Proceedings Paper:
Bunce, A., Hester, D. and Brennan, D.S. (2024) Where is the end of a Bridge (model)? In: Journal of Physics: Conference Series. XII International Conference on Structural Dynamics, 03-05 Jul 2023, Delft, Netherlands. IOP Publishing. Article no: 252024. ISSN: 1742-6588. EISSN: 1742-6596.

https://doi.org/10.1088/1742-6596/2647/25/252024

Journal of Physics: Conference Series

Where is the end of a Bridge (model)?

To cite this article: Andrew Bunce *et al* 2024 *J. Phys.: Conf. Ser.* **2647** 252024

Where is the end of a Bridge (model)?

Andrew Bunce[1], David Hester[1], Daniel S. Brennan[2]

1 Civil Engineering, School of Natural and Built Environment, Queens University Belfast, Stranmillis Road, Belfast, BT7 1NN, Northern Ireland

2 Dynamics Research Group, Department of Mechanical Engineering, University of Sheffield, Mappin Street, Sheffield, S1 3TD, UK

abunce01@qub.ac.uk

Abstract. Bridge SHM solutions have been developed to assist with the assessment and monitoring of bridges. State of the art bridge SHM solutions tend to be data based, where machine learning algorithms are trained using large, historical bridge datasets, and outlier analysis is subsequently used for anomaly detection. However, most bridges lack the required healthy state data for machine learning approaches to be considered, and many bridges are not in a healthy condition to collect the required data from. A population based structural health monitoring (PBSHM) approach has recently been proposed that seeks to facilitate knowledge transfer between similar structures. The approach proposes that if two structures are similar enough, there could be scope to make SHM inferences between the structures. The ability to make inferences between bridges, which are currently monitored in isolation, would be highly valuable as a bridge management tool, particularly if datasets could be leveraged between bridges through transfer learning. However, before knowledge can be shared between bridges, there is first a need to identify bridges that are similar enough for inferences to be made. The PBSHM approach proposes the use of Irreducible Element (IE) models to describe structures, which allows Attributed Graphs (AG) to generated and compared for similarity using graph theory techniques. The general method for comparing structures was trialled on bridges previously, however the resulting similarity metrics were for the whole bridge as opposed to particular common zones of interest e.g. the deck. This paper instead proposes that bridges be modelled as subsections of structures that interact via shared boundaries (i.e., points of articulation such as bearings), as opposed to whole structures. Bridge datasets are often limited to the part of the bridge that was investigated, i.e., datasets particular to bridge decks, abutments etc. Therefore, the extents of the IE models proposed in this paper are set to only include elements that would pertain to a particular dataset. In particular, two beam and slab bridges are each described with bridge deck, abutment and pier IE models to trial the concept. The revised extents of the bridge IE models reduced the number of elements being compared, resulting in increased resolution graph comparisons being carried out and more meaningful similarity metrics between the bridge parts being achieved.

1. Introduction

Bridges are valuable infrastructure assets; however, they are challenging and expensive to maintain. Currently, there is an estimated bridge maintenance backlog of over 3000 bridges in Great Britain where the bridges were identified as being substandard [1]. Of the substandard bridges deemed unable to carry the heaviest traffic on roads today, approximately ~10% are intended to be returned to full load carrying capacity over the next few years. Data driven approaches to bridge condition assessment are preferable to visual inspections and can lead to an increase in load carrying capacity from assessment [2], [3], potentially alleviating the maintenance requirement pressures. The motivation to improve the longevity of infrastructure, with the advancement of technology and computing resources available, has seen a large focus on data-based bridge SHM solutions being researched.

Monitoring systems for bridge SHM data collection have been proposed for both temporary and permanent installations with increasingly available bridge datasets over time. This increase in data availability has led to the growing development of bridge SHM tools that utilise Machine Learning (ML) algorithms to support data-informed bridge SHM decisions [4]. In principle, these approaches use large volumes of known-condition data to train a system and create operational benchmarks for a given data type. Subsequent monitoring data is then compared to the benchmark for assessment, where anomalies and outliers have been used to qualify conditions as well as detect, and even locate, damage on bridge structures. To date, several candidate systems and approaches have been put forward, and a comprehensive review of state-of-the-art ML-based bridge SHM systems can be found in [5], [6]. Whilst the use of ML has shown great potential in providing bridge managers with the ability to make data-informed bridge maintenance decisions, the capabilities of the ML-based systems are limited by the quality and quantity of historical data available.

Recently, a population-based structural health monitoring (PBSHM) approach was proposed that seeks to enhance the monitoring of structures by looking at what insights may be gained from other structures in a SHM context [7]. As a bridge management tool, a PBSHM approach means a given bridge with missing (or no) historical data, may be able to be monitored using some amount of transfer-learnt data from a population of compatible structures. Transfer learning approaches have been limited in application to real world bridge structures to date, though there are several proposed methods in the bridge SHM literature. A Structural State Translation knowledge transfer approach was trialed between numerical bridge models for pair of similar prestressed bridge decks [8]. The authors demonstrated that despite subtle differences in topology, the bridge responses were similar enough for knowledge transfer to be carried out between the pair of real bridges. There are examples of transfer learning occurring between real bridges and numerical models of those bridges, where simulated data from the models are used to create synthetic training datasets for machine learning algorithms [9], [10]. However, sharing information between real-world bridge structures has been significantly limited as there are challenges associated with environmental variance, sensor placement and noise, as well as subtle variation that exists even between similar bridges [11]. Therefore, in terms of transfer learning between real bridge structures, knowledge transfer has been limited to pairs of known identical elements [12], [13]. There is currently a lack of understanding of the degree of similarity between structures, and the level of knowledge transfer likely achievable between those structures for a wider uptake of the approach.

The PBSHM approach therefore has two main themes: (i) identifying similar structures (or parts of structures), and (ii) investigating what, if any, data may be shareable between the structures. For a homogeneous population of structures that can be represented by a single model [14], such as with wind farms, transfer learning has been utilised with success where data from the population of wind turbines proved useful in enabling performance monitoring for individual turbines within the population [15]. For a population of heterogeneous structures however, that is, structures that may be subtly or substantially different to one another, transfer learning may not be feasible. If two structures that are not compatible were to share SHM data, negative transfer could occur, where negative transfer refers to inferences being made that could harm the performance of the SHM system. To prevent negative transfer from occurring, there is a need to identify structures, or parts of structures, that would be similar enough to one-another for some level of knowledge transfer to be sensible [16], [17]. The PBSHM approach has proposed the use of Irreducible Element (IE) models to capture information about a structure that would be significant to that structure's response [18]. Attributed Graphs (AGs) are then generated from IE models, allowing graphs of structures to be compared for similarity using graph theory techniques. Similarity metrics are evaluated from the AG comparisons, informing of the likelihood of there being transferrable knowledge between the compared structures.

The use of IE models to describe bridges has been explored once (with real bridges) already, using an initial population of eight bridges of five different types [19]. The work introduced the largest and most complicated structures to be described with IE Models (so far), with the results showing promise in the AG comparisons being able to identify identical, and completely different bridges apart. However, for bridges that had partial matches, limited information was available without further interrogation and the similarity metrics between bridges calculated using the Jaccard Index (JI) proved sensitive to the size of graphs (of structures) being compared. To that end, this paper presents problem-driven IE models for bridges. Problem-driven IE models were introduced in [20] where wings of an aircraft were modelled for a damage detection and localization problem. Details of the structures deemed irrelevant to the knowledge transfer problem were omitted. In this paper, the same concept is applied to bridges, where common datasets are often limited to subsections of bridges. Therefore, if one were creating IE models for bridges with a view to transfer learning potential between parts of bridges (with datasets), where would the extents of IE model for the bridges and parts of bridges be?

IE models describing two beam and slab bridges are presented, using the same two beam and slab bridges from the work in [19]. Subsections of the original IE models are modelled to a higher granularity demonstrating improved informativity and relevance of partial matches between bridge parts, whilst maintaining a minimum level of information within the IE models to prevent false matching occurring [21]. Section 2 presents a summary of the work carried out in [19], reviewing the process of describing a bridge with IE's and limitations of the resulting comparisons. Section 3 presents purpose-driven IE models for bridges (parts), reducing the information contained in each model to achieve more meaningful AG comparisons. Section 4 then investigates the effectiveness of the problem-driven IE models for comparing reduced AGs of the two beam and slab bridges featured in [19], compared to the original similarity metrics obtained.

2. Irreducible Element models and Attributed Graphs (for bridges)

The PBSHM approach works by first comparing structures, or parts of structures, for similarity. The (parts of) structures are described with an Irreducible Element Model and then converted to an Attributed Graph (AG) for comparison. In the first application to bridges, the IE Models were used to describe the topology of whole bridges at a high level of detail, including all deck and support elements within the models. Figure 1 (a) is a photo of the first beam and slab bridge (B&S 1), and Figure 1 (b) is a photo of the second beam and slab bridge (B&S 2). The second beam and slab bridge is largely similar to that of the first beam and slab bridge, with two main variations:

- The first bridge features four beams per span, supporting the bridge deck, where the second bridge features five beams per span, and
- The first bridge's abutments are skeletal abutments, made up of four columns supporting a wall, where the second bridge's abutments are bank seats with little flexure in their design.

(a) (b)

Figure 1 photos of beam and slab bridges, (a) B&S 1, (b) B&S 2.

Attributed graphs were then generated from IE Models (of each bridge). Figure 2 (a) shows the AG representation of the first beam and slab bridge (B&S 1) and Figure 2 (b) is the AG representation of

Journal of Physics: Conference Series **2647** (2024) 252024 doi:10.1088/1742-6596/2647/25/252024

the second beam and slab bridge (B&S 2). Both graphs show the respective bridges in their Canonical Forms (CF) [22], where the red circles represent regular elements, blue circles ground elements, and the black lines between the various elements represent the relationships between those elements. Groups of elements have been circled (magenta) and annotated for clarity.

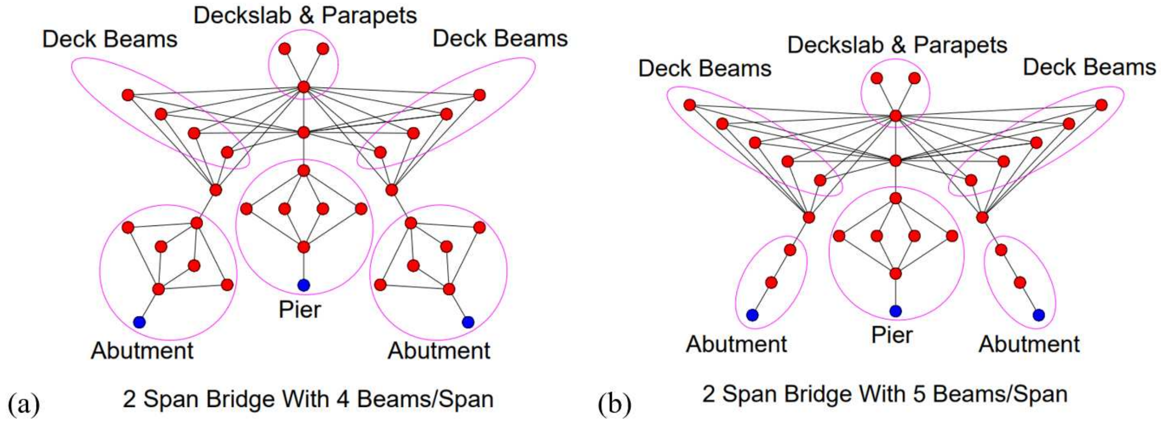

Figure 2 Attributed Graph representations of beam and slab bridges, (a) B&S 1, (b) B&S 2.

In layman's terms, the graph comparison approach used in this paper, seeks out the largest matching graph that would exist in both of the two graphs (of structures) called the maximum common subgraph (MCS). The Jaccard Index was used to calculate the level of similarity between two graphs, numerically representing the size of the MCS relative to the largest AG being compared. Results range from 0 to 1, where 0 is no match between two graphs, and 1 is a complete match. Figure 3 (a) shows the resulting similarity metrics between the two beam and slab bridges, and (b) shows the annotated maximum common subgraph (MCS) between the two bridges, with the unmatched elements and connections at 50% transparency. The JI similarity metric between the two bridges was calculated as 0.51, where the 0.49 of unmatched elements are mostly from the abutment structures (bottom left and right portions of Figure 3 (b)). Considering only the bridge decks, two deck-supporting beams, one per span, did not match (with associated relationships).

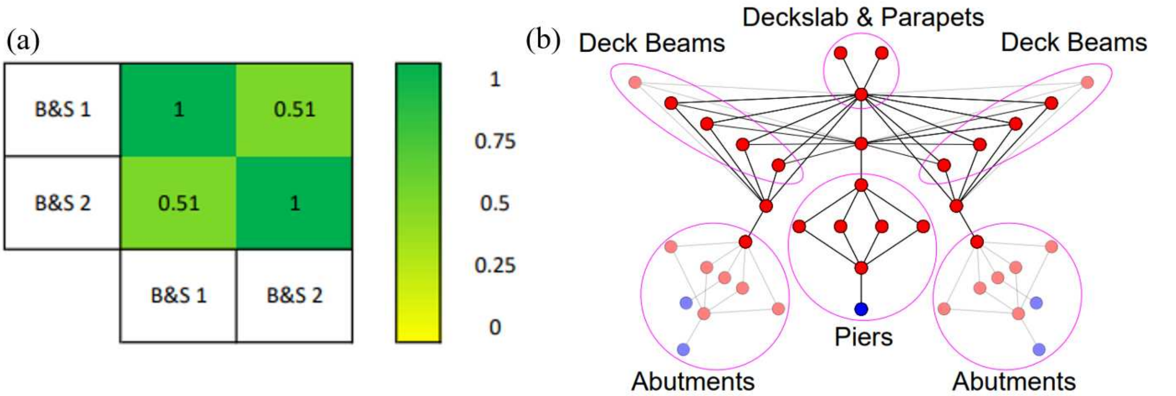

Figure 3 Results from Attributed Graph comparisons, (a) Jaccard Index similarity metrics, (b) annotated maximum common subgraph shared between two beam and slab bridge attributed graphs.

Whilst the results obtained from the first bridge AG comparisons were indicatively positive in detecting a partial match, the partial matches lacked any immediate meaningfulness. That is, where the two beam and slab bridges have similar deck structures, the JI similarity metric is relative to the MCS size compared to the largest AG, therefore variations between the bridge supports causes a substantial reduction in what would otherwise be expected to be a reasonably large match. The large matching deck portion is only noticed with interrogation of the MCS constituents.

The beam and slab bridges produce easy digestible graphs for visual interpretation; however, this is not always true. Particularly when element and relationship numbers of IE models are much larger, Hypergraphs become the more sensible, but less human-interpretable, representation of the bridge. The lack of intuition can cause substantial confusion in interpreting JI similarity metrics as including larger element and relationship numbers reduce the value, and therefore impact, of each matched (and unmatched) element and relationship.

3. Revised extents of bridge IE Models

Considering larger bridges and other bridge-like structures that would be described with large numbers of elements (and relationships), the increased element numbers reduce the value of each matched (and unmatched) element using the JI similarity metric to calculate the similarity. This limited value of matches between larger AGs presents potential limitations in identifying similar structures, or parts of structures. Therefore, there needs to be some consideration about the amount of information included within IE models when using the JI similarity metric. Figure 4 shows three linking corridors between buildings. The corridors could be considered as bridges, with buildings where, traditionally, one would find bridge supports. If the structures at the support locations were to be included in the IE model detail, the linking corridors would become difficult to identify similar structures from the similarity metrics due to the complexities of the support structures. There is an array of examples of bridges that are ancillary to much larger structures, much like bridge supports are (generally) ancillary to bridge decks.

Figure 4 Linking corridors between buildings.

To overcome the challenge of comparing bridges as AG's, using the JI similarity metric to quantify similarities, there needs to be a revision of the extents of bridge IE Models. Whilst [20] noted there is need for minimum information contained within an IE model for sensible comparisons to be achieved, the previous implementation on bridges demonstrated problems associated with too much information creating vague partial matches. Therefore, rather than comparing AGs for whole bridges, problem-driven models should be considered that capture subsections of a bridge IE model, relative to common datasets that may be used for SHM transfer learning. Datasets for bridge decks are often used to assess and diagnose the bridge deck (and elements), and likewise for bridge supports. Information beyond the shared connections of these structures is significantly reduced, where forces are grounded, or articulation occurs, or two system responses compete rather than harmonize. For bridge deck data, limits of useful information would typically be at the bridge bearings, located at the support structures, where it would then make sense to condense the information about the support structure elements within a bridge deck IE model. Inversely, bridge support structures can then be modelled with the bearings (supporting the bridge deck) set as the extents of the model, condensing bridge deck information (and other supports) to the ground elements. Figure 5 demonstrates the revised extents for the first beam and slab bridge model. Previously, the whole bridge was captured where each red line denoted a ground (extent) of the structure, below each support. The blue lines at the top of each support, where the supports meet the bridge deck elements, represent the internal ground locations introduced for the problem-driven models. The one beam and slab bridge IE model can then be described with four purpose-driven IE models (for the bridge deck, intermediate pier, and two abutments, respectively). The original grounds, for whole bridge models, were considered below the support structures; the revised extents are effectively additional grounds set at the top of the support structures. For the bridge deck IE model, the three new ground elements would be representative of the two abutments and intermediate pier structures, respectively. For each of the abutments and the intermediate pier IE models, the introduced grounds would be representative of the bridge deck.

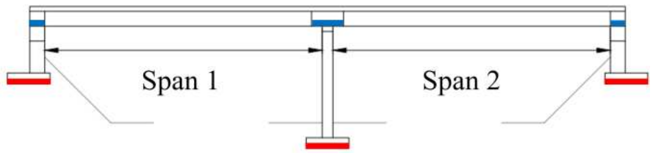

Figure 5. Beam and Slab bridge elevation showing whole bridge with original extents (red lines) below each support and new extents proposed atop each support (blue lines).

The reduction of ancillary information is not new to engineering and features as common practice during various aspects of design and assessment of bridge structures, from FE Models with “reactions” at supports, to structural force calculations using Method of Sections approach. A similar concept is adopted here, where the ancillary structures are not ignored, but their detail is condensed, and consideration is reserved for their inputs on the system in question without penalising similarity metrics.

4. Effects of revised extents on bridge IE Models and AG comparisons

Remodelling the two beam and slab bridges from Section 2 with the considerations of Section 3, three models are used to describe each bridge:

- one model describes the two-span bridge deck structure,
- one model describes the intermediate pier, and
- one model describes the abutment structures.

As the two abutments of the bridge can be considered homogeneous, complete topological matches to one another, only one IE model is required to represent both abutments. The extents of the models (ground elements) have been set at the bearings of the bridge, where, typically, information about the adjoining structure would be reduced in most datasets. Where in an FE model, one could have multiple reactions to represent multiple points of contact with an adjacent structure, within an IE model, a ground element represents the adjoining third-party structure. As such, only one would be required per adjoining structure. Figure 6 shows the (CF) AGs for the first beam and slab bridge Deck, Pier and Abutment(s), respectively. Figure 7 shows the (CF) AGs for the second beam and slab bridge Deck, Abutment and Pier structures, respectively. As highlighted with the previous AGs, the red circles are regular elements, and the blue circles are ground elements. The lines between elements represent (and contain information for) the relationships between those elements. The location of bearings between the deck elements and support elements for both deck and support models are described as ground elements, where one could expect inputs on the structure from the adjoining structure.

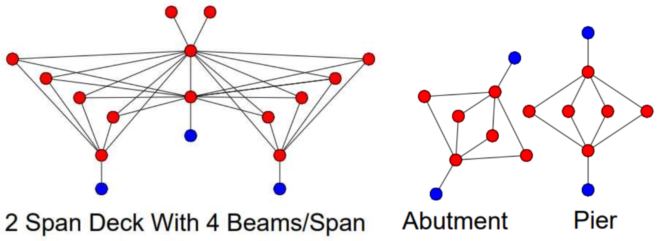

Figure 6 Attributed Graph representation of bridge parts for first beam and slab bridge.

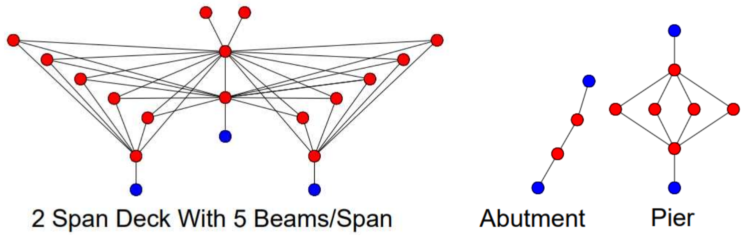

Figure 7 Attributed Graph representation of bridge parts for second beam and slab bridge.

The IE Models and AGs are much smaller than the previous models and graphs in [19], and as a result, the value of each matched (or unmatched) element is increased. Figure 8 shows the AG comparison results between the two beam and slab bridge decks. Figure 8 (a) shows the similarity metrics between the bridge decks, and Figure 8 (b) shows the (annotated) MCS with unmatched elements set to 50% transparency. The similarity metric increases from 0.51 to 0.89 with the condensing of the ancillary structures. The 0.89 is more indicative of the level of similarity between the two bridge decks than the original 0.51, using engineering judgement, as the only difference between the bridge deck AGs is an additional beam per span supporting the deck in B&S 2.

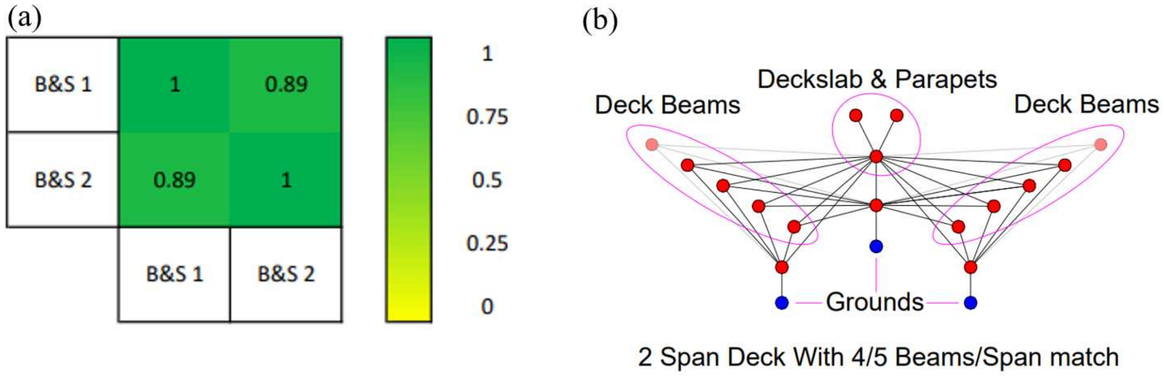

Figure 8 Similarity metrics between beam and slab bridge decks.

For the bridge supports, the IE Models and AGs for each of the support structures were smaller than those for the bridge decks. Figure 9 shows AG results comparing the four bridge supports. B&S 1 A and B&S 1 P refer to the first beam and slab bridge abutment and pier structures, respectively. B&S 2 A and B&S 2 P are also Abutment and Pier structures, respectively, but for the second beam and slab bridge. The comparisons were able to distinguish differences between two different abutments (0.2), whilst identifying the two notionally identical piers (1.0). The perfect match makes sense as the only difference between the two piers is the number of beams supported (for the bridge deck), hence a perfect match where changes to the bridge deck properties would be captured by changing ground element properties. Finally, whilst B&S 1 abutment and pier graphs look and are topologically identical, the embedded attributes within the graphs, particularly contextual attributes such as element type, allow distinction between the two structures resulting in a similarity of 0.6. The partial similarity between the piers and B&S 1 abutment largely makes sense as the abutment features a wall in place of the pier structures' cap beams. Overall, the JI similarity metrics between the bridge supports are in line with what one would expect when comparing the support structures. The similarities and differences of the support structures are much clearer than if they were to be considered within the whole bridge models, where they pose minority representation.

Journal of Physics: Conference Series **2647** (2024) 252024 doi:10.1088/1742-6596/2647/25/252024

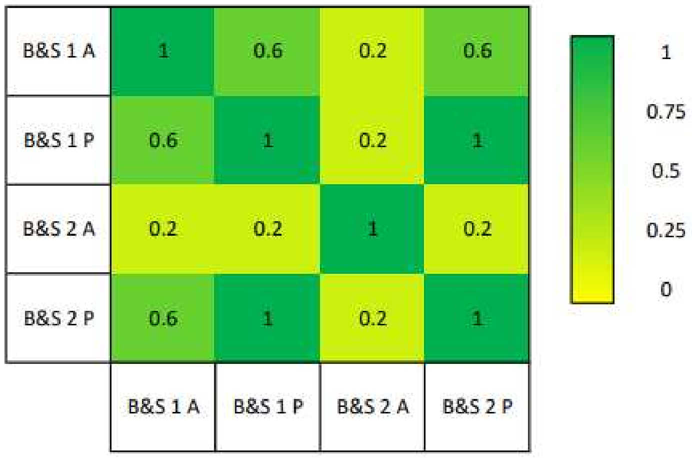

Figure 9 Similarity metrics between bridge support structures.

5. Conclusions

Irreducible Elements (IE) models have been shown to be capable of describing bridge structures previously, allowing Attributed Graphs (AG) of bridges to be generated and compared for similarity. However, the similarity metrics between bridges compared as AGs, calculated using the Jaccard Index, are sensitive to both the size of the structures and the level of detail included in the IE model. Bridges are often large structures and therefore JI similarity metrics for partial matches between whole bridges in their Canonical Forms (CF) are of limited informativity owing to the size of the IE models required to describe whole bridges.

Using two beam and slab bridges for demonstration, this paper considers revised extents for bridge IE Models, compared to the original work in [19]. To improve the informativeness of the JI similarity metrics for partial matches between bridges, the two beam and slab bridges are described as a series of (part) structures that interact, rather than a single (whole) structure. The extents of the bridge IE Models are set to mimic extents of information typically contained in common bridge datasets, in line with the aims of PBSHM to facilitate information sharing between similar structures. Bridge bearings are considered as common ground elements between bridge deck and support structures, leading to JI similarity metrics between the bridges (parts) that are more reflective of the actual similarity between the bridges (parts), than achieved when comparing whole bridges.

The PBSHM approach proposes two main themes of work, identifying structures that are similar, and then, investigating what information may be sharable between the (similar) structures. The revision of bridge model extents is of particular importance for comparisons featuring larger, more complex bridges when using the JI similarity metric. These include long span or multi-span bridges, as well as bridge structures within or between buildings, where the general method for comparing whole structures results in too low a resolution of graph comparison to achieve meaningful similarity metrics. The work in this paper therefore provides an approach for modelling and comparing parts of bridges (or other large structures), where model extents are governed by the extents of information in common datasets. Whilst a positive step towards *how* PBSHM can be applied to bridges, there is still a need to check that similar bridges have similar responses, that transfer learning would be appropriate. There is also further investigation required as to what elements should be contained in a problem driven IE model, and to what detail should they be described to reflect their associated datasets most accurately. The work from this paper proposes it is more sensible and economical to approach this from a problem driven perspective, by comparing bridge decks, or supports, and associated datasets.

Acknowledgements
The authors would like to thank the UK EPSRC for funding through grant EP/R513118/1 and the Established Career Fellowship EP/R003645/1. The authors of this paper also gratefully acknowledge the support of the UK Engineering and Physical Sciences Research Council (EPSRC) via grant reference EP/W005816/1. For the purpose of open access, the authors have applied a Creative Commons Attribution (CC BY) licence to any Author Accepted Manuscript version arising.

References
[1] RAC Foundation, "Proportion of substandard road bridges falls," 2023. https://www.racfoundation.org/media-centre/proportion-of-substandard-road-bridges-falls (accessed Mar. 17, 2023).
[2] B. Bakht and A. Mufti, "Evaluation of one hundred and one instrumented bridge," Struct. Innov. Monit. Resour. Centre, Univ. Manitoba, no. August, 2017.
[3] B. Bakht and A. Mufti, "Evaluation of one hundred and one instrumented bridges suggests a new level of inspection should be established in the bridge design codes," J. Civ. Struct. Heal. Monit., vol. 8, no. 1, p. 3, 2018, doi: 10.1007/s13349-017-0256-1.
[4] Flah, M., Nunez, I., Ben Chaabene, W., & Nehdi, M. L. (2020). Machine Learning Algorithms in Civil Structural Health Monitoring: A Systematic Review. Archives of Computational Methods in Engineering, 0123456789. https://doi.org/10.1007/s11831-020-09471-9
[5] Y. Bao, Z. Chen, S. Wei, Y. Xu, Z. Tang, and H. Li, "The State of the Art of Data Science and Engineering in Structural Health Monitoring," Engineering, vol. 5, no. 2, pp. 234–242, 2019, doi: 10.1016/j.eng.2018.11.027.
[6] R. Niyirora, W. Ji, E. Masengesho, J. Munyaneza, F. Niyonyungu, and R. Nyirandayisabye, "Intelligent damage diagnosis in bridges using vibration-based monitoring approaches and machine learning: A systematic review," Results Eng., vol. 16, no. October, p. 100761, 2022, doi: 10.1016/j.rineng.2022.100761.
[7] K. Worden et al., "A Brief Introduction to Recent Developments in Population-Based Structural Health Monitoring," Front. Built Environ., vol. 6, no. September, pp. 1–14, 2020, doi: 10.3389/fbuil.2020.00146.
[8] Luleci, F., & Necati Catbas, F. (2023). Condition transfer between prestressed bridges using structural state translation for structural health monitoring. AI in Civil Engineering, 2(1). https://doi.org/10.1007/s43503-023-00016-0
[9] Teng, S., Chen, X., Chen, G., & Cheng, L. (2023). Structural damage detection based on transfer learning strategy using digital twins of bridges. Mechanical Systems and Signal Processing, 191(January), 110160. https://doi.org/10.1016/j.ymssp.2023.110160
[10] Figueiredo, E., Omori Yano, M., da Silva, S., Moldovan, I., & Adrian Bud, M. (2023). Transfer Learning to Enhance the Damage Detection Performance in Bridges When Using Numerical Models. Journal of Bridge Engineering, 28(1). https://doi.org/10.1061/(asce)be.1943-5592.0001979
[11] Pan, Q., Bao, Y., & Li, H. (2023). Transfer learning-based data anomaly detection for structural health monitoring. Structural Health Monitoring, 22(5), 3077–3091. https://doi.org/10.1177/14759217221142174
[12] Vamvoudakis-Stefanou, K. J., & Fassois, S. D. (2017). Vibration-based damage detection for a population of nominally identical structures via Random Coefficient Gaussian Mixture AR model based methodology. Procedia Engineering, 199, 1888–1893. https://doi.org/10.1016/j.proeng.2017.09.123
[13] Vamvoudakis-Stefanou, K. J., Sakellariou, J. S., & Fassois, S. D. (2014). Random vibration response-only damage detection for a set of composite beams. Proceedings of ISMA 2014 - International Conference on Noise and Vibration Engineering and USD 2014 - International Conference on Uncertainty in Structural Dynamics, 3839–3854.

[14] L. A. Bull et al., “Foundations of population-based SHM, Part I: Homogeneous populations and forms,” Mech. Syst. Signal Process., vol. 148, p. 107141, 2021, doi: 0.1016/j.ymssp.2020.107141.

[15] E. Papatheou, N. Dervilis, A. E. Maguire, I. Antoniadou, and K. Worden, “A Performance Monitoring Approach for the Novel Lillgrund Offshore Wind Farm,” IEEE Trans. Ind. Electron., vol. 62, no. 10, pp. 6636–6644, 2015, doi: 10.1109/TIE.2015.2442212.

[16] J. Gosliga, P. A. Gardner, L. A. Bull, N. Dervilis, and K. Worden, “Foundations of Population-based SHM, Part II: Heterogeneous populations – Graphs, networks, and communities,” Mech. Syst. Signal Process., vol. 148, p. 107144, 2021, doi: 10.1016/j.ymssp.2020.107144.

[17] P. Gardner, L. A. Bull, J. Gosliga, N. Dervilis, and K. Worden, “Foundations of population-based SHM, Part III: Heterogeneous populations – Mapping and transfer,” Mech. Syst. Signal Process., vol. 149, p. 107142, 2021, doi: 10.1016/j.ymssp.2020.107142.

[18] D. S. BRENNAN, J. GOSLIGA, E. J. CROSS, and K. WORDEN, “On Implementing An Irreducible Element Model Schema for Population-Based Structural Health Monitoring,” Mar. 2022, doi: 10.12783/shm2021/36342.

[19] J. Gosliga, D. Hester, K. Worden, and A. Bunce, “On Population-based structural health monitoring for bridges,” Mech. Syst. Signal Process., vol. 173, 2022, doi: 10.1016/j.ymssp.2022.108919.

[20] D. S. Brennan, J. Gosliga, P. Gardner, R. S. Mills, and K. Worden, “On the application of population-based structural health monitoring in aerospace engineering,” Front. Robot. AI, vol. 9, 2022, doi: 10.3389/frobt.2022.840058.

[21] K. Worden, D. Hester, A. Bunce, and J. Gosliga, “When is a Bridge not an Aeroplane ?,” no. July, pp. 1–8, 2021.

[22] Daniel S. Brennan, Timothy J. Rogers, Elizabeth J. Cross, Keith Worden, “On Quantifying the Similarity of Structures via a Graph Neural Network for Population-based Structural Health Monitoring”, ISMA 2022.