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Hierarchical model verification and validation for structural health monitoring using dynamic substructuring *

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Abstract. Despite the success of data-based methods in structural health monitoring (SHM), these approaches often suffer from a lack of training data, which can be difficult to acquire for several reasons: damage-state data acquisition can be infeasible, structures may be unique and only tested *in situ*, sensor placement can cause issues, certain structures cannot be tested in controllable laboratory conditions and representative environmental conditions can be difficult to simulate. Training data can be simulated using physics-based models. However, this is dependent on model verification and validation (V&V), meaning assembly-level data is still required.
Hierarchical V&V is a novel technique in the field of SHM. The aim of hierarchical V&V is to remove the necessity for assembly-level validation data. Instead, the process entails the V&V of subassembly-level models, which are then combined to produce an assembly-level model using dynamic substructuring (DS). This simplifies the data acquisition process in order to reduce the associated difficulties and costs.
This paper focuses on the role of DS in the hierarchical V&V process for SHM. DS allows substructures to be used to create an assembly model, and for simultaneous uncertainty propagation. This allows confidence to be established in the assembled models without requiring assembly-level data.

Keywords: V&V · Hierarchical model · Dynamic substructuring

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1 Introduction

The goal of structural health monitoring (SHM) is to infer health-state information in some way, ranging from detection to prognosis [15]. In order to validate a model for this purpose, damage-state data must be acquired for comparison with damage-state predictions. This process can often lead to a series of difficulties in an SHM context, summarised below:

1. Target structures of high value represent a significant expenditure when attempting to carry out invasive or damaging data acquisition processes
2. If the target structure is unique, usage requirements and other factors may restrict the data acquisition process
3. The target structure may be difficult to scale or transport in such a way as to allow well-designed, controllable laboratory tests
4. The design or operating environment of the target structure may make sensor placement difficult
5. The operating condition of the structure may be difficult to replicate in order to acquire representative validation data

Notwithstanding some successes [11] [16] [18], due to the issues outlined above, model-driven SHM has been significantly handicapped in its applicability in industry. This motivates further research into the advancement of verification and validation (V&V) techniques to mitigate the current difficulties.

Verification refers to the efforts to ensure that the model is accurate in its attempts to estimate a given solution, and therefore considers factors such as discretisation errors and errors of numerical model design [14]. Validation refers to the efforts to ensure that the model is an accurate estimation of reality and that the solutions it attempts to derive are representative of real-life observations; it therefore considers model discrepancies or biases and random errors [14]. Uncertainty can be separated intoaleatoric and epistemic uncertainty. Aleatoric uncertainty, also known as irreducible uncertainty, pertains to uncertainties inherent to a problem that cannot be reduced with additional knowledge. Aleatoric uncertainty is generally unbiased and random, and can usually be captured and estimated in the V&V process. Epistemic uncertainty, also known as systematic uncertainty, pertains to uncertainties due to actual lack of knowledge, for example in the case of model simplification leading to certain physics being neglected from the problem. These issues can lead to biased uncertainties; however, these cannot be accounted for solely by model discrepancy, as numerical error can also contribute significantly to overall bias [17]. Error refers to the difference between an estimation and the true solution, and is unavoidable when any level of uncertainty is present. Random errors manifest in a scattering of predictions around a mean value that can be described by some statistical model. Systematic errors, which are also referred to as model discrepancy or model bias, cause a repeated offset between the predicted and the true value.

Hierarchical V&V offers the potential for a model to be validated without the need for assembly-level data. This is achieved by using subassembly data to validate a series of submodels separately and then constructing an assembly-level model from the validated submodels. The uncertainty can be quantified at the assembly level by propagating the uncertainty from the subassembly levels upwards, thereby establishing quantifiable confidence in the predictions of the assembly-level model. Potential benefits of hierarchical V&V are as follows:

1. The method avoids the need to acquire damage-state data from assemblies that represent high capital investment
2. Testing can make use of repeated sub-assemblies or components, particularly in the case of modularity or symmetry of components
3. The testing of simpler structures could increase ease of sensor placement and control of experimental conditions

For successful implementation of hierarchical V&V, a robust method must be identified for the assembly of a validated set of subassembly models. The method pursued in this study is dynamic substructuring (DS), which enables the assembly or disassembly of physics-based models used for dynamic analyses [1].

2 Structural health monitoring: An overview

Structural health monitoring aims to assess the damage state of a target structure using live data taken from the structure over a period of time [7]. This assessment is used to inform decisions concerning the safety and operating condition of the structure, and is therefore key to making efficient, risk-managed use of structures. Damage detection has existed in some form throughout history [7], but has only begun to be formalised and modernised through academic research in recent decades [5] [3]. Before the advent of SHM, damage detection was generally restricted to *offline* (in which discrete sets of measurements are taken on some pre-defined basis) methods of non-destructive testing (NDT) such as visual inspection [7]. However, recent academic research has focused more on the use of *online* methods, in which continuous recording of data enables damage tracking in real-time. Online methods enable *condition-based* health monitoring, which offers significant cost-saving potential by reducing down-time during inspection procedures and man-hours on unnecessary callouts [7] [3].

Research in the field of *data-driven* SHM has made significant progress in recent years thanks to the development of sophisticated machine learning algorithms coupled with increased computational power. These methods do not depend on physical knowledge of the target structure, but instead monitor data taken from the structure itself in order to infer damage [2]. Damage detection can be carried out as a novelty detection exercise, and requires training data from the structure in its normal operating condition only; this is an *unsupervised* learning problem [7]. Attempts to localise or classify damage in a structure are more complex, and require training data from the structure in its damage states; this is a *supervised* learning problem [7]. The functionality of data-driven methods for SHM is limited by the data available, and they can generally not be used for prognostic purposes without the addition of physical knowledge [6]. The dependence on rich data sets is a significant limiting factor in the implementation of data-driven SHM. Physics-based models can be used to augment their application; this could be to simulate training data, to constrain the learning process, or to perform further SHM tasks separately [2].

A key method for online SHM is *model-driven* SHM [2]. These methods make use of a physics-based model of a target structure to infer the health state of the structure using model predictions. They have been enabled in the last half-century by modern computing and the finite-element (FE) technique for modelling of complex systems [9]. The most common method for model-driven SHM is model updating SHM, referred to in this paper as inverse model-driven (IMD) SHM. In this method, a physics-based model of a target structure is constructed with a discrete set of inputs, generally representing certain physical properties of the structure, which will make predictions that are comparable with live data obtained from the structure [9] [8]. As the target structure changes in health condition, the monitored data changes, and the model inputs are adjusted to match the model predictions to the data; alternatively, the updating process can be applied directly to the characteristic matrices of the model [2] [8]. This updating process is then used to infer the health state of the structure. IMD-SHM requires a well-validated model which, in addition, must be suitable to problems that are constrained in such a way that the updating procedure is computationally feasible and is able to offer unique solutions for a range of damage scenarios.

A novel method for model use in this context is forward model-driven (FMD) SHM [2] [10]. This is a 'hybrid' approach in that it makes use of both data-driven and model-based SHM techniques: a physics-based model is used to generate training data for a machine learning-based damage identification system [2]. The method offers greater feasibility in an SHM context than IMD methods in that it avoids the difficulties present in developing a stable updating problem, with predictions being used in the forward direction only. This also has the added benefit of concentrating computational resource at the development stage: once the data-driven system is trained, no further computation is required beyond the running of the statistical model. Compared to data-driven SHM, FMD-SHM is less reliant on data availability, and the use of physics-based predictions allows for more advanced SHM tasks such as damage prognosis to be carried out.

3 Dynamic substructuring: Theory and background

It is a generally accepted paradigm that to solve a complex, multi-level engineering problem, the problem domain can be broken down into more basic sub-domains [1]. As such, assembly-level models can be approached through substructuring, allowing for the simplification of many problems and more efficient allocation of computational resource. Dynamic substructuring refers to a group of methods designed for the assembly or disassembly of predictive models of dynamic systems [1]. The foremost resource in the field is the work by Allen et al. [1], which covers the available techniques for DS as well as detailing the circumstances for their implementation. The Craig–Bampton method for DS [4], which is a widely used method for assembly of substructures, is demonstrated in [13]. Voormeeren et al. presented DS as a technique for aiding the analysis of offshore wind turbines, which allowed for the capture of local dynamic effects and their contribution to the global system [21]. The step to apply UQ to frequency-based substructuring (FBS) showed that the DS methodology was receptive to uncertainty integration, underlining its credentials in an SHM context [19]. DS has also been applied to problems in the automotive industry [20] [12]. The former work focuses on use of experimental FBS in analysis of the gear train of BMW cars. The latter concerned uncertainty propagation within experimental FBS for system identification.

In DS, conditions need to be defined which describe the interface behaviour at the joints between the substructures. The two key defining conditions are degree of freedom (DOF) compatibility and interface force equilibrium. The simplest compatibility constraint is for the responses to be equal at the interface (as in the case of rigid joints); however, this can be loosened in order to improve accuracy in modelling of joints. The compatibility condition is enforced by defining a matrix, B. B is a signed Boolean matrix; in the case of rigid connections, it is defined such that its product with x (the response vector of the substructures) is zero. The dimensions of B are the number of interface connections in the assembly by the number of unassembled DOFs. The interface forces are constrained by the matrix L (the localisation matrix), which is defined (in the case of rigid joints) such that the product of its transpose with g, the vector of interface forces, is equal to zero. L is an unsigned Boolean matrix whose dimensions are the number of unassembled DOFs by the number of DOFs in the assembly. L will also map the global vector of assembled responses, x_{global}, to x (Eqn. 1). It can therefore be used to remove redundant information from the assembled equation of motion, and it can also be shown that the product of B and L is the null space.

$$x = Lx_{global} \tag{1}$$

A substructuring problem under the rigid-joint assumption can be represented using the three-field formulation, which couples the assembled equation of motion with the constraint conditions (see Eqn. 2). The mass (M), damping (C) and stiffness (K) matrices are assembled from their substructure constituents in a block-diagonal form, while the force (f and g) and response (x) vectors are concatenated vertically.

$$\begin{cases} M\ddot{x} + C\dot{x} + Kx = f + g \\ Bx = 0 \\ L^T g = 0 \end{cases} \tag{2}$$

There are two processes for DS: primal and dual assembly; these are equivalent to each other mathematically, but each lends itself to different techniques and situations [1]. In primal assembly, Eqn. 1 is substituted into Eqn. 2, eliminating redundant response entries in the equation of motion. Following this, the equation is premultiplied by L^T to eliminate the vector g. This yields Eqn. 3. This process is similar to the assembly of submodels in finite element modelling.

$$\tilde{M}\ddot{x}_{global} + \tilde{C}\dot{x}_{global} + \tilde{K}x_{global} = L^T f \tag{3}$$

In dual assembly, also known as the Lagrange multiplier method [1] [20] [12], a new vector, λ, is defined containing the magnitudes of g according to Eqn. 4. When this is applied to the three-field formulation, it can be reduced to Eqn. 5 as the interface force equilibrium is satisfied by definition.

$$g = -B^T \lambda \tag{4}$$

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \ddot{x} \\ \lambda \end{pmatrix} + \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{x} \\ \lambda \end{pmatrix} + \begin{bmatrix} K & B^T \\ B & 0 \end{bmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix} \tag{5}$$

Frequency-based substructuring (FBS) refers to the use of DS methods in the frequency domain, and is commonly used in experimental substructuring processes [19] [20] [12]. The modal domain can also be used for DS methods. This allows modellers to reduce the number of modes used in assembly as the higher order modes contribute less energy to the full solution. Therefore, an accurate solution can be estimated with significant potential reductions in computational load. For further details on DS methods, the reader is referred to [1].

4 Case study: Assembly of a plate by dynamic substructuring

This case study investigates the assembly of a plate constrained at opposite ends from two substructures representing each half of the plate in the presence of damage. The study uses primal assembly in the physical domain. The propagation of uncertainty of the parameters of the substructures, as well as model discrepancy at the assembly level are taken into account.

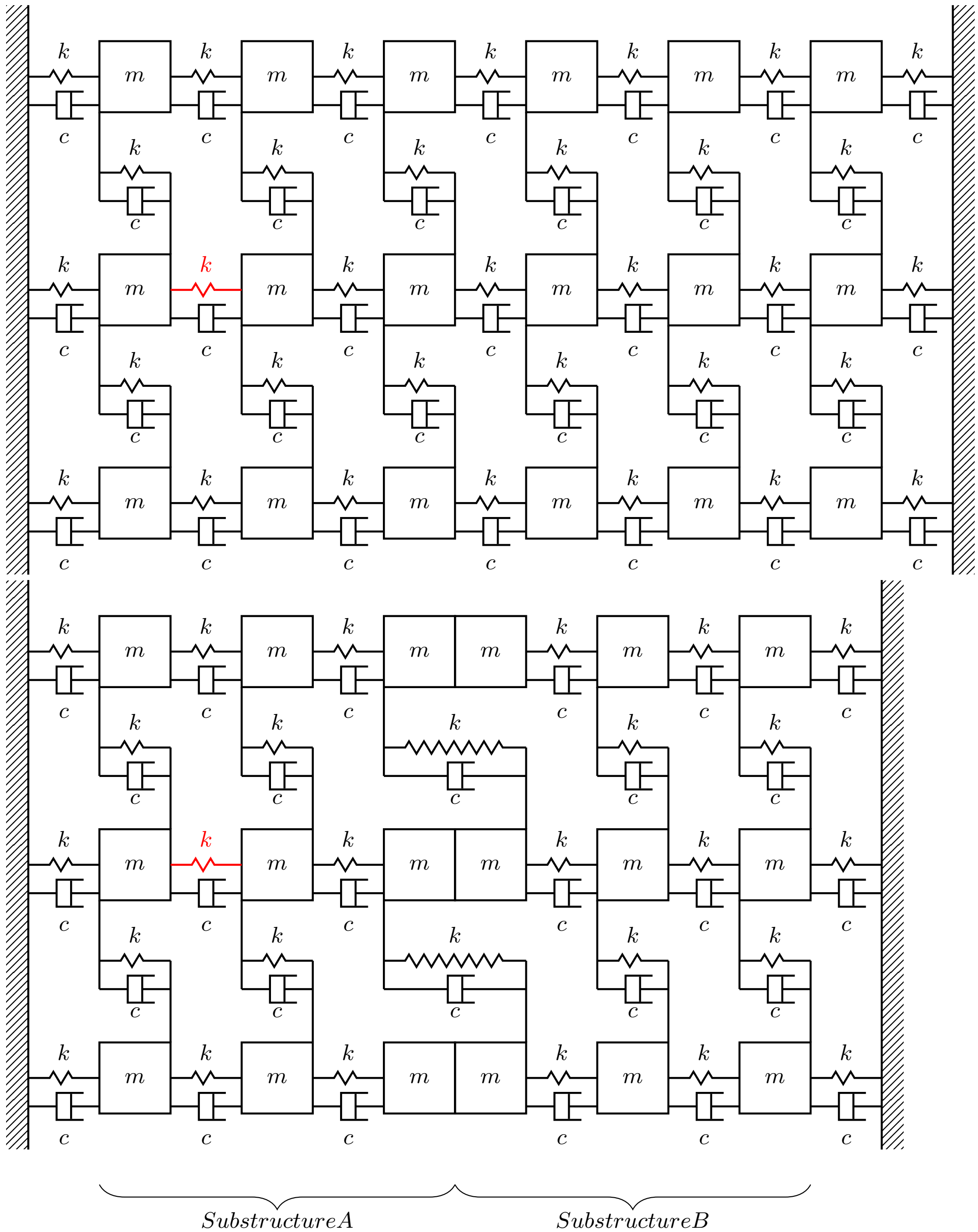

Fig. 1. The true structure (above); the assembled substructures A and B (below); damage location highlighted in red

The plates were modelled as mass-spring-damper structures with nine DOFs, with m set to 1 kg, k set to 10000 N/m and c set to 10 Ns/m. These parameters were sampled from normal distributions with 1% standard deviation using Latin Hypercube sampling to create 100 substructure samples. This was intended to reflect a scenario in which V&V had been carried out to construct posterior distributions for the key parameters of the submodels. The 'true' assembly was modelled using the mean values for each parameter; this was used to evaluate the performance of the assembled substructure model (Figure 1). The DS assembly was carried out by using the assumption of rigid joints, which made definition of the B matrix trivial – the L matrix could then be derived numerically. This discrepancy to the benchmark model was intended to reflect that in a more realistic scenario some model bias would be unavoidable. Damage was simulated in the model by reducing the stiffness value of the spring connecting masses four and five in Substructure A; the associated damper was reduced accordingly. The stiffness was reduced in increments of 20% from the undamaged condition to full damage.

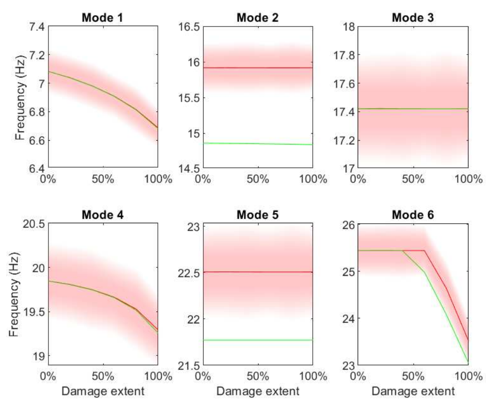

Fig. 2. The effect of damage extent on the first six modal frequencies; true solution in green, DS solution in red (with $\pm 3\sigma$ indicated for the DS solution)

The results of the DS approach are compared with the true solution in Figure 2, which shows how the first six modal frequencies develop with increasing damage in the structure. For both the DS and the benchmark solutions, modes 1, 4 and 6 were damage-sensitive, which indicates that a damage detector could be trained using these modal frequencies. However, mode 6 only shows sensitivity to damage at a certain damage level. Further investigations showed that the modal behaviour changed at this damage level, which was observable in a change in the mode shapes. Modes 2, 3 and 5 were not damage-sensitive, indicating that the damaged spring was not under stress for their associated mode shapes. The model discrepancy is noticeable for modes 2 and 5, showing that the reduced fidelity in the DS process did not capture a particular element of the true solution. This set of results shows that DS interfaces easily with SHM in that it allows for the development of a probabilistic model that can be used either in an updating procedure, or to generate training data for statistical model.

5 Conclusions

This paper presented a novel method for the V&V of physics-based models for SHM: hierarchical V&V. In this method, the V&V of a series of substructures of a given assembly-level model is carried out. The substructures are then combined to allow predictions to be made at the assembly level, and their associated uncertainties are likewise propagated upwards. Hierarchical V&V was demonstrated via a case study of a plate in this paper using DS in conjunction with a Latin-Hypercube sampling scheme. The results indicate that DS is a promising tool for hierarchical V&V, and could provide the means for an assembly-level model to be created and validated from its substructures. This would allow for the development of physics-based SHM strategies that do not require assembly-level validation data, which have historically proven to be very difficult to acquire.

The next steps would entail the investigation of how appropriate features and validation metrics can be propagated or compared at multiple levels and how to reduce the computational load that will be required for more complex models, as well as demonstrations of the methods using real-life datasets. In addition, efforts to incorporate model discrepancy functions into the UQ and UP processes are critical to ensuring the robustness of the hierarchical V&V process.

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