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Gong, C., Siraj, S., Yu, L. et al. (2024) A generalized form of the distance-induced OWA operators – Demonstrating its use for evaluation indicator system in China. Expert Systems with Applications, 247. 123257. ISSN: 0957-4174

https://doi.org/10.1016/j.eswa.2024.123257

A generalized form of the distance-induced ordered weighted averaging operator – Demonstrating its use for the evaluation of science and technology development level in Chinese provinces

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Abstract

Using multi-criteria decision making (MCDM) techniques to rank alternatives is a well-known area of study in which aggregation operators, such as ordered weighted averaging (OWA), play an important role in merging information and producing an overall ranking. The distance measures from ideal argument values in aggregation operators have gained attention in recent literature. Distance measures are traditionally used as argument variables, which leads to the depiction that the attributes cannot be aggregated directly. In this paper, a generalized form of distance-induced OWA (DIOWA) operators is proposed with distance measures used as order-inducing variables. A distinctive benefit of DIOWA operators is that they permit us to consider ideal argument values while simultaneously also taking the attribute values as argument variables. Three variants of DIOWA operators are proposed and investigated, namely a) the Hamming-distance-induced OWA operator, b) the normalized Hamming-distance-induced OWA operator, and c) the weighted Hamming-distance-induced OWA operator. We highlight their important properties and provide proofs to necessary theorems, and also suggest the determination methods for calculating their associated weights. We discuss further extensions of the proposed DIOWA operators with the help of generalized and quasi-arithmetic means. We discuss the use of our proposed family of operators for two different decision making situations, and demonstrate their validity by an illustrative numerical example. Finally, we apply the proposed operators to a real-life problem of ranking Chinese provinces for their science and technology (S&T) development levels. The proposed operators are shown to be a useful addition to the aggregation toolbox for decision analysts.

Keywords: distance measures, ideal argument values, MCDM, DIOWA operators, science and technology development level.

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1. Introduction

Multi-criteria decision making (MCDM) techniques provide various ways to evaluate multiple alternatives concerning multiple criteria to help a decision maker (DM) to gain insights into the problem at hand (Boukezzoula and Coquin, 2020; Ishizaka and Siraj, 2018; Yager and Alajlan, 2019; Rao et al., 2022; Aggarwal, 2019). However, how all these individual evaluations can be aggregated to produce a final recommendation is a much-debated issue within the MCDM community. A number of methods have been proposed in the literature to address this issue (Xu and Wang, 2020; Kumar and Chen, 2022; Petry and Yager, 2022; Zeng et al., 2018; Gadomer and Sosnowski, 2019). Among these methods, the use of the ordered weighted averaging (OWA) operator (Yager, 1988) has been widely discussed and applied in this domain. The OWA operator provides a parameterized family of aggregation operators, including the average, maximum, minimum, and many other special cases. As the name implies, the weights in the OWA operator are associated with the ordered positions of aggregated values and not the aggregated values themselves. An influential generalized variant of the OWA operator is termed the induced OWA (IOWA) operator (Yager and Filev, 1999). In this variant, the ordered positions of the attribute values to be aggregated are assigned by using some other order-inducing variables. Considering the different combinations of these parameters and contextual situations, several variants for the OWA operator have been proposed in this field. Some of those that have been widely discussed in the literature are the generalized OWA (GOWA), induced GOWA (IGOWA), probabilistic OWA (POWA), induced POWA (IPOWA), heavy OWA (HOWA), induced HOWA (IHOWA), uncertain OWA (UOWA), and uncertain IOWA (UIOWA) operators (Wei and Tang, 2012; Merigo and Gil-Lafuente, 2009; Merigo, 2012; Yager, 2002; Xu and Da, 2002; Merigo, 2015; Xu, 2006; Merigo and Casanovas, 2011d). In particular, how the order-inducing variables are derived is an important problem when we use the IOWA operator and some of its extensions to aggregate argument variables. Lots of corresponding research articles have discussed this issue and further extended the IOWA operator, which is beneficial to researchers and DMs (Chiclana et al., 2004, 2007; Beliakov and James, 2011).

When making decisions, DMs assess the similarities and differences in the attributes of decision alternatives and, for that, some sorts of distance metric are required to quantify these similarities and differences. Some of the most frequently used distance measures include Hamming distance, Euclidean distance, Chebyshev distance, and Minkowski distance (Hamming, 1950; Xu and Xia, 2011; Singh, 2017; Balopoulos et al.,

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2007; Liao and Xu, 2015; Hussian and Yang, 2019). Combining these distance measures with the OWA operators has attracted much attention in recent years. Merigo and Gil-Lafuente (2010) proposed the OWA distance (OWAD) operator, in which the distance measures are regarded as aggregated variables in the OWA operator. By contrast, the IOWA variant is called the induced OWAD (IOWAD) operator (Merigo and Casanovas, 2011b). Both these operators provide a parameterized family of distance aggregation operators that range from the minimum to the maximum distance.

The rest of this paper is organized as follows. The related literature is analyzed in Section 2. The basics and related work are reviewed in Section 3. In Section 4, we first introduce a generalized form of the DIOWA operators with the Hamming distance and then propose three basic types of the DIOWA operators with different Hamming distances. For those DIOWA operators, we also analyze their main properties, three determination methods of associated weights, and their families. The DIOWA operators are also generalized by using generalized and quasi-arithmetic means. In Section 5, we study MCDM frameworks with the proposed DIOWA operators by considering two situations, and an illustrative example is given to illustrate the application of the DIOWA operators and compare them with other operators. In Section 6, the WHDIOWA operator is applied to evaluate the science and technology (S&T) development levels of 31 provinces in China. Section 7 concludes this work along with some suggestions for possible future work.

2. Literature review

2.1. Distance aggregation operators

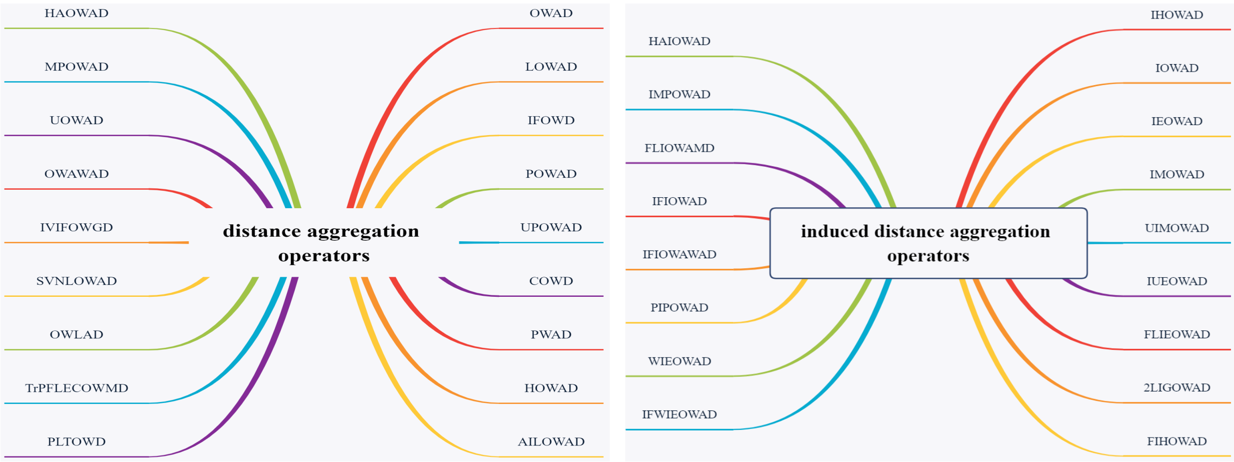
Inspired by the OWAD and IOWAD operators, several distance operators have been proposed in the literature. A summary of 35 of these operators is provided in Table 1, ordered according to their publication date, which shows that combining distance measures in aggregation operators have been studied since 2010.

We divide these 35 operators into two categories of a) distance aggregation operators and b) induced distance aggregation operators. This is shown in Fig. 1. The main feature of all distance aggregation operators (shown in Fig. 1a) is that the reordering of the argument variables is determined by the argument variables themselves. By contrast, the main feature of induced distance aggregation operators (shown in Fig. 1b) is that the reordering of argument variables is determined by other variables.

The feature of the operators, such as the OWAD, IPOWAD, and UOWAD operators, in Fig. 1a, is that the weight of each criterion is determined by its corresponding distance measure value. Therefore, if the weights corresponding to these operators are fixed, the aggregation results by using these operators are also fixed. However, the feature of the operators, such as the IOWAD, HIOWAD, and WIEOWAD operators, in

Table 1: Basic information of 35 distance aggregation operators

No.	Operator	Author and Publication Time	Journal
1	OWA distance (OWAD) operator	Merigo and Gil-Lafuente (2010)	Inform Sciences
2	linguistic OWAD (LOWAD) operator	Merigo and Casanovas (2010a)	Int J Fuzzy Syst
3	induced heavy OWAD (IHOWAD) operator	Merigo and Casanovas (2010b)	J Syst Eng Electron
4	induced OWAD (IOWAD) operator	Merigo and Casanovas (2011b)	Comput Ind Eng
5	intuitionistic fuzzy ordered weighted distance (IFOWD) operator	Zeng and Su (2011)	Knowl-Based Syst
6	induced Euclidean OWAD (IEOWAD) operator	Merigo and Casanovas (2011a)	Expert Syst Appl
7	induced Minkowski OWAD (IMOWAD) operator	Merigo and Casanovas (2011c)	Int J Comput Int Sys
8	probabilistic OWAD (POWAD) operator	Merigo (2013)	Knowl-Based Syst
9	uncertain OWAD (UOWAD) operator	Zeng (2013)	Appl Math Model
10	uncertain induced Minkowski OWAD (UIMOWAD) operator	Zeng et al. (2013)	Kybernetes
11	induced uncertain Euclidean OWAD (IUEOWAD) operator	Su et al. (2013)	Technol Econ Dev Eco
12	continuous ordered weighted distance (COWD) operator	Zhou et al. (2013)	Group Decis Negot
13	probabilistic weighted averaging distance (PWAD) operator	Merigo et al. (2013)	Kybernetes
14	heavy OWAD (HOWAD) operator	Merigo et al. (2014)	Appl Math Model
15	Atanassov's intuitionistic LOWAD (AILOWAD) operator	Su et al. (2014)	Int J Fuzzy Syst
16	Fuzzy linguistic IEOWAD (FLIEOWAD) operator	Xian and Sun (2014)	Int J Intell Syst
17	2-tuple linguistic induced generalized OWAD (2LIGOWAD) operator	Li et al. (2014)	J Comput Syst Sci
18	fuzzy IHOWAD (FIHOWAD) operator	Zeng et al. (2014)	Int J Fuzzy Syst
19	Hungarian algorithm with the OWAD (HAOWAD) operator	Vizuete-Luciano et al. (2015)	Technol Econ Dev Eco
20	Hungarian algorithm with the IOWAD (HAIOWAD) operator	Vizuete-Luciano et al. (2015)	Technol Econ Dev Eco
21	Minkowski POWAD (MPOWAD) operator	Casanovas et al. (2016)	Cybernet Syst
22	induced POWAD (IPOWAD) operator	Casanovas et al. (2016)	Cybernet Syst
23	uncertain OWAD (UOWAD) operator	Zeng (2016)	Cybernet Syst
24	fuzzy linguistic IOWA Minkowski distance (FLIOWAMD) operator	Xian et al. (2016)	Pattern Anal Appl
25	OWA weighted averaging distance (OWAWAD) operator	Merigo et al. (2017)	Appl Soft Comput
26	intuitionistic fuzzy IOWAD (IFIOWAD) operator	Zeng et al. (2017)	Int J Fuzzy Syst
27	intuitionistic fuzzy induced OWAWAD (IFIOWAWAD) operator	Zeng et al. (2017)	Int J Fuzzy Syst
28	interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) operator	Liu and Peng (2017)	Informatica
29	single-valued neutrosophic LOWAD (SVN-LOWAD) operator	Chen et al. (2018)	Int J Env Res Pub He
30	prioritized induced POWAD (PIPOWAD) operator	Aviles-Ochoa et al. (2018)	Int J Fuzzy Syst
31	ordered weighted logarithmic averaging distance (OWLAD) operator	Alfaro-Garcia et al. (2018)	Int J Intell Syst
32	weighted IEOWAD (WIEOWAD) operator	Wang et al. (2019)	Transform Bus Econ
33	intuitionistic fuzzy WIEOWAD (IFWIEOWAD) operator	Wang et al. (2019)	Transform Bus Econ
34	trapezoidal Pythagorean fuzzy linguistic entropic combined ordered weighted Minkowski distance (TrPFLECOWMD) operator	Xian et al. (2019)	Int J Intell Syst
35	probabilistic linguistic term ordered weighted distance (PLTOWD) operator	Liu et al. (2022)	J Control Decis



(a) Distance Aggregation Operators

(b) Induced Distance Aggregation Operators

Figure 1: Two classification methods of distance aggregation operators

Fig. 1b, is that the weight for each criterion is derived by the associated order-inducing variable. This means that experts or DMs can determine different order-inducing variables by considering different preferences or factors. Therefore, the aggregating results by using these operators will be changed based on the order-inducing variables determined. In other words, if we hope that the decision making is objective enough, the operators in Fig. 1a are more suitable. If we hope that more information, including objective information and subjective information, is considered in the decision making, the operators in Fig. 1b are more suitable.

We also divide these aggregation operators based on the crispness of the information they use. The two types are given in Table 2 and Table 3, where the operators are used to aggregate crisp information, and in Table 3, where they are used to aggregate uncertain information.

Operators such as IMOWAD, HOWAD, and WIEOWAD were developed to aggregate crisp numbers (see Table 2). The IMOWAD operator is the generalized version of the IEOWAD and IOWAD operators because the Euclidean distance and the Hamming distance can be derived from the Minkowski distance. These operators are developed by considering different distance measure methods, decision making information, aggregation operators, and algorithms in the OWAD and IOWAD operators. The POWAD operator is obtained by further considering probabilistic information in the OWAD operator. Therefore, these operators are suitable for different decision situations. With further effort, operators such as LOWAD, IUEOWAD, and IFIOWAD, in Table 3, further consider different fuzzy numbers, such as interval numbers, linguistic fuzzy numbers, and intuitionistic fuzzy numbers, in the OWAD and IOWAD operators, to address uncertain decision making problems. For example, the LOWAD operator is obtained by introducing linguistic information in the OWAD operator, whereas the UPOWAD is obtained by considering interval numbers and

Table 2: Basic information of 35 distance aggregation operators

Operator	Basic Information
OWAD	Hamming distance, distance measures, OWA
IHOWAD	Hamming distance, distance measures, IHOWA
IOWAD	Hamming distance, distance measures, IOWA
IEOWAD	Euclidean distance, IOWA
IMOWAD	Minkowski distance, IOWA
POWAD	Hamming distance, probability, OWA
COWAD	distance measures, interval distance, continuous, OWA
PWAD	Hamming distance, probability, WA
HOWAD	Hamming distance, HOWA
HAOWAD	Hungarian algorithm, OWAD
HAIOWAD	Hungarian algorithm, OWAD
MPOWAD	Minkowski distance, probability, OWA
IMPOWAD	Minkowski distance, probability, OWA
OWAWAD	Hamming distance, WA operator, OWA
PIPOWAD	Hamming distance, prioritized operator, probability, IOWA
OWLAD	Hamming distance, logarithm, OWA
WIEOWAD	weighted Euclidean distance, IOWA

Table 3: Distance aggregation operators for uncertain information

Operator	Basic Information
LOWAD	linguistic information, OWAD
IFOWD	intuitionistic information, distance measures, OW
UPOWAD	interval numbers, Hamming distance, probability, OWA
UIMOWAD	interval numbers, Minkowski distance, IOWA
UIMOWAD	interval numbers, Euclidean distance, IOWA
AILOWAD	Atanassov's intuitionistic linguistic information, Hamming distance, OWA
FLIEOWAD	Atanassov's intuitionistic linguistic information, Hamming distance, OWA
2LIGOWAD	2-tuple linguistic information, distance measures, generalized means, IOWA
FIHOWAD	triangular fuzzy numbers, Hamming distance, IHOWAD
UOWAD	interval numbers, OWAD
FLIOWAMD	fuzzy linguistic information, Minkowski distance, IOWA
IFIOWAD	intuitionistic fuzzy information, distance measures, IOWA
IFIOWAWAD	intuitionistic fuzzy information, distance measures, weighted average, IOWA
IVIFOWGD	interval-valued intuitionistic fuzzy information, distance measures, OWG
SVNLOWAD	single-valued neutrosophic linguistic information, distance measures, OWA
IFWIEOWAD	intuitionistic fuzzy information, weighted Euclidean distance, distance measures, IOWA
TrPFLECOWMD	trapezoidal Pythagorean fuzzy linguistic information, Minkowski distance, entropic measure, combined ordered weighted
PLTOWD	probabilistic linguistic term information, Hamming distance, OWA

probabilistic information in the OWAD operator.

Most of the distance aggregation operators mentioned above are applied to solve MCDM problems. Some of them are also applied in real applications. For example, the OWAD operator is used in the selection of financial products and the UEOWAD operator is used for a group decision making problem about the selection of new artillery weapons under uncertainty.

The superiority of these distance operators in an MCDM problem is that they allow us to consider ideal argument values that might not exist in reality. They can also alleviate or enlarge the influence of unduly large or small deviations, respectively. The presentation of these distance operators can make DMs consider many scenarios according to their interests and preferences. And the distance measures are regarded as aggregated argument variables, which is their common point.

2.2. Motivation and contributions

An issue with these distance operators is that they will fail to deal with situations where DMs want original attribute values to be argument variables. Therefore, it will further expand their applicability if some new operators are designed with distance measures that allow DMs to introduce ideal argument values and, at the same time, regard original attribute values as argument variables.

To achieve this goal, we first find that, if the distance measures are also regarded as argument variables in new operators, this issue cannot be solved. Therefore, we plan to consider other parameters that are represented in the form of distance measures in the research process of new operators. These parameters are not taken as argument variables, so other variables (such as attribute values) can be regarded as argument variables. Accordingly, this issue can be solved. To introduce other parameters, i.e., distance measures, which are not used as argument variables in an operator, we find that this idea can be achieved by extending the IOWA operator. Namely, in the IOWA operator, we consider distance measures to be order-inducing parameters. Therefore, the issue described in the previous paragraph can be solved.

Based on this argument, Gong et al. (2020) first selected the Hamming distance to determine distance measures and put forward the distance-induced OWA (DIOWA) operators. Their most remarkable feature is that the order-inducing variables are obtained by distance measures. However, that paper just proposed a simple type of DIOWA operator to consider crisp or accurate attribute information in decision making. It is necessary to research the DIOWA operator further to make better decisions in certain situations. The contributions of our study are shown below.

In this paper, we first propose the general form of the DIOWA operator, and then the formulations of the HDIOWA, NHDIOWA, and WHDIOWA operators are given. Their properties are also analyzed. We further give three weighting methods with distance measures for the DIOWA operators and then illustrate the families of those DIOWA operators and generalize the DIOWA operators with generalized and quasi-arithmetic means. Then, looking at how the DIOWA operators are applied, the framework of MCDM methods with the DIOWA operators is constructed, and finally we use the WHDIOWA operator for a strategy selection problem for a vehicle enterprise. We find that DIOWA operators can alleviate or enlarge the influence of unduly large or small deviations, respectively. The biggest advantage of DIOWA operators is that DMs can consider ideal argument values and at the same time take other parameters as argument variables. In addition, other ideal argument values can also be considered in the DIOWA operators if we use another distance measure method to determine their weighting vectors. Moreover, the DIOWA operators will afford more options for DMs to select according to their interests and preferences, so that they can make better decisions.

3. Preliminaries

Here, we will briefly review some concepts about the Hamming distance, the IOWA operator, the IGOWA operator, and the quasi-IOWA operator, which support our research.

3.1. The Hamming distance

Distance measures are very important in several fields, including MCDM, supply chain management, and machine learning. Lots of distance measure methods have been studied in the literature; among these, the Hamming distance (Hamming, 1950), which is a basic distance measure method, plays an important role and is the most used. There are three types of Hamming distance: the generalized Hamming distance (HD), the normalized Hamming distance (NHD), and the weighted Hamming distance (WHD).

For two sets $X = (x_1, x_2, \cdots, x_n)$ and $Y = (y_1, y_2, \cdots, y_n)$, a generalized Hamming distance of dimension n is a mapping HD: $R^n \times R^n \to R$ such that

$$HD\,(X, Y) = \sum_{j=1}^{n} |x_i - y_i| \tag{1}$$

in which x_i and y_i are the ith arguments in X and Y, respectively.

An NHD of dimension n between X and Y is a mapping NHD: $[0,1]^n \times [0,1]^n \to [0,1]$ such that

$$NHD\,(X, Y) = \frac{1}{n} \sum_{i=1}^{n} |x_i - y_i| \tag{2}$$

in which x_i and y_i are the ith arguments in X and Y, respectively.

A WHD of dimension n between X and Y is a mapping WHD: $[0,1]^n \times [0,1]^n \rightarrow [0,1]$ with an associated weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$, such that

$$WHD\,(X, Y) = \sum_{i=1}^{n} \omega_i \left| x_i - y_i \right| \tag{3}$$

in which x_i and y_i are the ith arguments in X and Y, respectively.

3.2. The IOWA operator

The IOWA operator was first proposed by Yager and Filev (1999) and it can be seen as a generalized form of the OWA operator. It provides a parameterized family of aggregation operators including the maximum, the minimum, the average, and the OWA operator. Its main feature is that the argument variables are reordered by order-inducing variables.

An IOWA operator of dimension n is defined as a mapping IOWA: $R^n \times R^n \rightarrow R$ having an associated weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$, according to the following formula:

$$IOWA\,(\langle a_1, x_1 \rangle, \langle a_2, x_2 \rangle, \cdots, \langle a_n, x_n \rangle) = \sum_{j=1}^{n} \omega_j b_j \tag{4}$$

in which b_j is the x_i value with the jth largest order-inducing variable a_i in the IOWA pair $\langle a_i, x_i \rangle$, and x_i is the argument variable.

3.3. The IGOWA operator

The IOWA operator is further extended with generalized means to obtain the IGOWA operator, which is a more general form of the IOWA operator.

An IGOWA operator (Merigo and Gil-Lafuente, 2009) of dimension n is a mapping IGOWA: $R^n \times R^n \rightarrow R$ with a related weighting vector W having the properties $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$, such that

$$IGOWA\,(\langle a_1, x_1 \rangle, \langle a_2, x_2 \rangle, \cdots, \langle a_n, x_n \rangle) = \left(\sum_{j=1}^{n} \omega_j \left(b_j \right)^{\lambda} \right)^{\frac{1}{\lambda}} \tag{5}$$

in which b_j is the x_i value with the jth largest order-inducing variable a_i in the IGOWA pair $\langle a_i, x_i \rangle$, x_i is the argument variable, and $\lambda \in (-\infty, +\infty)$.

Some special cases of the IGOWA operator can be obtained. When $\lambda = 1$, we obtain the IOWA operator. When $\lambda \to 0$, we obtain the IOWG operator. When $\lambda = 2$, we get the IOWQA operator, and when $\lambda = -1$ we get the IOWHA operator.

3.4. The quasi-IOWA operator

Researchers further generalize the IOWA operator by using quasi-arithmetic means and get the quasi-IOWA operator. A quasi-IOWA operator (Merigo and Gil-Lafuente, 2009) of dimension n is defined as a mapping QIOWA: $R^n \times R^n \to R$ with an associated weighting vector W having the properties $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0, 1]$ for all $i = 1, 2, \cdots, n$, such that

$$QIOWA\left(\langle a_1, x_1\rangle, \langle a_2, x_2\rangle, \cdots, \langle a_n, x_n\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} \omega_j g\left(b_j\right)\right) \tag{6}$$

in which b_j is the x_i value with the jth largest order-inducing variable a_i in the QIOWA pair $\langle a_i, x_i\rangle$, x_i is the argument variable, and g is a strictly continuous monotonic function.

If $g(b) = b^\lambda$, we get the IGOWA operator. If $g(b) = b$, we get the IOWA operator. If $g(b) = b^2$, we get the IOWQA operator. If $g(b) = b^{-1}$, we get the IOWHA operator.

It should be noted that all the operators mentioned in Section 3 can also be categorized into descending type or ascending type. The Hamming distance and the IOWA, IGOWA, and quasi-IOWA operators are a little more "theoretical." If the information existing in practical problems is considered in them, the application value will be further improved.

4. The distance-induced OWA operators

We will study the distance-induced OWA (DIOWA) operators in this section. The main feature of the DIOWA operators is that some ideal argument values are introduced into an MCDM problem and, at the same time, the distance measures between two sets are used as order-inducing variables.

4.1. The DIOWA operators

The DIOWA operators are extensions of the IOWA operator, which combines Hamming distances with the IOWA operator. A generalized form of the DIOWA operators is constructed as follows. A generalized form of the DIOWA operators of dimension n by using any type of Hamming distance measures is defined as a mapping DIOWA: $R^n \times R^n \rightarrow R$ with a corresponding weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$, according to the formula

$$DIOWA\left(\langle a_1, x_1\rangle, \langle a_2, x_2\rangle, \cdots, \langle a_n, x_n\rangle\right) = \sum_{j=1}^{n} \omega_j c_j \tag{7}$$

in which $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, a_i represents the distance with any type of Hamming distance measures between x_i and y_i, and c_j is the x_i value of the DIOWA pair $\langle a_i, x_i\rangle$ with the jth largest a_i.

Based on different types of Hamming distance measure used, three basic types of DIOWA operators are obtained.

A DIOWA operator of dimension n by using the HD is defined as a mapping HDIOWA: $R^n \times R^n \rightarrow R$ with an associated weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$; we have

$$HDIOWA\left(\left\langle a_1^{'}, x_1\right\rangle, \left\langle a_2^{'}, x_2\right\rangle, \ldots, \left\langle a_n^{'}, x_n\right\rangle\right) = \sum_{j=1}^{n} \omega_j c_j^{'} \tag{8}$$

in which $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, $a_i^{'}$ represents the HD between x_i and y_i, and $c_j^{'}$ is the x_i value with the jth largest $a_i^{'}$ in the HDIOWA pair $\left\langle a_i^{'}, x_i\right\rangle$.

An example is given below to illustrate the aggregation process of the HDIOWA operator.

Example 1. Assume the set to be aggregated is $X = (3.45, 6.28, 8.44, 7.62)$, the set of ideal argument variables is $Y = (2.76, 6.51, 5.81, 3.29)$, and the weighting vector is $W = (0.20, 0.30, 0.35, 0.15)$. The detail aggregation process by using the HDIOWA operator is shown as follows.

First, calculate the generalized Hamming distances between x_i and y_i for all $i = 1, 2, 3, 4$; according to Eq. (1), the results are

$$a_1^{'} = HD\left(3.45, 2.76\right) = 0.68,$$

$$a_2^{'} = HD\left(6.28, 6.51\right) = 0.23,$$

$$a_3^{'} = HD\left(8.44, 5.81\right) = 2.63,$$

$$a_4^{'} = HD\left(7.62, 3.29\right) = 4.33.$$

Then, on rearranging the four arguments aggregated according to the descending order of the Hamming distances obtained, we obtain

$$c_1^{'} = 7.62, c_2^{'} = 8.44, c_3^{'} = 3.45, c_4^{'} = 6.28.$$

Therefore, the aggregation result is

$$HDIOWA\,(3.45, 6.28, 8.44, 7.62) = 0.20 \times 7.62 + 0.30 \times 8.44 + 0.35 \times 3.45 + 0.15 \times 6.28 = 6.21.$$

We also obtain the NHD-induced OWA (NHDIOWA) operator by using the NHD in DIOWA operators.

An NHDIOWA operator of dimension n with the NHD is a mapping NHDIOWA: $R^n \times R^n \to R$ with a related weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$; we have

$$NHDIOWA\,(\langle a_1^*, x_1\rangle, \langle a_2^*, x_2\rangle, \cdots, \langle a_n^*, x_n\rangle) = \sum_{j=1}^{n} \omega_j c_j^* \tag{9}$$

in which $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, a_i^* represents the NHD between x_i and y_i, and c_j^* is the x_i value with the jth largest a_i^* in the NHDIOWA pair $\langle a_i^*, x_i\rangle$.

It should be noted that, with the same argument variables and ideal argument variables, the aggregation results calculated by using the HDIOWA and NHDIOWA operators are the same. Although the values of distance measures with the HDIOWA operator are different from those with the NHDIOWA operator, the ranks of order-inducing variables remain the same. The change of distance measures does not change the positions of the argument variables aggregated, and therefore we obtain the same aggregation result. The process of proof is omitted.

We also consider the WHD in DIOWA operators to get the WHDIOWA operator.

A WHDIOWA operator of dimension n with the WHD is a mapping WHDIOWA: $R^n \times R^n \to R$ with an associated weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$; we have

$$WHDIOWA\left(\left\langle a_1^{\#}, x_1\right\rangle, \left\langle a_2^{\#}, x_2\right\rangle, \cdots, \left\langle a_n^{\#}, x_n\right\rangle\right) = \sum_{j=1}^{n} \omega_j c_j^{\#} \tag{10}$$

in which $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, $a_i^{\#}$ represents the WHD between x_i and y_i, and $c_j^{\#}$ is the x_i value of the WHDIOWA pair $\left\langle a_i^{\#}, x_i\right\rangle$ with the jth largest $a_i^{\#}$.

An example is also given below to explain how to use the WHDIOWA operator.

Example 2. For comparison, we use the same data in this example as in Example 1. The detailed aggregation process with the WHDIOWA operator is shown below.

First, calculate the WHDs between x_i and y_i for all $i = 1, 2, 3, 4$; according to Eq. (3), the results are

$$a_1^{\#} = WHD\,(3.45, 2.76) = 0.14,$$

$$a_2^{\#} = WHD\,(6.28, 6.51) = 0.07,$$

$$a_3^{\#} = WHD\,(8.44, 5.81) = 0.92,$$

$$a_4^{\#} = WHD\,(7.62, 3.29) = 0.65.$$

Then, the order of these four arguments to be aggregated is

$$c_1^{\#} = 8.44, c_2^{\#} = 7.62, c_3^{\#} = 3.45, c_4^{\#} = 6.28.$$

So, the value of the WHDIOWA operator is

$$WHDIOWA\,(3.45, 6.28, 8.44, 7.62) = 0.20 \times 8.44 + 0.30 \times 7.62 + 0.35 \times 3.45 + 0.15 \times 6.28 = 6.12.$$

Comparing the aggregation results of the HDIOWA operator and the WHDIOWA operator, we see that the results are not the same. The reason is that the rank of distance measures with the generalized Hamming distance is different from the one with the WHD. The rank of distance measures with the HD or the NHD is not always the same as the rank of distance measures with the WHD.

4.2. The weighted DIOWA operators

Another circumstance that needs to be considered is that the weights associated with the WHD are independent. At this time, we define the WHDIOWA operator as follows. This is a more generalized form of the WHDIOWA operator.

A WHDIOWA operator of dimension n with the WHD is defined as a mapping WHDIOWA: $R^n \times R^n \to R$ having two related weighting vectors $W_1 = (w_1, w_2, \cdots, w_n)$ and $W_2 = (\omega_1, \omega_2, \cdots, \omega_n)$ such that $\sum_{i=1}^{n} w_i =$

$1, w_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1, \omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$, according to the formula

$$WHDIOWA\left(\left\langle a_1^{\wedge}, x_1\right\rangle, \left\langle a_2^{\wedge}, x_2\right\rangle, \cdots, \left\langle a_n^{\wedge}, x_n\right\rangle\right) = \sum_{j=1}^{n} \omega_j c_j^{\wedge} \tag{11}$$

in which $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, $a_i^{\wedge}$ represents the WHD between x_i and y_i with the weighting vector W_1, and $a_j^{\wedge}$ is the x_i value of the WHDIOWA pair $\langle a_i^{\wedge}, x_i\rangle$ having the jth largest $a_i^{\wedge}$ and $a_i^{\wedge}$ such that

$$a_i^{\wedge} = w_i \left|x_i - y_i\right| \tag{12}$$

Another circumstance that needs to be considered is that the sum of w_i does not equal 1 and the sum of ω_i does not equal 1. At this time, we use the following formula

$$WHDIOWA\left(\left\langle a_1^{\wedge}, x_1\right\rangle, \left\langle a_2^{\wedge}, x_2\right\rangle, \cdots, \left\langle a_n^{\wedge}, x_n\right\rangle\right) = \frac{1}{W_2} \sum_{j=1}^{n} \omega_j c_j^{\wedge} \tag{13}$$

and $a_i^{\wedge}$ such that

$$a_i^{\wedge} = \frac{w_i}{W_1} \left|x_i - y_i\right| \tag{14}$$

Gong et al. (2020) have also conducted research on the HDIOWA and WHDIOWA operators as two different special types of the generalized form of the DIOWA operators.

4.3. The properties of DIOWA operators

All the DIOWA operators mentioned above in this section are commutative, monotonic, bounded, and idempotent. Assume $\langle e_i, x_i\rangle$ represents the DIOWA pair with respect to f; f represents any of the types of DIOWA operator mentioned above.

THEOREM 1. (Commutativity) Let $\left\langle \widetilde{e}_1, \widetilde{x}_1\right\rangle, \left\langle \widetilde{e}_2, \widetilde{x}_2\right\rangle, \cdots, \left\langle \widetilde{e}_n, \widetilde{x}_n\right\rangle$ be any permutation of the DIOWA pairs $\langle e_1, x_1\rangle, \langle e_2, x_2\rangle, \cdots, \langle e_n, x_n\rangle$, then

$$f\left(\left\langle \widetilde{e}_1, \widetilde{x}_1\right\rangle, \left\langle \widetilde{e}_2, \widetilde{x}_2\right\rangle, \cdots, \left\langle \widetilde{e}_n, \widetilde{x}_n\right\rangle\right) = f\left(\langle e_1, x_1\rangle, \langle e_2, x_2\rangle, \cdots, \langle e_n, x_n\rangle\right).$$

THEOREM 2. (Commutativity) Let $\langle D(x_1, y_1), x_1\rangle, \langle D(x_2, y_2), x_2\rangle, \cdots, \langle D(x_n, y_n), x_n\rangle$ be the DI-

OWA pairs, then

$$f(\langle D(y_1,x_1),x_1\rangle,\langle D(y_2,x_2),x_2\rangle,\cdots,\langle D(y_n,x_n),x_n\rangle)=f(\langle D(x_1,y_1),x_1\rangle,\langle D(x_2,y_2),x_2\rangle,\cdots,\langle D(x_n,y_n),x_n\rangle).$$

THEOREM 3. (Monotonicity) Let $\langle \hat{e}_1,\hat{x}_1\rangle,\langle \hat{e}_2,\hat{x}_2\rangle,\cdots,\langle \hat{e}_n,\hat{x}_n\rangle$ be the DIOWA pairs, if the reordered position of $\hat{e}_i$ is the same as the reordered position of e_i and $x_i \geq \hat{x}_i$ for all $i=1,2,\cdots,n$, then

$$f(\langle e_1,x_1\rangle,\langle e_2,x_2\rangle,\cdots,\langle e_n,x_n\rangle) \geq f(\langle \hat{e}_1,\hat{x}_1\rangle,\langle \hat{e}_2,\hat{x}_2\rangle,\cdots,\langle \hat{e}_n,\hat{x}_n\rangle).$$

THEOREM 4. (Idempotency) Let $\langle e_1,x_1\rangle,\langle e_2,x_2\rangle,\cdots,\langle e_n,x_n\rangle$ be the DIOWA pairs, if $x_i=a$ for all $i=1,2,\cdots,n$, then

$$f(\langle e_1,x_1\rangle,\langle e_2,x_2\rangle,\cdots,\langle e_n,x_n\rangle)=a.$$

THEOREM 5. (Boundary) Let $\langle e_1,x_1\rangle,\langle e_2,x_2\rangle,\cdots,\langle e_n,x_n\rangle$ be the DIOWA pairs, then

$$\min_i\{x_i\} \leq f(\langle e_1,x_1\rangle,\langle e_2,x_2\rangle,\cdots,\langle e_n,x_n\rangle) \leq \max_i\{x_i\}.$$

All the properties are easy to prove, and the process of proof is omitted.

Note that any of the DIOWA operators mentioned above in this section can be divided into the descending type or the ascending type, such as the descending WHDIOWA (DWHDIOWA) and ascending WHDIOWA (AWHDIOWA) operators. The relation of the weights of these operators is $\omega_j=\omega^*_{n-j+1}$, where ω_j represents the jth weight of one descending type of any DIOWA operator and ω^*_{n-j+1} represents the jth weight of its relevant ascending type.

When the ties of order-inducing variables happen, their average is used to replace the argument variables with the same order-inducing variables (Merigo and Gil-Lafuente, 2009).

It should be noted that the IOWA operator belongs to a special case of the DIOWA operators and the OWA operator belongs to the IOWA operator. So, we have $OWA \subseteq IOWA \subseteq DIOWA$.

4.4. Weights determination methods for DIOWA operators

An interesting issue is how we can determine the weights of the DIOWA operators. We can refer to a lot of literature covering this research (Rao et al., 2022; Ahn, 006a,b; Wu et al., 2015). Based on the methodologies of Xu and Xia (2011), Casanovas et al. (2016), Xu and Chen (2008), and Gong et al. (2019), we can also use the following methods to obtain the weights of the DIOWA operators. Also, assume $D(x_j,y_j)$ represents any type of Hamming distance.

(1) Let

$$\omega_j = \frac{D(x_j, y_j)}{\sum\limits_{j=1}^{n} D(x_j, y_j)}, \quad j = 1, 2, \cdots, n. \tag{15}$$

we can see ω_j such that $\omega_j \leq \omega_{j-1}, \ j = 2, 3, \cdots, n$ and $\sum_{j=1}^{n} \omega_j = 1$.

(2) Let

$$\omega_j = \frac{e^{-D(x_j, y_j)}}{\sum\limits_{j=1}^{n} e^{-D(x_j, y_j)}}, \quad j = 2, \cdots, n \tag{16}$$

we can see ω_j such that $\omega_j \geq \omega_{j-1}, \ j = 2, 3, \cdots, n$ and $\sum_{j=1}^{n} \omega_j = 1$.

(3) Let

$$d = \max_j D(x_j, y_j) \tag{17}$$

$$\dot{D}(X, Y) = \frac{1}{n} \sum_{j=1}^{n} D(x_j, y_j) \tag{18}$$

and

$$\ddot{D}\left(D(x_j, y_j), \dot{D}(X, Y)\right) = \left| D(x_j, y_j) - \dot{D}(X, Y) \right| \tag{19}$$

then

$$\omega_j = \frac{d - \ddot{D}\left(D(x_j, y_j), \dot{D}(X, Y)\right)}{\sum\limits_{j=1}^{n} \left(d - \ddot{D}\left(D(x_j, y_j), \dot{D}(X, Y)\right)\right)} \tag{20}$$

we can see ω_j such that $\omega_j \geq 0, \ j = 1, 2, \cdots, n$ and $\sum_{j=1}^{n} \omega_j = 1$.

It is easy to find that the weights obtained from Eq. (15) are a monotonic decreasing sequence, the weights obtained from Eq. (16) are a monotonic increasing sequence, and the weights obtained from Eq. (20) combine the above two cases, i.e., the closer the value of $D(x_j, y_j)$ to the mean $\frac{1}{n} \sum_{j=1}^{n} D(x_j, y_j)$, the larger the weight ω_j is.

It should be noted that we can replace the ideal argument vector Y with another ideal argument vector such as Y^* to determine weights so that we can consider two different groups of ideal argument values in a DIOWA operator at the same time.

4.5. Families of the DIOWA operators

There is a wide range of aggregation operators as special cases included in the DIOWA operators.

REMARK 1. If $Y = (0, 0, \cdots, 0)$, the OWA operator is obtained from the HDIOWA operator and the

NHDIOWA operator.

REMARK 2. If $Y = X + A$, and $A = (a_1, a_2, \cdots, a_n)$, $a_i \in (-\infty, +\infty)$, the IOWA operator is obtained from the WHDIOWA operator.

REMARK 3. If $Y = X + a$, and $a \in (-\infty, +\infty)$, the arithmetic mean is obtained from the HDIOWA and NHDIOWA operators.

REMARK 4. If $Y = X$, the arithmetic mean is obtained from the WHDIOWA operator.

REMARK 5. If Y is an ordered vector, the WA operator is derived from the HDIOWA and NHDIOWA operators.

Using different manifestations in Y and W, the maximum and the minimum can be derived from the DIOWA operators.

REMARK 6. If the distance measure value of $\max_i \{x_i\}$ is the largest, and $W = (1, 0, \cdots, 0)$, then we get the maximum from the HDIOWA operator and the NHDIOWA operator.

REMARK 7. If the distance measure value of $\min_i \{x_i\}$ is the largest, and $W = (0, 0, \cdots, 1)$, then we get the minimum from the HDIOWA operator and the NHDIOWA operator.

REMARK 8. Assume $x_{\sigma(i)} = \max_i \{x_i\}$, if $Y = \Big(x_1, x_2, \cdots, x'_{\sigma(i)}, \cdots, x_{n-1}, x_n\Big)$, $x'_{\sigma(i)} \neq x_{\sigma(i)}$ and $W = (1, 0, \cdots, 0)$, then we get the maximum from the WHDIOWA operator.

REMARK 9. Assume $x_{\tau(i)} = \min_i \{x_i\}$, if $Y = \Big(x_1, x_2, \cdots, x'_{\tau(i)}, \cdots, x_{n-1}, x_n\Big)$, $x'_{\tau(i)} \neq x_{\tau(i)}$ and $W = (0, 0, \cdots, 1)$, then we get the minimum from the WHDIOWA operator.

REMARK 10. If $W = \left(\frac{1}{n}, \frac{1}{n}, \cdots, \frac{1}{n}\right)$, we get the arithmetic mean from the DIOWA operators.

REMARK 11. If $\omega_k = 1$ and $\omega_j = 0$ for all $j \neq k$, we get the step-DIOWA operators.

REMARK 12. If $\omega_{\frac{(n+1)}{2}} = 1$ and $\omega_j = 0$ for $j \neq \frac{(n+1)}{2}$ when n is odd, and if $\omega_{\frac{n}{2}} = \omega_{\left(\frac{n}{2}\right)+1} = 0.5$ and $\omega_j = 0$ for all others when n is even, we get the median-DIOWA operators.

REMARK 13. An interesting particular case is the Olympic-DIOWA operators which can be derived from DIOWA operators when $\omega_1 = \omega_n = 0$ and $\omega_j = \frac{1}{(n-2)}$ for all others. Note that if $n = 3$ or $n = 4$, the Olympic-DIOWA operators become the median-DIOWA operators.

REMARK 14. The more general forms of the Olympic-DIOWA operators are presented when $\omega_j = 0$ for $j = 1, 2, \cdots, k$ and $j = n - k + 1, \cdots, n - 1, n$, and $\omega_j = \frac{1}{(n-2k)}$ for all others, where $k < \frac{n}{2}$. The Olympic-DIOWA operators are derived when $k = 1$. The median-DIOWA operators are derived when $k = \frac{(n-1)}{2}$.

REMARK 15. Contrary cases of general Olympic-DIOWA operators are presented when $\omega_j = \frac{1}{2k}$ for $j = 1, 2, \cdots, k$ and $j = n - k + 1, \cdots, n - 1, n$, and $\omega_j = 0$ for all others, where $k < \frac{n}{2}$. The median-DIOWA operators are obtained when $k = 1$.

REMARK 16. Centered-DIOWA operators are obtained if $\omega_p \leq \omega_q$ when $p < q \leq \frac{(n+1)}{2}$ and if $\omega_p \geq \omega_q$ when $p > q \geq \frac{(n+1)}{2}$, ω_j such that $\omega_j = \omega_{j+n-1}$ and $\omega_j > 0$.

REMARK 17. Other interesting families of the DIOWA operators are generalized S-DIOWA operators when $\omega_p = (1/n)(1-(\mu+\eta)) + \mu$ with $x_p = \max_i \{x_i\}$ and $\omega_p = (1/n)(1-(\mu+\eta)) + \eta$ with $x_q = \min_i \{x_i\}$, and $\omega_j = (1/n)(1-(\mu+\eta))$ for $j \neq p, q$, where $\mu, \eta \in [0,1]$ and $\mu + \eta \leq 1$. Especially, if $\mu = 0$, we obtain the "andlike" S-DIOWA operators, and if $\eta = 0$, we obtain the "orlike" S-DIOWA operators. If $\mu + \eta = 1$, we obtain the distance-induced Hurwicz criteria.

4.6. Generalized and quasi-DIOWA operators

The DIOWA operators can also be generalized by using generalized means, and the distance-induced generalized OWA (DIGOWA) operators will be obtained. These can be defined as follows.

A DIGOWA operator of dimension n is a mapping DIGOWA: $R^n \times R^n \to R$ with a related weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$; we have

$$DIGOWA\left(\left\langle a_1^{\Delta}, x_1\right\rangle, \left\langle a_2^{\Delta}, x_2\right\rangle, \cdots, \left\langle a_n^{\Delta}, x_n\right\rangle\right) = \left(\sum_{j=1}^{n} \omega_j \left(c_j^{\Delta}\right)^{\lambda}\right)^{\frac{1}{\lambda}} \tag{21}$$

where $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, a_i^{Δ} represents the distance between x_i and y_i by using any type of Hamming distance measures, and c_j^{Δ} is the x_i value of the DIGOWA pair $\left\langle a_i^{\Delta}, x_i\right\rangle$ having the jth largest a_i^{Δ}, and $\lambda \in (-\infty, +\infty)$ is a parameter.

REMARK 18. If we combine the generalized Hamming distance and the DIGOWA operator, we get the Hamming-distance-induced generalized OWA (HDIGOWA) operator. If we combine the NHD and the DIGOWA operator, we get the NHD-induced generalized OWA (NHDIGOWA) operator. If we use the WHD in the DIGOWA operator, we get the WHD-induced generalized OWA (WHDIGOWA) operator.

REMARK 19. When $\lambda = 1$, the DIOWA operators are obtained. When $\lambda \to 0$, the DIOWG operators are obtained. When $\lambda = 2$, the DIOWQA operators are obtained. When $\lambda = -1$, the DIOWHA operators are obtained. Some other types of DIGOWA operator can also be obtained.

REMARK 20. All the operators mentioned above in Section 4 can be obtained from the DIGOWA operators, by choosing different W, Y and λ.

Referring to the quasi-IOWA operator, we can obtain a more generalized form of the DIGOWA operators by using quasi-arithmetic means.

A quasi-DIOWA operator of dimension n is a mapping QDIOWA: $R^n \times R^n \to R$ with a related weighting vector W such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \in [0,1]$ for all $i = 1, 2, \cdots, n$; we have

$$QDIOWA\left(\left\langle a_1^{\nabla}, x_1\right\rangle, \left\langle a_2^{\nabla}, x_2\right\rangle, \cdots, \left\langle a_n^{\nabla}, x_n\right\rangle\right) = g^{-1}\left(\sum_{j=1}^{n} \omega_j g\left(c_j^{\nabla}\right)\right) \tag{22}$$

where $X = (x_1, x_2, \cdots, x_n)$ is the set of argument variables to be aggregated, $Y = (y_1, y_2, \cdots, y_n)$ is the set of ideal argument variables, a_i^{∇} represents the distance between x_i and y_i by using any type of Hamming distance measures, c_j^{∇} is the x_i value of the QDIOWA pair $\left\langle a_i^{\nabla}, x_i\right\rangle$ having the jth largest a_i^{∇}, and g is a strictly continuous monotonic function.

REMARK 21. If we use the generalized Hamming distance in a quasi-DIOWA operator, we get the quasi-Hamming-distance-induced OWA (quasi-HDIOWA) operator. If we use the NHD in a quasi-DIOWA operator, we get the quasi-NHD-induced OWA (quasi-NHDIOWA) operator. If we use the WHD in a quasi-DIOWA operator, we get the quasi-WHD-induced OWA (quasi-WHDIOWA) operator.

REMARK 22. When $g(b) = b^{\lambda}$, the quasi-DIOWA operators are reduced to the DIGOWA operators.

REMARK 23. When we use trigonometric functions such as $g(b) = sin\left(\left(\frac{\pi}{2}\right)b\right)$, $g(b) = cos\left(\left(\frac{\pi}{2}\right)b\right)$ and $g(b) = tan\left(\left(\frac{\pi}{2}\right)b\right)$, some trigonometric -DIOWA operators are obtained from the quasi-DIOWA operators.

REMARK 24. All the operators mentioned above in Section 4 can be obtained from the quasi-DIOWA operators by choosing different W, Y, and function g.

It should be noted that we can also use another strictly continuous monotonic function h to calculate the values of a_i^{∇} for all $i = 1, 2, \cdots, n$ such that $a_i^{\nabla} = h\left(D\left(x_i, y_i\right)\right)$, where $D\left(x_j, y_j\right)$ represents any type of Hamming distance measure.

5. Frameworks of MCDM methods with the DIOWA operators

The DIOWA operators are applicable in a wide range of situations such as statistics, pattern recognition, engineering, and decision making. In this section, a framework of MCDM methods with the DIOWA operators is given to introduce how the DIOWA operators can be used to make a decision. Under this framework, numerous MCDM problems, such as the evaluation of strategies, best candidates, and investments, could be solved when a number of ideal argument values for these problems are considered. The frameworks afford DMs a lot of options so that they can make better decisions according to their preferences, interests, and so on.

In the MCDM context, we can have two possible scenarios related to the attribute values, one where all attribute values are fixed (situation 1) and the other where these attribute values are not fixed (situation 2). The steps involved in these two scenarios will differ slightly and therefore the details of the proposed framework are provided below in two separate versions.

Framework for situation 1: Every attribute value of every alternative is fixed.

In this situation, every attribute value of every alternative is a unique value.

Step 1: Obtain the decision making matrix. Let $O = \{o_1, o_2, \cdots, o_p\}$ be a set of finite alternatives and $X = \{x_1, x_2, \cdots, x_n\}$ be a set of finite attributes. According to the available information construct a decision making matrix which is shown as follows, where x_{ij} is the attribute value of alternative o_i concerning attribute x_j and $i = 1, 2, \cdots, p$, $j = 1, 2, \cdots, n$, $p, n \geq 3$. Assume that the better alternative is the one with a higher value of x_{ij}.

$$E = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_p \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{p1} & x_{p2} & \cdots & x_{pn} \end{bmatrix} \end{array} \tag{23}$$

Step 2: Construct the set of ideal argument values. Let $o^* = \{x_1^*, x_2^*, \cdots, x_n^*\}$ be the set; x_j^* represents the attribute value of the set o^* concerning attribute x_j.

Step 3: Choose appropriate DIOWA operators. One or more of the DIOWA operators can be chosen by different DMs to solve the same decision making problem.

Step 4: Calculate the order-inducing variables. According to the DIOWA operators selected, use relevant types of Hamming distance measures with the information of the ideal argument values to determine all the values of order-inducing variables.

Step 5: Calculate the weights associated with the DIOWA operators selected. Weights associated with the DIOWA operators selected can be determined through existing methods in literature or Eq. (15), Eq. (16), or Eq. (20).

Step 6: Determine the individual evaluation result of every alternative based on the DIOWA operators selected in Step 3. Let $y_i^{(l)}$ be the evaluation result of alternative o_i based on the lth DIOWA operator selected, and $l \in N_+$.

Step 7: Determine the final evaluation result of every alternative. Aggregate all the individual evaluation results of every alternative to a final evaluation result. Let y_i be the final evaluation result of alternative o_i, which can be obtained by $y_i = \sum_l d_l y_i^{(l)}$, where $d_l (l \in N_+)$ are parameters such that $\sum_l d_l = 1$ and $d_l \geq 0$.

Step 8: Determine the best alternative(s). Sort all the final evaluation values of all alternatives in ascending order. The best selection is the alternative with the smallest final evaluation value.

Framework for situation 2: Every attribute value of every alternative is not fixed.

In this situation, every attribute value of every alternative is given by every DM.

Step 1: Construct individual decision making matrices by all DMs. Let $O = \{o_1, o_2, \cdots, o_p\}$ be a set of finite alternatives, $X = \{x_1, x_2, \cdots, x_n\}$ be a set of finite attributes, and $S = \{s_1, s_2, \cdots, s_m\}$ be a set of finite DMs. According to the available information construct individual decision making matrices shown as follows, where x_{ij}^k represents the attribute value of alternative o_i concerning attribute x_j given by DM s_k, and $i = 1, 2, \cdots, p$, $j = 1, 2, \cdots, n$, $k = 1, 2, \cdots, m$, $p, n, m \geq 3$. Assume that the better alternative is the one with a higher value of x_{ij}^k.

$$E = \begin{array}{c} \\ o_1 \\ o_2 \\ \vdots \\ o_p \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \left[\begin{array}{cccc} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ x_{p1} & x_{p2} & \cdots & x_{pn} \end{array} \right] \end{array} \tag{24}$$

Step 2: Obtain the individual set of ideal argument values from all DMs. Let $o_k^* = \{x_1^{k*}, x_2^{k*}, \cdots, x_n^{k*}\}$ be the set of the ideal argument values obtained from DM s_k; x_j^{k*} is the value of the set o_k^* concerning attribute x_j given by DM s_k. All the DMs can also construct only one set of the ideal argument values.

Step 3: Choose one or more DIOWA operator for each DM. One or more of the DIOWA operators can be chosen by each DM in the same decision making process.

Step 4: Use Hamming distance measure methods to determine all order-inducing variables. According to the DIOWA operators selected and the ideal argument values constructed, choose different types of Hamming distance to determine all the values of order-inducing variables.

Step 5: Determine all the weights associated with the DIOWA operators selected by every DM. Refer to the existing methods in literature or Eq. (15), Eq. (16), or Eq. (20) to obtain all the values of corresponding weights.

Step 6: Determine the evaluation results of all alternatives of every DM. Let y_i^k be the evaluation result of alternative o_i obtained from DM s_k. The determination process of y_i^k refers to steps 6–7 in situation 1.

Step 7: Calculate the weights of DMs. The weights of DMs can be obtained through existing methods in the literature or given directly.

Step 8: Determine the final evaluation results of all alternatives. Aggregate all the evaluation results y_i^k of every alternative from all DMs with weights of DMs to a final evaluation result y_i. Lots of existing operators such as the OWA operator can be considered in this aggregation process.

Step 9: Make sure the best alternative(s). Sort all the final evaluation values of all alternatives in ascending order. The best selection is the alternative with the smallest final evaluation value.

6. Illustrative example

The DIOWA operators are applicable in a wide range of situations. In this section, we illustrate the application of DIOWA operators from a numerical example.

Energy is a huge issue for almost all countries in the world. The selection of energy strategy is a crucial decision that will affect a country's competitiveness and social development. In recent years, to reduce energy consumption and improve air pollution, new energy vehicles have attracted more and more attention, and cars with multiple different driven power forms have appeared. However, which form or forms should be selected as the development direction of a vehicle manufacturing company or a country is a selection problem in energy strategy. Below, an example is illustrated to solve this selection problem.

Suppose a vehicle manufacturing company wants to determine its development strategy for the next 10 years after deep consideration. Now, there are four alternative strategies: conventional vehicles (o_1), electric vehicles (o_2), fuel cell vehicles (o_3), and hybrid-powered vehicles (o_4) are regarded as the main objects to be considered, and only one will be selected to be company's future development strategy. To make the best decision, four experts, s_1, s_2, s_3, and s_4, are invited to participate in this process, and their four strategies will be evaluated by five attributes: technology reserves (x_1), expected customer acceptance (x_2), expected investment (x_3), expected earning capacity (x_4), and expected risk (x_5). The weights of the four experts are 0.20, 0.30, 0.25, and 0.25, respectively. Every expert constructs matrices E_k based on the available information and their judgments, which are shown as follows. An ideal strategy determined by the four experts is also built and this is shown in Table 4. In this numerical example, we use the WHDIOWA operator, in which $W_1 = (0.24, 0.13, 0.25, 0.12, 0.26)$, to calculate weighted Hamming distances to analyze the detailed process of decision making in this numerical example.

Table 4: Attribute values of the constructed ideal strategy

	x_1	x_2	x_3	x_4	x_5
o^*	99	97	96	96	98

$$E_1 = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 85 & 89 & 94 & 85 & 79 \\ 77 & 73 & 82 & 76 & 83 \\ 89 & 93 & 76 & 87 & 78 \\ 95 & 95 & 94 & 85 & 86 \end{bmatrix} \end{array}, \quad E_2 = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 92 & 78 & 94 & 76 & 87 \\ 76 & 76 & 85 & 81 & 78 \\ 88 & 71 & 85 & 78 & 88 \\ 81 & 80 & 77 & 73 & 87 \end{bmatrix} \end{array}$$

$$E_3 = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 88 & 84 & 94 & 86 & 94 \\ 82 & 94 & 86 & 80 & 89 \\ 79 & 73 & 84 & 85 & 92 \\ 79 & 84 & 81 & 93 & 95 \end{bmatrix} \end{array}, \quad E_4 = \begin{array}{c} \\ o_1 \\ o_2 \\ o_3 \\ o_4 \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & x_4 & x_5 \end{array} \\ \begin{bmatrix} 87 & 72 & 84 & 89 & 79 \\ 83 & 71 & 85 & 71 & 90 \\ 82 & 72 & 90 & 84 & 88 \\ 78 & 91 & 79 & 83 & 79 \end{bmatrix} \end{array}$$

6.1. Decision making procedure using the framework for situation 2

Step 1: Determine the order-inducing variables of the WHDIOWA operator. Use the weighting vector $W_1 = (0.24, 0.13, 0.25, 0.12, 0.26)$ to calculate weighted Hamming distances $u_{ij}^{\wedge}$ for $j = 1, 2, 3, 4, 5$, where $u_{ij}^{(k)\wedge}$ represents the order-inducing variable of attribute x_j concerning candidate o_i in matrix E_k. The results are shown as follows.

$$E_1' = \left(u_{ij}^{(1)}, x_{ij}^{(1)}\right) = \begin{bmatrix} (3.36, 85) & (1.04, 89) & (0.50, 94) & (1.32, 85) & (4.94, 79) \\ (5.28, 77) & (3.12, 73) & (3.50, 82) & (2.40, 76) & (3.90, 83) \\ (2.40, 89) & (0.52, 93) & (5.00, 76) & (1.08, 87) & (5.20, 78) \\ (0.96, 95) & (0.26, 95) & (0.50, 94) & (1.32, 85) & (3.12, 86) \end{bmatrix},$$

$$E_2' = \left(u_{ij}^{(2)}, x_{ij}^{(2)}\right) = \begin{bmatrix} (1.68, 92) & (2.47, 78) & (0.50, 94) & (2.40, 76) & (2.86, 87) \\ (5.52, 76) & (2.73, 76) & (2.75, 85) & (1.80, 81) & (5.20, 78) \\ (2.64, 88) & (3.38, 71) & (2.75, 85) & (2.16, 78) & (2.60, 88) \\ (4.32, 81) & (2.21, 80) & (4.75, 77) & (2.76, 73) & (2.86, 87) \end{bmatrix},$$

$$E_3' = \left(u_{ij}^{(3)}, x_{ij}^{(3)}\right) = \begin{bmatrix} (2.64, 88) & (1.69, 84) & (0.50, 94) & (1.20, 86) & (1.04, 94) \\ (4.08, 82) & (0.39, 94) & (2.50, 86) & (1.92, 80) & (2.34, 89) \\ (4.80, 79) & (3.12, 73) & (3.00, 84) & (1.32, 85) & (1.56, 92) \\ (4.80, 79) & (1.69, 84) & (3.75, 81) & (0.36, 93) & (0.78, 95) \end{bmatrix},$$

$$E_4' = \left(u_{ij}^{(4)}, x_{ij}^{(4)}\right) = \begin{bmatrix} (2.88, 87) & (3.25, 72) & (3.00, 84) & (0.84, 89) & (4.94, 79) \\ (3.84, 83) & (3.38, 71) & (2.75, 85) & (3.00, 71) & (2.08, 90) \\ (4.08, 82) & (3.25, 72) & (1.50, 90) & (1.44, 84) & (2.60, 88) \\ (5.04, 78) & (0.78, 91) & (4.25, 79) & (1.56, 83) & (4.94, 79) \end{bmatrix},$$

in which E_k' is the decision making matrix of expert s_k consisting of WHDIOWA pairs.

Step 2: Calculate the weights of five attributes associated with every decision making matrix. Let $W_i^{(k)} = \left(\omega_{i1}^{(k)}, \omega_{i2}^{(k)}, \cdots, \omega_{in}^{(k)}\right)$ be the weighting vector of attributes of strategy o_i associated with expert s_k. Let $\omega_{ij}^{(k)}$ be the weight of attribute x_j of strategy o_i calculated based on Eq. (16) according to the decision making matrix of expert s_k. The results are shown as follows.

$$W_1^{(1)} = \left(\omega_{11}^{(1)}, \cdots, \omega_{15}^{(1)}\right) = (0.01, 0.03, 0.21, 0.28, 0.48),$$

$$W_2^{(1)} = \left(\omega_{21}^{(1)}, \cdots, \omega_{25}^{(1)}\right) = (0.03, 0.11, 0.16, 0.23, 0.48),$$

$$W_3^{(1)} = \left(\omega_{31}^{(1)}, \cdots, \omega_{35}^{(1)}\right) = (0.01, 0.01, 0.09, 0.33, 0.57),$$

$$W_4^{(1)} = \left(\omega_{41}^{(1)}, \cdots, \omega_{45}^{(1)}\right) = (0.02, 0.13, 0.18, 0.29, 0.37).$$

$$W_1^{(2)} = \left(\omega_{11}^{(2)}, \cdots, \omega_{15}^{(2)}\right) = (0.06, 0.08, 0.09, 0.18, 0.59),$$

$$W_2^{(2)} = \left(\omega_{21}^{(2)}, \cdots, \omega_{25}^{(2)}\right) = (0.01, 0.02, 0.21, 0.21, 0.54),$$

$$W_3^{(2)} = \left(\omega_{31}^{(2)}, \cdots, \omega_{35}^{(2)}\right) = (0.09, 0.18, 0.20, 0.21, 0.32),$$

$$W_4^{(2)} = \left(\omega_{41}^{(2)}, \cdots, \omega_{45}^{(2)}\right) = (0.03, 0.05, 0.23, 0.25, 0.43).$$

$$W_1^{(3)} = \left(\omega_{11}^{(3)}, \cdots, \omega_{15}^{(3)}\right) = (0.05, 0.12, 0.20, 0.23, 0.40),$$

$$W_2^{(3)} = \left(\omega_{21}^{(3)}, \cdots, \omega_{25}^{(3)}\right) = (0.02, 0.08, 0.09, 0.14, 0.66),$$

$$W_3^{(3)} = \left(\omega_{31}^{(3)}, \cdots, \omega_{35}^{(3)}\right) = (0.01, 0.08, 0.09, 0.36, 0.46),$$

$$W_4^{(3)} = \left(\omega_{41}^{(3)}, \cdots, \omega_{45}^{(3)}\right) = (0.01, 0.02, 0.13, 0.33, 0.51).$$

$$W_1^{(4)} = \left(\omega_{11}^{(4)}, \cdots, \omega_{15}^{(4)}\right) = (0.01, 0.07, 0.09, 0.10, 0.74),$$

$$W_2^{(4)} = \left(\omega_{21}^{(4)}, \cdots, \omega_{25}^{(4)}\right) = (0.07, 0.12, 0.17, 0.22, 0.42),$$

$$W_3^{(4)} = \left(\omega_{31}^{(4)}, \cdots, \omega_{35}^{(4)}\right) = (0.03, 0.07, 0.13, 0.38, 0.40),$$

$$W_4^{(4)} = \left(\omega_{41}^{(4)}, \cdots, \omega_{45}^{(4)}\right) = (0.01, 0.01, 0.02, 0.30, 0.66).$$

Step 3: Calculate the evaluation results of all strategies with the WHDIOWA operator according to the decision making matrix of each expert. The results are shown as follows.

$$y_1^{(1)} = 79 \times 0.01 + 85 \times 0.03 + 85 \times 0.21 + 89 \times 0.28 + 94 \times 0.48 = 90.38,$$

$$y_2^{(1)} = 77 \times 0.03 + 83 \times 0.11 + 82 \times 0.16 + 73 \times 0.23 + 76 \times 0.48 = 77.03,$$

$$y_3^{(1)} = 78 \times 0.01 + 76 \times 0.01 + 89 \times 0.09 + 87 \times 0.33 + 93 \times 0.57 = 90.50,$$

$$y_4^{(1)} = 86 \times 0.02 + 85 \times 0.13 + 95 \times 0.18 + 94 \times 0.29 + 95 \times 0.37 = 93.23.$$

$$y_1^{(2)} = 87 \times 0.06 + 78 \times 0.08 + 76 \times 0.09 + 92 \times 0.18 + 94 \times 0.59 = 90.33,$$

$$y_2^{(2)} = 76 \times 0.01 + 78 \times 0.02 + 85 \times 0.21 + 76 \times 0.21 + 81 \times 0.54 = 80.65,$$

$$y_3^{(2)} = 71 \times 0.09 + 85 \times 0.18 + 88 \times 0.20 + 88 \times 0.21 + 78 \times 0.32 = 82.64,$$

$$y_4^{(2)} = 77 \times 0.03 + 81 \times 0.05 + 87 \times 0.23 + 73 \times 0.25 + 80 \times 0.43 = 79.78.$$

$$y_1^{(3)} = 88 \times 0.05 + 84 \times 0.12 + 86 \times 0.20 + 94 \times 0.23 + 94 \times 0.40 = 90.91,$$

$$y_2^{(3)} = 82 \times 0.02 + 86 \times 0.08 + 89 \times 0.09 + 80 \times 0.14 + 94 \times 0.66 = 90.67,$$

$$y_3^{(3)} = 79 \times 0.01 + 73 \times 0.08 + 84 \times 0.09 + 92 \times 0.36 + 85 \times 0.46 = 86.45,$$

$$y_4^{(3)} = 79 \times 0.01 + 81 \times 0.02 + 84 \times 0.13 + 95 \times 0.33 + 93 \times 0.51 = 92.17.$$

$$y_1^{(4)} = 79 \times 0.01 + 72 \times 0.07 + 84 \times 0.09 + 87 \times 0.10 + 89 \times 0.74 = 87.13,$$

$$y_2^{(4)} = 83 \times 0.07 + 71 \times 0.12 + 71 \times 0.17 + 85 \times 0.22 + 90 \times 0.42 = 82.99,$$

$$y_3^{(4)} = 82 \times 0.03 + 72 \times 0.07 + 88 \times 0.13 + 90 \times 0.38 + 84 \times 0.40 = 85.93,$$

$$y_4^{(4)} = 78 \times 0.01 + 79 \times 0.01 + 79 \times 0.02 + 83 \times 0.30 + 91 \times 0.66 = 88.10.$$

Step 4: Determine the final aggregation results of all strategies. In this step, the OWA operator is used to aggregate individual evaluation results of every strategy obtained from all experts to a final evaluation result. The details are shown as follows.

$$y_1 = 0.3 \times 90.91 + 0.25 \times 90.38 + 0.25 \times 90.33 + 0.20 \times 87.13 = 89.87,$$

$$y_2 = 0.3 \times 90.67 + 0.25 \times 82.99 + 0.25 \times 80.65 + 0.20 \times 77.03 = 84.24,$$

$$y_3 = 0.3 \times 90.50 + 0.25 \times 86.45 + 0.25 \times 85.93 + 0.20 \times 82.64 = 84.61,$$

$$y_4 = 0.3 \times 93.23 + 0.25 \times 92.17 + 0.25 \times 88.10 + 0.20 \times 79.78 = 85.58.$$

Step 5: Rank the strategies and make a decision. The rank of these four strategies is $o_2 \succ o_3 \succ o_4 \succ o_1$, so the best choice of strategy is electric vehicles (o_2).

6.2. Comparative analysis

We also analyze the evaluation and rank results of these four strategies by changing the WHDIOWA operator to the NHDIOWA, Median-NHDIOWA, Median-WHDIOWA, Olympic-NHDIOWA, Olympic-WHDIOWA, NHDIOWQA, WHDIOWQA, NHDIOWHA, and WHDIOWHA operators, which are shown in Table 5 and Table 6.

Table 5: Evaluation results 1

	NHDIOWA	Median-NHDIOWA	Median-WHDIOWA	Olympic-NHDIOWA	Olympic-WHDIOWA	NHDIOWQA	WHDIOWQA	NHDIOWHA	WHDIOWHA
o_1	91.26	85.95	78.20	86.50	83.60	91.28	89.88	91.22	89.84
o_2	86.06	81.65	83.40	80.67	80.47	86.14	84.35	85.90	84.03
o_3	87.67	86.20	87.20	84.27	85.28	87.67	84.62	87.67	84.57
o_4	86.93	81.50	84.65	81.58	82.23	87.01	85.75	86.78	85.23

As we can see, the best strategy for most cases is o_2. However, we may find other best choices. Therefore, it is of interest to rank all candidates for each particular case. We can also see that using different DIOWA operators may lead to different ranks of candidates. Therefore, different decisions may be obtained by using different DIOWA operators.

Table 6: Ranks of strategies 1

	Rank
NHDIOWA	$o_2 \succ o_4 \succ o_3 \succ o_1$
WHDIOWA	$o_2 \succ o_3 \succ o_4 \succ o_1$
Median-NHDIOWA	$o_4 \succ o_2 \succ o_3 \succ o_1$
Median-WHDIOWA	$o_1 \succ o_2 \succ o_4 \succ o_3$
Olympic-NHDIOWA	$o_2 \succ o_4 \succ o_3 \succ o_1$
Olympic-WHDIOWA	$o_2 \succ o_4 \succ o_1 \succ o_3$
NHDIOWQA	$o_2 \succ o_4 \succ o_3 \succ o_1$
WHDIOWQA	$o_2 \succ o_3 \succ o_4 \succ o_1$
NHDIOWHA	$o_2 \succ o_4 \succ o_3 \succ o_1$
WHDIOWHA	$o_2 \succ o_3 \succ o_4 \succ o_1$

Table 7: Evaluation results 2

	Max	Min	OWA	OWAD	OWAWAD	NHD	WHD
o_1	93.00	77.60	80.27	1.26	1.22	12.06	10.92
o_2	88.95	76.20	77.71	2.32	2.16	16.40	15.91
o_3	89.70	71.85	77.86	2.04	2.21	14.81	13.48
o_4	90.40	74.40	77.06	2.19	1.91	16.04	16.30

For comparison, we also further use the Max, Min, OWA, OWAD, OWAWAD, NHD, and WHD operators in this example. The evaluation results and corresponding ranks are listed in Table 7 and Table 8.

We can see that the ranks in Table 8 are different from the ranks in Table 6. This shows that DIOWA operators provide DMs with more scenarios to consider for making better decisions. Another phenomenon is that the ranks of the DIOWA operators in Table 6 are different from the ranks of the distance operators in Table 8. This shows that it is necessary to research order-inducing variables represented by distance measures in aggregation operators.

Furthermore, we also use the IOWA, IOWAD, IOWAWD, DIOWA, and NHDIOWA operators in this example. The evaluation results and corresponding ranks are listed in Table 9 and Table 10.

In Table 8, we can see that the rank of the IOWA operator is different from the ranks of the other five operators, which all consider the distance measures in the aggregation process. So, it is further shown that

Table 8: Ranks of strategies 2

	Rank
Max	$o_2 \succ o_3 \succ o_4 \succ o_1$
Min	$o_3 \succ o_4 \succ o_2 \succ o_1$
OWA	$o_4 \succ o_2 \succ o_3 \succ o_1$
OWAD	$o_1 \succ o_3 \succ o_4 \succ o_2$
OWAWAD	$o_1 \succ o_4 \succ o_2 \succ o_3$
NHD	$o_1 \succ o_3 \succ o_4 \succ o_2$
WHD	$o_1 \succ o_3 \succ o_2 \succ o_4$

Table 9: Evaluation results 3

	IOWA	IOWAD	IOWAWD	DIOWA	NHDIOWA	WHDIOWA
o_1	90.94	1.57	1.37	92.99	91.50	89.87
o_2	88.54	2.98	2.47	88.32	86.36	84.24
o_3	85.99	2.26	1.92	90.96	89.13	84.61
o_4	89.85	2.06	1.72	91.89	90.47	85.58

Table 10: Ranks of strategies 3

	Rank
IOWA	$o_1 \succ o_3 \succ o_4 \succ o_2$
IOWAD	$o_1 \succ o_4 \succ o_3 \succ o_2$
IOWAWD	$o_1 \succ o_4 \succ o_3 \succ o_2$
DIOWA	$o_2 \succ o_3 \succ o_4 \succ o_1$
NHDIOWA	$o_2 \succ o_3 \succ o_4 \succ o_1$
WHDIOWA	$o_2 \succ o_3 \succ o_4 \succ o_1$

considering distance measures in aggregation operators will afford more choices for DMs to select. We can also see that the ranks of operators in which the distance measures are regarded as aggregated argument variables are different from the ranks of operators in which they are regarded as order-inducing variables. Therefore, it is further verified that it is necessary to research the distance-induced aggregation operators, as our research does.

7. Evaluation indicator system in China - a case study

S&T is the key factor in the strength and sustainable development of a country. With China's increasing development, S&T has also been continuously improved. As an effective tool for S&T management, S&T evaluation plays an important role and is always used to evaluate the level of S&T from different aspects for different regions. Researchers often use different evaluation methods for the activities of S&T evaluation, and this is also the case in China. Therefore, based on the operators proposed in this paper and corresponding decision making frameworks, we also hope that our proposed operators will contribute to S&T evaluation problems. So, this paper takes the evaluation of the S&T development level as the evaluation target and takes the 31 provinces in China as the evaluated objects. The S&T development levels of 31 provinces in China are evaluated and analyzed in this section. The evaluation framework and processes are shown in Fig. 2.

7.1. Construction of evaluation index system

Based on the combined consideration of the existing research results about the evaluation of S&T (Shi et al., 2016; Chi et al., 2011; Gu et al., 2010; Chi et al., 2008; National Bureau of Statistics, 2021a), an

Table 11: Evaluation index system of development level of provincial S&T

No.	Criterion	Indicator	Unit
1	S&T input intensity X_1	R&D investment intensity A1	%
2		R&D personnel input intensity A2	%
3		R&D full-time staff input intensity A3	%
4		New product development investment intensity A4	%
5		R&D Personnel project approval intensity A5	Items per person
6		R&D Project funding intensity A6	Ten thousand yuan per item
7	S&T output intensity X_2	Domestic invention patent application intensity B1	%
8		Domestic invention patent approval intensity B2	%
9		Output intensity of technology market B3	%
10		New product development intensity B4	Items of per enterprise
11		New product revenue intensity B5	%
12		Intensity of papers published in foreign retrieval journals B6	Papers for per S&D staff
13	S&T service intensity X_3	Qualified rate of product quality C1	%
14		Service intensity of unit maker space C2	Enterprises
15		Maker space to absorb employment intensity C3	Person for per maker space
16		Service intensity of teachers and students in Colleges and Universities C4	%
17		Input intensity of science popularization personnel per thousand people C5	Persons
18		Unit business incubator incubation intensity C6	Numbers

evaluation indicator system was developed in China by selecting indicators from three aspects: S&T input intensity, S&T output intensity, and S&T service intensity. In this way, a set of 18 indicators was developed and these are shown in Table 11.

All 18 indicators in Table 11 are benefit indicators.

We evaluated the S&T development level of 31 provinces in 2020, because the newest data we could obtain were only updated to 2020. The data of all the indicators or their related indicators were collected from the China Statistical Year Book (2021) (National Bureau of Statistics, 2021a), the China Statistical Yearbook on Science and Technology (2021) (National Bureau of Statistics, 2021b), and the website of the National Bureau of Statistics (http://www.stats.gov.cn).

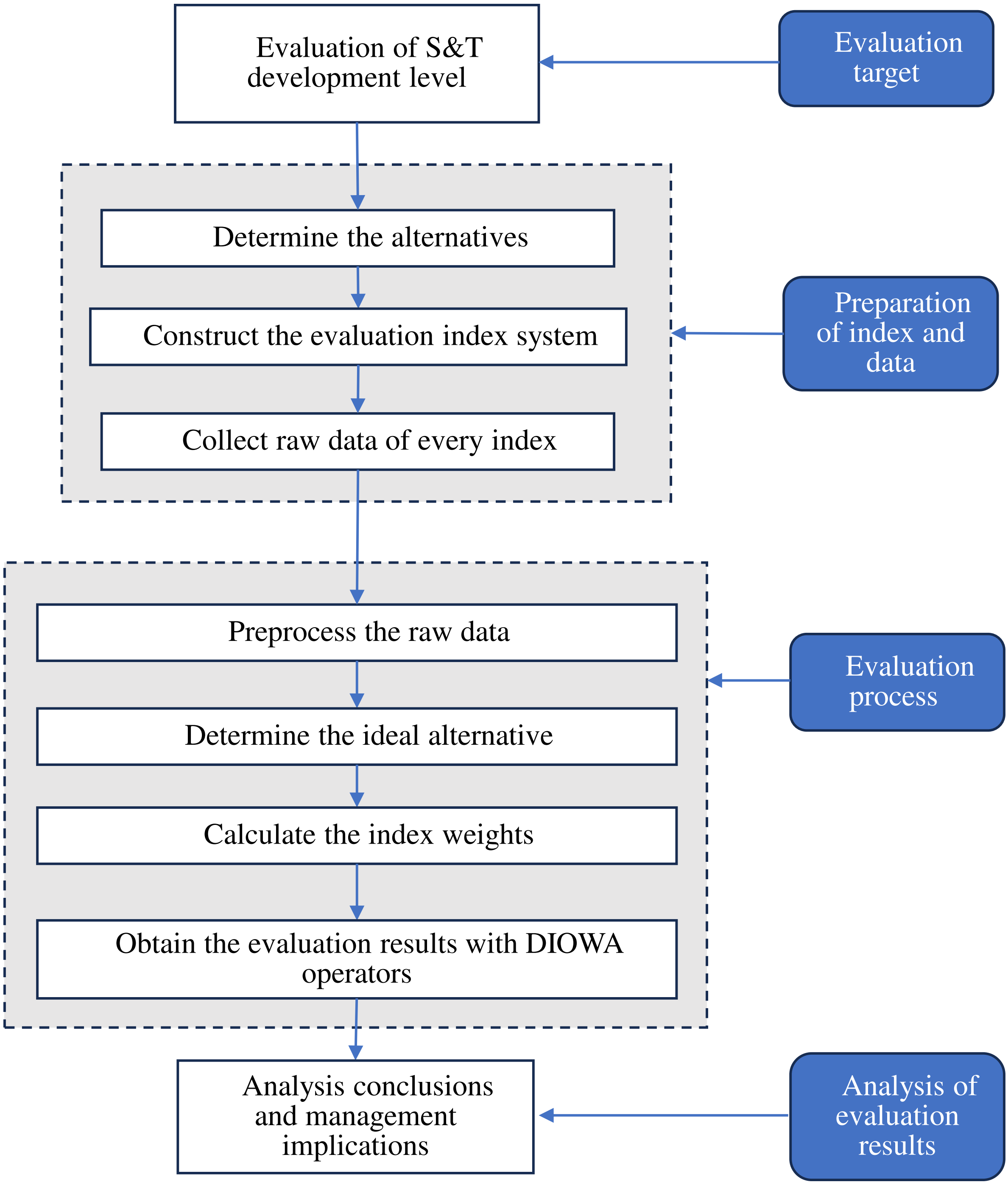


Figure 2: Evaluation framework of S&T development level in China

Table 12: Indicator weights for every criterion

	A1	A2	A3	A4	A5	A6
ST input intensity X_1	0.116	0.099	0.345	0.141	0.147	0.152
	B1	B2	B3	B4	B5	B6
ST output intensity X_2	0.176	0.171	0.139	0.176	0.168	0.169
	C1	C2	C3	C4	C5	C6
ST service intensity X_3	0.180	0.163	0.160	0.171	0.174	0.152

7.2. Evaluation procedure of S&T development level

Below, we evaluate the S&T development level of 31 provinces in China according to the decision making framework for situation 1 proposed in Section 5.

Step 1: Preprocess the original data of the 18 indicators to eliminate the influence of different dimensions with the following formula for normalization

$$x_{ij}^{k} = \frac{x_{ijk}}{\max\limits_{i}\{x_{ijk}\}} \tag{25}$$

where x_{ijk} represents the original value of the jth ($j = 1, 2, \cdots, 6$) indicator about the ith ($i = 1, 2, \cdots, 31$) province corresponding to the kth ($k = 1, 2, 3$) criterion.

Step 2: Choose the highest value for every indicator (of 31 provinces in China) to be treated as the ideal argument values, so all the ideal argument values are assigned the value of 1.

Step 3: After processing the ideal argument values, calculate the weights for the six indicators of every criterion with the following formula

$$\omega_j^{(k)} = \frac{\sum\limits_{i=1}^{n}\left(1 - \left|x_{ij}^{(k)} - \max\left\{x_{ij}^{(k)}\right\}\right|\right)}{\sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}\left(1 - \left|x_{ij}^{(k)} - \max\left\{x_{ij}^{(k)}\right\}\right|\right)} \tag{26}$$

where $\omega_j^{(k)}$ for $j = 1, 2, \cdots, 6$ representing the weights of six indicators of the kth ($k = 1, 2, 3$) criterion. It can be found that the sum of the six indicators of every criterion is 1. The weights of the indicators of every criterion are shown in Table 12.

Step 4: Once the weights are calculated, use the WHDIOWA operator to calculate the evaluation results of the three criteria for every province. The results are shown in Table 13.

Step 5: Also use the proposed WHDIOWA operator to determine the overall S&T development level of

Table 13: Evaluation results of three criteria for 31 provinces in China

Province	X_1	X_2	X_3	Province	X_1	X_2	X_3
Beijing	0.736	0.853	0.947	Hubei	0.387	0.456	0.560
Tianjin	0.499	0.488	0.567	Hunan	0.407	0.375	0.552
Hebei	0.374	0.335	0.483	Guangdong	0.609	0.367	0.492
Shanxi	0.277	0.463	0.504	Guangxi	0.225	0.477	0.479
Inner Mongolia	0.261	0.314	0.542	Hainan	0.234	0.464	0.622
Liaoning	0.355	0.502	0.563	Chongqing	0.362	0.380	0.537
Jilin	0.285	0.573	0.475	Sichuan	0.326	0.399	0.542
Heilongjiang	0.227	0.616	0.525	Guizhou	0.221	0.305	0.506
Shanghai	0.575	0.577	0.633	Yunnan	0.254	0.287	0.550
Jiangsu	0.575	0.347	0.519	Xizang	0.208	0.432	0.527
Zhejiang	0.572	0.352	0.575	Shaanxi	0.367	0.569	0.555
Anhui	0.434	0.493	0.498	Gansu	0.292	0.338	0.515
Fujian	0.382	0.265	0.525	Qinghai	0.238	0.337	0.556
Jiangxi	0.403	0.236	0.586	Ningxia	0.271	0.344	0.534
Shandong	0.446	0.357	0.514	Xinjiang	0.249	0.313	0.531
Henan	0.357	0.327	0.545				

these 31 provinces. The weights of the three criteria are determined according to the following formula

$$w_k = \frac{\sum_{i=1}^{n} \left(1 - \left|x_{ik} - \max_i \{x_{ik}\}\right|\right)}{\sum_{k=1}^{p} \sum_{i=1}^{n} \left(1 - \left|x_{ik} - \max_i \{x_{ik}\}\right|\right)} \tag{27}$$

where w_k for $k = 1, 2, 3$ are the weights of three criteria, x_{ik} is the evaluation result of the kth criterion of the ith province. The weights for the three criteria are 0.351, 0.314, and 0.335, respectively.

Step 6: Apply the WHDIOWA operator to aggregate the evaluation results of the three criteria to obtain the final evaluation score of the S&T development level of each province. The evaluation results are listed in Table 14.

7.3. Analysis of evaluation results

We illustrate the evaluation results of every criterion in Fig. 3 to Fig. 5 (data in Table 13).

From Fig. 3 we can see that Beijing, Guangdong, and Jiangsu are the top three provinces for the development level of S&T input intensity and Guangxi, Guizhou, and Xizang are the worst three provinces. Their development levels of S&T input intensity are only about one third of that of Beijing.

From Fig. 4, we can see that Beijing, Heilongjiang, and Shanghai are the top three provinces for the development level of S&T output intensity. Less than half of the 31 provinces (13 provinces) have a development level of S&T output intensity higher than the average. Yunnan, Fujian, and Jiangxi are the

Table 14: Evaluation results of S&T development level of 31 provinces in China

Province	Evaluation result	Rank	Province	Evaluation result	Rank	Province	Evaluation result	Rank
Beijing	0.843	1	Hunan	0.446	12	Guangxi	0.389	23
Shanghai	0.595	2	Shandong	0.441	13	Xizang	0.385	24
Tianjin	0.519	3	Jilin	0.439	14	Ningxia	0.382	25
Zhejiang	0.504	4	Hainan	0.436	15	Gansu	0.381	26
Guangdong	0.494	5	Chongqing	0.426	16	Qinghai	0.376	27
Shaanxi	0.493	6	Sichuan	0.421	17	Inner Mongolia	0.372	28
Jiangsu	0.484	7	Jiangxi	0.412	18	Xinjiang	0.364	29
Anhui	0.474	8	Shanxi	0.411	19	Yunnan	0.364	30
Liaoning	0.471	9	Henan	0.411	20	Guizhou	0.343	31
Hubei	0.466	10	Hebei	0.398	21			
Heilongjiang	0.449	11	Fujian	0.393	22			

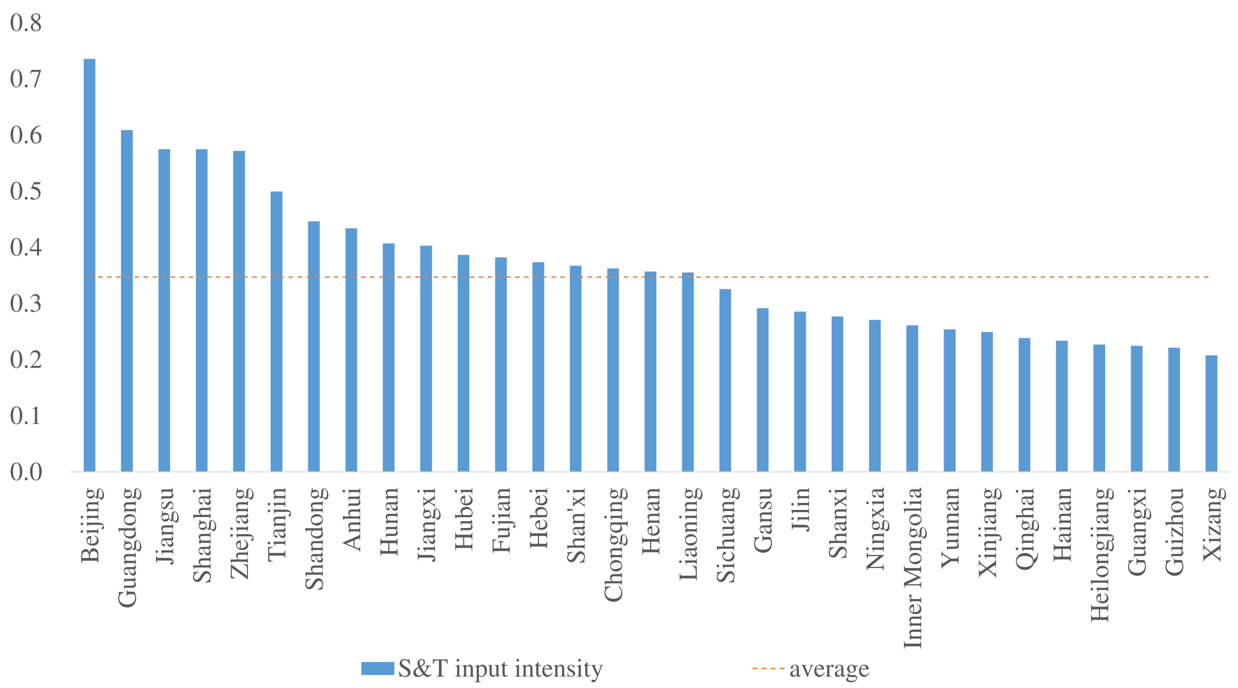


Figure 3: Evaluation results of S&T input intensity of 31 provinces in China

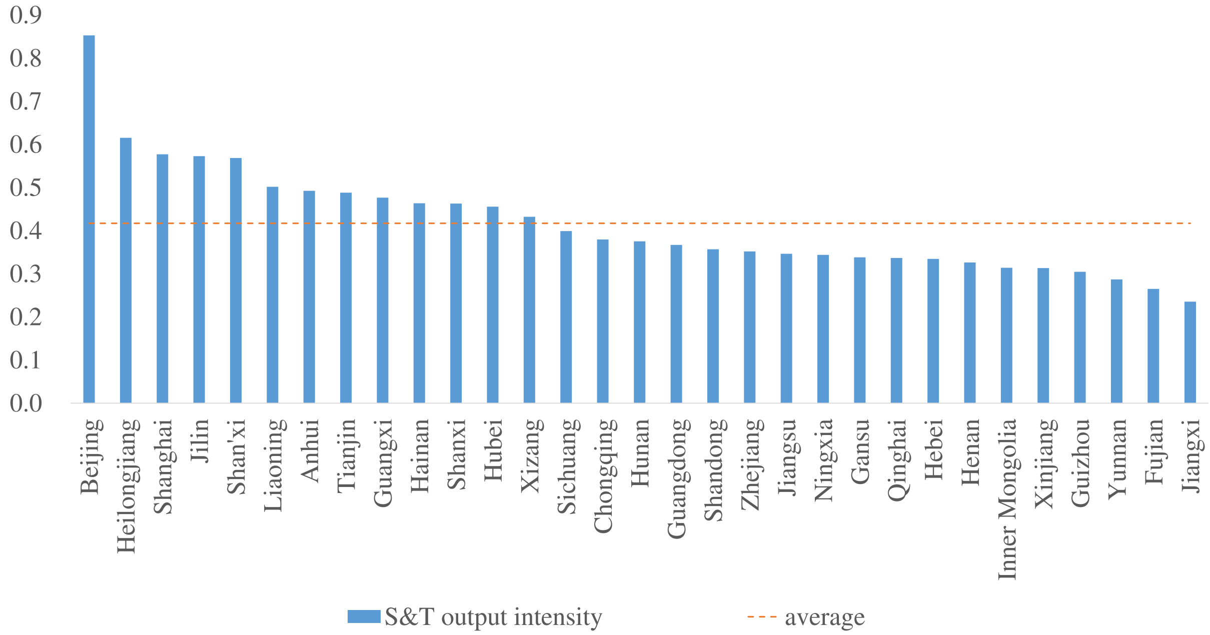


Figure 4: Evaluation results of S&T output intensity of 31 provinces in China

worst three provinces: their development levels of S&T output intensity are even less than a third of that of Beijing.

From Fig. 5 we can see that Beijing, Shanghai, and Hainan are the top three provinces for the development level of S&T service intensity. Less than half of the 31 provinces (11 provinces) have a development level of S&T service intensity higher than the average. Hebei, Guangxi, and Jilin are the worst three provinces: their development levels of S&T service intensity are about half that of Beijing.

We convert Table 14 to Fig. 6 to show the evaluation results of the S&T development level of 31 provinces in China.

In Fig. 6, the line in green is the average S&T development level of the corresponding area, i.e., the east, the midlands, the northeast, and the west of China. The line in red is the average S&T development level of all 31 provinces.

From Fig. 6 we find that the top three provinces with the best S&T development level are Beijing, Shanghai, and Tianjin. Less than half of the 31 provinces (12 provinces) have an S&T development level higher than the average level across all 31 provinces. The three provinces with the worst S&T development levels are Xinjiang, Yunnan, and Guizhou. Their S&T development levels are less than half that of Beijing.

From the viewpoint of the regional development level of S&T, the ranking of the S&T development

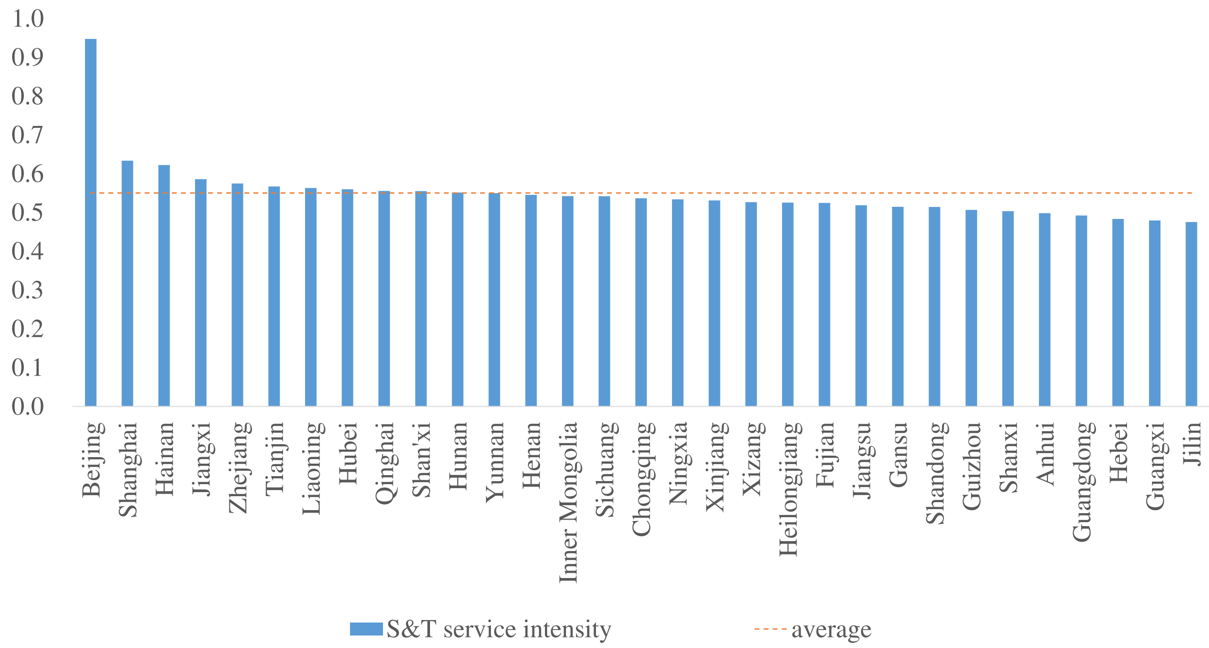


Figure 5: Evaluation results of S&T service intensity of 31 provinces in China

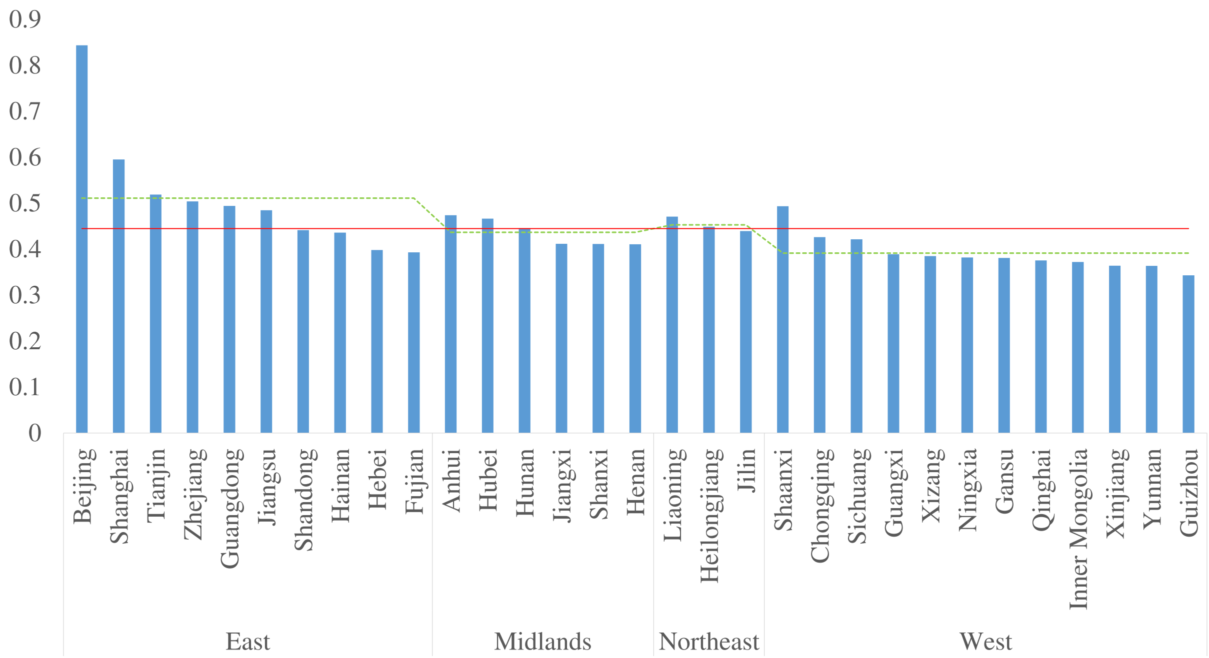


Figure 6: S&T development level of 31 provinces in China

level for the four regions is, from best to worst, east, northeast, midlands, and west. The average S&T development level of the east and the northeast of China is higher than the average level across all 31 provinces. The average S&T development level of the midlands and the west of China is lower than the average level across all 31 provinces. Specifically, the provinces in the east of China with the best and worst S&T development levels are, respectively, Beijing and Fujian. Only 3 in 7 provinces with an S&T development level less than the average level of the total 31 provinces. However, only 3 in 6 provinces with an S&T development level higher than the average level of the east of China. The provinces with the best and the worst S&T development levels in the midlands of China are, respectively, Anhui and Henan. Here, half of the six provinces have an S&T development higher than the average level across all 31 provinces and also higher than the average level of the midlands. In northeast China, the provinces with the best and worst S&T development levels are, respectively, Heilongjiang and Jilin. Only Jilin has an S&T development level lower than both the average across all 31 provinces and the average of the northeast of China.

7.4. Management implications

Looking at the evaluation results, some useful suggestions for improving the development of S&T in China are given as follows:

(1) More attention should be given to the provinces besides Beijing to narrow the gap between other provinces and Beijing. In particular, more attention should be given to the provinces in the west of China.

(2) In the east of China, more attention should be given to Hannan, Hebei, and Fujian to improve their S&T development levels. Specifically, more action should be taken to improve the S&T input intensity of Hannan, and more action should be taken to improve the S&T input intensity and the S&T output intensity of Hebei and Fujian.

(3) In the midlands of China, more attention should be given to Jiangxi, Shanxi, and Henan to improve their S&T development levels. Specifically, more action should be taken to improve the S&T output intensity of Jiangxi and the S&T input intensity of Shanxi.

(4) In the northeast of China, more attention should be given to Jilin to improve its S&T development level. More action should be taken to improve its S&T input intensity.

(5) More attention should be given to almost all the provinces of the west of China. More action should be taken to improve their S&T input intensity and S&T output intensity.

Looking at the evaluation process, some useful suggestions to further improve the usefulness of evaluating the development level of S&T in China are given as follows:

(1) Extending the proposed DIOWA operators to group decision making to combine different types of proposed DIOWA operator and invite more DMs will be beneficial to scientifically evaluate the development level of S&T in China.

(2) Different evaluation backgrounds and evaluation demands should be considered, which require DMs to combine the proposed DIOWA operators with other information aggregation methods or decision making methods, to better illustrate the development level of S&T of China.

(3) Explore the construction of an evaluation indicator system that considers the difference between different provinces of China will be beneficial to assess the development level of S&T in China.

8. Conclusions

8.1. Summary of the study

Most existing operators considering distance such as the OWAD and IOWAD operators usually regard distance measures as aggregated argument variables. The most distinctive benefit of these distance operators is that the ideal argument values could be introduced to a decision making problem. Yet these distance operators will fail to deal with situations where we consider attribute values rather than distance measures as aggregated argument variables.

In this paper, we have determined order-inducing variables by distance measures and have proposed a generalized form of the DIOWA operators with the Hamming distance. This provides a parameterized family of aggregation operators that ranges from the minimum to the maximum. Three basic types of DIOWA operators – the HDIOWA, NHDIOWA, and WHDIOWA operators – with different types of Hamming distance measure have also been proposed. We have studied some main properties of the DIOWA operators and suggested three methods to determine their associated weights. We have also analyzed families of DIOWA operators with different manifestations of associated weighting vectors and their ideal argument values. We have further generalized DIOWA operators by using generalized means and quasi-arithmetic means. These distance-induced operators can also consider some ideal argument values for a decision making problem. And, because we regard distance measures as order-inducing variables, attribute values can also be taken as aggregated argument variables at the same time. Moreover, we have also suggested two frameworks of MCDM methods with the DIOWA operators that considered two situations. We have also applied this framework to a decision making problem about the selection of the best strategy for a vehicle manufacturing company to verify that the operators proposed can lead to different rankings aiming at the same decision making problem and accordingly afford more scenarios for DMs to select. At the end of this study, we presented a case study on the problem of evaluating the S&T development levels of 31 Chinese provinces by using our proposed WHDIOWA operator and decision making framework for situation 2.

8.2. Future research

In this study, we only considered the Hamming distance to calculate the order-inducing variables. This will fail in some decision situations where we need to consider other distance measure methods. With increasing decision complexity, the proposed operators will not be suitable for uncertain decision problems. Furthermore, although we give three weighting methods, how the weights for the proposed operators can be determined, which will have great effects on the decision process, is also an issue that needs to be solved.

Therefore, for future research, we will continue to study distance-induced operators from three aspects to make more contributions to decision making problems. First, we will obtain more generalized distance-induced operators for the decision situation where the decision making information is represented by certain information, i.e., crisp numbers, with different distance measure methods, such as the Euclidean distance, the Minkowski distance, and the Hausdorff distance. Therefore, our research in this paper will be a special case for this future research. We will also use these new distance-induced operators to solve decision making problems in the fields, such as the selection of electric vehicle charging station sites (Seikh and Mandal, 2022), the evaluation of bio-medical waste management (Seikh and Mandal, 2023), and performance evaluation. Second, to solve the uncertain decision making problems, we will propose several distance-induced operators to aggregate uncertain information which will be represented by various types of fuzzy numbers, including hesitant fuzzy numbers, intuitionistic fuzzy numbers, linguistic fuzzy numbers, fuzzy numbers with probability, and trapezoidal Pythagorean fuzzy numbers, based on their existing distance measure methods. Additionally, we will develop some new fuzzy numbers to represent uncertain information based on the demand of real applications and then propose corresponding distance and similarity measure methods. We will further develop distance-induced operators for these newly built fuzzy numbers based on the research above to present more selections for DMs or experts to make better decisions. We will also apply these operators to deal with some real applications in the fields illustrated above. Third, weights are very important to distance-induced operators, so we will also pay more attention to the determination methods of weights for both the operators proposed in this paper and new operators that will be built in the future. We will focus on how distance information can be combines with preference information to determine weights and on how the weights of DMs or experts and the weights of criteria can be determined in group decision making when using distance-induced operators.

9. Acknowledgments

This work was supported by the project of the National Natural Science Foundation of China (71901079).

References

Aggarwal, M. (2019). Soft information set for multicriteria decision making. *International Journal of Intelligent Systems*, 34:3241–3259.

Ahn, B. S. (2006a). On the properties of owa operator weights functions with constant level of orness. *IEEE Transactions on Fuzzy Systems*, 14:511–515.

Ahn, B. S. (2006b). The uncertain owa aggregation with weighting functions having a constant level of orness. *International Journal of Intelligent Systems*, 10:469–483.

Alfaro-Garcia, V. G., Merigo, J. M., Gil-Lafuente, A. M., and Kacprzyk, J. (2018). Logarithmic aggregation operators and distance measures. *International Journal of Intelligent Systems*, 10:1488–1506.

Aviles-Ochoa, E., Leon-Castro, E., Perez-Arellano, L. A., and Merigo, J. M. (2018). Government transparency measurement through prioritized distance operators. *International Journal of Fuzzy Systems*, 34:2783–2794.

Balopoulos, V., Hatzimichailidis, A. G., and Papadopoulos, B. K. (2007). Distance and similarity measures for fuzzy operators. *Information Sciences*, 177:2336–2348.

Beliakov, G. and James, S. (2011). Induced ordered weighted averaging operators. In Yager, R. R., Kacprzyk, J., and Beliakov, G., editors, *Recent Developments in the Ordered Weighted Averaging Operators: Theory and Practice*, pages 29–47. Springer Berlin Heidelberg, Berlin, Heidelberg.

Boukezzoula, R. and Coquin, D. (2020). A decision-making computational methodology for a class of type-2 fuzzy intervals, an interval-based approach. *Information Sciences*, 510:256–282.

Casanovas, M., Torres-Martinez, A., and Merigo, J. M. (2016). Decision making in reinsurance with induced owa operators and minkowski distances. *Cybernetics and Systems*, 47:460–477.

Chen, J., Zeng, S. Z., and Zhang, C. H. (2018). An owa distance-based, single-valued neutrosophic linguistic topsis approach for green supplier evaluation and selection in low-carbon supply chains. *International Journal of Environmental Research and Public Health*, 15:1439–1453.

Chi, G. T., Gu, X. S., and Wang, W. (2011). The evaluation model of science and technology based on relational analysis and empirical study on typical provinces. *Science Research Management*, 32:68–78.

Chi, G. T., Zhu, Z. C., and Zhang, Y. L. (2008). The science and technology evaluation model based on entropy and g1 and empirical research of china. *Studies in Science of Science*, 26:1210–1220.

Chiclana, F., Herrera-Viedma, E., Herrera, F., and Alonso, S. (2004). Induced ordered weighted geometric operators and their use in the aggregation of multiplicative preference relations. *International Journal of Intelligent Systems*, 19:233–255.

Chiclana, F., Herrera-Viedma, E., Herrera, F., and Alonso, S. (2007). Some induced ordered weighted averaging operators and their use for solving group decision-making problems based on fuzzy preference relations. *European Journal of Operational Research*, 182:383–399.

Gadomer, L. and Sosnowski, Z. A. (2019). Knowledge aggregation in decision-making process with c-fuzzy random forest using owa operators. *Soft Computing*, 23:3741–3755.

Gong, C. J., Su, Y., Liu, W. W., Hu, Y., and Zhou, Y. (2020). The distance induced owa operator with application to multi-criteria group decision making. *International Journal of Fuzzy Systems*, 22:1624–1634.

Gong, C. J., Yi, P. T., and Li, W. W. (2019). Research on evaluation method of provincial science and technology development level based on dual driven. *Studies in Science of Science*, 37:1589–1597.

Gu, X. S., Chi, G. T., and Cheng, H. (2010). The establishment of science and technology evaluation indicators system based on cluster - factor analysis. *Studies in Science of Science*, 28:508–514.

Hamming, R. W. (1950). Error-detecting and error-correcting codes. *The Bell System Technical Journal*, 29:147–160.

Hussian, Z. and Yang, M. S. (2019). Distance and similarity measures of pythagorean fuzzy sets based on the hausdorff metric with application to fuzzy topsis. *International Journal of Intelligent Systems*, 34:2633–2654.

Ishizaka, A. and Siraj, S. (2018). Are multi-criteria decision-making tools useful? an experimental comparative study of three methods. *European Journal of Operational Research*, 264:462–471.

Kumar, K. and Chen, S. M. (2022). Group decision making based on advanced intuitionistic fuzzy weighted heronian mean aggregation operator of intuitionistic fuzzy values. *Information Sciences*, 601:306–322.

Li, C. G., Zeng, S. Z., and Pan, T. J. (2014). A method based on induced aggregation operators and distance measures to multiple attribute decision making under 2-tuple linguistic environment. *Journal of Computer and System Sciences*, 80:1339–1349.

Liao, H. C. and Xu, Z. S. (2015). Approaches to manage hesitant fuzzy linguistic information based on the cosine distance and similarity measures for hfltss and their application in qualitative decision making. *Expert Systems with Applications*, 42:5328–5336.

Liu, C. P. and Peng, B. (2017). A method for group decision making based on interval-valued intuitionistic fuzzy geometric distance measures. *Informatica*, 28:453–470.

Liu, X., Tao, Z. F., Chen, H. Y., and Zhou, L. G. (2022). Probabilistic linguistic term ordered weighted distance operator and its applications in public eco-environment satisfaction evaluation. *Journal of Control and Decision*, 9:58–70.

Merigo, J. M. (2012). Probabilities in the owa operator. *Expert Systems with Applications*, 10:11456–11467.

Merigo, J. M. (2013). The probabilistic weighted averaging distance and its application in group decision making. *Kybernetes*, 42:686–697.

Merigo, J. M. (2015). Decision-making under risk and uncertainty and its application in strategic management. *Journal of Business Economics and Management*, 16:93–116.

Merigo, J. M. and Casanovas, M. (2010a). Decision making with distance measures and linguistic aggregation operators. *International Journal of Fuzzy Systems*, 12:190–198.

Merigo, J. M. and Casanovas, M. (2010b). Induced and heavy aggregation operators with distance measures. *Journal of Systems Engineering and Electronics*, 21:431–439.

Merigo, J. M. and Casanovas, M. (2011a). Decision-making with distance measures and induced aggregation operators. *Computers & Industrial Engineering*, 60:66–76.

Merigo, J. M. and Casanovas, M. (2011b). Induced aggregation operators in the euclidean distance and its application in financial decision making. *Expert Systems with Applications*, 38:7603–7608.

Merigo, J. M. and Casanovas, M. (2011c). Induced and uncertain heavy owa operators. *Computers & Industrial Engineering*, 60:106–116.

Merigo, J. M. and Casanovas, M. (2011d). A new minkowski distance based on induced aggregation operators. *International*

Journal of Computational Intelligence Systems, 4:123–133.

Merigo, J. M., Casanovas, M., and Zeng, S. Z. (2014). Distance measures with heavy aggregation operators. *Applied Mathematical Modelling*, 38:3142–3153.

Merigo, J. M. and Gil-Lafuente, A. M. (2009). The induced generalized owa operator. *Information Sciences*, 179:729–741.

Merigo, J. M. and Gil-Lafuente, A. M. (2010). New decision-making techniques and their application in the selection of financial products. *Information Sciences*, 180:2085–2094.

Merigo, J. M., Palacios-Marques, D., and Soto-Acosta, P. (2017). Distance measures, weighted averages, owa operators and bonferroni means. *Applied Soft Computing*, 50:356–366.

Merigo, J. M., Xu, Y. J., and Zeng, S. Z. (2013). Group decision making with distance measures and probabilistic information. *Knowledge-Based Systems*, 40:81–87.

National Bureau of Statistics (2021a). *2021 China statistical yearbook*. China Statistic Press.

National Bureau of Statistics (2021b). *2021 China statistical yearbook on science and technology*. China Statistic Press.

Petry, F. E. and Yager, R. R. (2022). Interval-valued fuzzy sets aggregation and evaluation approaches. *Applied Soft Computing*, 124:Article 108887.

Rao, C. J., Gao, M. Y., Wen, J. H., and Goh, M. (2022). Multi-attribute group decision making method with dual comprehensive clouds under information environment of dual uncertain z-numbers. *Information Sciences*, 10:106–127.

Seikh, M. R. and Mandal, U. (2022). Multiple attribute group decision making based on quasirung orthopair fuzzy sets: Application to electric vehicle charging station site selection problem. *Engineering Applications of Artificial Intelligence*, 115:Article 105299.

Seikh, M. R. and Mandal, U. (2023). Interval-valued fermatean fuzzy dombi aggregation operators and swara based promethee ii method to bio-medical waste management. *Expert Systems with Applications*, 226:Article 120082.

Shi, B. F., Cheng, Y. Q., and Wang, J. (2016). A combination empowerment model based on variation coefficient weighted and its empirical study of s&t evaluation. *Science Research Management*, 37:122–131.

Singh, P. (2017). Distance and similarity measures for multiple-attribute decision making with dual hesitant fuzzy sets. *Computational & Applied Mathematics*, 36:111–126.

Su, W. H., Li, W., Zeng, S. Z., and Zhang, C. H. (2014). Atanassov's intuitionistic linguistic ordered weighted averaging distance operator and its application to decision making. *Journal of Intelligent & Fuzzy Systems*, 26:1491–1502.

Su, W. H., Zeng, S. Z., and Ye, X. J. (2013). Uncertain group decision-making with induced aggregation operators and euclidean distance. *Technological and Economic Development of Economy*, 19:431–447.

Vizuete-Luciano, E., Merigo, J. M., Gil-Lafuente, A. M., and Boria-Reverter, S. (2015). Decision making in the assignment process by using the hungarian algorithm with owa operators. *Technological And Economic Development of Economy*, 21:684–704.

Wang, J. S., Cao, C. D., Zeng, S. Z., and Balezentis, T. (2019). Weighted induced aggregation euclidean distance operators for the decision making of robot selection. *Transformations in Business & Economics*, 18:81–94.

Wei, C. P. and Tang, X. J. (2012). Generalized prioritized aggregation operators. *International Journal of Intelligent Systems*, 27:578–589.

Wu, L. F., Liu, S. F., and Yang, Y. J. (2015). A model to determine owa weights and its application in energy technology evaluation. *International Journal of Intelligent Systems*, 30:798–806.

Xian, S. D. and Sun, J. W. (2014). Fuzzy linguistic induced euclidean owa distance operator and its application in group linguistic decision making. *International Journal of Intelligent Systems*, 29:478–491.

Xian, S. D., Sun, W. J., Xu, S. H., and Gao, Y. Y. (2016). Fuzzy linguistic induced owa minkowski distance operator and its application in group decision making. *Pattern Analysis and Applications*, 19:325–335.

Xian, S. D., Xiao, Y., Li, L., and Yu, D. X. (2019). Trapezoidal pythagorean fuzzy linguistic entropic combined ordered weighted minkowski distance operator based on preference relations. *International Journal of Intelligent Systems*, 34:2196–2224.

Xu, Y. and Wang, X. S. (2020). Three-way decision based on improved aggregation method of interval loss function. *Information Sciences*, 508:214–233.

Xu, Z. S. (2006). Induced uncertain linguistic owa operators applied to group decision making. *Information Fusion*, 7:231–238.

Xu, Z. S. and Chen, J. (2008). Ordered weighted distance measure. *Journal of Systems Science and Systems Engineering*, 17:432–445.

Xu, Z. S. and Da, Q. L. (2002). The uncertain owa operator. *International Journal of Intelligent Systems*, 17:569–575.

Xu, Z. S. and Xia, M. M. (2011). Distance and similarity measures for hesitant fuzzy sets. *Information Sciences*, 181:2128–2138.

Yager, R. R. (1988). On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Transactions on Systems, Man, and Cybernetics*, 18:183–190.

Yager, R. R. (2002). Heavy owa operators. *Fuzzy Optimization and Decision Making*, 1:379–397.

Yager, R. R. and Alajlan, N. (2019). Importance-based multicriteria decision making with interval valued criteria satisfactions. *International Journal of Intelligent Systems*, 34:3336–3344.

Yager, R. R. and Filev, D. P. (1999). Induced ordered weighted averaging operators. *IEEE Transactions on Systems Man And Cybernetics Part B - Cybernetics*, 29:141–150.

Zeng, S. Z. (2013). Decision making with the uncertain imowad operator. *Kybernetes*, 42:785–799.

Zeng, S. Z. (2016). An extension of owad operator and its application to uncertain multiple-attribute group decision-making. *Cybernetics and Systems*, 47:363–375.

Zeng, S. Z., Llopis-Albert, C., and Zhang, Y. H. (2018). A novel induced aggregation method for intuitionistic fuzzy set and its application in multiple attribute group decision making. *International Journal of Intelligent Systems*, 33:2175–2188.

Zeng, S. Z., Merigo, J. M., Palacios-Marques, D., Jin, H. H., and Gu, F. J. (2017). Intuitionistic fuzzy induced ordered weighted averaging distance operator and its application to decision making. *Journal of Intelligent & Fuzzy Systems*, 32:11–22.

Zeng, S. Z., Merigo, J. M., and Su, W. H. (2013). The uncertain probabilistic owa distance operator and its application in group decision making. *Applied Mathematical Modelling*, 37:6266–6275.

Zeng, S. Z. and Su, W. H. (2011). Intuitionistic fuzzy ordered weighted distance operator. *Knowledge-Based Systems*, 24:1224–1232.

Zeng, S. Z., Su, W. H., and Chen, J. (2014). Fuzzy decision making with induced heavy aggregation operators and distance measures. *Journal of Intelligent & Fuzzy Systems*, 26:127–135.

Zhou, L. G., Chen, H. Y., and Liu, J. P. (2013). Continuous ordered weighted distance measure and its application to multiple attribute group decision making. *Group Decision and Negotiation*, 22:739–758.