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On sequential single-pion production in double-pionic fusion

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Abstract

Recently a two-step process has been proposed for the double-pionic fusion to deuterium $pn(I=0) \rightarrow d\pi^+\pi^-$. Its calculation is solely based on total cross section data for the two sequential single-pion production steps $pn(I=0) \rightarrow pp\pi^-$ followed by $pp \rightarrow d\pi^+$. Though this sequential process was aimed to explain the dibaryon resonance $d^*(2380)$ peak in double-pionic fusion, we demonstrate that this is not the case. It rather fits to a possible broad bump at 2.31 GeV in the energy dependence of the $pn \rightarrow d\pi^0\pi^0$ reaction, which was recently interpreted as a consequence of dibaryonic excitations in isoscalar single-pion production.

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Two-step processes are well-known in nuclear physics and have been studied there intensively for decades in a variety of nuclear reactions. In general their cross section is smaller than a competing direct process by an order of magnitude. Hence two-step processes are usually important, if the direct process is suppressed for some reason.

Recently it has been proposed by Molina, Ikeno and Oset [1,2] that a two-step process in form of two successive single-pion production processes may happen for the basic double-pionic

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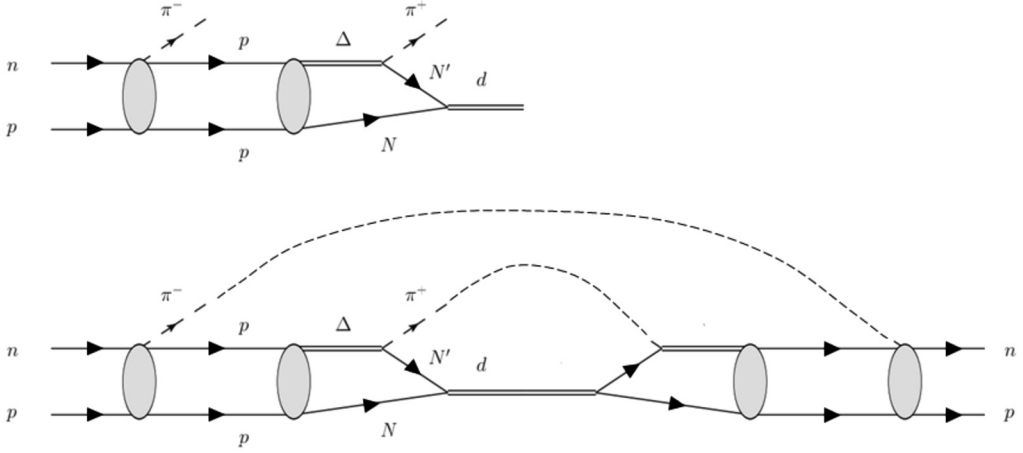


Fig. 1. Diagrammatical representation of the two-step process for the $np \rightarrow d\pi^+\pi^-$ reaction (top) suggested in Refs. [1,2] and its continuation as a four-step process to affect elastic np scattering as suggested in Ref. [1] (bottom).

reaction $pn \rightarrow d\pi\pi$. In particular the isoscalar reaction sequence $pn(I=0) \rightarrow (pp)\pi^- \rightarrow (d\pi^+)\pi^-$ has been considered. The two-step process with explicit Δ excitation in the second step is depicted diagrammatically in Fig. 1(top). In Ref. [1] it was argued that this two-step process could produce even a circle in the Argand plot of a specific partial wave in elastic np scattering, which is a necessary condition for a true resonance. It is well known that the chain $pp \rightarrow \Delta^+ p \rightarrow d\pi^+ \rightarrow \Delta^+ p \rightarrow pp$ can reproduce at least part of the loop in the Argand plot [3,4]. As illustrated in Fig. 1(bottom) the situation in our case is more complicated, since the chain $pp \rightarrow \Delta^+ p \rightarrow d\pi^+ \rightarrow \Delta^+ p \rightarrow pp$ is preceded by the further step $pn(I=0) \rightarrow (pp)\pi^-$.

The appealing beauty of the presented formalism is that the total cross section in the final $d\pi^+\pi^-$ channel is claimed to be calculable by just the knowledge of the total cross sections of the participating reactions $pn(I=0) \rightarrow pp\pi^-$ (step-1 reaction) and $pp \rightarrow d\pi^+$ (step-2 reaction), which both have been studied experimentally, phenomenologically and theoretically. The drawback of the formalism presented in Ref. [1], of course, is that no differential cross sections can be calculated and that no spin-parity quantum numbers are selected. Hence one has to be very careful in the interpretation of the results.

In Ref. [1] the intention was to present an alternative explanation of the $d^*(2380)$ dibaryon resonance structure with $I(J^P) = 0(3^+)$ in the $np \rightarrow d\pi^+\pi^-$ reaction by use of the formalism for the sequential single-pion production. The pole of the $d^*(2380)$ resonance has been identified at 2.38 GeV both in polarized [5,6] and unpolarized [7] elastic neutron-proton scattering by use of the full SAID database, albeit the critical contribution came from the polarization data [5,6]. The $d^*(2380)$ resonance has been observed in the isoscalar part of all the various $NN\pi\pi$ channels [8–14] exhibiting there a pronounced narrow Lorentzian of width 70 MeV. Hence the two-step process should undergo such a narrow structure around 2.38 GeV at least in one of the two participating reactions (in principle one can get a peak also, if one process is rising, whereas the other one is falling). The total cross section of the step-2 reaction, the $pp \rightarrow d\pi^+$ reaction, exhibits only a broad resonance structure due to the $\Delta(1232)$ excitation.¹ Therefore the desired structure must be found in the step-1 reaction, the $pn(I=0) \rightarrow pp\pi^-$ reaction.

¹ For recent interpretations see, e.g., Refs. [4,15].

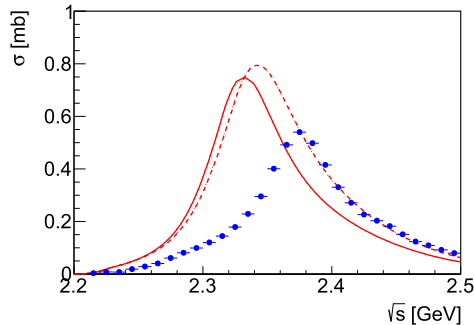


Fig. 2. The isoscalar part of the total $pn \rightarrow d\pi^+\pi^-$ cross section in the region of the dibaryon resonance $d^*(2380)$. Blue filled circles represent the experimental results from WASA-at-COSY [9,11], the horizontal bars give the binning width used. Red solid and dotted curves show the calculations of Ref. [1]. (For interpretation of the colors in the figures, the reader is referred to the web version of this article.)

From Ref. [18].

The experimental isoscalar cross section exhibits indeed a bump structure around 2.31 GeV, but again only a broad one, which was fitted in Ref. [16] by a Gaussian of width 150 MeV. However, by increasing the error bars of the WASA data by an order of magnitude and ignoring recent high-precision data from Gatchina [17], Ref. [1] succeeded to achieve a seemingly alternative description with $\chi^2 \ll 1$ providing now Breit-Wigner shapes peaking at 2.33 - 2.34 GeV and having a width of 70 - 80 MeV. In Ref. [1] this could be achieved only by enlarging the uncertainties of the WASA-at-COSY results enormously by adding in quadrature a large systematic error arguing that is due to the neglect of isospin violation in the derivation of the $pn(I=0) \rightarrow pp\pi^-$ data in Ref. [16]. Such a procedure of handling systematic errors as presented in Ref. [1] is by no means justified, since the isospin violation is not fluctuating randomly from energy point to energy point and hence does not behave like statistical uncertainties. Therefore it cannot be added to them. Isospin violation rather affects just the absolute scale of the isoscalar cross section shifting the data solely in common up or down in scale. It was also shown in Ref. [11] that isospin violation strongly affects the shape of differential $M_{\pi\pi}$ distributions due to different thresholds for $M_{\pi^0\pi^0}$ and $M_{\pi^+\pi^-}$, see, *e.g.*, Ref. [4]. Hence, inability to correctly reproduce the differential observables would unavoidably lead to an incorrect isospin violation prediction.

In Ref. [1] the results of the two-step calculations were not confronted with experimental data. Hence we display both the calculations and the WASA-at-COSY data [11] in Fig. 2 for the isoscalar part of the $d\pi^+\pi^-$ channel. Despite of tuning the fit on the cross section for the step-1 process the calculated peak structure comes out too low in energy by 30 - 40 MeV, which is far outside experimental uncertainties [9,11]. The fact that the peak calculated for the $d\pi^+\pi^-$ channel misses the measured peak by about 40 MeV is associated in Ref. [1] with a pretended experimental resolution of 20 MeV in \sqrt{s} . However, here the authors of Ref. [1] mix up the experimental resolution with the bin width used for the presentation of differential distributions in Ref. [11]. Furthermore, a finite experimental energy resolution affects the width of a resonance structure, but not its position. The binning used for the presentation of total cross section was 10 MeV in Refs. [9,11] — see Fig. 2 — and the high precision COSY beam had a resolution in the sub-MeV range.

Since no specific angular momenta are considered in the formalism of Ref. [1], the calculated structure in the final channel contains a priori a variety of spin-parity combinations. From partial-

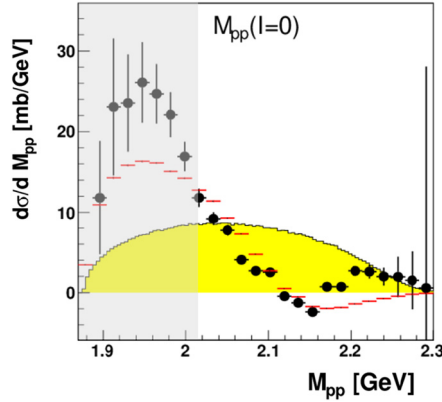


Fig. 3. Isoscalar pp invariant mass spectrum $M_{pp}(I=0)$ obtained from the difference of the corresponding distributions in the $pp \rightarrow pp\pi^0$ and $pn \rightarrow pp\pi^-$ reactions by use of eq. (1) in Refs. [16,23,24]. The phase-space distribution is indicated by the (yellow) shaded region. The red dashed histogram gives a conventional t -channel calculation for Roper excitation [16,19]. The gray shaded region is below the threshold of 2.015 GeV for the $pp \rightarrow d\pi^+$ reaction, *i.e.* not available for the step-2 reaction.

wave analyses [20,21] of the step-2 reaction, the $pp \rightarrow d\pi^+$ reaction, we know that 62% of its total cross section is due to the 1D_2 partial wave between the incident proton pair (which leads to a $^3S_1 - ^3D_1$ nucleon pair in the outgoing channel associated with an emerging pion in relative P -wave, often abbreviated as 1D_2P partial-wave channel). Therefore the proton pair emerging from the step-1 reaction, *i.e.* the $pp(I=0) \rightarrow pp\pi^-$ reaction, should be predominately just in this 1D_2 partial wave, in order to transport most part of the total step-1 reaction cross section to the step-2 part and form a structure with $I(J^P) = 0(3^+)$. However, the 1D_2 partial wave between the emerging protons is only marginal if existent at all, as has been demonstrated in a recent partial-wave analysis [22] of both the $pp \rightarrow pp\pi^0$ and the $np \rightarrow pp\pi^-$ reaction.

The finding of this partial-wave analysis [22] is in accord with the isoscalar proton-proton invariant-mass spectrum deduced from the WASA experiment [16,23], which is displayed in Fig. 3. As we can see there, the strength is concentrated just at lowest pp -masses. Approximately 2/3 of the strength is situated below 2.105 GeV, which is the threshold for the step-2 reaction, the $pp \rightarrow d\pi^+$ process. Hence only 1/3 of the total step-1 reaction cross section is kinematically available for the step-2 reaction. In addition we know from the partial-wave analyses results for isoscalar single-pion production [17,22] that there are practically only S - and P -waves between the proton pair emitted from the step-1 reaction. The $pn(^3D_3) \rightarrow pp(^1D_2)\pi$ partial wave contributes only with a few percent to the total cross section of the step-1 reaction, as may be seen in Fig. 4, where the results of the partial-wave analysis of Ref. [22] for this partial wave are indicated by the green horizontal bars. Taking these facts into account, the calculated two-step cross section for the $d\pi^+\pi^-$ channel drops by nearly two orders of magnitude – already on a qualitative level.

As has been demonstrated in a recent publication [23] the world pool of data [16,17,23,24, 26–30] for isoscalar single-pion production does not support the Breit-Wigner fits of Ref. [1] of having a peak at 2.33 - 2.34 GeV with a width of 70 - 80 MeV, but rather supports the result of Ref. [16] of having a peak at 2.31(1) GeV and a width of 150(10) MeV. Fig. 4 shows the energy dependence of the isoscalar single-pion production cross section based on data from WASA-at-COSY (solid dots) [16,23,24] and partial-wave analysis results from Ref. [17] (hatched band)

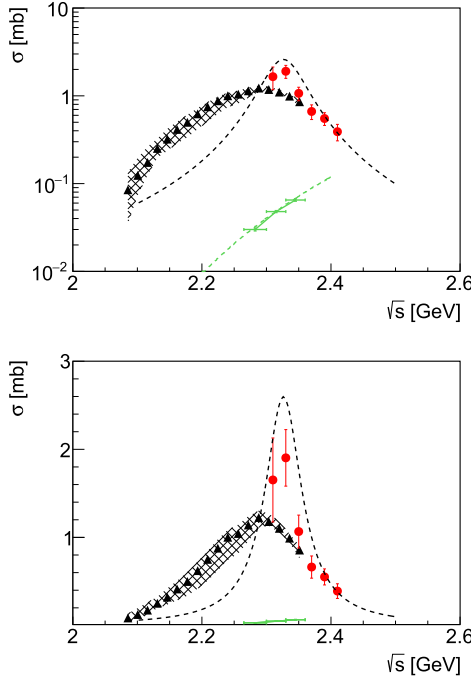


Fig. 4. Energy dependence of the isoscalar cross section for single-pion production in NN collisions in logarithmic (top) and linear (bottom) scale. Shown are the experimental results from WASA-at-COSY [16,23,24] (solid red dots) as well as the results of partial-wave analyses of Ref. [17] (hatched black band). The dotted curve shows the fit of Ref. [1], the green markers represent the partial-wave analysis results of Ref. [22] for the $np(^3D_3) \rightarrow pp(^1D_2)\pi$ contribution with a fit curve shown as a green dashed line.

together with the fit of Ref. [1] (dotted curve) and the partial-wave analysis results of Ref. [22] for the $np(^3D_3) \rightarrow pp(^1D_2)$ contribution (horizontal bars). We note in passing that the excursion of the WASA-at-COSY data point at $\sqrt{s} = 2.32$ GeV seen in Fig. 4 and which was focused on in Ref. [1] could, indeed, suggest a tiny narrow structure on top of the broad isoscalar Lorentzian. However, the neighboring data points are low – both those from WASA and from other experiments [27–29]. Hence the 3σ excursion at $\sqrt{s} = 2.32$ GeV appears to be of no particular significance as discussed in more detail in Ref. [23].

In conclusion we find that the two-step ansatz of Ref. [1] is far away from giving any explanation for the $d^*(2380)$ peak in double-pionic fusion. But we may ask what kind of prediction delivers the two-step ansatz, if we feed it with correct experimental information. To parametrize the $pp \rightarrow d\pi^+$ reaction the authors of Ref. [1] took very old data from Ref. [31] with large error bars. This reaction has been studied meanwhile in details [20], so the database contains more than thirty thousand points leading to extremely small uncertainties in the total cross-section. The SAID partial wave analysis [32] claims a 2% error [33] for an energy dependent solution. As we can see from Fig. 5 the fit of Ref. [1] has substantial deviations with regard to the latest cross-section parametrization. In order to perform a calculation of the sequential process cross-section, one should, however, not use the total cross-section, but only the cross-section of the 1D_2P partial wave. Though the 1D_2P partial wave is dominant in the $pp \rightarrow d\pi^+$ reaction, it is far from covering 100% of the total cross-section and also substantially different from the

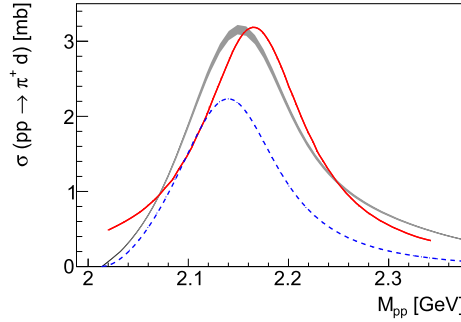


Fig. 5. Energy dependence the total $pp \rightarrow d\pi^+$ cross section plotted in dependence of the pp -invariant mass M_{pp} . Shown are the solution of the SAID partial-wave analysis (gray) [32], the parametrization of Ref. [1] (red) as well as the SAID solution for the single 1D_2 partial wave (blue dashed).

Ref. [1] parametrization. Ideally, to calculate the sequential process correctly, one would need to redo an integration with a proper parametrization. For simplicity we will just account for the difference in strength by

$$\frac{\int_{2015}^{2300} \sigma(pp \rightarrow d\pi^+)^{^1D_2}}{\int_{2015}^{2300} \sigma(pp \rightarrow d\pi^+)^{total}} = 0.62, \quad (1)$$

where the index 1D_2 refers to the 1D_2 partial wave in the initial pp channel.

From the partial-wave analysis of Ref. [22] we know the proper cross section $\sigma(pn(^3D_3) \rightarrow NN(^1D_2)\pi)$. So we can unfold the $\sigma(pn \rightarrow d\pi\pi)$ prediction of Ref. [1] from its unreasonably narrow Lorentzian for the isoscalar single-pion total cross-section $\sigma(NN\pi)_{I=0}^{narrow}$ and replace it by the correct cross-section of the proper partial wave, which can mimic the $d^*(2380)$ resonance. In a simplified manner this can be accomplished by

$$\sigma(d\pi\pi)^{cor} = \sigma(d\pi\pi)^{original} \cdot \frac{\sigma(pn(^3D_3) \rightarrow NN(^1D_2)\pi)}{\sigma(NN\pi)_{I=0}^{narrow}} \cdot 0.62. \quad (2)$$

The result of such unfolding is presented in Fig. 6 by the green dashed line. For the cross section $\sigma(pn(^3D_3) \rightarrow NN(^1D_2)\pi)$ we took a fit of the Ref. [22] data based on a phase-space distribution weighted with the fourth power of the beam momentum, $(P_{beam}^{CMS})^4$, in order to account for the D -wave dependence. In Fig. 4 the fit is shown by the green dashed line. For the range of interest $\sqrt{s} \in [2.25, 2.4]$ GeV the fit seems to be reasonable.

As can be seen in Fig. 6 the removal of the unreasonably narrow Lorentzian for the isoscalar single-pion production channel automatically removes the peaking from the $\sigma(d\pi\pi)$ cross-section. A smoothly rising $\sigma(pn(^3D_3) \rightarrow NN(^1D_2)\pi)$ dependence replicates itself in the $\sigma(d\pi\pi)$ cross-section. The overall contribution of such a process is in the order of 5% for the $d^*(2380)$ peak or smaller. However, one needs to be cautious here and consider this prediction as an upper limit, since it still does not account for the exact M_{pp} distribution for a $(pn(^3D_3) \rightarrow NN(^1D_2)\pi)$ partial wave. Also its cross-section cannot rise infinitely for higher energies and should be damped. As can be seen on Fig. 4, such effects should be sizable already at 2.4 GeV.

Whereas the two-step process cannot reproduce the $d^*(2380)$, a resonance in a 3D_3 partial wave, we may ask if it can give any meaningful contributions in the other partial waves. From the partial-wave analysis of Ref. [17] we know that the $^3S_1 - ^3D_1$ and 1P_1 partial waves in the initial

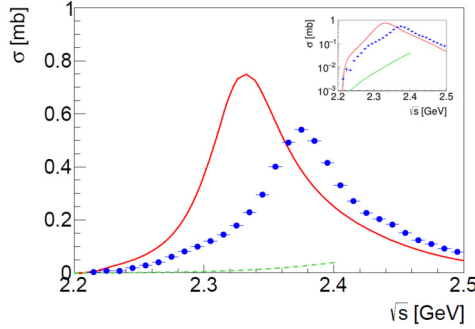


Fig. 6. The same as Fig. 2, but with the additional green dashed curve, which shows the corrected calculation, where the $pn(I=0) \rightarrow NN\pi$ cross-section of Ref. [1] was substituted by the PWA extracted $pn(^3D_3) \rightarrow NN(^1D_2)\pi$ cross-section of Ref. [22] and the $NN(^1D_2) \rightarrow d\pi^+$ cross section was properly scaled, see Fig. 5. The inset shows the same distribution in log scale.

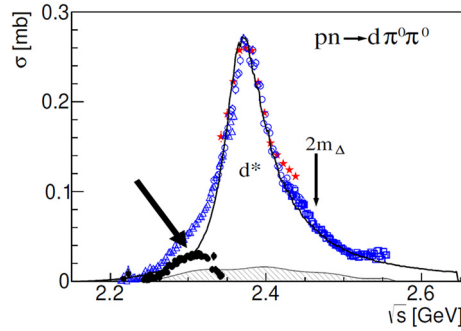


Fig. 7. Energy dependence of the total cross section for the $pn \rightarrow d\pi^0\pi^0$ reaction as measured by WASA-at-COSY. The blue open symbols represent the data of Ref. [9] normalized to the data (red stars) of Ref. [11]. The hatched area gives an estimate of systematic uncertainties. The solid curve displays a calculation of the d^* resonance with momentum-dependent widths [25]. It includes both Roper and $\Delta\Delta$ t -channel excitations as background reactions. The black filled dots show the difference between data and this calculation in the low-energy tail of d^* (2380). The thick arrow points to the resulting bump structure.

From Ref. [23].

pn system dominate the $pn(I=0) \rightarrow pp\pi^-$ reaction with the consequence of having only S - and P -waves between the emerging proton pair. From the partial-wave analyses of Refs. [20,21] we learn that initial S - and P -waves contribute to only about 10% to the total $pp \rightarrow d\pi^+$ cross section. Putting these facts together with the Breit-Wigner fit result of $\sigma_{peak} = 1.4$ mb (instead of 2.6 mb), $m = 2.31(1)$ GeV (instead of 2.33 - 2.34 GeV) and width $\Gamma = 150(10)$ MeV (instead of 70 - 80 MeV), we obtain with the two-step ansatz again a prediction for a resonance-like structure, but now around 2.32 GeV with a width of about 150 MeV and a peak cross section of about 0.06 mb. If we in addition account for the fact that only 1/3 of the strength in the M_{pp} spectrum is above the threshold for the step-2 reaction, then the peak cross section from the two-step ansatz decreases further to about 0.02 mb.

Indeed, we may associate this prediction with a possible peak reported [23] recently for the $d\pi^0\pi^0$ channel, which by isospin relation has a factor two smaller cross section than the isoscalar part of the $d\pi^+\pi^-$ channel. In Fig. 7, which has been taken from Ref. [23], the experimental

total cross section is displayed for the $np \rightarrow d\pi^0\pi^0$ reaction. If a Breit-Wigner ansatz with a momentum-dependent width [25] is used for the description of the $d^*(2380)$ resonance, then the data on the low-energy side of the $d^*(2380)$ resonance are underpredicted. The difference between data and resonance description yields a small bump around 2.32 GeV with a width of about 150 MeV and a peak cross section of about 0.03 mb, *i.e.* an order of magnitude smaller than the neighboring peak cross section of $d^*(2380)$. Amazingly, these features fit very well to the prediction of the two-step ansatz, if fed with the proper cross section data.

In Ref. [23] the small bump was interpreted as consequence of possible dibaryon resonances with $I(J^P) = 0(1^+)$ and $0(1^-)$, which produce the bump in isoscalar single-pion production due to the $^3S_1 - ^3D_1$ and 1P_1 partial waves between the incident pn pair. From the fact that the isoscalar proton-pion invariant-mass distribution exhibits strength only in the region of the Roper excitation (see Fig. 6 in Ref. [16]) it was concluded that these dibaryon resonances must have a N^*N structure. And since the Roper resonance decays by single- and by two-pion emission, this structure must be present both in single- and double-pion production. In Ref. [23] it was shown that the known branchings of the Roper decay fit to the relative size of the bumps observed in isoscalar single- and double-pion production.

Actually, it is not surprising that the two-step ansatz of Ref. [1] fits to the interpretation of Ref. [23]. By describing the isoscalar single-pion production with a Breit-Wigner form the authors of Ref. [1] implicitly assume a resonance in the isoscalar pn system – without stating that explicitly. Thus the two-step ansatz simulates in essence at least part of the dibaryon interpretation of Ref. [23] for the small bump in the $d\pi^0\pi^0$ channel, if one keeps in mind that the upper leg of the two-step graph after the first interaction blob can be reinterpreted as the sequential Roper decay $N^* \rightarrow \pi \Delta \rightarrow \pi\pi N$.

Finally we would like to comment on the situation of $d^*(2380)$ in photon absorption on the deuteron, where according to Ref. [1] a sequential process should not occur and hence would be decisive for the interpretation of the prominent peak in the $pn \rightarrow d\pi^0\pi^0$ reaction. According to the theoretical prediction of Ref. [34] the branching $d^*(2380) \rightarrow d\gamma$ is only in the order of 10^{-5} and hence a detection of a signal in γd -induced reactions is very difficult. It is true that the current experimental hints [35–37] for $d^*(2380)$ in the $\gamma d \rightarrow d\pi^0\pi^0$ reaction are not yet conclusive due to smallness of total cross section and large backgrounds, though the differential $M_{\pi^0\pi^0}$ spectrum in Ref. [37] is in favor of a $d^*(2380)$ contribution. More conclusive results are expected to come from MAMI, where dedicated measurements of $\gamma d \rightarrow d\pi^0\pi^0$ will be performed with active deuteron target and deuterium-TPC setups. In the situation, where background due to conventional reaction processes is large, measurements of polarization observables are known to be very helpful. In pn elastic scattering, where the contribution of $d^*(2380)$ to the total cross section is only marginal, it was demonstrated that polarization measurements reveal a pronounced effect of $d^*(2380)$ in the analyzing power, whereas its effect in the unpolarized differential cross section is small and quite unspecific [5–7]. Similarly, evidence of $d^*(2380)$ photoexcitation is found in polarization observables of deuteron photodisintegration $\gamma d \rightarrow pn$ [38–40], however, more polarization measurements (*e.g.* T, E, F, G observables) and a partial wave analysis would be highly desirable to settle the question of the $d^*(2380)$ photoproduction.

Summary

In conclusion the two-step ansatz of Ref. [1] has been shown to be not able to provide an alternative interpretation of $d^*(2380)$, if fed with proper experimental data for the step-1 and step-2 cross sections and if the constraints for proper partial waves are taken into account. However, we

have demonstrated that the two-step ansatz may give reasonable results for a possible small, but broad structure below the $d^*(2380)$ peak, which was recently interpreted as a consequence of a possible dibaryonic excitation in isoscalar single-pion production.

CRediT authorship contribution statement

We certify that both authors have contributed to all the different issues of the manuscript.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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