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A holistic investigation of fraction learning: examining the hierarchy of fraction skills, misconceptions, mathematics anxiety and response confidence

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ABSTRACT

This study aimed to holistically investigate fraction knowledge by considering pupils' fraction performance on various tasks (i.e. mapping, equivalence, comparison, and arithmetic) and in relation to whole number arithmetic, fraction misconceptions, mathematics anxiety, and confidence in responses. Northern Irish pupils ($N = 123$; 77 girls; $M_{age} = 11.1$ years) demonstrated a strong understanding of fraction magnitudes with few pupils showcasing misconceptions across magnitude tasks. In contrast, fraction arithmetic knowledge, which was uniquely predicted by whole number arithmetic and fraction comparison, was still developing with many pupils incorrectly applying whole number strategies to arithmetic problems. Mathematics anxiety was low, but negatively correlated with performance, and confidence reports were more closely aligned with performance on whole number arithmetic compared to fraction tasks. Overall, researchers and educators need to consider not just fraction performance, but also foundational skills, the presence of misconceptions, and pupils' feelings and beliefs to better understand fraction learning.

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KEYWORDS

Arithmetic; fraction knowledge; fraction misconceptions; mathematics anxiety; mathematics confidence



Fraction learning is notoriously challenging. After years of whole number mathematics instruction pupils are introduced to fractions and need to transition from an established understanding of whole numbers to a more abstract rational number system (Ni & Zhou, 2005; Vosniadou, 1994). Although challenging, this acquisition of fraction knowledge is essential, particularly because it serves as a “gatekeeper” for more advanced mathematics (Bailey et al., 2012; Barbieri et al., 2021; Booth et al., 2014; Booth & Newton, 2012; Siegler et al., 2011). Unfortunately, challenges with fractions appear to be a global phenomenon, hindering pupils' advancement in mathematics (Chan et al., 2007; Di Lonardo Burr et al., 2022; Gabriel et al., 2013; Meert et al., 2010; Siegler & Lortie-Forgues, 2017).

Research on fractions is often focussed on fraction performance, but not necessarily on other factors that may provide insights into the challenges pupils face when learning fractions. To deepen our understanding of fraction learning, we need to consider its relation to whole number skills, the types of misconceptions pupils hold for different fraction tasks, and the role of achievement emotions and barriers. In the

present study, we aim to holistically investigate fraction knowledge by considering fraction performance on various tasks and in relation to whole number arithmetic, misconceptions pupils hold about fractions, pupils' mathematics anxiety, and pupils' confidence in their responses across natural number and fraction tasks.

The development of fraction skills

When it comes to learning fractions, pupils need to acquire various skills, including mapping pictorial representations of magnitude to fraction symbols (Hecht & Vagi, 2010; Xu et al., 2022), determining the equivalence of the magnitude of two fractions presented in different forms (Boyer & Levine, 2012; Pedersen & Bjerre, 2021), comparing the magnitudes of two fraction symbols (Meert et al., 2010; Rinne et al., 2017), and performing arithmetic involving fraction symbols (Braithwaite et al., 2019; Lortie-Forgues, 2015). During the early stages of fraction instruction, pupils learn how to represent fraction magnitudes graphically and connect those magnitudes with fraction symbols (e.g. knowing that a circle divided into four equal parts with

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one part shaded should be represented as $\frac{1}{4}$; Hecht et al., 2007; Siegler, 2016). They must develop an understanding of the part-whole relations of fractions, that is, a fraction represents a relation between part(s) of an equally partitioned unit and the total number of parts (Charalambous & Pitta-Pantazi, 2007). Before formal schooling, pupils already gain experience in creating equal units by dividing wholes into parts in everyday activities, such as folding a piece of paper or sharing food (Hunting & Sharpley, 1988). However, even though this early exposure to the intuitive understanding of part-whole relations may contribute to the future formal learning of fractions (Pitkethly & Hunting, 1996), some pupils continue to struggle with these concepts even after instruction. For example, Xu et al. (2022) found that approximately one-fifth of 10-year-old pupils consistently made errors on a fraction mapping task; it was their proficiency in whole number division that distinguished those who could versus could not accurately map fractions.

In addition to connecting a fraction symbol and the magnitude it represents, pupils need to recognise that different fractions can represent the same magnitude, as demonstrated by the equivalence of $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and so forth (Behr et al., 1984; Kamii & Clark, 1995; Pedersen & Bjerre, 2021). Understanding fraction equivalence can be challenging for pupils because it requires them to move beyond mere perceptual understanding to learn that the same magnitude can be represented by fractions with different numerators and denominators simultaneously (Kamii & Clark, 1995). For example, when pupils are presented with a rectangle with two out of four equal-sized parts shaded, they should recognise that $\frac{2}{4}$ can be scaled down (i.e. $\frac{1}{2}$) or scaled up (e.g. $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$). For young pupils (ages 6–10), accuracy in visually matching equivalences shown in proportional relations was found to decline as the scaling factor increased (Boyer & Levine, 2012). Around the ages of 11 and 12, however, pupils were able to accurately map pictorial representations of non-reduced fractions to symbolic notations (Ni, 2001).

To further develop their understanding of fraction magnitudes, pupils must broaden their understanding of the number system, learning to view fractions as unified magnitudes instead of interpreting the numerator and denominator as distinct whole numbers (Chi et al., 1994; Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). Initially, this concept can be challenging. For example, when 10 year-old pupils were asked to

compare the magnitudes of two fractions, they had near-perfect scores on congruent problems where the relative magnitudes of the components (numerator and/or denominator) aligned with the fraction's magnitude (e.g. $\frac{6}{7}$ vs. $\frac{5}{7}$; $\frac{5}{7}$ vs. $\frac{1}{3}$); however, they showed extremely poor performance when this alignment was absent (e.g. $\frac{2}{3}$ vs. $\frac{2}{7}$; $\frac{4}{7}$ vs. $\frac{5}{6}$; Xu et al., 2022). Similarly, pupils between 10 and 12 years old were slower and more error prone when comparing fractions with common numerators (e.g. $\frac{2}{3}$ vs. $\frac{2}{7}$) than those with common denominators (e.g. $\frac{6}{7}$ vs. $\frac{5}{7}$; Meert et al., 2010). In a longitudinal study, pupils demonstrated substantial improvement in fraction comparison between the ages of 10 and 12, evidenced by their strategic shift from discretely processing numerators and denominators to consistently using appropriate strategies that consider both simultaneously (Rinne et al., 2017).

Moving beyond fraction magnitudes, pupils must learn how to perform operations with fractions. Like other aspects of fraction learning, fraction arithmetic, which is typically introduced later in primary school, is challenging to master (Lortie-Forgues, 2015). Introducing fraction arithmetic with visual representations may be initially helpful, so that pupils can see not only the relations between the symbolic and nonsymbolic fraction representations, but also link these nonsymbolic representations to symbolic operations (Cramer et al., 2008). For example, visualisations can help pupils understand that fractions are composed of unit fractions and also link this concept to why the denominators need to be equal before they can proceed with adding or subtracting numerators (Braithwaite & Siegler, 2021; Fazio & Siegler, 2010). Linking the symbolic and non-symbolic representations ensures that pupils are not just memorising a procedure, but that they have a conceptual understanding of fraction arithmetic which in turn can help pupils make the learning process of fractional arithmetic meaningful (Carpenter, 1986; Silver, 1986).

When pupils first begin to tackle symbolic fraction arithmetic problems, they often make errors that arise from the overgeneralisation of whole number arithmetic rules to fraction arithmetic and omission of crucial steps (Braithwaite & Siegler, 2023). For example, pupils might erroneously apply operations to both numerators and denominators (e.g. $\frac{1}{4} + \frac{2}{4} = \frac{3}{8}$) or skip crucial steps such as only multiplying the denominators when finding a common denominator (e.g. $\frac{1}{5} + \frac{1}{6} = \frac{2}{30}$). A possible

source of errors in fraction arithmetic is the interference between pupils' prior knowledge of whole number arithmetic and the newly introduced fraction arithmetic (DeWolf & Vosniadou, 2015; Ni & Zhou, 2005). This whole number bias decreases as fraction skills develop (Braithwaite & Siegler, 2018), however, many, including adults, misapply whole number arithmetic procedures to fraction tasks (Braithwaite et al., 2019; Di Lonardo Burr et al., 2020). The intrinsic difficulty of certain fraction arithmetic problems, such as those involving adding or subtracting fractions with uncommon denominators, add to these challenges (Braithwaite & Siegler, 2023). The amount of practice and instruction pupils receive, both in the classroom and at home, also contribute to the number of and types of errors pupils make on fraction arithmetic problems (Braithwaite & Siegler, 2023). Without adequate exposure and practice, pupils are more prone to make errors.

The hierarchy of whole number arithmetic and fraction skills

Although many pupils struggle with fractions in general, fraction arithmetic is one of the most difficult fraction concepts to master (Lortie-Forgues, 2015; Siegler & Lortie-Forgues, 2017). Prior research has shown that fraction arithmetic performance is strongly related to pupils' prior whole number arithmetic skills (see a meta-analysis by Lin & Powell, 2021). Fraction arithmetic fundamentally relies on whole number arithmetic skills. For example, when adding fractions with different denominators, such as $\frac{2}{5}$ and $\frac{1}{6}$, pupils need to use whole number multiplication to find a common denominator (i.e. $5 \times 6 = 30$) and update the numerators (i.e. $2 \times 6 = 12$; $1 \times 5 = 5$). They must then combine numerators using whole number addition to get the correct answer (i.e. $\frac{12}{30} + \frac{5}{30} = \frac{17}{30}$). Therefore, after pupils grasp the procedures for solving fraction arithmetic, those with stronger whole number arithmetic skills should outperform their peers with weaker skills.

Beyond whole number skills, what role do other fraction skills play in predicting fraction arithmetic? In general, studies that have investigated the correlations amongst fraction tasks find weak relations suggesting that mastery of one task may not lead to mastery of another (Di Lonardo Burr et al., 2022; Xu et al., 2024). Potentially, these weak correlations reflect the different conceptual pieces of fraction understanding that pupils need to master, including part-whole interpretations, measurement interpretations, and arithmetic (Charalambous & Pitta-Pantazi, 2007). Thus, in trying to uncover

which tasks might best predict fraction arithmetic it may be important to consider the characteristics of the task. On that view, tasks such as fraction comparison may be particularly important for fraction arithmetic because they not only require measurement understanding of fractions but also, like fraction arithmetic, the manipulation of fraction symbols. Moreover, these two tasks often co-develop (Bailey et al., 2017) as pupils begin to move beyond just conceptual fraction understanding into procedural knowledge (Faulkenberry, 2013). In the present study, we investigate whether earlier-learned fraction concepts (i.e. mapping, equivalence, comparison) predict more advanced fraction knowledge (i.e. fraction arithmetic) and whether these various fraction tasks reflect a hierarchy of fraction knowledge such that later-learned tasks that tap into symbolic and magnitude knowledge of fractions may uniquely predict this more advanced fraction knowledge.

Navigating challenges in learning fractions: anxiety and confidence

Given the inherent challenges associated with learning fractions, some pupils might develop mathematics anxiety—negative feelings or apprehension toward mathematics—that hinders their ability to solve mathematical problems in both everyday life and academic contexts (Ashcraft, 2002; Hembree, 1990; Maloney, 2016; Richardson & Suinn, 1972). Although the origins of mathematics anxiety remain unclear, it has been suggested that some may have a cognitive predisposition towards such anxiety, while others may be influenced by negative attitudes surrounding mathematics, including negative parental or teacher influence (Beilock & Maloney, 2015). Similar to mathematical competence, mathematical anxiety has been found to be a stable construct and thus can be seen as a trait (Liebert & Liebert, 1988; Luttenberger et al., 2018). While both mathematical competence and anxiety can be generalised across situations, the strength of the relation between the two is not always consistent, with findings from meta-analyses demonstrating that mathematics anxiety is more strongly correlated with more advanced tasks (i.e. arithmetic, algebra, geometry, multi-step problems; Caviola et al., 2022; Namkung et al., 2019). Notably, however, fractions were not considered in these meta-analyses. Indeed, few individual studies have examined how mathematics anxiety may be differentially related to fraction tasks. For example, there is some evidence to suggest that older pupils and adults have more negative attitudes towards fractions (Sidney et al., 2021), but the relation between attitudes and performance is not understood. Moreover, Starling-Alves et al. (2022) found that anxiety may be differentially associated with

symbolic but not nonsymbolic fraction skills but nevertheless found consistently weak correlations between mathematics anxiety and outcomes across whole number and fraction tasks. Overall, the relation between fractions and mathematics anxiety is poorly understood and thus in the present study we investigate pupils' feelings towards mathematics and the relations with performance on both whole number and arithmetic tasks.

In the context of learning fractions, it is also essential to consider pupils' confidence, or their self-perception of their performance on a given task. One way to assess this is by asking participants to report their confidence in their responses (Di Lonardo Burr & LeFevre, 2020; Fitzsimmons et al., 2020; Fitzsimmons & Thompson, 2022; Morony et al., 2013). For 6- and 7-year-old Chinese and British pupils, Dowker et al. (2019) found that self-ratings of performance were significantly correlated with actual performance on an arithmetic task, suggesting that pupils knew whether they were doing well on a mathematical assessment. However, with fractions, strong misconceptions may erroneously lead pupils to believe they are providing the correct responses to problems. For example, pupils (ages 10–12) with whole number bias have reported high confidence in their incorrect responses on fraction assessments (González-Forte et al., 2023) and low levels of state anxiety after completing fraction assessments (Halme et al., 2023). Thus, self-ratings of confidence in responses may provide additional insights into the misconceptions pupils have with respect to fractions.

The present study

In the present study we aimed to holistically examine fraction understanding by considering performance and the types of misconceptions pupils hold for tasks that tap into different types of fraction knowledge (Research Question 1), the relations between whole number arithmetic and fraction skills (Research Question 2), and the role of mathematics anxiety and confidence in responses (Research Question 3). Pupils from Northern Ireland completed whole number arithmetic, fraction magnitude (mapping, equivalence, comparison), and fraction arithmetic (addition and subtraction) tasks. Additionally, they completed a mathematics anxiety measure and were asked to rate their confidence in their responses for each of the mathematical tasks. These self-report measures allowed us to understand how pupils perceived mathematics, particularly when grappling with challenging topics like fractions.

In Northern Ireland formal mathematics education begins in Year 1 (age 4), following guidelines from the Education Authority (<https://www.eani.org.uk/>). Arithmetic

with whole numbers is introduced in Year 2, with a focus on addition and subtraction up to Year 4. Multiplication and division are typically introduced in Year 5, and pupils continue to refine these skills through Year 7. In contrast, fraction topics are first introduced in Year 3 (age 6). As pupils progress through their primary school years, a diverse range of fraction topics is covered, with each year's instruction building upon the foundation of previous knowledge. Specifically, pupils learn basic fraction concepts like "half" and "quarter" through hands-on activities like cutting cakes or paper folding. In Years 4 and 5, pupils continue to build their fraction knowledge, focusing on mapping between fraction notations and magnitude, as well as comparing and ordering fractions. In Years 6 and 7, pupils expand their fraction knowledge by learning about fraction equivalence and fraction arithmetic (addition and subtraction), both with and without common denominators. In the present study, we recruited pupils in Year 7 (aged 10 and 11 years). This age was selected because pupils had received instruction on numerous fraction concepts including fraction mapping, fraction comparison, and fraction addition and subtraction with common denominators (Years 4–7) but were still in the process of learning how to add and subtract fractions with uncommon denominators.

Research question 1: how do pupils perform on various fraction tasks and what types of errors do they make?

First, we assessed pupils' performance on timed fraction magnitude and fraction arithmetic tasks. Beyond considering accuracy, we explored the errors pupils made on fraction tasks to provide deeper insights into potential misconceptions pupils might possess for conceptual and/or procedural aspects of fractions. Based on curricular guidelines in Northern Ireland, we hypothesised that pupils would demonstrate a strong understanding of fraction magnitude knowledge, making minimal errors on the three fraction magnitude tasks (Hypothesis 1a). In contrast, because pupils were still in the process of learning and acquiring the complex procedures associated with fraction arithmetic, we hypothesised that their performance on this task would be weak, especially on challenging problems involving uncommon denominators (Hypothesis 1b).

Research question 2: what are the hierarchical relations among whole number arithmetic, fraction magnitude and fraction arithmetic tasks?

Second, we examined the hierarchical relations among whole number arithmetic, fraction magnitude, and

fraction arithmetic skills. In accordance with the findings from previous research (Hecht et al., 2003; Jordan et al., 2013; Siegler et al., 2011; Siegler & Pyke, 2013; Xu et al., 2024) and a meta-analysis by Lin and Powell (2021), we hypothesised that fraction arithmetic would be uniquely predicted by whole number arithmetic skills (Hypothesis 2a) and fraction comparison, with more variability in this later-learned symbolic fraction skill than fraction mapping and equivalence (Hypothesis 2b).

Research question 3: how do mathematics anxiety and confidence in responses relate to pupils' mathematical performance?

Lastly, we investigated self-reports of mathematics anxiety and confidence related to the mathematical tasks presented in this study. Consistent with previous literature (Barroso et al., 2021), we hypothesised negative correlations between pupils' reported mathematics anxiety and their performance on mathematical tasks (Hypothesis 3a). With respect to confidence, which was measured for each task, we hypothesised that pupils would report high confidence in their performance on whole number arithmetic and that their confidence would be consistent with their actual performance (Hypothesis 3b). For fractions, however, because misconceptions may erroneously lead pupils to believe they have provided a correct response (González-Forte et al., 2023), we hypothesised that for many pupils with poor performance on the fraction assessments, their self-rated confidence would be inconsistent with their actual performance (Hypothesis 3c).

Method

Participants

The present study was approved by the Institutional Review board of Queen's University Belfast. Participants included 123 Year 7 pupils in Northern Ireland (77 girls, 43 boys, 2 identified as other), with a mean age of approximately 11 years (ranging from 10.6 to 11.6). These pupils were recruited from five schools with varying socioeconomic status within the region.

Procedure

Group testing was conducted during school hours in each classroom, facilitated by five undergraduate experimenters pursuing degrees in Psychology. After obtaining written consent from both pupils and parents, pupils received instructions related to task timing and the importance of maintaining a silent working

environment. The order of tasks was the same for all pupils: Pupils first completed the mathematics anxiety inventory, followed by whole number arithmetic (addition, subtraction, multiplication, and division), fraction tasks (mapping, equivalence, comparison, and arithmetic), and finally the self-rated confidence scale. The testing session lasted a maximum of 25 min and was followed by a debriefing session.

Measures

Data were collected in December 2022 and January 2023. The anonymized data for the measures and materials used in the current study are available for download at Open Science Framework: <https://osf.io/82dps/>. All reliabilities were computed using the present sample and are reported below.

Whole number arithmetic tasks

Pupils solved four subsets (addition, subtraction, multiplication, division) of problems derived from the modified version of the *Arithmetical Ability* subscale from the *Heidelberg Rechen Test* (Haffner et al., 2005), as adapted by Wu and Li (2006). For each operation, pupils were given 1 min to answer as many problems as possible. Problems were presented in two columns and became increasingly more difficult; pupils were instructed to answer the problems in order. Scoring for each operation was the total number of correct answers (with a maximum score of 40). The reliabilities based on individual items for addition, subtraction, multiplication, and division were high, Cronbach's α s = .88, .90, .92, and .95, respectively.

Addition. The first column included problems with single- and double-digit addends (e.g. $2 + 8 = _$, $12 + 3 = _$) with no sums greater than 20. The second column included problems with single-, double-, and triple-digit addends (e.g. $17 + 15 = _$, $177 + 623 = _$).

Subtraction. The first column included single- and double-digit minuends and subtrahends (e.g. $7 - 5 = _$, $15 - 13 = _$), with no minuends greater than 20. The second column included problems with double- and triple-digit minuends, and single-, double- and triple-digit subtrahends (e.g. $55 - 23 = _$, $155 - 66 = _$).

Multiplication. The first column included single-digit multiplicands and multipliers (e.g. $4 \times 4 = _$, $4 \times 8 = _$). The second column included problems with single- and double-digit multiplicands and multipliers, all less than 20 (e.g. $7 \times 13 = _$, $19 \times 9 = _$).

Division. The first column included single- and double-digit dividends and single-digit divisors (e.g. $6 \div 2 = \underline{\quad}$, $28 \div 4 = \underline{\quad}$). The second column included problems with double- and triple-digit dividends and single-digit divisors (e.g. $40 \div 8 = \underline{\quad}$, $192 \div 8 = \underline{\quad}$).

Fraction tasks

For all four fraction tasks, pupils were given one minute to complete a maximum of 20 problems, in order. The scoring for each task was the total number of correct responses.

Fraction mapping. Each trial consisted of a shape partitioned into equal-sized segments, some of which were shaded. Pupils were instructed to write down the fraction that accurately represented the shaded part of the figure (e.g. a circle showing 2 out of 5 portions shaded corresponds to the fraction $\frac{2}{5}$). The reliability based on individual items was high, Cronbach's $\alpha = .81$.

Fraction equivalence. Each trial consisted of a fraction alongside a shape partitioned into equal-sized segments, some of which were shaded. Pupils were instructed to mark a "✓" when the fraction and the depicted shape represented the same magnitude and an "×" when they did not. For the first ten problems, the shapes depicted non-reducible fractions (e.g. a rectangle showing 2 out of 5 portions shaded alongside the fraction $\frac{2}{5}$); the last ten problems had shapes depicting reducible fractions (e.g. a rectangle showing 2 out of 8 portions shaded alongside the fraction $\frac{1}{4}$). For both reducible and non-reducible problems, half of the fractions aligned with pictorial representations of the same magnitude, whereas the other half did not. The reliability based on individual items was good, Cronbach's $\alpha = .73$.

Fraction comparison. Each trial consisted of two fractions. Pupils were asked to circle the fraction that had a greater magnitude. There were 11 congruent and 9 incongruent problems. For the congruent problems, the relative magnitude of components (numerator and/or denominator) matched the relative magnitude of the whole fractions (e.g. $\frac{3}{4} > \frac{1}{4}$, $\frac{5}{8} > \frac{4}{7}$); for the incongruent problems, the relative magnitude of components (numerator and/or denominator) did not match the relative magnitude of the whole fractions (e.g. $\frac{3}{4} > \frac{3}{5}$, $\frac{2}{3} > \frac{4}{9}$). The reliability based on individual items was good, Cronbach's $\alpha = .71$.

Fraction arithmetic. For both addition and subtraction tasks, the first 5 problems had common denominators (e.g. $\frac{5}{8} + \frac{2}{8} = \underline{\quad}$; $\frac{2}{4} - \frac{1}{4} = \underline{\quad}$). The next 5 problems had common numerators (e.g. $\frac{1}{4} + \frac{1}{5} = \underline{\quad}$; $\frac{1}{2} - \frac{1}{3} = \underline{\quad}$). Subsequently, the next 10 problems had neither common denominators nor numerators (e.g. $\frac{1}{6} + \frac{2}{3} = \underline{\quad}$; $\frac{3}{4} - \frac{1}{8} = \underline{\quad}$). Pupils were instructed to use the margin of the testing sheet for any rough work. The reliabilities based on individual items for addition and subtraction were good, Cronbach's $\alpha = .83$ and $.76$, respectively.

Mathematics anxiety

Mathematics anxiety was measured using the modified *Abbreviated Math Anxiety Scale* (Carey et al., 2017). Pupils were asked to rate their anxiety levels on a five-point scale (ranging from 1 = low anxiety/feeling calm to 5 = high anxiety/feeling stressed or worried) for a range of mathematics-related scenarios (e.g. taking a mathematics test, encountering a challenging mathematics topic). The reliability based on individual items was high, Cronbach's $\alpha = .87$.

Self-rated confidence

Pupils were asked to reflect on the tasks they had completed and provide self-assessments of their performance for each task at the end of testing session. Specifically, they were instructed to consider how well they thought they performed on each task, focusing on the questions they were able to answer within the time limit and not worrying about those they did not have time to complete. They were prompted to circle their perceived accuracy among three options: "All or most correct", "some correct", and "most of my answers were wrong". Scores were reverse coded so that higher values indicated greater confidence in their responses. The reliability based on individual items was high, Cronbach's $\alpha = .82$.

Analytical plan

To investigate Research Question 1, we conducted repeated measures analysis of variance (ANOVA) and *t*-tests to assess pupils' performance on fraction magnitude and fraction arithmetic tasks. In accordance with the recommendations of Field (2013), we assumed unequal variance for *t*-tests and that the assumption of sphericity was violated for ANOVAs, which were subsequently Greenhouse-Geisser corrected. We also

examined frequency of correct and incorrect responses to infer misconceptions that pupils might hold about fraction magnitude and arithmetic. Next, to investigate Research Question 2, we conducted a hierarchical linear regression to investigate the unique contributions of whole number arithmetic and each of the fraction magnitude tasks (mapping, equivalence, and comparison) to their performance in fraction arithmetic. Lastly, to investigate Research Question 3, we examined the correlations between pupils' mathematics anxiety and mathematics performance. For confidence ratings, we examined the consistency between pupils' self-reported confidence and their overall performance for each mathematics task. Performance for this analysis was the percentage of correct answers, excluding blank responses (i.e. if a pupil completed 12 problems and answered 9 of them correctly, their score was 75%). Performance was divided into three categories (> 80% correct; 50–79% correct; < 50% correct) and the proportion of pupils who responded with low (1), medium (2), or high (3) confidence across the categories of performance was computed. The percentages were chosen to closely align with the wording from the inventory.

Results

Descriptive statistics and correlations

No data were missing in any of the tasks. Gender differences were observed in only two measures: Girls ($M = 2.2$, $SD = 0.7$) reported higher levels of mathematics anxiety than boys ($M = 1.9$, $SD = 0.6$), $t(118) = 2.35$, $p = .021$, Cohen's $d = 0.45$, and boys ($M = 23.5$, $SD = 4.4$) performed better on whole number addition than girls ($M = 21.7$, $SD = 4.0$), $t(118) = -2.22$, $p = .028$, Cohen's $d = -0.42$.

Tables 1 and 2 show the descriptive statistics and correlations among the measures. With the exception of the

Table 1. Descriptive statistics among mathematics tasks and self-rated confidence on the tasks ($N = 123$).

Variable	Mathematics Tasks			Self-rated Confidence		
	M (SD)	Min, Max	Skew	M (SD)	Min, Max	Skew
Whole Number Arithmetic						
Addition	22.4 (4.2)	8, 31	-0.6	2.8 (0.3)	2, 3	-2.1
Subtraction	22.7 (5.0)	9, 32	-0.6	2.7 (0.5)	1, 3	-1.4
Multiplication	21.3 (6.0)	5, 32	-0.3	2.7 (0.5)	1, 3	-1.5
Division	20.9 (8.1)	1, 34	-0.4	2.5 (0.6)	1, 3	-0.9
Fraction Magnitude						
Mapping	13.0 (2.8)	2, 19	-0.8	2.7 (0.5)	1, 3	-1.3
Equivalence	12.1 (2.8)	4, 20	-0.5	2.4 (0.6)	1, 3	-0.5
Comparison	12.2 (3.0)	2, 20	-0.7	2.3 (0.7)	1, 3	-0.4
Fraction Arithmetic						
Fraction	4.1 (2.4)	0, 9	-0.6	1.9 (0.7)	1, 3	0.1
Addition						
Fraction	4.4 (1.8)	0, 8	-1.1	1.7 (0.6)	1, 3	0.4
Subtraction						

relation between mathematics anxiety and fraction equivalence ($p = .057$), all measures were significantly correlated with each other. The distributions of the scores varied across the measures, as shown in the violin plots in Figures 1 and 2. Notably, there was substantial variability in scores observed across all tasks. Extreme outliers, defined by cases with $|z\text{-scores}| > 3.29$ (Field, 2013), were identified in a few tasks: whole number addition ($n = 1$), fraction mapping ($n = 1$), and fraction comparison ($n = 1$). Through sensitivity analyses, the pattern of results remained consistent with and without these outliers and thus all data were included in the analyses.

Research question 1: how do pupils perform on various fraction tasks and what types of errors do they make?

Fraction magnitude tasks

As shown in Figure 1, pupils had proficient performance on the fraction magnitude assessments (i.e. mapping, equivalence, and comparison). The total correct scores were analysed in a repeated measures ANOVA. Performance varied with task type, $F(1.96, 238.76) = 6.12$, $p = .003$, $\eta_p^2 = .05$. Using the Bonferroni adjustment, post hoc pairwise comparisons revealed that pupils performed better on the mapping task compared to the equivalence and comparison tasks, $ps < .05$, but no significant difference was found between the latter two, $p = .999$.

Next, we conducted a more detailed analysis of individual trial performance for each of the tasks. First, we examined the specifics of error types on the mapping task using the coding scheme developed by Di Lonardo Burr et al. (2022). Across all problems the most common response was accurate (91.1%), followed by no response (8.9%). On average, among problems where pupils provided responses, errors were rare throughout all problems (3.3%). The identified errors included providing the fraction for the unshaded rather than shaded portion (1.2%; e.g. writing $\frac{1}{3}$ for the fraction $\frac{2}{3}$), careless mistakes (0.9%; e.g. writing $\frac{2}{6}$ for fraction $\frac{2}{7}$), using the unshaded portion as the denominator (0.5%; e.g. writing $\frac{2}{3}$ for the fraction $\frac{3}{5}$), miscellaneous (0.5%; e.g. writing $\frac{3}{5}$ for fraction $\frac{5}{6}$), and inverting (0.2%; e.g. writing $\frac{3}{1}$ for fraction $\frac{1}{3}$). Notably, no instances were found where pupils made whole number errors (i.e. providing a whole number response equal to

Table 2. Correlations among measures ($N = 123$).

Variable	1	2	3	4	5	6	7	8	9
1. Whole Number Addition	–								
2. Whole Number Subtraction	.83***	–							
3. Whole Number Multiplication	.80***	.83***	–						
4. Whole Number Division	.80***	.80***	.88***	–					
5. Fraction Mapping	.57***	.57***	.62***	.62***	–				
6. Fraction Equivalence	.42***	.43***	.30***	.32***	.38***	–			
7. Fraction Comparison	.54***	.53***	.41***	.46***	.28***	.37***	–		
8. Fraction Addition	.50***	.46***	.43***	.49***	.30***	.30***	.42***	–	
9. Fraction Subtraction	.45***	.41***	.37***	.44***	.36***	.27**	.38***	.76***	–
10. Mathematics Anxiety	–.40***	–.38***	–.39***	–.41***	–.20*	–.17	–.24**	–.25*	–.19*

$p < .05^*$, $p < .01^{**}$, $p < .001^{***}$.

either the numerator or denominator), and none of the pupils exhibited consistent errors in the mapping task. These findings suggest that pupils have mastered the fraction mapping skill.

We then examined accuracy on the equivalence and comparison tasks, which involved binary decisions. For the equivalence task, focusing on problems where responses were provided, pupils performed better on the problems that did not require reduction ($M = 0.94$, $SD = 0.14$) compared to those that did ($M = 0.62$, $SD = 0.32$), $t(109) = 11.32$, $p < .001$, Cohen's $d = 1.08$. Further examination of the distribution revealed a negatively skewed pattern for the former and a normal distribution for the latter problems. These findings suggest that most pupils performed exceptionally well when matching pictorial representations of fractions to fraction symbols when reduction was not required. However, challenges arose for some of the pupils when matching such representations to symbols in problems requiring reduction.

For the comparison task, among problems where pupils provided responses, their performance was better on the congruent problems ($M = 0.86$, $SD = 0.18$)

compared to incongruent problems ($M = 0.76$, $SD = 0.34$), $t(122) = 2.71$, $p = .009$, Cohen's $d = 0.24$. A closer inspection of the distribution revealed a negatively skewed pattern for both types of problems, indicating that most pupils provided a correct response to the attempted problems. These results suggest that, as a group, pupils demonstrated proficient performance in fraction comparison, a task assessing fraction magnitude without relying on visual representations.

Fraction arithmetic tasks

As expected, of the 20 fraction arithmetic problems for each of addition and subtraction, pupils were only able to correctly solve the initial five problems, which involved common denominators ($Mdn = 5$; see Figure 1). Excluding blank responses, the three most common response types for addition and subtraction were: (i) correct responses (69.4% and 84.9% for addition and subtraction, respectively), (ii) erroneously adding both the numerators and denominators (25.6% and 7.6%) and (iii) miscellaneous errors that could not be categorised (3.3% and 6.7%).

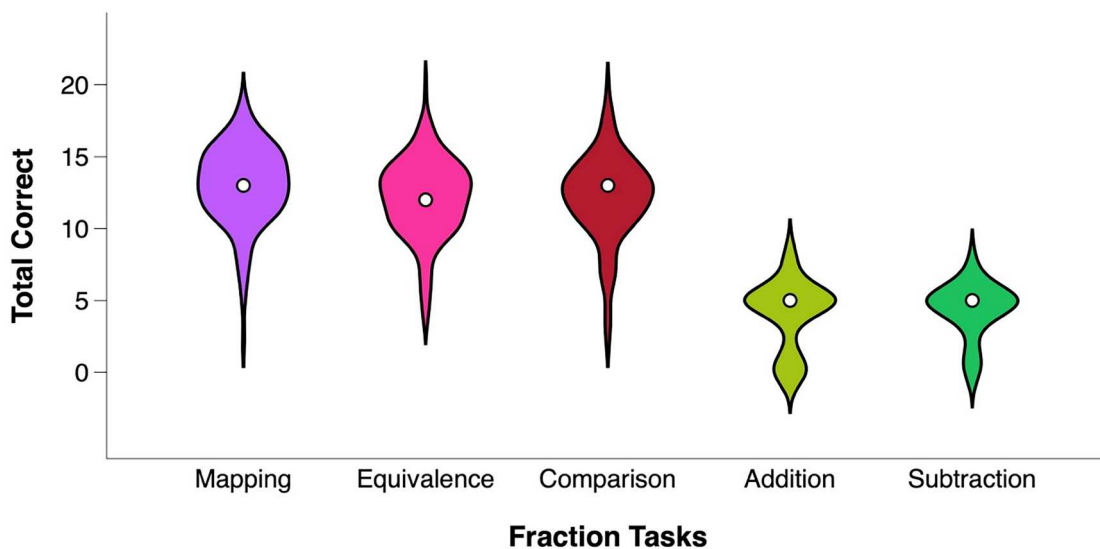


Figure 1. Violin plots of scores on fraction tasks for mapping, equivalence, comparison, addition and subtraction.

Note. The white dot is the median. The width represents the density of the data, with wider sections indicating more data points and narrower sections indicating fewer data points.

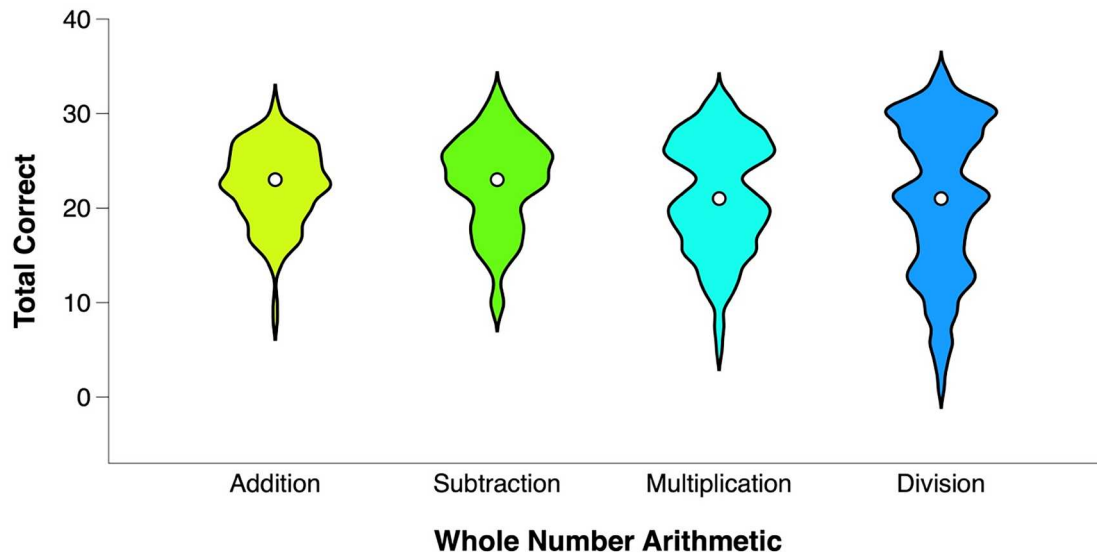


Figure 2. Violin plots of scores on whole number arithmetic for addition, subtraction, multiplication and division tasks.

Note. The white dot is the median. The width represents the density of the data, with wider sections indicating more data points and narrower sections indicating fewer data points.

We further examined error types using the coding scheme developed by Di Lonardo Burr et al. (2020). Figure 3 shows that most pupils accurately solved the first five problems involving common denominators; however, there was a substantial decline in performance starting from the sixth trial, which introduced uncommon denominators. Notably, pupils began committing various errors, with the most common being adding/subtracting both numerators and denominators, suggesting that they applied whole number arithmetic rules to fraction arithmetic. Less common errors included using the bigger/smaller denominator as the denominator (e.g. $\frac{1}{6} + \frac{2}{3} = \frac{2}{6}$; $\frac{1}{5} - \frac{1}{6} = \frac{0}{5}$), keeping the numerator and only adding/subtracting the denominators (e.g. $\frac{1}{3} + \frac{1}{3} = \frac{1}{6}$; $\frac{1}{5} - \frac{1}{6} = \frac{1}{1}$), adding whole number components (e.g. $\frac{1}{4} + \frac{2}{4} = 11$; $\frac{3}{4} - \frac{1}{6} = 14$), only multiplying denominators when finding a common denominator (e.g. $\frac{1}{5} + \frac{1}{6} = \frac{2}{30}$; $\frac{1}{5} - \frac{1}{6} = \frac{0}{30}$), and miscellaneous.

In summary, supporting Hypothesis 1, pupils performed well on the fraction magnitude tasks, excelling particularly on the mapping task compared to the fraction equivalence and comparison tasks. Many pupils demonstrated competence in fraction arithmetic problems involving common denominators; however, challenges emerged when working with uncommon denominators. This finding was anticipated, given that pupils were still in the process of learning fraction arithmetic procedures.

Research question 2: what are the hierarchical relations among whole number arithmetic, fraction magnitude and fraction arithmetic tasks?

As shown in Figure 2, pupils performed well across all arithmetic operations, with increasing score variability from addition to division. As shown in Table 2, the intercorrelations among the four operations were high (all $r_s \geq .80$). Moreover, all four whole number arithmetic operations were positively correlated with all five fraction tasks (r_s ranged from .30 to .57). Because of the high correlation amongst the four whole number operations, we created a *whole number arithmetic* factor using principal component analysis. The factor accounted for 86.7% of the total variance, accompanied by an eigenvalue of 3.47, with factor loadings of .92, .93, .94, and .94 for addition, subtraction, multiplication, and division, respectively.

With respect to fraction performance, as shown in Figures 1 and 3, there was a high degree of similarity in performance patterns between fraction addition and subtraction tasks. Moreover, the correlation between the two tasks was high ($r = .76$; see Table 2), thus, we created a *fraction arithmetic* factor using principal component analysis. The factor accounted for 87.9% of the total variance, accompanied by an eigenvalue of 1.76, with factor loadings of .94 for both tasks. For the hierarchical regression analysis, the whole number arithmetic and fraction arithmetic factor scores were used.

Hierarchical linear regression analysis was conducted to examine the unique contributions of whole number arithmetic and fraction magnitude tasks to fraction arithmetic.

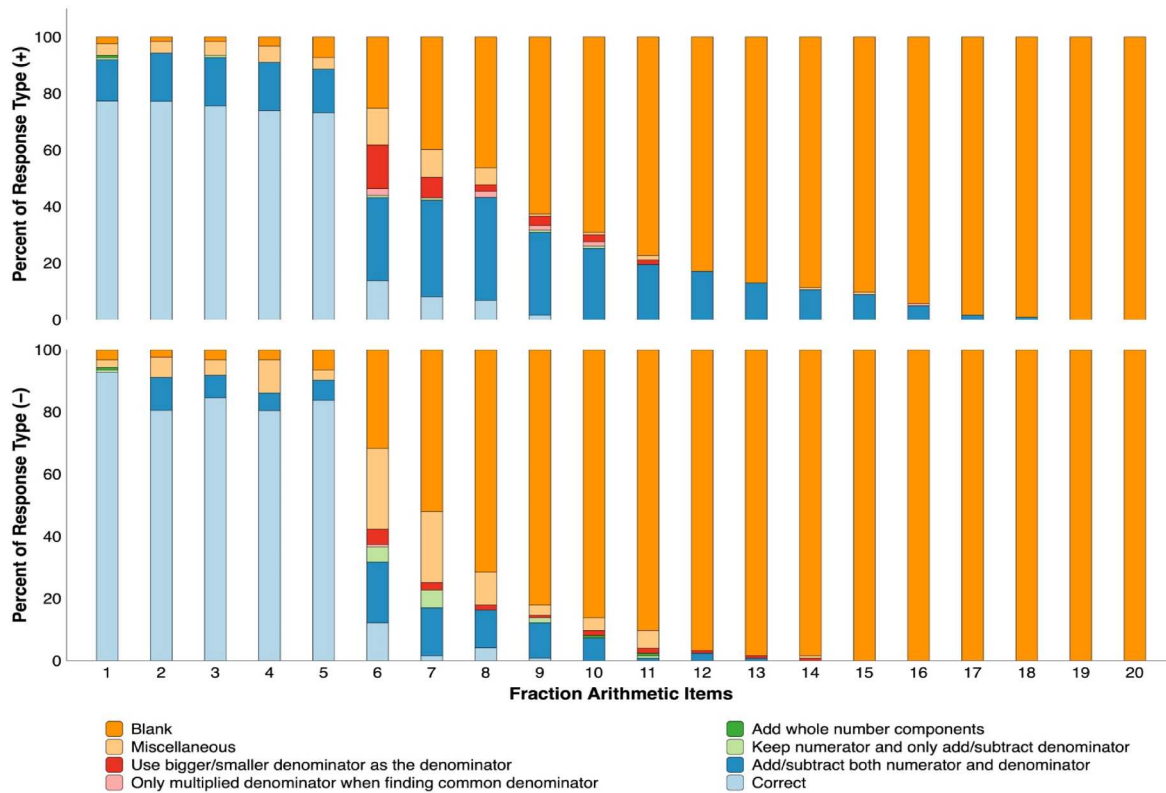


Figure 3. Percentage of each response type by item for fraction addition (top) and fraction subtraction (bottom).

In the first model, only whole number arithmetic was included. Fraction mapping, equivalence, and comparison tasks were included as predictors, one-by-one, in each subsequent model.

The results are presented in Table 3. In Model 1, pupils' whole number arithmetic skills explained approximately

Table 3. Hierarchical linear regression showing whole number arithmetic, fraction mapping, fraction equivalence and fraction comparison predicting fraction arithmetic ($N = 123$).

Variable	B	SE	β	t	p	Unique r^2
Block 1						
Whole Number Arithmetic	.51	.08	.51	6.49	<.001***	.258
Block 2						
Whole Number Arithmetic	.48	.10	.48	4.70	<.001***	.136
Fraction Mapping	.02	.04	.05	0.44	.663	.001
Total R^2						.260
Block 3						
Whole Number Arithmetic	.45	.10	.45	4.32	<.001***	.114
Fraction Mapping	.01	.04	.02	0.16	.877	<.001
Fraction Equivalence	.05	.03	.13	1.47	.145	.013
Total R^2						.273
Block 4						
Whole Number Arithmetic	.34	.11	.34	3.02	.003**	.054
Fraction Mapping	.02	.04	.04	0.43	.667	.001
Fraction Equivalence	.03	.03	.08	0.94	.350	.005
Fraction Comparison	.07	.03	.21	2.22	.028*	.029
Total R^2						.302

Note. Unique r^2 represents the squared semi-partial correlations within that specific model tested. $p < .05^*$, $p < .01^{**}$, $p < .001^{***}$.

25.8% of the variance in fraction arithmetic. The addition of fraction mapping and fraction equivalence in Model 2 ($\Delta R^2 = .001$) and Model 3 ($\Delta R^2 = .013$) did not explain additional variance in fraction arithmetic, $ps > .05$. In Model 4, fraction comparison significantly explained additional variance in fraction arithmetic, $\Delta R^2 = .029$, $p = .028$. Consistent with the correlations between whole number arithmetic and fraction magnitude tasks (see Table 2), the two jointly accounted for approximately 24% of the variance in fraction arithmetic. However, only whole number arithmetic and fraction comparison remained as unique predictors of fraction arithmetic.

In summary, these results support Hypothesis 2: Pupils' whole number arithmetic skill, which is required for accurate fraction arithmetic, uniquely predicted fraction arithmetic. Furthermore, the unique prediction of fraction arithmetic by fraction comparison suggests that the fraction comparison task may involve both a conceptual understanding of fraction magnitude and the application of procedural knowledge.

Research question 3: how do mathematics anxiety and confidence in responses relate to pupils' mathematical performance?

Mathematical anxiety

The average mathematics anxiety score was 2.1 ($SD = 0.7$) on a scale that ranged from 1 (indicating low

anxiety/feeling calm) to 5 (indicating high anxiety/feeling stressed or worried). Thus, on average, pupils reported moderately low levels of anxiety. Nonetheless, in support of Hypothesis 3a, pupils' mathematics anxiety scores were negatively correlated with their mathematics performance (see Table 2), suggesting that higher levels of anxiety were associated with lower performance in mathematics. Specifically, the correlations between mathematics anxiety and whole number arithmetic demonstrated a very large effect size, while the effect sizes for other fraction tasks ranged from small to medium (see the evaluation criteria in Funder & Ozer, 2019).

Self-rated confidence on mathematical tasks

As shown in Table 1, as task difficulty increased, pupils' self-rated confidence tended to decline. Using the Bonferroni adjustment, a one-way repeated measures ANOVA examining pupils' confidence across the nine mathematical tasks showed that pupils reported lower confidence in their performance on fraction arithmetic tasks compared to any of the whole number arithmetic and fraction magnitude tasks, $ps < .001$, supporting Hypothesis 3b.

To further examine the consistency between confidence ratings and actual performance, the proportion of pupils' who responded with low (1), medium (2), and high (3) confidence for each of the tasks was calculated and compared to their actual performance, which was divided into three categories corresponding to "all or most correct" ($> 80\%$ correct), "some correct" (50–79%), and "most of my answers were wrong" ($< 50\%$ correct). As shown in Figure 4, for whole number arithmetic almost all pupils provided correct responses (accuracy $> 80\%$) for the problems they attempted

(i.e. excluding blank responses). Across all four operations most pupils' confidence ratings were consistent with their performance. More specifically, among the pupils with high performance (accuracy $> 80\%$), 86%, 75%, 73%, and 62% reported high confidence in their responses for addition, subtraction, multiplication, and division, respectively. Pupils who had high accuracy but reported medium to low confidence were those who, although their responses were correct, answered fewer problems and thus likely factored the speed of their responses into their confidence ratings.

For fraction magnitude tasks, the confidence ratings were not as straightforward. As shown in Figure 5, the pattern for fraction mapping, a fraction task which pupils were expected to have mastered, was similar to that of whole number arithmetic. Consistent with their actual performance, 71% of pupils with high performance (accuracy $> 80\%$) had high confidence in their responses. For equivalence and comparison, although 66% and 61% of pupils had high performance, respectively, only about 50% of pupils had high confidence in their responses. Similar to whole number arithmetic, pupils with high performance but medium to low confidence only attempted about 60% of the problems during the time limit, suggesting speed factored into their confidence ratings. Notably, the mean number of incorrect responses was 0.88 and 1.24 for equivalence and comparison, respectively, suggesting that these pupils were not making inaccurate guesses. Contrary to whole number arithmetic patterns, there were some pupils whose confidence ratings were higher than their actual performance. We speculate that these pupils might hold misconceptions about fraction magnitude. Consistent with this speculation, pupils provided incorrect responses for 21% and 40% of attempted problems for equivalence and

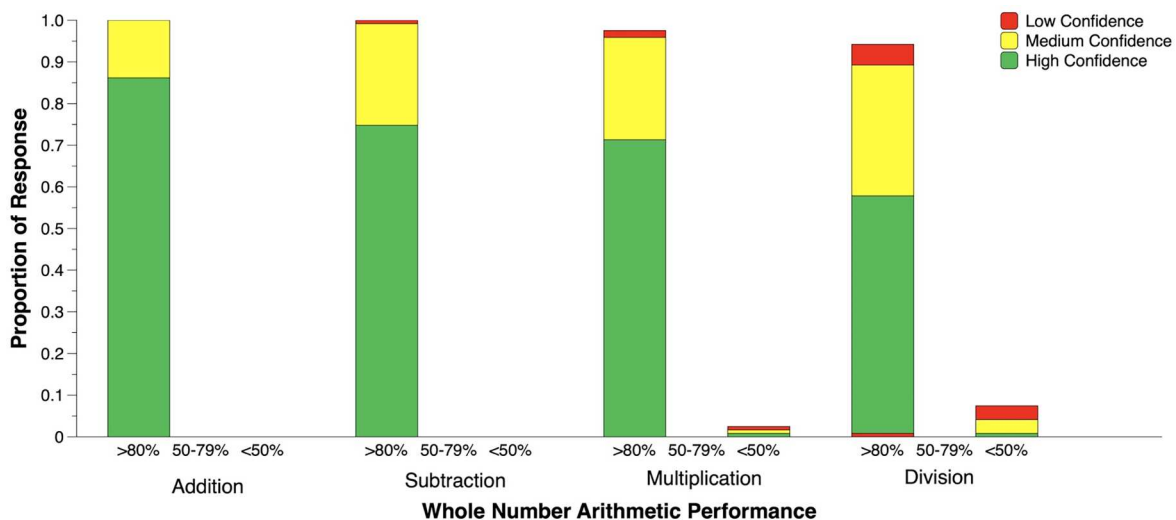


Figure 4. Pupils' confidence ratings and actual performance for whole number arithmetic.

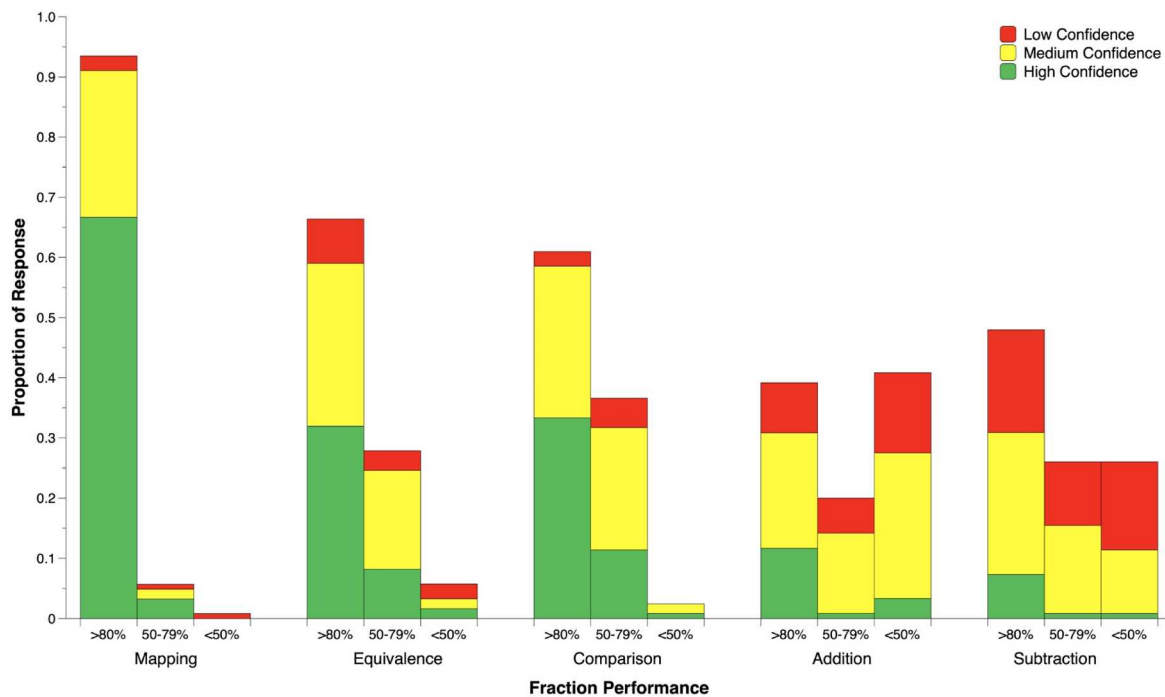


Figure 5. Pupils' confidence ratings and actual performance for fraction tasks.

comparison, respectively, and these incorrect responses were often made on specific types of trials (e.g. problems that required reduction for equivalence and incongruent trials for comparison), suggesting that an incorrect strategy was consistently selected.

For fraction arithmetic tasks, as shown in Figure 5, 70% and 85% of pupils who had high performance on fraction addition and subtraction problems, respectively, rated their confidence as either medium or low. Similar to whole number arithmetic, they likely factored speed into their confidence ratings, on average only attempting six problems for each of addition and subtraction during the allotted time. Like the fraction magnitude tasks, some pupils' confidence ratings were higher than their actual performance. We again speculate that these pupils might hold misconceptions about fraction arithmetic procedures. Consistent with this speculation, pupils provided incorrect responses for 78% and 77% of attempted problems for addition and subtraction, respectively, and these incorrect responses reflected a conceptual misunderstanding (e.g. applying natural number rules to rational number problems) as opposed to an arithmetic error (e.g. incorrectly adding or subtracting numerators).

Discussion

Although it has been well established that pupils find it challenging to learn about fractions, research studies often focus on a single aspect of fraction learning or a single fraction task. However, fraction learning may be

influenced by pupils' whole number skills and feelings towards mathematics (Siegler et al., 2011; Starling-Alves et al., 2022). Moreover, learning fractions involves various aspects of fraction understanding (Charalambous & Pitta-Pantazi, 2007; Thoma et al., 2023) and as such pupils may hold misconceptions about one aspect of fractions but not another, or they may hold different misconceptions depending on their stage of learning (Di Lonardo Burr et al., 2022). Because prior whole number knowledge interferes with rational number knowledge (DeWolf & Vosniadou, 2015; Ni & Zhou, 2005) pupils may be unaware they hold such misconceptions, reporting high levels of confidence in incorrect responses (González-Forte et al., 2023). The present study aimed to holistically investigate fraction knowledge by considering fraction performance on multiple fraction tasks; closely examining errors and misconceptions on fraction tasks; examining the hierarchical relations among whole number arithmetic, fraction magnitude, and fraction arithmetic performance; and exploring how anxiety and confidence relate to fraction performance for Year 7 pupils in Northern Ireland.

Performance and misconceptions across fraction magnitude and arithmetic tasks

There are many different fraction concepts that pupils must master to have a solid understanding of fractions. One of the earliest introduced concepts is mapping wherein pupils map pictorial representations of

magnitude to fraction symbols (Hecht & Vagi, 2010; Xu et al., 2022). As early as Year 3, pupils in Northern Ireland are exposed to fraction-to-symbol mapping and thus by Year 7, with repeated practice and exposure, we anticipated high performance on a fraction mapping task. Consistent with this expectation and the findings of a large study with pupils of a similar age (Di Lonardo Burr et al., 2022), pupils made few errors on the fraction mapping task. Variability on this task mostly reflected speed as opposed to accuracy. In the few instances where errors did occur, they were more consistent with careless mistakes (i.e. reporting the fraction for unshaded regions as opposed to shaded regions; miscounting the number of segmented pieces within the shape). Notably, contrary to other studies in which pupils made errors consistent with whole number bias (e.g. stating that a fraction with three out of eight equally-partitioned sections shaded is equal to “3”; Di Lonardo Burr et al., 2022; Mack, 1995), there was no evidence of whole number bias among this group of Northern Ireland pupils in the fraction mapping task.

Beyond mapping, pupils can demonstrate fraction magnitude understanding through fraction equivalence (Boyer & Levine, 2012; Pedersen & Bjerre, 2021) and comparison tasks (Meert et al., 2010; Rinne et al., 2017). In Northern Ireland, fraction comparison is introduced in Years 4 and 5 whereas fraction equivalence is introduced in Year 6. The level of fraction understanding required to successfully solve these types of magnitude tasks can be challenging for pupils because new information about fractions conflicts with their prior knowledge of whole numbers (Chi et al., 1994; Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017). Moreover, pupils can no longer discretely process the whole number components of fractions for these tasks, like they can for fraction mapping, and thus misconceptions that are not present in mapping tasks, such as whole number bias, may reappear (Di Lonardo Burr et al., 2022).

In the present study, although error analyses revealed few errors amongst the fraction equivalence and comparison tasks, some aspects of fraction magnitude were not as well mastered. For example, consistent with the conceptual challenges previously noted in the literature regarding recognition that different fractions can represent the same magnitude (Kamii & Clark, 1995; Pedersen & Bjerre, 2021), pupils made more errors on the equivalence task when problems required them to reduce fractions. Furthermore, while most pupils showed no signs of whole number bias on the fraction comparison task, challenges arose when they were tasked with efficiently comparing more complex fraction pairings. For example, consider $\frac{4}{7}$ vs. $\frac{5}{9}$, a fraction

pairing where reliable, efficient strategies, such as recognising that the fraction with the larger numerator is greater when the two fractions have equal denominators (e.g. $\frac{4}{7}$ vs. $\frac{5}{7}$); recognising that the fraction with the larger denominator is smaller when the two fractions have equal numerators (e.g. $\frac{4}{7}$ vs. $\frac{4}{9}$); and comparing fractions to common benchmarks, such as 0, $\frac{1}{2}$, or 1 (e.g. $\frac{4}{7}$ vs. $\frac{2}{9}$ one fraction is greater than $\frac{1}{2}$ and the other is less than $\frac{1}{2}$), are not applicable (Fazio et al., 2016). For such pairings, participants may have to rely on alternative strategies, such as converting both fractions into decimals or percentages (Fazio et al., 2016) or cross-multiplying (Faulkenberry, 2013). Such strategies, while effective, are more time-consuming and susceptible to errors. Thus, we speculate that the relatively modest performance on certain problems from the fraction comparison task likely reflects the intrinsic complexity of these problems.

After developing a strong understanding of fraction magnitude, pupils begin to manipulate fractions through tasks such as fraction arithmetic. In Northern Ireland, pupils are introduced to fraction arithmetic in Year 6. In the present study, performance on the fraction arithmetic task was weaker than performance on the fraction magnitude tasks, supporting the view that the understanding of fraction operations emerges later than the understanding of fraction magnitudes (Van Hoof et al., 2015). Error analyses showed that pupils performed well on simpler fraction arithmetic problems, such as those involving common denominators, but either struggled with or did not attempt more challenging problems, such as those involving uncommon denominators. Consistent with the findings of Braithwaite and Siegler (2023), most errors on the problems involving uncommon denominators resulted from overgeneralisation of whole-number arithmetic rules. While omissions of essential steps were also observed, they occurred less frequently. These results support the view that the intrinsic complexity of these problems contributes to the errors observed in fraction arithmetic tasks (Braithwaite & Siegler, 2023).

Predicting fraction arithmetic performance

The educational system in Northern Ireland follows a spiral curriculum in which pupils are introduced to a broad range of topics each year which are revisited in later years in greater depth (Snider, 2004). By Year 7, pupils have had years of exposure to and practice with

whole number arithmetic, and thus they are expected to solve these types of problems quickly and accurately. Indeed, in the present study pupils demonstrated excellent proficiency in all operations of whole number arithmetic, indicating a solid foundation in arithmetic skills.

Foundational whole number arithmetic skills have been found to correlate with various fraction tasks (see meta-analysis by Lin & Powell, 2021). Consistent with this meta-analysis, moderate correlations were found between whole number arithmetic and our fraction tasks. Whole number arithmetic skills may be particularly important for fraction arithmetic wherein pupils must use these skills to accurately determine and compute common denominators, combine numerators, or present solutions in a reduce fraction format. On this view, once pupils understand the procedures for solving fraction arithmetic problems, those with better whole number arithmetic skills are likely to solve fraction arithmetic problems more efficiently and effectively than their peers with weaker skills. Indeed, in the present study whole number arithmetic uniquely predicted performance on fraction arithmetic.

Fraction arithmetic performance was also uniquely predicted by performance on the fraction comparison task. Notably, whole number arithmetic skills may be beneficial in fraction comparison tasks, too, wherein strong performance on such tasks reflects not only an understanding of fraction magnitude, but also procedural knowledge (Faulkenberry, 2013). Specifically, for more complex fraction comparison problems, pupils may rely on their whole number arithmetic skills, possibly converting fractions to decimals or percentages, or using a cross-multiplication strategy (Faulkenberry & Pierce, 2011; Fazio et al., 2016). Above and beyond whole number arithmetic, the unique contribution of fraction magnitude in predicting fraction arithmetic performance may reflect the co-development of these skills (Bailey et al., 2017).

Interestingly neither fraction mapping nor equivalence uniquely predicted fraction arithmetic. Contrary to foundational whole number skills, stronger performance on an earlier-learned fraction skill may not predict performance on more advanced fraction skills, such as fraction arithmetic. Previous studies on fractions among Chinese pupils have suggested that mastery of one fraction skill may not lead to mastery of another (Di Lonardo Burr et al., 2022; Xu et al., 2024). Potentially our mapping and equivalence tasks, both of which involve taking nonsymbolic representations and mapping them to fraction notation, tap into part-whole understanding which does not transfer to procedural fraction knowledge. In contrast, fraction comparison taps into measurement understanding which may be

particularly beneficial for pupils who are developing their fraction arithmetic skills.

Overall, our results show that whole number arithmetic and fraction magnitude skills explain approximately one-third of the variance in fraction arithmetic performance, highlighting the importance of whole number arithmetic in both tasks. Consistent with previous research, once pupils practice and refine the newly learned skill, individual differences in performance reflect the integration of these newly acquired skills into their expanding mathematical system knowledge (Braithwaite & Siegler, 2023; Fazio et al., 2016; Siegler et al., 2011; Xu et al., 2021; Xu & LeFevre, 2021).

Relations between perceived mathematical attitudes and pupils' performance

As demonstrated in the present study, fractions are challenging and mastery of one fraction concept does not necessarily translate to mastery of another. For many pupils, poor attitudes towards mathematics may develop in response to limited understanding of a topic in mathematics (Jennison & Beswick, 2010). More negative attitudes towards fractions compared to whole number mathematics skills have been found (Sidney et al., 2021), possibly because fractions are notoriously challenging and many pupils and adults do not overcome these challenges.

In the present study we did not find stronger correlations between mathematics anxiety and fraction tasks compared to whole number arithmetic. More specifically, our study revealed small to medium negative correlations between mathematics anxiety and performance on various mathematical tasks, with two exceptions: A large negative correlation was observed for whole number arithmetic, and the correlation was not statistically significant for fraction equivalence tasks. In their meta-analysis Barroso et al. (2021) found that the strength of this correlation was dependent on grade level, with weaker relations appearing in the later years of primary school compared to middle and high school. Interestingly, these are the years when pupils often receive more intensive instruction on fractions, a topic known to be challenging. Notably, as a group, pupils in the present study reported low levels of mathematics anxiety and also had strong foundational mathematics skills. In a study by Song et al. (2021), whole number arithmetic predicted the change in mathematics anxiety over the course of one year, such that weaker fluency in Grade 2 was related to an increase in mathematics anxiety from Grade 2 to Grade 3 (mean ages of 7 and 8 years), whereas mathematics anxiety in Grade 2 did not predict the change in whole

number arithmetic from Grade 2 to 3. On this view, the strong foundational skills in whole number arithmetic observed in pupils in the present study may explain their generally low levels of mathematics anxiety. These findings support the view that strong foundational mathematical skills may be the key to reducing mathematics anxiety as pupils are presented with more complex topics, such as fractions (Gunderson et al., 2018).

Prior studies have found that mathematics anxiety is more strongly linked to tasks requiring advanced mathematical skills rather than foundational ones (see meta-analyses in Caviola et al., 2022; Namkung et al., 2019). In contrast, our findings indicated a reverse pattern: The correlations between mathematics anxiety and fraction tasks were generally weaker compared to that of whole number arithmetic. Notably, the two meta-analyses did not include studies of fractions. In fact, few studies have investigated the relation between mathematics anxiety and fraction performance (cf. Starling-Alves et al., 2022). Thus, more research is needed to understand how mathematics anxiety may differentially relate to fraction performance compared to other advanced mathematics skills.

One possibility is that, although mathematics anxiety is considered to be a relatively stable trait (Liebert & Liebert, 1988; Luttenberger et al., 2018), perhaps pupils feel less anxious when completing fraction assessments because they are confident in their responses, even when their responses are incorrect. Consistent with this view, a study found that middle school pupils (ages 10–12) with whole number bias reported low levels of state anxiety after completing fraction assessments likely because they were unaware that they were providing incorrect responses (Halme et al., 2023). Similarly, pupils of a similar age had high confidence in their incorrect responses on fraction assessments (González-Forte et al., 2023). In the present study, although we observed weaker correlations between fraction tasks and mathematics anxiety compared to whole number arithmetic and mathematics anxiety, we only had a trait mathematics anxiety measure and thus could not monitor changes in state anxiety as pupils completed different fraction assessments. However, pupils did rate their confidence in responses for each task. Consistent with their actual performance, pupils reported that they felt confident in their responses on the whole number arithmetic tasks. Expanding the research by Dowker et al. (2019) to older pupils in the UK, the congruency between confidence and performance suggests that pupils are capable of accurately assessing their mathematical abilities. Pupils who reported lower levels of confidence despite high performance answered fewer problems during the allotted

time than pupils who had higher levels of confidence. Notably, these pupils did not have more incorrect responses than their peers with high confidence.

For fractions, however, the patterns of confidence were not as straightforward. While, similar to whole number arithmetic, there were some pupils who reported lower levels of confidence because they were slower to respond to problems, there were other pupils who reported medium to high levels of confidence despite low performance. These pupils often had numerous incorrect responses to problems suggesting that, consistent with González-Forte et al. (2023), they had fraction misconceptions that led them to believe they were providing the correct answers. The fraction misconceptions may also be related to their differential experiences with fraction components. For example, Fitzsimmons and Thompson (2022) found that in a fraction number line task, pupils (ages 8–12) and adults reported higher levels of confidence with equivalent fractions that had smaller components rather than larger components, after accounting for their estimation precision. This finding suggests that differential experiences with fraction component size may also contribute to the inconsistent patterns of confidence ratings. Overall, confidence ratings provided important insights into how fraction misconceptions may lead to inflated confidence and possibly lower levels of mathematics anxiety.

Limitations and future directions

In the present study, we found evidence supporting the hierarchical relations among whole number arithmetic and fraction skills for pupils who were still refining their fraction arithmetic skills. However, because this was a single-timepoint study, we can only speculate about the development of this hierarchy. Moreover, we only assessed a few different fraction tasks which may not adequately capture pupils' conceptual fraction understanding (e.g. although we had pupils map pictorial representations to fraction notation, we did not assess their understanding that the "whole" needs to be partitioned into equal sized pieces). According to Northern Ireland's spiral curriculum, more advanced fraction topics are introduced in Years 6 and 7. Therefore, longitudinal research is crucial for gaining a nuanced understanding of how curriculum design influences the development and mastery of advanced fraction skills. Future studies that follow pupils from Years 6 to 7 could provide invaluable insights into how these fraction skills become integrated into a hierarchy of mathematical knowledge.

The present study focused on the hierarchical relations among whole number arithmetic and fraction

skills, without accounting for potential shared links with domain-general cognitive skills. Because domain-general skills correlate with mathematics (De Smedt, 2022), some observed relations between fraction skills might be attributed to a shared variance in these domain-general skills. Thus, future studies should consider incorporating measures such as reasoning (Peng et al., 2019), working memory, and executive functions (Peng et al., 2016).

Pupils self-reported their general feelings towards mathematics through the mathematics anxiety inventory. While these trait anxiety reports provide useful insights into stable feelings surrounding mathematics, they do not capture changes in anxiety as a result of completing mathematics tasks of varying difficulty. Moreover, they do not allow for the examination of relations between feelings towards mathematics in the moment and ratings of confidence. Thus, in the future, state math anxiety ratings collected after each task would deepen our understanding of how anxiety may influence performance and confidence and shed light on the development and maintenance of fraction misconceptions.

Finally, in the present study pupils provided their confidence ratings at the end of the testing session, after all mathematical tasks were completed. While this method allowed pupils to reflect on their performance and potentially eliminated biases from the order of presentation—such as increased confidence as they settled into the testing session—it also introduced the possibility that confidence ratings could be affected by comparisons between tasks. As a result, the ratings may reflect pupils' confidence for one task in relation to another. In future research, pupils could provide confidence ratings at the end of each assessment and the order of assessments could be randomised. Moreover, presenting items for timed tasks one at a time, perhaps on a tablet, so that pupils do not know the total number of items on an assessment may reduce the likelihood of pupils factoring speed of response into their confidence ratings.

Conclusion

Our study aimed to holistically investigate fraction knowledge by considering fraction performance on various tasks and in relation to whole number arithmetic, misconceptions pupils hold across various fraction tasks, the relations between mathematics anxiety and fraction knowledge, and pupils' confidence in their responses across natural number and fraction tasks. We found that 11-year-old pupils in Northern Ireland had a solid understanding of fraction magnitude, with

little evidence of misconceptions across mapping, equivalence, and comparison tasks. In contrast, they were still developing their fraction arithmetic skills and thus procedural errors and misconceptions were present for fraction arithmetic problems involving uncommon denominators. Consistent with a view of hierarchical mathematical development, fraction arithmetic performance was uniquely predicted by pupils' foundational whole number arithmetic skills and more advanced symbolic magnitude skills (i.e. fraction comparison). Despite the known challenges associated with fraction learning, in Year 7 pupils reported low levels of mathematics anxiety. Moreover, they were confident in their whole number arithmetic skills but less confident in their fraction skills, likely because although most pupils provided correct responses to attempted problems, they were inefficient and thus could only provide answers for a handful of problems for the more advanced fraction tasks. Although confidence ratings generally aligned with performance, some pupils had high confidence in their fraction skills despite low performance, suggesting that they might hold misconceptions about fractions which have led to false confidence in their abilities. In summary, we suggest that effective learning, both with respect to performance and attitudes towards mathematics, may be best achieved when foundational skills are thoroughly acquired before progressing to more advanced topics and when misconceptions are quickly identified and corrected.

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Disclosure statement

No potential conflict of interest was reported by the author(s).

Geolocation statement

The data were collected and analysed in Northern Ireland, United Kingdom.

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