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The Role of Mathematical Vocabulary in the Development of Mathematical Skills for Spanish-speaking Students

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Abstract

Does mathematical vocabulary predict the change in students' performance on mathematical tasks from one academic year to the next? Chilean Spanish-speaking students (*N* = 87) completed measures of mathematical vocabulary, mathematical skills (i.e., arithmetic fluency, calculation, and applied problems), receptive vocabulary, and working memory in Grade 2 (T1, M_{age} = 7:11 years:months, *SD* = 0:5, 46% girls). One year later (T2) they completed the same mathematical measures. Concurrent relations were found between mathematical vocabulary and the three mathematical skills at both time points. Together, general and mathematical vocabulary in T1 explained significant unique variance in the change in applied problems and calculation from T1 to T2. For calculation however, only mathematical vocabulary predicted significant unique variance in the change from T1 to T2. Change in arithmetic fluency was only predicted by working memory. These results address the roles of general and mathematical vocabulary in students' mathematical development in elementary school.

Word Count: 150

Keywords: mathematical language, mathematical vocabulary, mathematics, math growth, Chile

The Role of Mathematical Vocabulary in the Development of Mathematical Skills for Spanish-speaking Students

The development of students' mathematical knowledge and skills is crucial for their future academic achievement (Davis-Kean et al., 2021; Duncan et al., 2007; Watts et al., 2014) and economic success (Ritchie & Bates, 2013). Mathematical development is supported by students' domain-general skills such as receptive vocabulary, working memory, and relational reasoning (Alexander et al., 2016; Di Lonardo Burr et al., 2022; LeFevre et al., 2010; Peng et al., 2020; Sowinski et al., 2015). Given the hierarchical nature of mathematical skill development however (Xu & LeFevre, 2021; Xu et al., 2021b), over the school years, students' developing domain-specific mathematical skills may become more proximal predictors of mathematical performance than their general cognitive and domain-general language skills (Lin et al., 2021; Peng & Lin, 2019; Ye et al., 2016). Accordingly, in the present study we focused on the development of a domain-specific aspect of mathematical learning: mathematical vocabulary (Powell et al., 2021).

Mathematical vocabulary comprises technical terms (e.g., hexagon, equal sign) that have mathematical meaning and are used in mathematical contexts (Powell et al., 2019). Students need to master an extensive number of mathematical terms over the school years (Powell et al., 2021; Powell et al., 2017). Although mathematical vocabulary has been concurrently linked to mathematics performance in elementary school students (Lin et al., 2021), whether mathematical vocabulary supports the development of mathematical performance is not well understood. Thus, in the current study we explored the predictive role of mathematical vocabulary in Chilean students' mathematical development from ages 8 to 9 years (Grade 2 to Grade 3), focusing on terms that they are expected to acquire as part of the curriculum.

Relations Between Language Skills and Mathematical Performance

Students use language skills to successfully perform mathematical tasks (Kleemans et al., 2018; Purpura & Ganley, 2014; Zhang et al., 2017). A meta-analysis indicated that students' general language skills are related to their concurrent mathematical performance and predict growth in subsequent mathematical skills (Peng et al., 2020). As they progress through school, the language skills that students need for mathematical tasks become more specific to the domain of mathematics (Peng & Lin, 2019; Ufer & Bochnik, 2020). Thus, although general language skills continue to predict individual differences in mathematics (Cirino et al., 2018; Peng et al., 2020), as students' knowledge develops in elementary and middle grades, concurrent mathematical language skills are more highly correlated with mathematical skills than are concurrent general language skills (Fuchs et al., 2015; Peng & Lin, 2019; Powell et al., 2017; Powell et al., 2019; Schleppegrell, 2007). Students' knowledge of mathematical vocabulary is correlated with their concurrent mathematical performance in the elementary grades on tasks such as word problem solving (Lin et al., 2021), arithmetic fluency (Xu et al., 2021a), and arithmetic computations (Lin et al., 2021).

Lin et al. (2021) conducted a meta-analysis of correlational studies on mathematical vocabulary and mathematical performance. Across 40 studies, most of which examined concurrent relations, the average correlation between mathematical vocabulary and mathematical tasks was .49 (ranging from .31 for fractions to .58 for word problems). Age was not a significant moderator of these relations. Although this meta-analysis captured the relations between mathematical vocabulary and mathematical performance, most of the available studies were done with U.S. students (78%). The findings, therefore, may not generalize to students from other countries. Moreover, 25% of the studies were with preschool children (i.e., before they started formal schooling) and included mathematical terms such as “more” or “fewer,” and thus may not generalize to older students, who need to acquire mathematical terms that are very specific to school-based learning (i.e., difference, hundreds, column; Rubenstein & Thompson, 2001). Most critically, none of the studies (as far as we could tell) included longitudinal data covering both mathematical vocabulary and mathematical performance. Thus, additional research is needed to expand the findings on the relations between mathematical vocabulary and mathematical performance beyond concurrent correlations, and to shed light onto the question of whether knowledge of mathematical vocabulary facilitates the development of mathematical concepts and knowledge (Powell et al., 2021).

The Importance of Generalization Beyond English-Speaking Students

Although most of the research on mathematical vocabulary has been conducted with English-speaking students based in the U.S., the limited amount of research with students in other countries also showed the same pattern of positive relations between mathematical vocabulary and mathematical performance (Lin et al., 2021). For example, Vanluydt et al. (2021) found that Belgian students’ knowledge of proportional reasoning terms in their first year of elementary school (mean age of 6 years) predicted their proportional reasoning skills in the second year of elementary school (mean age of 7 years). However, general vocabulary remained a significant predictor of proportional reasoning abilities after considering proportional reasoning vocabulary. Similarly, Peng and Lin (2019) reported that both mathematical vocabulary and general vocabulary predicted unique variance in word problem solving skills for Chinese students in Grade 4 (mean age of 10 years). For Grade 8 students in Turkey (mean age of 14 years), the relation between mathematical vocabulary and mathematical performance varied based on mathematics achievement (Ünal et al., 2021). For the higher-achieving mathematics group, mathematical vocabulary predicted mathematical performance (i.e., computation) whereas for the lower-achieving group, only general vocabulary predicted mathematical performance. In summary, there is a similar pattern of positive association between mathematical vocabulary and mathematical performance in several countries and with different languages.

Only a few researchers have studied the relation between mathematics and language for Spanish-speaking students (Pina et al., 2015; Pina et al., 2014; Singer et al., 2019; Stelzer et al., 2019), focusing on general language skills (Singer et al., 2019) or on general vocabulary growth with younger students (i.e., 5-year-olds, Tamis-LeMonda et al., 2014). Singer et al. (2019) found relations between general language skills (phonological and semantic) and arithmetic outcomes in Uruguayan students in Grades 3 to 6 (ages 8 to 11), whereas Stelzer et al. (2019) found that domain-general abilities and division knowledge in Argentinian students in Grade 4 (i.e., ages 9 to 10) predicted their fraction knowledge in Grade 5. In the meta-analysis by Lin et al. (2021), only two of 27 peer-reviewed studies were focused on Spanish-speaking students (Pina et al., 2015; Pina et al., 2014). Pina and colleagues (Pina et al., 2015; Pina et al., 2014) assessed knowledge of mathematical concepts, symbols, and vocabulary of Spanish-speaking children from Spain in a single standardized assessment (i.e., Concepts subtest of the Spanish version of the Woodcock Johnson-III Tests of Achievement, Muñoz-Sandoval et al., 2005). Performance on this test was correlated with math fluency and arithmetic. Neither of these two studies, however, included longitudinal information about students' mathematical language. In this study we explored the relations between mathematical vocabulary and mathematical outcomes in Spanish-speaking students from one school year to the next (i.e., ages 8 to 9).

Associations between General Vocabulary, Mathematical Vocabulary, and Mathematical Performance

One reason that students' may need mathematical vocabulary to accomplish mathematical tasks is because these tasks require students use oral language skills to learn mathematics and think about mathematical problems (Lin et al., 2021). Consistent with the view that, with development, mathematical language becomes increasingly predictive of mathematical performance, knowledge of mathematical vocabulary partially mediates the relation between general language skills and mathematics. For preschool children (average age of 5 years), mathematical vocabulary (i.e., spatial vocabulary, but not quantitative vocabulary) partially mediated the relation between general vocabulary skills and early numeracy skills (Turan & De Smedt, 2023). For students in Grade 2 (7-year-olds, Fuchs et al., 2015; Vanluydt et al., 2021) and Grade 4 (10-year-olds, Pen & Lin, 2019), mathematical vocabulary also partially mediated the relation between general language skills and mathematical word problem solving (Fuchs et al., 2015; Peng & Lin, 2019) or proportional reasoning (Vanluydt et al., 2021). These findings suggest that both general and mathematical vocabulary are related to students' mathematical performance in the elementary grades.

Peng et al. (2020) posited a Developmental Function Hypothesis of Language for Mathematics, where language has two different functions in supporting mathematics. First, language functions as the *medium* for foundational mathematical tasks that rely on retrieval of domain-specific mathematical knowledge that is stored in verbal codes (e.g., number facts and calculation procedures). Second, Peng et al. proposed that language serves a *thinking* function, especially for higher-order mathematical tasks that require complex cognitive processing and have significant working memory demands (e.g., word problems). According to Peng et al.'s model, the relevance of language skills for mathematical tasks will increase with age because both medium and thinking functions continue to support mathematical learning and performance.

Thus, the development of fluent mathematical vocabulary reduces cognitive resource demands, thereby supporting students' performance on advanced mathematical tasks (Peng et al., 2020).

The meta-analysis by Lin et al. (2021) provided some support for the Developmental Function Hypothesis because findings from the studies that controlled for comprehension and/or cognitive skills suggested that mathematical vocabulary was both a medium for building and retrieving knowledge and also assisted comprehension and thinking processes in mathematical performance. However, in contrast to the developmental predictions of Peng et al. (2020)'s model, Lin et al. (2021) did not find that age moderated the associations between mathematical vocabulary and mathematical tasks. In some prior research, mathematical vocabulary was differently related to mathematical performance depending on the grade of the students (Lin, 2021; Powell et al., 2017). These studies, however, were not longitudinal and thus did not address the question of development. In a few studies, researchers collected longitudinal data on students' understanding of mathematical vocabulary (Warren, 2006) or on mathematical vocabulary as a longitudinal predictor of mathematical skills (Vanluydt et al., 2021). However, no one has studied students' mathematical vocabulary as a predictor of change in mathematical performance, which is key to understanding how mathematical vocabulary supports mathematical development in elementary school (Powell et al., 2021).

Given that mathematical vocabulary increases in complexity with each subsequent grade and is cumulative (Powell et al., 2021; Powell et al., 2017), the mathematical vocabulary knowledge students gain from one year to the next may both support the development of mathematics in the following year and be related to mathematical performance concurrently in both years. Longitudinal data can help frame models of how mathematical vocabulary develops in relation to mathematical skills (Powell et al., 2021) and address the developmental question of whether the relation between mathematical vocabulary and mathematical performance remains stable or changes with age (Lin et al., 2021).

Specificity of the Relations between Mathematical Vocabulary and Mathematical Skills

Mathematical vocabulary may relate to mathematical skills differently depending on the processes required in those skills (Lin et al., 2021). Peng and Lin (2019) evaluated the associations between different types of mathematical vocabulary (i.e., numerical operations, measurement, and geometry) and mathematical performance for students in Grade 4 (average age of 10 years). They found that mathematical vocabulary terms for measurement and geometry concepts predicted unique variance in word problem solving, but not calculation. Numerical operations vocabulary, however, was not directly related to mathematical outcomes (i.e., calculation and word problems). Thus, different types of mathematical vocabulary made different contributions to mathematical skills, suggesting topic-specific associations.

In their meta-analysis, Lin et al. (2021) found that students' performance on mathematical tasks that require multistep processes and integration of concepts (i.e., higher order mathematical tasks, such as solving word problems) were more strongly correlated with mathematical vocabulary than was performance on foundational mathematical tasks that rely primarily on procedural accuracy and fluency. According to Lin et al. (2021), more extensive knowledge of mathematical vocabulary might help students manage the cognitive demands of complex mathematical tasks by facilitating working memory and reasoning, supporting the Developmental Function Hypothesis of Language for Mathematics proposed by Peng et al. (2020). In summary, mathematical vocabulary may have a more important role in complex tasks and those that involve language, such as word problems, compared to other foundational tasks that are less demanding of cognitive (Lin et al., 2021) or language processes (Xu et al., 2021a). These specific predictions, however, have not been tested with longitudinal data.

Mathematical Vocabulary and Cognitive Skills

According to the Pathways to Mathematics model, several cognitive skills (i.e., linguistic, attentional, and quantitative) support students' mathematical development (Kalra et al., 2020; LeFevre et al., 2010; Raghubar et al., 2010; Sowinski et al., 2015). Therefore, domain-general language (Peng et al., 2020), domain-specific mathematical language (Lin et al., 2021), and attentional skills, particularly working memory (Cragg & Gilmore, 2014; Raghubar et al., 2010; Spiegel et al., 2021), may all be related to students' mathematical performance in the primary grades. In their meta-analysis, Lin et al. (2021) evaluated the correlations between mathematical vocabulary and mathematical performance in studies where certain domain-general cognitive knowledge and skills such as vocabulary, working memory, and/or nonverbal reasoning were included as covariates. In general, adding covariates reduced the correlations between mathematical vocabulary and mathematical performance by about 10%, but moderate correlations remained. However, in the seven studies that included a full range of cognitive skills as covariates, the partial correlation between mathematical vocabulary and mathematical skills was only .17, substantially lower than the uncorrected correlation of .49. These results highlight the shared variance among domain-general comprehension and cognitive skills and mathematical vocabulary in predicting mathematical outcomes.

Lin et al. (2021) concluded that mathematical vocabulary captures a range of cognitive skills (i.e., language, thinking, and mathematics), and it is involved both in the direct retrieval of conceptual knowledge from long-term memory and in mathematical thinking and comprehension. In the present study, we tested whether mathematical vocabulary and mathematical performance for Chilean 8-year-old students (Grade 2, T1) predicted their mathematical vocabulary and mathematical performance a year later (Grade 3, T2), taking into account the variance explained by general language (i.e., general vocabulary) and cognitive (i.e., working memory) skills at Time 1.

The Current Study

The study was conducted with Spanish-speaking students in Chile. In the Chilean educational system, students in Grades 1 to 6 (primary school) learn mathematics in five content areas: numbers and operations, patterns and algebra, geometry, measurement, and data and probabilities. Schools follow national curricular guidelines that outline learning goals for these content areas (MINEDUC, 2018). Additionally, the Chilean Ministry of Education provides schools with curricular materials that detail the purpose, required prior knowledge, and key vocabulary, knowledge, skills, and attitudes students should learn in each unit per grade. In the curricular documents, the importance of subject-specific vocabulary is mentioned as a central aspect of students' learning. Although mathematical vocabulary terms are included in the learning units as key vocabulary, there is no explicit focus on teaching mathematical vocabulary terms in the Chilean mathematics curriculum.

We had two specific goals in the current study. The first goal was to describe the concurrent relations among mathematical vocabulary and mathematical skills (i.e., arithmetic fluency, calculation, and applied problems) of Spanish-speaking students at two time points one year apart. The second goal was to evaluate the change in students' mathematical vocabulary and mathematical skills from Time 1 (8 years old; Grade 2) to T2 (9 years old; Grade 3), controlling for language and cognitive skills at Time 1. We analyzed three mathematical skills that varied in their language requirements and cognitive demands: (a) untimed, verbally-presented applied problems (e.g., If you had two books and got two more, how many books would you have?), (b) timed written single-digit arithmetic to assess fluency (e.g., 1 + 7, 9 – 4, 7 × 4), and (c) untimed written multi-digit calculation (e.g., 18 – 9, 8 × 7, 96 ÷ 3).

Research Question 1: Is the mathematical vocabulary of Spanish-speaking students related to their mathematical skills from one school year to the next?

To extend existing research to Spanish-speaking students in Chile, we described students' knowledge of mathematical vocabulary and evaluated the concurrent relations among mathematical vocabulary and measures of general vocabulary, working memory, and mathematical skills (i.e., arithmetic fluency, calculation, and applied problems) at ages 8 (T1) and 9 (T2) years. We hypothesized that there would be significant concurrent relations at both time points between mathematical vocabulary and the three mathematical skills (Hypothesis 1a), but that the relative strength of these relations would depend on the demands of the tasks. Specifically, based on the cognitive demands of the mathematical tasks, we hypothesized that the correlation between mathematical vocabulary and students' mathematical performance would be stronger for applied problems than for computational skills, that is, calculation and arithmetic fluency (Hypothesis 1b, Lin et al., 2021).

Research Question 2: Does mathematical vocabulary predict the change in students' performance on mathematical tasks from one school year to the next?

To investigate the role of mathematical vocabulary in the change in mathematical performance, we evaluated whether mathematical vocabulary in 8-year-olds (T1) predicted their performance on mathematical skills (i.e., arithmetic fluency, calculation, and applied problems) one year later (T2), above and beyond general vocabulary, working memory, concurrent mathematical vocabulary, and prior mathematical skills. Consistent with Lin et al. (2021), we hypothesized that both general language skills (i.e., receptive vocabulary) and domain-specific mathematical vocabulary at Time 1 would predict significant unique variance in mathematical performance at Time 2 (Hypothesis 2). Figure 1 summarizes the hypothesized associations between predictors and outcomes.

According to previous research, relations between mathematical language and mathematical skills may depend on the specific language and cognitive demands of each task (Lin et al., 2021; Peng & Lin, 2019). In this view, the relations between specific mathematical skills and language could vary, depending on the requirements of the mathematical skills, and thus result in different predictions for each mathematical skill.

First, we hypothesized (Hypothesis 2a) that the development of arithmetic fluency would be least strongly related to language skills. Fluency involves direct retrieval of facts or procedures from long-term memory. Arithmetic fluency and calculation were presented as written problems (using only Arabic digits) and students wrote down their answers. Language may have been the medium for retrieval of arithmetic facts and procedures, but there was presumably little demand on the thinking function of language because the operation sign in each problem provided the appropriate procedure. In previous work, attentional skills predicted the development of fluency but not of math knowledge (i.e., calculation; LeFevre et al., 2013).

Second, for calculation, the emphasis is on activation and implementation of procedural knowledge (i.e., arithmetic knowledge that guides the performance of procedures for solving calculations) which may be supported by language skills, either through the medium or the thinking functions of language (e.g., Caviola et al., 2012; Peng & Lin, 2019). Accordingly, we hypothesized (Hypothesis 2b), that the development of calculation will be related to language skills, especially those most closely related to mathematical knowledge.

Third, because the applied problems were presented verbally, students needed to use their general oral language skills to encode the problem information and activate an appropriate solution strategy (Fuchs et al., 2015; Passolunghi & Siegel, 2001). They also had to implement that strategy and any required calculations. These requirements suggest that applied problem solving will involve both the thinking and the medium functions of language, including domain-general skills (i.e., interpreting the wording of the problem) and domain-specific skills (i.e., retrieving the appropriate schema, for example, addition vs. multiplication) (Hypothesis 2c). In summary, by including outcomes with different cognitive demands and variable language requirements, we were able to analyze whether there were different developmental associations between vocabulary knowledge and three mathematical outcomes (Lin et al., 2021).

Figure 1.

Summary of Predictors and Outcomes in Regression Analyses (i.e., Research Question 2).

Predictors		T2 Outcomes			
		Mathematical Vocabulary	Arithmetic Fluency	Calculation	Applied Problems
T1	Spatial Span	✓	✓	✓	✓
	Receptive Vocabulary	✓	✓	✓	✓
	Mathematical Vocabulary	✓	✓	✓	✓
	Arithmetic Fluency		✓		
	Calculation			✓	
	Applied Problems				✓
T2	Mathematical Vocabulary		✓	✓	✓

Note. Checkmarks identify whether a predictor was included in the regression analysis.

Method

Transparency and Openness

We report how we determined the sample size of the larger project from which this study was derived, criteria for data exclusion, and all measures involved in this study. The data are not available because a sharing policy was not included in the consent forms approved by the Institutional Ethics Committee. Data were analyzed in IBM SPSS Statistics (Version 28) (IBM Corp, 2021) and R (Version 4.2.1) (R Core Team, 2022) using the *semTools* (Jorgensen et al., 2022) and *lavaan* (Rosseel, 2012) packages. This study design and its analyses were not preregistered. We follow Journal Article Reporting Standards (JARS) for reporting quantitative methods (Appelbaum et al., 2018).

Project Overview

This correlational study was conducted within the context of a larger project focused on the development of mathematical skills in Chilean students from Kindergarten to Grade 3 (Authors, 2021, 2022). The project received ethics approval by the Scientific Ethics Committee of Social Sciences, Arts and Humanities of the [institution blinded].

Principals of schools in the urban metropolitan area of Santiago were invited to participate in the study. The schools that were selected for the study served students from families of either high or low socioeconomic status (SES), given the highly-segregated Chilean educational system (Mizala & Torche, 2012). Parents of students from Kindergarten to Grade 3 were sent letters inviting them and their children to participate in the study. Parents who agreed to participate were subsequently contacted by phone by the research team to arrange a home visit. The larger study involved home visits and the participation of two primary caregivers; at the home visit both parents signed consent forms and answered demographic and home learning environment questionnaires, but students were tested at schools. The final sample included approximately 30% of the students enrolled in the invited Kindergarten to Grade 3 classrooms from five schools (three low-SES and two high-SES). Each parent received a small monetary reward for their participation (i.e., a gift card of CLP 10,000, worth approximately 14 USD at the time), and students received a book. After the home visit, those students with parental consent who also gave their own verbal assent were tested in two sessions in a private space at their schools. The sample size of the larger study ($N = 367$) was chosen based on an a priori power analysis (estimated $N = 370$, $p < .05$, power of 80%, and small expected effect sizes [i.e., $d = 0.3$]) for the associations between numerical predictors and arithmetic performance of children in Grades 1-3 (Lyons et al., 2014).

Parental education was assessed on a scale ranging from incomplete primary school education to doctoral degree. The median level of education for both mothers and fathers was equivalent to some community college or other non-university post-secondary education, with a range from incomplete primary school to a master's degree (for mothers) or doctoral degree (for fathers). Students attended either high-SES ($n = 35$) or low-SES ($n = 52$) schools. High-SES schools were private and did not receive any monetary subsidy, whereas low-SES schools received monetary support from the Chilean Government. There were differences in the median educational attainment of parents whose children attended low- versus high-SES schools. The median educational level for both mothers and fathers in low-SES schools was a high school diploma. In contrast, the median educational level for both mothers and fathers of high-SES schools was a university degree. These educational levels reflect the segregation of the Chilean educational system in terms of socio-economic and educational background of the families (Mizala & Torche, 2012).

Participants

The mathematical vocabulary task was administered to students in Grades 2 (mean age of 8 years) and 3 (mean age of 9 years) only (see a detailed description of the tasks below). Therefore, the sample for the current analyses includes students who were in Grade 2 at the first time point of the study (N = 87; M = 7:11 years: months, SD = 0:5; 46% girls) and were followed up with in Grade 3 at the second time point of the study (N = 77; 8:11 years: months, SD = 0:5; 48% girls). All students were Spanish speaking. For one student, we only had data corresponding to Time 2, due to student absence at Time 1. Grades 2 and 3 were chosen because we focused on evaluating the growth of mathematical language skills and the relations among general language, mathematical language skills, and students' mathematical outcomes after students had some years of experience with formal mathematical instruction.

Procedure

Testing took place in two sessions of approximately 20-30 minutes each in a quiet room in the school. Students received a sticker as appreciation for their participation. Assessments were presented in the same order for all students. Testing was conducted by research assistants who had participated in a three-hour training session led by the first author, which took place each year of the data collection period. A research coordinator was present at the school sites to closely supervise the testing during the entire assessment period. Students in Grade 2 were tested over a four-month period in the second half of the 2018 school year (August-November 2018). A year later, in the second semester of 2019 (August-December 2019), the participating students who were in Grade 3 at that time were tested again, following the same procedure. The original plan for two more testing time points (years 2020 and 2021) had to be abandoned due to the COVID-19 pandemic that led to generalized school closures in Chile.

Measures

Students were administered the same measures of mathematical vocabulary and mathematical outcomes at Time 1 (8 years of age; Grade 2) and Time 2 (9 years of age; Grade 3). They also completed tests of receptive vocabulary and working memory at Time 1.

Receptive Vocabulary

Students completed the Hispanic-American adaptation of the Peabody Picture Vocabulary Test–Revised: Test de Vocabulario en Imágenes Peabody (Dunn et al., 1986b), a measure of receptive vocabulary, as an index of general language skills. In this test, students are presented with four images, the experimenter says a word, and students are asked to point to the picture that best represents the target word. The starting point of the task depends on the age of the student, but the experimenter needs to ensure a basal criterion, as per the test's administration procedures. Testing ends after the student makes six errors in a set of eight items. The total score was the number of correct responses. Reported split-half reliability for 8- to 9-year-old students ranged from .91 to .94 (Dunn et al., 1986a).

Spatial Span

A spatial span task was administered using an iPad© to students at Time 1 (i.e., Path Span: https://hume.ca/ix/pathspan/) as an index of working memory. In this task, students are presented with sequences of 2–9 green circles on the screen, with each circle lighting up for approximately 200 ms. Students are asked to watch and remember the sequence of locations (starting with sequence lengths of two dots) and then reproduce it by touching the locations (green circles) in the same order. Three sequences of locations for each length are presented, and if students correctly reproduce at least one of those sequences, the task proceeds to the next level, increasing in length by one dot. At each length level, if students incorrectly reproduce all three sequences, the task is terminated. This task is scored as the total number of sequences completed correctly. Reliability was calculated based on the sum scores of the first, second, and third trials for each sequence (Cronbach's $\alpha = .82$).

Mathematical Vocabulary

Students were administered the Spanish-language version of a mathematical language task that has been previously administered in both English and French by the Authors (blinded) with students in Grades 2 and 3 (i.e., ages 8 and 9 years). For the Spanish-language version, minor adjustments were made to ensure that the items were appropriate for Spanish-speaking Chilean students when direct translations were not sufficient. The complete list of vocabulary terms is presented in Spanish, along with its corresponding English translation, in Table A in the Appendix. These words are included in the contents of the National Study Programs of the curriculum used by Chilean students to learn mathematics (MINEDUC, 2012).

For this task, students are shown some pictures or symbols and are asked to point to the picture or symbol that best represent the target word or expression that is said by the experimenter. For example, the experimenter asks the student, "Point to the picture that shows half of a pizza" while the student is shown three images: one with three quarters of a pizza, one with half a pizza, and another with a fifth of a pizza. The task comprised 22 target vocabulary terms including some related to each of numbers and operations (e.g., fewer, tens column, less than), spatial terminology (e.g., between, nearest), measurement (e.g., mass, pesos), and geometry (e.g., cube, cylinder). The score for this task is the total number of correct responses. Ordinal alpha reliability (Zumbo et al., 2007) was .82 and .86 at Time 1 and Time 2, respectively.

Outcomes

Students were administered three mathematics subtests of the Batería III Woodcock-Muñoz Pruebas de Aprovechamiento (WJ III Tests of Achievement, Muñoz-Sandoval et al., 2005)—Math Fluency, Calculation, and Applied Problems. These are standardized assessments with specific administration protocols, as described below. The median reliability coefficients (internal consistency) based on the Spanish calibration for the group of 9-year-old students (Schrank et al., 2005) for the non-timed tests were: .90 (Calculation) and .92 (Applied Problems). For the Math Fluency subtest, Rasch procedures were employed to calculate the reliability. The reliability reported in the Technical Manual is .94 for the group of 9-year-old children (McGrew et al., 2007).

Arithmetic Fluency. The *Fluidez en Matemáticas* (Math Fluency) subtest of the Batería III Woodcock-Muñoz Pruebas de Aprovechamiento (Muñoz-Sandoval et al., 2005) was administered to students. This subtest assesses arithmetic fluency via a paper-and-pencil task in which students solve operations involving numbers between 0 and 10 (i.e., simple addition and subtraction in the first 60 items, and a mixture of simple addition, subtraction, and multiplication in the remaining items) within three minutes. There are 160 items in total, distributed over two pages of 10 items per row. Students are instructed to answer as quickly and accurately as possible, moving from one row to the next, and then to the second page. If they do not know how to solve one of the operations, they are told to move to the next one in the row. Testing is stopped after three minutes. The score is the number of correct responses within three minutes.

Calculation. In the *Cálculo* (Calculation) paper-and-pencil subtest of the Batería III Woodcock-Muñoz Pruebas de Aprovechamiento (Muñoz-Sandoval et al., 2005), students are presented with two pages of 45 written mathematical calculations that increase in difficulty. Calculations start with single-digit addition and subtraction and then move to double-digit addition and subtraction, single-digit multiplication, three-digit addition, division, and more complex arithmetic operations. Students continue solving mathematical calculations until they make six consecutive errors or omissions, in which case the experimenter stops the testing, as per the administration protocol (Mather & Woodcock, 2005). The score is the total number of correct responses.

Applied Problems. The *Problemas Aplicados* (Applied Problems) subtest of the Batería III Woodcock-Muñoz Pruebas de Aprovechamiento (Muñoz-Sandoval et al., 2005) was included to assess students' ability to solve mathematical word problems. Students hear the problem, see a visual stimulus or a written version of the problem, decide on the required mathematical operations, perform simple calculations, and provide an oral answer to the problem. This subtest assesses mathematical skills and concepts, including arithmetic operations, money, time, measurement, geometry, fractions and percentages, probability, and algebra. This subtest includes 62 items, with the starting point determined by the grade of the student, and testing is stopped after six consecutive errors or omissions (Mather & Woodcock, 2005). The score is the total number of correct responses, assuming the questions below the established basal level are correct, as per protocol instructions.

Analytic Strategy

The first goal of the current study was to describe the concurrent relations among mathematical vocabulary and mathematical skills (i.e., arithmetic fluency, calculation, and applied problems) in Spanish-speaking students from one academic year (8 years; T1) to the next (9 years; T2). We first report students' performance on the mathematical vocabulary task at both time points. Then, we describe change in mathematical vocabulary between Time 1 and Time 2. Finally, we examine the correlations between mathematical vocabulary and mathematical performance scores by grade, as well as the associations between mathematical vocabulary and mathematical performance with working memory and general language skills.

To address the second goal of the study concerning whether mathematical vocabulary at Time 1 predicted mathematical vocabulary and change in mathematical performance (i.e., arithmetic fluency, calculation, and applied problems) at Time 2, four multiple regressions were conducted. Each of these regressions included working memory, general language skills, and prior mathematical performance at Time 1.

Results

Descriptive Statistics

Descriptive statistics are shown in Table 1. The means and standard deviations showed no evidence of ceiling or floor effects; no variables were significantly skewed (i.e., |skew values| were < 1.00). Correlations among the measures are also shown in Table 1. Except for spatial span at Time 1 with arithmetic fluency and mathematical vocabulary at Time 2, all the measures were significantly correlated with each other. For mathematical performance (i.e., arithmetic fluency, calculation, and applied problems), total raw scores were significantly higher at Time 2 than at Time 1, *p*s < .05, providing evidence that, as expected, students were improving over time.

Students' mathematical vocabulary scores improved from Time 1 (M = 15.8) to Time 2 (M = 17.6), $t(74) = -6.26$, $p < .001$, $d = -0.72$. The amount of growth was modest, in part because students performed well on many items. Nine of the 22 items had accuracy above 90%. Nevertheless, there was some room for growth (see Table B in the Appendix for the list of items, students' performance by time point, and the effect size of the change per item). The items on which students' scores increased significantly from Time 1 to Time 2 reflected their growing knowledge of place value (tens and hundreds column), operational symbols (difference), measurement (mass), and problem solving (fewer). Thus, mathematical vocabulary performance was stable from one grade to the next, but also showed improvement in some items from Time 1 to Time 2. The items that reflected this improvement included mathematical words students are learning in school between 8 and 9 years of age (Grade 2 and Grade 3).

Research Question 1: Is the mathematical vocabulary of Spanish-speaking students related to their mathematical skills from one school year to the next?

As shown in Table 1, mathematical vocabulary at Time 1 was positively correlated with all mathematical performance tasks at the same time point, ranging from .37 for the correlation with calculation to .58 for the correlation with applied problems. One year later (Time 2), mathematical vocabulary was positively correlated with all three mathematical performance tasks, ranging from .40 (arithmetic fluency) to .51 (applied problems). In support of Hypothesis 1a, these results indicate that Spanish-speaking students show similar patterns of correlation between mathematical vocabulary and mathematical performance as has been found in other populations (Lin et al., 2021).

For Hypothesis 1b, we predicted that mathematical vocabulary would be more strongly correlated with mathematical tasks that pose substantial demands on oral language (i.e., applied problems), compared to mathematical tasks that do not (i.e., calculation, arithmetic fluency). There was not much evidence for this hypothesis, however. At Time 1, mathematical vocabulary was more strongly correlated with applied problems than with calculation ($z = 2.59$, $p = .005$). However, the correlation between mathematical vocabulary and applied problems did not significantly differ in strength from the correlation between mathematical vocabulary and arithmetic fluency ($z = .40$, $p = .34$). Moreover, at Time 2, mathematical vocabulary was not significantly more strongly correlated with applied problems than with arithmetic fluency (z = 1.47, p = .07) or calculation (z = 1.35, p = .09). Thus, these data do not strongly support Lin et al.'s (2021) predictions that mathematical tasks with relatively greater language skills are more strongly related to mathematical vocabulary. Note however that the results reported in Lin et al. (2021) were similar to the present findings, in that the average correlations between mathematical vocabulary and both foundational tasks and higher-order mathematical tasks were all positive.

Research Question 2: Does mathematical vocabulary predict the change in students' performance on mathematical tasks from one school year to the next?

As shown in Table 1, the correlations between the same tasks at Time 1 and Time 2 were moderate to high, ranging from .50 (mathematical vocabulary) to .75 (applied problems). Mathematical outcomes at Time 2 were significantly correlated with mathematical vocabulary at Time 1 (ranging from .45 to .59) and Time 2 (ranging from .40 to .51). To investigate whether both general language skills (i.e., receptive vocabulary) and domain-specific mathematical vocabulary at Time 1 would predict significant unique variance in mathematical performance at Time 2 (Hypothesis 2), we conducted four multiple regression analyses.

Mathematical Vocabulary

First, we tested the predictive relations for mathematical vocabulary across grades. As shown in Table 2, together, receptive vocabulary, spatial span, and mathematical vocabulary at Time 1 explained approximately 29% of the variance in mathematical vocabulary at Time 2, $F(3, 69) = 9.31$, $p < .001$. As expected, mathematical vocabulary at Time 1 predicted unique variance in mathematical vocabulary at Time 2. Neither receptive vocabulary nor spatial span predicted significant unique variance.

Table 1

Descriptive Statistics and Correlations for All Variables

	Time 1						Time 2			
	1	2	3	4	5	6	7	8	9	10
1. Receptive Vocabulary T1	--	.29**	.45***	.30**	.30**	.40***	.41**	.31**	.30**	.48***
2. Spatial Span T1		--	.27*	.15	.37***	.32**	.18	.29*	.33**	.36**
3. Mathematical Vocabulary T1			--	.55***	.37***	.58***	.50***	.45***	.54***	.59***
4. Arithmetic Fluency T1				--	.55***	.63***	.54***	.71***	.62***	.68***
5. Calculation T1					--	.62***	.29**	.49***	.65***	.68***
6. Applied Problems T1						--	.49***	.62***	.62***	.75***
7. Mathematical Vocabulary T2							--	.40***	.43***	.51***
8. Arithmetic Fluency T2								--	.66***	.73***
9. Calculation T2									--	.83***
10. Applied Problems T2										--
N per measure	86	84	86	86	86	86	76	77	77	77
Mean	84.34	8.64	15.77	29.38	10.90	28.27	17.62	39.45	13.23	45.47
SD	10.24	3.66	2.65	10.03	2.78	3.81	2.34	11.76	2.42	6.00
Skew	0.13	-0.26	-0.36	0.57	0.20	0.27	-0.57	0.60	-0.29	0.27

Note. [a]Total correct. T1 = Time 1 (Grade 2), T2 = Time 2 (Grade 3).

* $p < .05$. ** $p < .01$. *** $p < .001$.

Table 2

Multiple Regression Predicting Mathematical Vocabulary at Time 2

Variable	*B*	*SE*	*β*	r^2_U
Constant	**7.73*** **	**2.07**		
Receptive Vocabulary T1	0.05	0.03	.23	.04
Spatial Span T1	0.01	0.07	.01	.00
Mathematical Vocabulary T1	**0.35** **	**0.10**	**.39** **	**.12** **
R^2	.29			

Note. N = 77. r^2_U = unique (semi-partial) r^2. Multiple regression analysis with receptive vocabulary, spatial span, and mathematical vocabulary at Time 1 predicting mathematical vocabulary at Time 2. Bolded text highlights significance.

$^{**}p < .01$. $^{***}p < .001$.

Mathematical Skills

To predict change in the three mathematical outcomes (i.e., arithmetic fluency, calculation, and applied problems), hierarchical multiple regressions were conducted. Model 1 included working memory (i.e., spatial span) and performance on the same mathematics task at Time 1. Model 2 incorporated language skills, both domain-general receptive vocabulary and domain-specific mathematical vocabulary, into the regression. Lastly, Model 3 added mathematical vocabulary at Time 2 to control for concurrent associations among mathematical vocabulary and mathematical skills.

For these three models, sensitivity analyses were conducted to determine the minimum effect size (f^2) that could be detected with our sample size (N = 77) and an alpha level of .05 at 80% power. Our hierarchical regressions consist of three models: In Model 1 two predictors are entered, in Model 2 an additional two predictors are added, and in Model 3 one additional predictor is added for a total of five predictors. Sensitivity analyses revealed the minimum effect size (f^2) that it would be possible to detect for the five-predictor model is $f^2 = 0.18$. The minimum effect sizes it would be possible to detect for the increase in variance explained by adding two predictors in Model 2 and one predictor in Model 3 are 0.13 and 0.10, respectively. Obtained effect sizes f^2 for each hierarchical regression analysis are reported below.

Arithmetic Fluency. In Model 1, spatial span and arithmetic fluency were entered as predictors. As shown in Table 3, this model explained approximately 54% of the variance in arithmetic fluency at Time 2, $F(2, 70) = 39.95$, $p < .001$, $f^2 = 1.14$, with both variables predicting unique variance. In Model 2, receptive vocabulary, spatial span, mathematical vocabulary, and arithmetic fluency at Time 1 explained approximately 54% of the variance in arithmetic fluency at Time 2, $F(4, 68) = 19.64$, $p < .001$, $f^2 = 1.16$, with spatial span and arithmetic fluency continuing to predict significant unique variance. Contrary to Hypothesis 2a, receptive vocabulary and mathematical vocabulary did not explain additional variance in arithmetic fluency at Time 2 ($f^2 = 0.006$) nor did either predictor explain unique variance. Additionally, adding mathematical vocabulary at Time 2 (Model 3) did not explain additional variance in the model ($f^2 = 0.001$). In summary, the final model explained approximately 54% of the variance in arithmetic fluency at Time 2, $F(5, 67) = 15.51$, $p < .001$, $f^2 = 1.16$.

Table 3

Hierarchical Multiple Regression Models Predicting Arithmetic Fluency at Time 2

Variable	Model 1					Model 2					Model 3				
	B	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U
Constant	**10.79**	**3.52**		**.003**		5.66	8.47		.506		6.78	9.61		.483	
Spatial Span T1	**0.62**	**0.27**	**.19**	**.022**	**.04**	**0.57**	**0.28**	**.18**	**.048**	**.03**	**0.57**	**0.28**	**.18**	**.049**	**.03**
Arithmetic Fluency T1	**0.79**	**0.10**	**.68**	**<.001**	**.45**	**0.77**	**0.12**	**.65**	**<.001**	**.30**	**0.78**	**0.13**	**.66**	**<.001**	**.27**
Receptive Voc. T1						0.06	0.11	.05	.569	.00	0.07	0.11	.06	.544	.00
Mathematical Voc. T1						0.07	0.48	.02	.879	.00	0.10	0.49	.02	.844	.00
Mathematical Voc. T2											-0.13	0.53	-.03	.801	.00
R^2	**.53**			**<.001**		**.54**			**<.001**		**.54**			**<.001**	
ΔR^2						.00			.80		.00			.80	

Note. N = 77. r^2_U = unique (semi-partial) r^2. Voc. = Vocabulary. In Model 1, we entered spatial span and arithmetic fluency at Time 1 to predict arithmetic fluency at Time 2. In Model 2, we entered receptive vocabulary and mathematical vocabulary at Time 1 as predictors. In Model 3, we entered mathematical vocabulary at Time 2 as a predictor. Bolded text highlights significance at $p < .05$.

Calculation. In Model 1, spatial span and calculation at Time 1 were entered as predictors. As shown in Table 4, this model explained approximately 43% of the variance in calculation at Time 2, $F(2, 70) = 26.55$, $p < .001$, $f^2 = 0.76$, with calculation predicting unique variance. In Model 2, receptive vocabulary and mathematical vocabulary at Time 1 were entered as predictors. This model explained approximately 53% of the variance in calculation at Time 2, $F(4, 68) = 18.80$, $p < .001$, $f^2 = 1.11$. In partial support of Hypothesis 2b, the inclusion of mathematical vocabulary and receptive vocabulary in Model 2 explained an additional 10% of the variance ($f^2 = 0.20$), but only mathematical vocabulary predicted significant unique variance. When analyzing the shared variance of the vocabulary measures, 8.5% was variance uniquely explained by mathematical vocabulary, indicating that domain-specific mathematical vocabulary, and not general vocabulary knowledge, predicted the change in calculation performance from Time 1 to Time 2. Adding mathematical vocabulary at Time 2 (Model 3) did not explain additional significant variance in the model ($f^2 = 0.04$). Overall, this model explained approximately 54% of the variance in calculation at Time 2, $F(5, 67) = 15.90$, $p < .001$, $f^2 = 1.19$.

Table 4

Hierarchical Multiple Regression Predicting Calculation at Time 2

Variable	Model 1					Model 2					Model 3				
	B	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U
Constant	**6.85**	**0.91**		**<.001**		**3.63**	**1.77**		**.044**		2.41	1.91		.211	
Spatial Span T1	0.07	0.06	.11	.278	.01	0.04	0.06	.06	.536	.00	0.04	0.06	.06	.516	.00
Calculation T1	**0.53**	**0.09**	**.61**	**<.001**	**.32**	**0.44**	**0.08**	**.51**	**<.001**	**.20**	**0.43**	**0.08**	**.49**	**<.001**	**.19**
Receptive Voc. T1						-0.01	0.02	-.02	.816	.00	-0.01	0.02	-.06	.562	.00
Mathematical Voc. T1						**0.31**	**0.09**	**.34**	**<.001**	**.09**	**0.26**	**0.09**	**.28**	**.008**	**.05**
Mathematical Voc. T2											0.16	0.10	.16	.114	.02
R^2	**.43**			**<.001**		**.53**			**<.001**		**.54**			**<.001**	
ΔR^2						**.09**			**.002**		.02			.114	

Note. $N = 77$. r^2_U = unique (semi-partial) r^2. Voc. = Vocabulary. In Model 1, we entered spatial span and calculation at Time 1 to predict calculation at Time 2. In Model 2, we entered receptive vocabulary and mathematical vocabulary at Time 1 as predictors. In Model 3, we entered mathematical vocabulary at Time 2 as a predictor. Bolded text highlights significance at $p < .05$.

Applied Problems. In Model 1, spatial span and applied problems at Time 1 were entered as predictors. As shown in Table 5, this model explained approximately 58% of the variance in applied problems at Time 2, $F(2, 70) = 48.78$, $p < .001$, $f^2 = 1.39$, with only applied problems predicting significant unique variance. In Model 2, receptive vocabulary and mathematical vocabulary at Time 1 were entered as predictors. This model explained approximately 63% of the variance, $F(4, 68) = 28.83$, $p < .001$, $f^2 = 1.70$. The inclusion of mathematical vocabulary and receptive vocabulary in Model 2 explained an additional 5% of the variance ($f^2 = 0.13$), with mathematical vocabulary and receptive vocabulary each uniquely predicting 1.7% of the variance. Contrary to Hypothesis 2c, neither language measure predicted significant unique variance in the change in applied problems from Time 1 to Time 2. Adding mathematical vocabulary at Time 2 (Model 3) did not explain additional variance in the model ($f^2 = 0.02$). The final model explained approximately 64% of the variance in applied problems at Time 2, $F(5, 67) = 23.38$, $p < .001$, $f^2 = 1.75$. Overall, these findings suggest that although general vocabulary and mathematical vocabulary do not separately explain unique variance in applied problems, together these vocabulary skills contribute to explaining the change in applied problems performance from Time 1 to Time 2. In summary, relatively little of this relation is specific to mathematical vocabulary.

Table 5

Hierarchical Multiple Regression Predicting Applied Problems at Time 2

Variable	Model 1					Model 2					Model 3				
	B	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U	*B*	*SE*	*β*	*p*	r^2_U
Constant	**11.95**	**3.48**		**<.001**		5.55	4.22		.193		4.09	4.42		.358	
Spatial Span T1	**0.22**	**0.13**	**.13**	**.107**	**.02**	0.15	0.13	.09	.260	.01	0.15	0.13	.09	.245	.01
Applied Problems T1	1.12	0.13	.71	<.001	.45	**0.89**	**0.15**	**.57**	**<.001**	**.20**	**0.85**	**0.15**	**.54**	**<.001**	**.17**
Receptive Voc. T1						0.09	0.05	.15	.083	.02	0.08	0.05	.13	.135	.01
Mathematical Voc. T1						0.38	0.22	.17	.082	.02	0.32	0.22	.14	.151	.01
Mathematical Voc. T2											0.26	0.23	.10	.275	.01
R^2	**.58**			**<.001**		**.63**			**<.001**		**.64**			**<.001**	
ΔR^2						**.05**			**.018**		.01			.275	

Note. $N = 77$. r^2_U = Unique (semi-partial) r^2. In Model 1, we entered spatial span and applied problems at Time 1 to predict applied problems at Time 2. In Model 2, we entered receptive vocabulary and mathematical vocabulary at Time 1 as predictors. In Model 3, we entered mathematical vocabulary at Time 2 as a predictor. Bolded text highlights significance at $p < .05$.

Discussion

We evaluated the relations among cognitive skills, mathematical vocabulary, and mathematical performance in early elementary school for Chilean students. To assess mathematical vocabulary, we created a Spanish-language version of a mathematical vocabulary task used in prior research with French- and English-speaking students (Authors, blinded). In the current study we addressed two research questions in a sample of Chilean students tested at ages 8 (Time 1; Grade 2) and 9 years (Time 2, Grade 3). First, we explored the concurrent relations among mathematical vocabulary and mathematical skills (i.e., arithmetic fluency, calculation, and applied problems). Second, we evaluated whether mathematical vocabulary at Time 1 uniquely predicted changes in mathematical performance from Time 1 to Time 2. Because researchers have not explored the longitudinal relation between mathematical vocabulary and mathematical performance, and few researchers have examined mathematical language skills in Spanish-speaking students (Pina et al., 2015; Pina et al., 2014), the present research addresses novel questions about the change in mathematical vocabulary and the relations between mathematical vocabulary and mathematical performance in the early years of elementary school for Spanish-speaking students.

Research Question 1: Is the mathematical vocabulary of Spanish-speaking students related to their mathematical skills from one school year to the next?

As expected, Chilean students showed improvement in their mathematical vocabulary from Time 1 to Time 2, supporting the assumption that mathematical vocabulary is learned in school (Powell et al., 2017). However, their improvement was modest. One plausible reason for this finding is that learning mathematical terms occurs over more extended periods and with explicit instruction (Powell et al., 2019). Students might not get the constant and direct instruction needed to master mathematical vocabulary terms. Another possibility is that our mathematical vocabulary task did not fully capture the mathematical terms students learned from Grade 2 to Grade 3. Therefore, because this is the first study on the change in mathematical vocabulary with Chilean Spanish-speaking students, additional empirical studies are needed to fully explore how mathematical vocabulary develops across the first years of primary school for these students.

Consistent with a meta-analysis by Lin et al. (2021), mathematical vocabulary was concurrently correlated with mathematical outcomes (i.e., arithmetic fluency, calculation, and applied problems) at both Time 1 and Time 2. Moreover, our findings partially align with the view that mathematical vocabulary is more strongly linked to higher-order outcomes involving language skills (i.e., word-problem solving) than to foundational mathematical tasks (i.e., those involving accuracy and fluency of arithmetic operations, Lin et al., 2021). Specifically, we found that applied problems showed stronger correlations with mathematical vocabulary than did other mathematical outcomes such as calculation and arithmetic fluency at Time 2, and calculation at Time 1. These results support their argument that mathematical language skills are important both because they reflect the role of mathematics-related knowledge in mathematical performance (Lin et al., 2021), and because language is an important component of problem solving, more generally (Fuchs et al., 2015). In Lin et al.'s (2021) meta-analysis, there were positive and significant correlations between mathematical vocabulary and both foundational and higher-order mathematical tasks. However, when controlling for covariates and moderators (including family socioeconomic status, publication type, and whether the sample was typically developing), the correlation between mathematical vocabulary and higher-order mathematical tasks was significantly larger than with foundational tasks. Thus, the unique relation between mathematical vocabulary and mathematical performance may be relatively small when these measures share variance with cognitive skills and other factors.

Our study contributes evidence from Spanish-speaking students of the moderate but consistent correlations between mathematical vocabulary and mathematical performance in both Grades 2 and 3 (i.e., 8 to 9 years-of-age). This extension to Spanish-speaking learners is important because it is possible that linguistic differences across languages might influence the extent to which language skills are related to mathematical learning. Rubenstein and Thompson (2002) noted that translation of mathematical terms from one language to another is not always straightforward because one mathematical word might be translated in different ways, and not all the terms that describe a mathematical concept (e.g., place value) have a specific mathematical meaning. Thus, mathematical vocabulary will vary across languages. For Spanish and English specifically, a comparison of the phonological complexity of mathematical vocabulary terms used in national standards and assessments indicated that a third of the assessed words were more complex in Spanish than in English (Purpura et al., 2018). Despite the linguistic differences between Spanish and English (Rubenstein & Thompson, 2002), our findings suggest that the significant associations between students' mathematical vocabulary and students' mathematical performance in Spanish-speaking students are similar to those reported for English-speaking students (Lin et al., 2021). Nevertheless, in future, researchers should test whether the language that students use for mathematics is differentially predictive of the relation between mathematical vocabulary and mathematical performance.

Research Question 2: Does mathematical vocabulary predict the change in students' performance on mathematical tasks from one school year to the next?

Some studies have looked at the growth of mathematical language skills in students in the early years of elementary school (Warren, 2006), but none have been focused on predictors of change in mathematical vocabulary and mathematical performance from one early elementary grade to another. The students in our study demonstrated growth in mathematical vocabulary from Time 1 to Time 2. Notably, this growth was modest, which we speculate is due to the need for more instructional focus on teaching mathematical vocabulary (Powell et al., 2019). This study did not explore instructional factors, however, and future research is needed to understand the role of instruction in the development of mathematical vocabulary.

In relation to Hypotheses 2a, b, and c, we found some support for the prediction that mathematical vocabulary is related to the growth of mathematical skills. There was diversity, however, in the specifics of these relations. For arithmetic fluency (Hypothesis 2a), growth was only predicted by spatial working memory, not mathematical vocabulary or general vocabulary. This pattern is consistent with other findings showing that development of fluency is driven by individual differences in memory and attentional processes (De Smedt, 2022; LeFevre et al., 2013; Raghubar et al., 2010; Spiegel et al., 2021). For the calculation task (Hypothesis 2b), mathematical vocabulary at Time 1 predicted unique variance at Time 2 whereas general vocabulary did not. This finding that mathematical vocabulary predicted change in calculation skills, after considering general language and working memory, is aligned with Lin et al. (2021)'s view that mathematical language serves as a medium for retrieving stored conceptual knowledge.

For applied problem solving (Hypothesis 2c), the story was more complex. Mathematical vocabulary and general vocabulary, together, predicted change in applied problem solving. This pattern is consistent with the view that the shift in the importance of language skills from general to more specific relations with mathematical learning occurs gradually throughout the elementary school years (Lin et al., 2021; Powell et al., 2021; Powell et al., 2017). This finding also provides some support for the thinking function of language for mathematics (Peng et al., 2020), because various aspects of language supported the development of mathematical performance in problem solving.

Similar to the gradual shift from domain-general to domain-specific knowledge, we found incremental growth in mathematical vocabulary (Powell et al., 2021). In the early years students are focused on learning foundational numeracy and mathematical skills (e.g., arithmetic and numeration). Thus, need to learn new mathematical vocabulary terms may be modest from Grades 2 to 3, because the same mathematical terms are commonly used (Rubenstein & Thompson, 2002). Consistent with this interpretation, we found that concurrent mathematical vocabulary at Time 2 did not explain additional variance in any of the mathematical outcomes above and beyond mathematical vocabulary at Time 1. Our findings could also be related to the extent to which there is shared variance between general language skills and mathematical vocabulary (Lin et al., 2021) in predicting mathematical skills. More generally, at this age, students might need to use vocabulary skills, both general and mathematical, to perform typical academic tasks (Ufer & Bochnik, 2020). More work is needed to determine the points in development at which domain-specific versus more general vocabulary knowledge becomes the dominant predictor in mathematical tasks.

Our results highlight how mathematical vocabulary differentially relates to changes in mathematical performance in the early grades (Lin et al., 2021). Specifically, mathematical vocabulary and receptive vocabulary together explained variance in the change in applied problems from Time 1 to Time 2, suggesting that both domain-general vocabulary and domain-specific mathematical vocabulary are key aspects of mathematical performance in tasks that have large language demands (Lin et al., 2021; Peng et al., 2020). For performing written calculations, however, mathematical vocabulary, not receptive vocabulary, explained change from Time 1 to Time 2, suggesting that mathematical vocabulary might capture variability in knowledge specific to mathematics. Finally, the change in arithmetic fluency was related to working memory skills and not to vocabulary. LeFevre et al. (2013) also found that growth in arithmetic fluency was predicted by working memory, whereas calculation was not. Arithmetic fluency is a timed task, whereas the other mathematical outcomes were not. For timed tasks, the attentional component of working memory is relevant in predicting growth, as students might need to recall and efficiently access mathematical information to quickly answer basic arithmetic questions. In brief, these findings highlight how domain-general and domain-specific skills differentially support mathematical development in early elementary school and support the domain-specific relations between mathematical vocabulary and mathematical performance (Peng & Lin, 2019).

Limitations, Future Directions, and Implications

The present study has some limitations. First, the data are correlational and therefore no causal associations can be inferred. Future research including students from a broader range of grades is necessary to inform the developmental trajectories of the associations between domain-general language skills, domain-specific mathematical vocabulary skills, and mathematical outcomes. Second, our measure of mathematical vocabulary did not exhaustively test all aspects of mathematical vocabulary required in the curriculum over the early elementary years. The mathematical words that were selected could only capture a subset of the educational curricula. Accordingly, we did not distinguish types of mathematical vocabulary (e.g., numerical operations or geometry mathematical vocabulary). In future, researchers should consider how to reliably assess mathematical vocabulary by tapping into different aspects of the construct (Peng & Lin, 2019), allowing for growth over the years (see Peng & Lin, 2019), and evaluating the differential role of specific types of mathematical vocabulary and mathematical outcomes at different stages of learning.

Third, we assessed mathematical performance using standardized tests that are intended to be very stable from one year to the next. Accordingly, correlations from one year to the next were high for these outcomes and thus the variance available to be explained by other predictors above and beyond the variance explained by the same outcome was small. Accordingly, these analyses were stringent tests of the role of mathematical vocabulary in predicting change in mathematical skills in the early elementary years. Despite this limitation, our findings indicated that mathematical vocabulary predicted significant variance in mathematical change from one school year to the next.

In addition to the implications for future research, our findings have several educational implications. First, the results highlight the role of mathematical-specific vocabulary for students' mathematical performance in Grades 2 and 3. Consistent with other studies (Lin et al., 2021), mathematical vocabulary was a predictor of students' mathematical performance in both grades. Second, after a few years of formal schooling, students' cognitive skills, general language vocabulary, and domain-specific mathematical vocabulary were related to mathematics performance for Spanish-speaking students. Moreover, knowledge of vocabulary specific to mathematics accounted for change in students' mathematical performance. This finding is consistent with the view that teachers should provide direct and explicit instruction of mathematical vocabulary terms so that students fully comprehend the meaning of these terms. (Powell et al., 2019). Teaching mathematical vocabulary using accurate mathematical language is recommended as an effective practice for improving students' mathematics performance in primary school (Fuchs et al., 2021).

Conclusion

The present research highlights the associations between language skills, in general, and mathematical vocabulary, in particular, to students' mathematical performance concurrently and longitudinally. Furthermore, it provides data on these associations for Spanish-speaking students, a group that has rarely been studied (Lin et al., 2021). Our findings for calculation versus applied problem solving were consistent with the view that language skills related to achievement in mathematics gradually become more specific as students move through the early elementary years (Powell et al., 2017). In summary, by providing longitudinal evidence of the relations between mathematical vocabulary and mathematical outcomes (Peng & Lin, 2019; Powell et al., 2017), the current study provided support for the role of mathematical vocabulary as a predictor of mathematical learning (Powell et al., 2021).

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