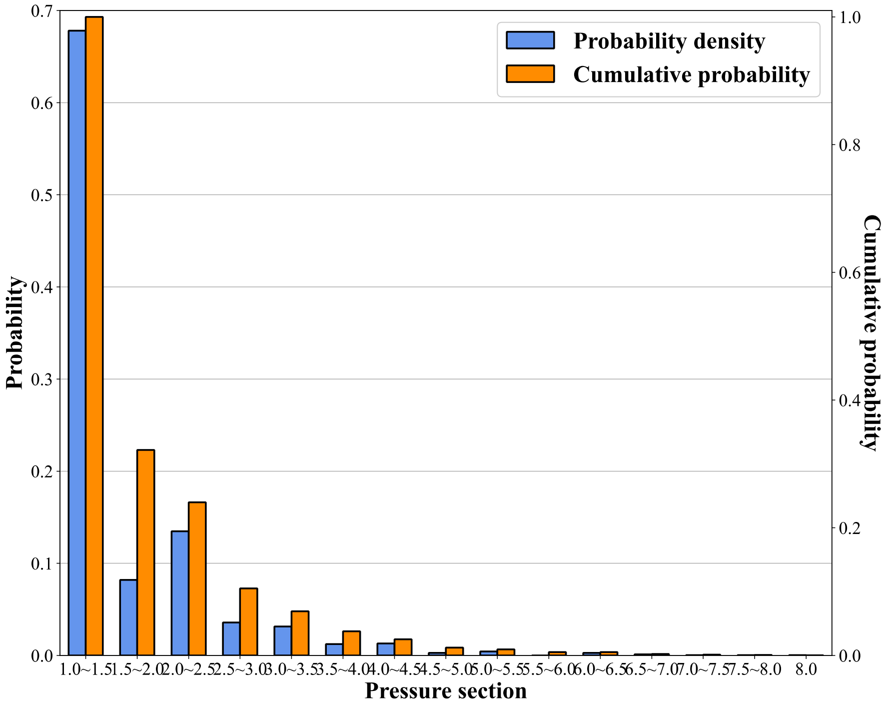


Figure 9. Pressure distribution and its cumulative distribution of Xuzhou UPTN.

Table 7. Partial stations with a high pressure of Xuzhou UPTN.

No.	Station Name	Pressure	No.	Station Name	Pressure
1	Ceramics Market	8.0	11	Shayuan	6.0
2	Yangzhuang	7.5	12	North Gate of Yujing Community	6.0
3	Health and Family Planning Commission	7.0	13	Mingshi Community	6.0
4	Procuratorate of Xuzhou city	6.7	14	No.4 Hospital of Xuzhou city	5.1
5	Hequn Bridge	6.5	15	No.24 Middle School	5.0
6	Hequn Bridge	6.5	16	Hequn Community	5.0
7	No.24 Middle School	6.4	17	Hequn Community	5.0
8	Bus Company of Pei County	6.0	18	Houshan Community	5.0
9	Tuolongshan Park	6.0	19	Menzhuang Community	5.0
10	West Bus Station of Feng County	6.0	20	Taishan Huijing Community	5.0

4.2.4. Clustering Coefficient Distribution of Bus Station

The clustering coefficient distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 10. The 20 stations with the highest clustering coefficient are shown in Table 8. There are 35 stations with a clustering coefficient up to 1, which means that the local network in the vicinity of them achieves full connectivity. Most stations have a clustering coefficient below 0.1, indicating that the overall connectivity of the network is low.

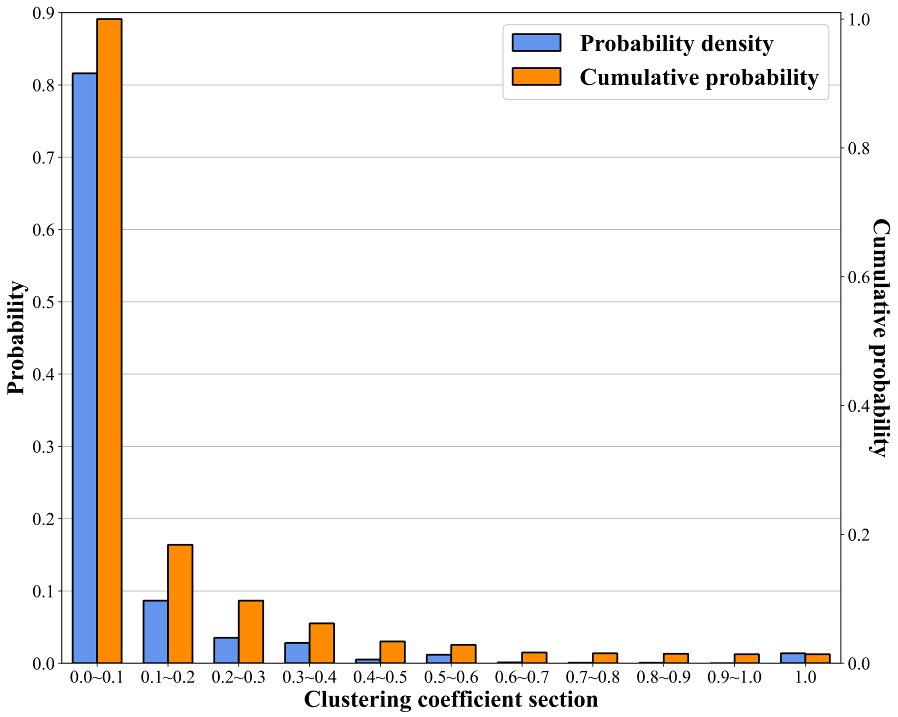


Figure 10. Clustering coefficient distribution and its cumulative distribution of Xuzhou UPTN.

Table 8. Partial stations with a high clustering coefficient of Xuzhou UPTN.

No.	Station Name	Clustering Coefficient	No.	Station Name	Clustering Coefficient
1	Hydraulic Part Factory	1.0	11	South Gate of Xuanwu Market	1.0
2	Dongge Community	1.0	12	Oriental Hospital	1.0
3	Xuzhou University of Technology Feihong Campus	1.0	13	Education Bureau of Tongshan District	1.0
4	Juntinghupan Community	1.0	14	Hanshan Bus Terminal	1.0
5	General Hospital of Xuzhou Mining Group	1.0	15	Procuratorate of Quanshan District	1.0
6	Ludishiji Community	1.0	16	Yanzhai Primary School	1.0
7	Intermediate Court of Xuzhou city	1.0	17	Xinpei Community East	1.0
8	Sudi North Crossing	1.0	18	Hanbangjing Community	1.0
9	Tongpei Crossing	1.0	19	Suning Appliance	1.0
10	Radio and Television Bureau of Tongshan District	1.0	20	Petrol Station (Zhaohu Station)	1.0

4.2.5. Betweenness Centrality Distribution of Bus Stations

The betweenness centrality distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 11. The 20 stations with the highest betweenness centrality are shown in Table 9. Almost 95% of stations have a betweenness centrality below 0.02, indicating that most stations do not play a key connectivity role in the network. While only eight stations have a betweenness centrality above 0.08, reflecting that few stations act as hubs in the network. It is evident that the connectivity of the network is vulnerable to disruption.

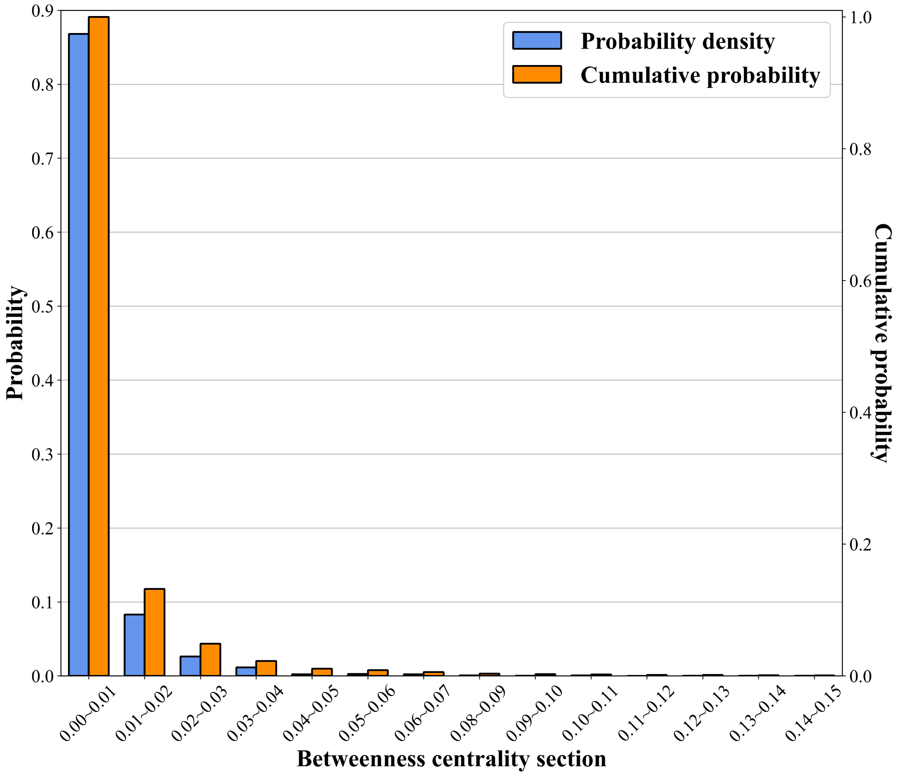


Figure 11. Betweenness centrality distribution and its cumulative distribution of Xuzhou UPTN.

Table 9. Partial stations with a high betweenness centrality of Xuzhou UPTN.

No.	Station Name	Betweenness Centrality	No.	Station Name	Betweenness Centrality
1	Sunzhuang	0.1451	11	Liuxin	0.0664
2	Vocational Education Centre	0.1336	12	Zhaishan	0.0648
3	Zhouzhuang	0.1222	13	Huaihai Cultural and Exposition Park (Food City)	0.064
4	Huaihai Cultural and Exposition Park (Food City)	0.1117	14	Meidi Community	0.0614
5	Mazhuang	0.0924	15	Malou	0.061
6	South Bus Station	0.0908	16	Jizhuang Company	0.0588
7	Yuanhe Road	0.0818	17	Tianqiao East	0.0586
8	Xuzhou Station (Chaoyang)	0.079	18	Xinxin Crossing	0.0545
9	Zhaishan Market	0.077	19	Xuanwu Market	0.0539
10	Examination and Approval Centre	0.0695	20	Dahuangshan Roundabout	0.051

4.2.6. Average Shortest Path Length Distribution of Bus Stations

The average shortest path length distribution and its cumulative distribution of Xuzhou UPTN are shown in Figure 12. The average shortest path length of the network is 14.3, i.e., it is necessary to pass through 14 stations to achieve connectivity between stations, indicating that the network is generally convenient for travel by residents. Meanwhile, the average clustering coefficient of the network is 0.0586, illustrating that Xuzhou UPTN has a small-world effect [7].

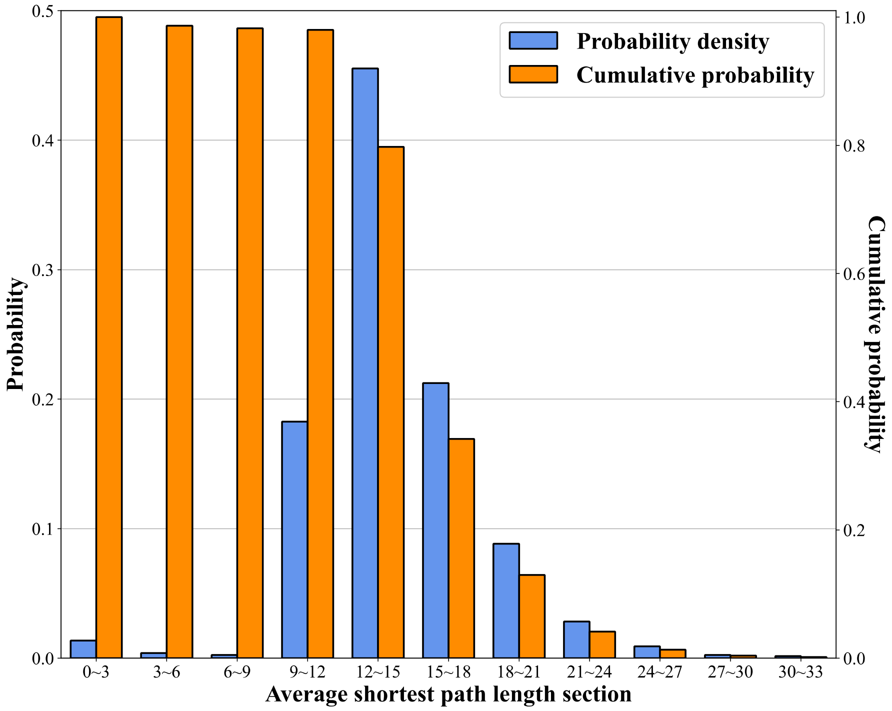


Figure 12. Average shortest path length distribution and its cumulative distribution of Xuzhou UPTN.

4.3. Analysis of Xuzhou UPTN Robustness

4.3.1. Analysis of Static Robustness

Critical stations are selected according to the significance evaluation indices in Section 2.1. Deliberate attack strategies are conducted on critical stations in descending order, and a random attack strategy is performed for a comparative experiment. The ratio of the maximum connected subgraph is calculated. The static robustness is shown in Figure 13.

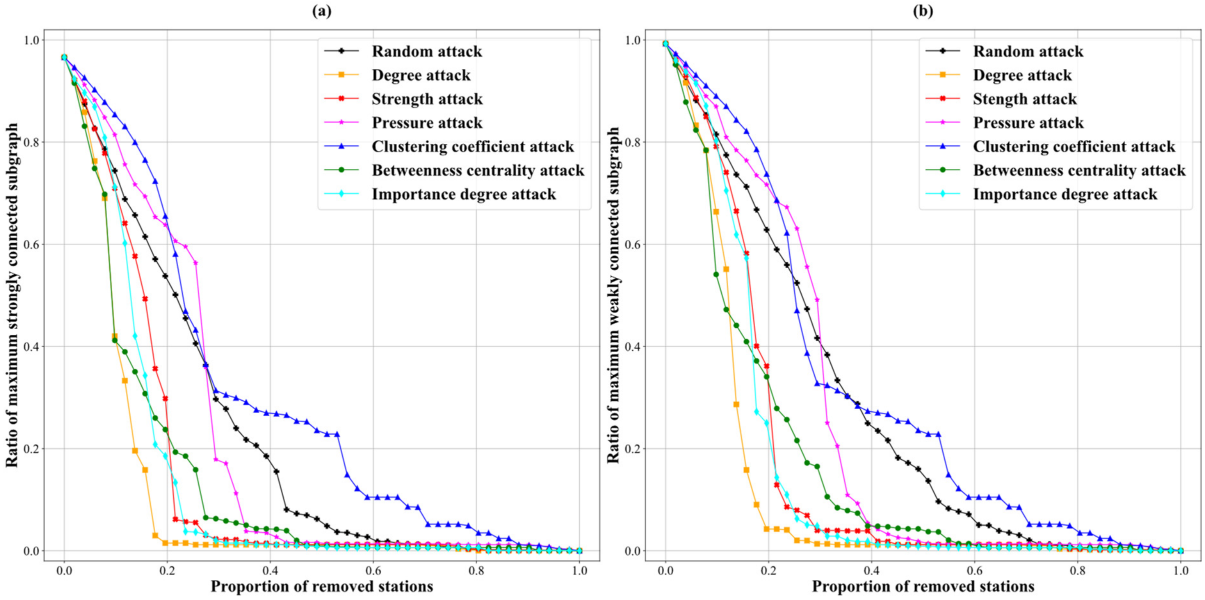


Figure 13. Static robustness of Xuzhou UPTN: (**a**) the maximum strongly connected subgraph ratio; (**b**) the maximum weakly connected subgraph.

Under the random attack strategy, the ratio of the maximum connected subgraph of the network decreases slowly. When 20% of stations are removed, its ratio can still be maintained at more than 50%, indicating that Xuzhou UPTN has high static robustness in the face of random attacks. For the pressure attack and clustering coefficient attack strategy, the decreasing trend of the ratio is close to or even slower than that for the random attack strategy. Therefore, the pressure and clustering coefficient do not represent the connectivity function of the station well. Under the degree attack, strength attack, betweenness centrality attack, and importance degree attack strategies, when 10% of stations are removed, the ratios of the maximum strongly connected subgraph are 69.04%, 77.82%, 69.79%, and 80.88%, and the ratios of the maximum weakly connected subgraph are 78.38%, 84.98%, 78.42%, and 87.08%, reflecting that the network still has high connectivity; when 20% of stations are removed, the ratios of the maximum strongly connected subgraph are 2.98%, 35.65%, 25.99%, and 20.83%, and the ratios of the maximum weakly connected subgraph are 9.02%, 40.06%, 37.16%, and 27.23%, indicating that the network connectivity has decreased dramatically; when 30% of stations are removed, the ratios of the maximum strongly connected subgraph are 1.19%, 3.06%, 6.48%, and 3.22%, and the ratios of the maximum weakly connected subgraph are 1.99%, 6.92%, 17.21%, and 5.09%, showing that the network has collapsed. The network has a relatively good static robustness against lower-strength attacks. However, as the intensity of the attack increases, the structure of the network gradually collapses.

4.3.2. Analysis of Dynamic Robustness

The capacity parameter α is initialized to 1.5 according to the cascading failure model in Section 2.2. Critical stations are selected based on the significance evaluation indices. The ratio of the maximum connected subgraph is calculated and compared under deliberate attack strategies and the random attack strategy. The dynamic robustness is shown in Figure 14 when the average strategy is adopted for different attack strategies.

Under the random attack strategy with the cascading failure phenomenon, the decreasing trend of the ratio of the maximum connected subgraph is apparently faster than that under the random attack strategy without the cascading failure phenomenon. When 20% of stations are removed, the ratio is close to 10%, illustrating the poor dynamic robustness of Xuzhou UPTN against random attacks. Under degree attack, strength attack, betweenness centrality attack, and importance degree attack strategies, when 20% of stations are removed, the ratios of the maximum strongly connected subgraph are 1.51%, 12.52%, 2.03%, and 3.38%, and the ratios of the maximum weakly connected subgraph are 3.86%, 15.46%, 2.54%, and 3.62%, reflecting the extremely fast loss of connectivity performance. The dynamic robustness of Xuzhou UPTN is quite poor, which is obviously different from the case of the static robustness. Deliberate attacks of lower intensity can lead to an overall collapse of the network.

Moreover, different load redistribution strategies are adopted for the importance degree evaluation index in the dynamic robustness analysis. The results are shown in Figure 15. It is observed that the ratio of the maximum connected subgraph shows a rapid decline and then tends to level off under different redistribution strategies. This trend is consistent with the dynamic robustness of Xuzhou UPTN, indicating that the importance degree can embody the connectivity function of a station in the network.

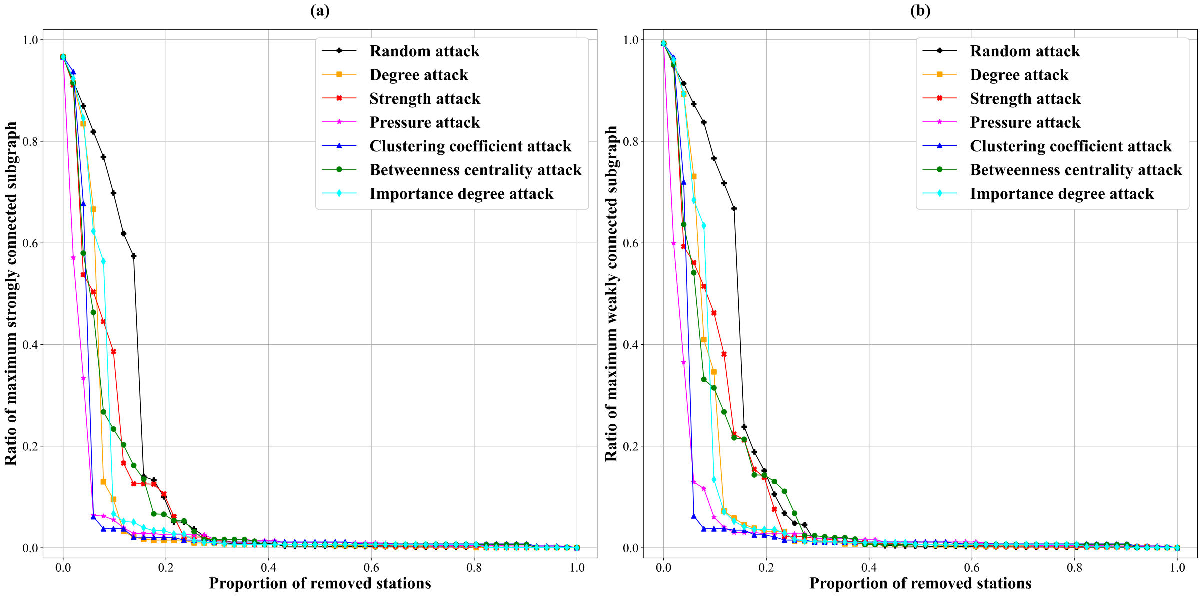


Figure 14. Dynamic robustness with different attack strategies of Xuzhou UPTN: (**a**) the maximum strongly connected subgraph ratio; (**b**) the maximum weakly connected subgraph.

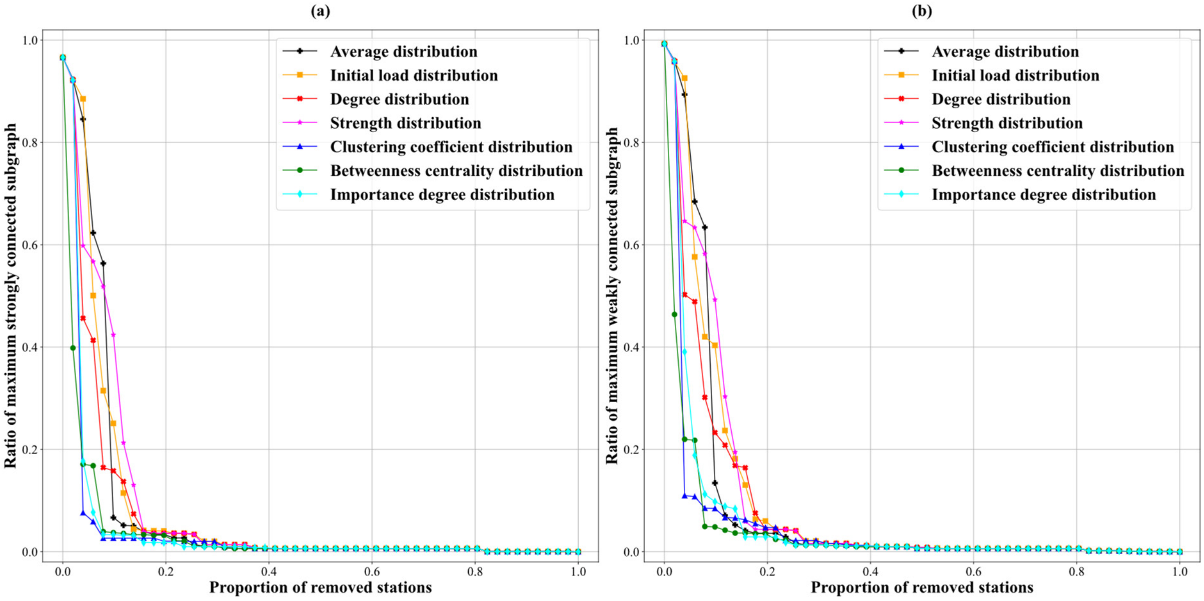


Figure 15. Dynamic robustness with different load redistribution strategies of Xuzhou UPTN: (**a**) the maximum strongly connected subgraph ratio; (**b**) the maximum weakly connected subgraph.

5. Discussions

The results of the above experiments lead to the discussions as follows:

1. According to the analysis of the topological properties, Xuzhou UPTN is verified to have small-world effects and scale-free characteristics. This feature is consistent with UPTNs in most cities, such as Beijing [9], Shanghai [12], and Chengdu [13]. UPTNs are probably constructed in this way because the small-world and scale-free network makes it easier for residents to travel together and resists random failures.
2. In the static robustness analysis, the network connectivity decreases slowly when the random attack strategy is adopted. Xuzhou UPTN has high robustness against unexpected station failures, which will not significantly affect the network function. When the deliberate attack strategy of the pressure and clustering coefficient is adopted, the trend of connectivity change is closer to that under the random attack strategy, indicating that the pressure and clustering coefficients of bus stations are not suitable for representing the network connectivity.
3. When introducing the cascading failure model for the dynamic robustness analysis, removing only 20% of the stations leads to a significant decrease in connectivity, indicating that Xuzhou UPTN shows poor robustness in the face of the cascading failure phenomenon, and needs to strengthen the load capacity of the stations.
4. When conducting dynamic robustness analysis of the importance degree, it is found that the change trend of the network connectivity is similar to that of other commonly used indices, which verifies the feasibility of the importance degree as a station significance evaluation index. This index is a comprehensive evaluation index, the use of which can avoid the one-sidedness of single evaluation indices and help maintain network security operation.

6. Conclusions

Based on complex network theory, this paper constructs the L-Space topological structure model of the UPTN; calculates and analyzes its various topological characteristics; combines the degree, strength, clustering coefficient, and betweenness centrality to propose a comprehensive station significance evaluation index; selects critical stations of the UPTN; improves the cascading failure model for the load redistribution strategy; and carries out the static and dynamic robustness analysis of the UPTN. The network properties and robustness of Xuzhou UPTN are analyzed as an example. The conclusions of this study are as follows:

1. Most UPTNs conform to the features of a small-world and scale-free network. The small-world effect increases the convenience of the UPTN for residents' travel, but causes the characteristics of poor connectivity and low tightness. The scale-free characteristic confers the UPTN good random contingency tolerance but worrying deliberate attack resistance, i.e., good static robustness but poor dynamic robustness.
2. Without the cascading failure phenomenon, the UPTN has a better robustness against station failures. When the load redistribution is taken into account, the network performance is overburdened. The failure of critical stations impacts nearby stations, giving rise to a significant drop in network performance.
3. This study helps select the key stations in the UPTN, which can provide the direction to enhance robustness and efficiency. Increasing the facility capacity of these stations can alleviate heavy traffic flow during peak periods. Furthermore, it is advantageous for stable operation of the public transport system to enhance the maintenance and upgrading of these stations.
4. Besides the expansion of facilities at key stations, optimizing the layout of bus routes is a necessary way to improve the robustness of the UPTN. Increasing the direct connections between key stations can effectively improve the interchange experience of residents and significantly enhance the overall connectivity of the network. Adding bus routes to share the traffic pressure at critical stations can reduce the incidence of station failure so as to ensure the safe operation of public transport systems.

5. These findings can provide empirical guidance for urban planning and public management. Relevant administrators can formulate targeted public transport policies to prioritize investment in the construction and maintenance of key stations and improve the urban grid structure. By optimizing the design of bus routes and the layout of bus stations, the maximum use of public resources and coordinated economic and environmental development can be achieved to realize the sustainability of the city.

Although this study deeply analyzes the topological properties and robustness of the UPTN, there are some limitations. The model mainly focuses on the physical topology, without adequate consideration of traffic flow, passenger travel patterns, and time variations. These factors are important because of their effect on network performance and robustness. Further research will be carried out combined with other multimodal information to capture dynamic change.

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References

1. Ma, F.; Shi, W.J.; Yuen, K.F.; Sun, Q.P.; Xu, X.B.; Wang, Y.J.; Wang, Z.H. Exploring the robustness of public transportation for sustainable cities: A double-layered network perspective. *J. Clean. Prod.* **2020**, *265*, 121747. [CrossRef]
2. Xiao, G.N.; Xiao, Y.; Shu, Y.Q.; Ni, A.N.; Jiang, Z.R. Technical and economic analysis of battery electric buses with different charging rates. *Transp. Res. Part D Transp. Environ.* **2024**, *132*, 104254. [CrossRef]
3. Yu, F.; Mei, D.Z.; Zhang, M.T. Transportation infrastructure, industrial structure change and economic convergence. *Econ. Sci.* **2021**, *43*, 52–67.
4. Hu, L.Y.; Yu, L.P.; Duan, Y.L. A research on the impact of traffic convenience level and information on gathering of high-level service industry. *Sci. Res. Manag.* **2020**, *41*, 101–109.
5. Yang, X.G.; Liu, L.W. Key problems and research progress of urban road traffic examination. *China J. Highw. Transp.* **2024**, *37*, 314–342.
6. Lu, N.C.; Wang, Y.G.; Zhou, Y.; Wu, C.Z. Review of road traffic safety analysis and evaluation methods. *China J. Highw. Transp.* **2023**, *36*, 183–201.
7. Watts, D.J.; Steven, S.H. Collective dynamics of 'small-world' networks. *Nature* **1998**, *393*, 440–442. [CrossRef] [PubMed]
8. Barabási, A.L.; Albert, R. Emergence of scaling in random networks. *Science* **1999**, *286*, 509–512. [CrossRef] [PubMed]
9. Zheng, X.; Chen, J.P.; Shao, J.L.; Bie, L.D. Analysis on topological properties of Beijing urban public transit based on complex network theory. *Acta Phys. Sin.* **2012**, *61*, 190510. [CrossRef]
10. Luo, Y.; Qian, D.L. Construction of subway and bus transport networks and analysis of the network topology characteristics. *J. Transp. Syst. Eng. Inf. Technol.* **2015**, *15*, 39–44.
11. Zhang, H.; Zhao, P.; Gao, J.; Yao, X.M. The analysis of the properties of bus network topology in Beijing basing on complex networks. *Math. Probl. Eng.* **2013**, *2013*, 694956. [CrossRef]
12. Zhang, C.; Zhang, N. Topology research on Shanghai public traffic network. *J. Univ. Shanghai Sci. Technol.* **2006**, *28*, 489–494.
13. Wang, Z.; Peng, Q.Y. Research on topological properties of Chengdu public traffic complex network. *J. Transp. Inf. Saf.* **2007**, *25*, 39–42.
14. Liu, Z.Y.; Li, R.M. The robustness research of Beijing public traffic network system. *Highw. Eng.* **2017**, *42*, 17–23.
15. Wang, J.E.; Mo, H.H.; Wang, F.H.; Jin, F.J. Exploring the network structure and nodal centrality of China's air transport network: A complex network approach. *J. Transp. Geogr.* **2011**, *19*, 712–721. [CrossRef]
16. Yang, J.F.; Zhu, D.P.; Zhao, R.L. Analysis on characteristics of urban rail transit network and robustness of cascading failure. *Comput. Eng. Appl.* **2022**, *58*, 250–258.
17. Wang, L.L.; Huang, M.; Gao, L. Methods of importance evaluation of traffic network node based on clustering algorithms. *J. Transp. Inf. Saf.* **2020**, *38*, 80–88.

18. Piraveenan, M.; Saripada, N.B. Transportation centrality: Quantifying the relative importance of nodes in transportation networks based on traffic modeling. *IEEE Access* **2023**, *11*, 142214–142234. [CrossRef]
19. Motter, A.E.; Lai, Y.C. Cascade-based attacks on complex networks. *Phys. Rev. E* **2002**, *66*, 2. [CrossRef] [PubMed]
20. Wang, S.C.; Qian, X.D. Complex network-based cascade failure model optimization and network robustness analysis—Taking the postal logistics network in Northwest China as an example. *Control Eng. China* 2024, *in press*.
21. Sonnam, J.; Gao, L.; Liu, F.; Li, M.H.; Shen, Z.S.; Xu, L.D.; Gao, Z.Y. Cascading failure with preferential redistribution on bus-subway coupled network. *Int. J. Mod. Phys. C* **2021**, *32*, 2150103.
22. Wang, Y.D.; Ma, L.; Guo, J.; Tang, X.P. Robustness analysis of complex network of railway freight based on maximum connectivity component of the network. *Railw. Stand. Des.* 2024, *in press*.
23. Yang, H.H.; Shi, A. Robustness evaluation for multi-subnet composited complex network of urban public transport. *Alex. Eng. J.* **2021**, *60*, 2065–2074. [CrossRef]
24. Müller-Hannemann, M.; Rückert, R.; Schiewe, A.; Schöbel, A. Estimating the robustness of public transport systems using machine learning. *arXiv* **2021**, arXiv:2106.08967.