Children's Home Numeracy Environment Predicts Growth of their Early Mathematical Skills in Kindergarten

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Abstract

This study investigated the longitudinal associations between children's early mathematics and their home numeracy environment (HNE). Chilean children from families that varied widely in socioeconomic status were assessed at the beginning and end of pre-kindergarten in 2016 (N = 419, M_{age} = 4:7 [years:months]), and at the end of kindergarten in 2017 (N = 368, M_{age} = 5:10). Children whose parents provided frequent operational numeracy activities (e.g., learning simple sums) at pre-kindergarten showed better arithmetic performance and growth in non-symbolic and symbolic number comparison at the end of kindergarten. Parents' knowledge of number-related games predicted children's arithmetic skills and growth in non-symbolic comparison. These findings underscore the persistent relations between the HNE and the development of children's mathematical skills.

Keywords: home numeracy, home environment, numeracy activities, children, preschool, early number skills, symbolic number comparison, early mathematics.

Children's cognitive and academic development is the responsibility of their parents: In addition to sending children to preschool and kindergarten, parents provide a home numeracy environment (HNE) that may foster their children's early numeracy skills (Kleemans, Peeters, Segers, & Verhoeven, 2012; Skwarchuk, Sowinski, & LeFevre, 2014). Children's early numeracy skills form a foundation for their success in school mathematics (Duncan et al., 2007). Later math skills, in turn, are related to college and career options, graduation rates, and future job earnings (National Mathematics Advisory Panel, 2008). Because individual differences in early math skills are remarkably stable (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009), it is important to develop a comprehensive understanding of how children's home numeracy environment supports the development of early mathematical skills *before* children enter kindergarten.

Although research supports the associations between the HNE and children's numeracy skills, few researchers have used longitudinal studies to examine whether these relations continue in subsequent years (Elliott & Bachman, 2017). Longitudinal studies are important to understand the links between the home environment and children's skills over time. In the present research, we examined the role of the home numeracy environment in predicting the growth of children's numeracy skills for Chilean children as they transitioned from pre-kindergarten to kindergarten.

The Home Numeracy Environment (HNE)

The numeracy experiences that children have at home relate to the development of children's early numerical skills (Anders et al., 2012) and are connected to parents' attitudes and beliefs about math (Blevins-Knabe, Austin, Musun, Eddy, & Jones, 2000; Skwarchuk et al., 2014). Home numeracy experiences and their links to the development of early math skills have been documented in many regions of the world including North America (LeFevre et al., 2009; Skwarchuk et al., 2014); Europe (Kleemans et al., 2012; Manolitsis, Georgiou, & Tziraki, 2013; Niklas & Schneider, 2014); Asia (Qi, Xiao, Yingyi, Wen, & Zhanmei, 2017); and some Latin American countries (Becerra Orellana, 2016; Susperreguy, Douglas, Xu, Molina-Rojas, & LeFevre, 2018). These researchers have found positive links between home numeracy activities and children's skills. Other studies, however, have not found links between the HNE and children's numeracy skills (see Dunst, Hamby, Wilkie, & Dunst, 2017, for a meta-analysis; Elliott & Bachman, 2017).

Elliott and Bachman (2017) outline several possible reasons for the variability of findings in the field of HNE, including a heavy reliance on parents' self-reports, failure to test the direction of the relations between children's performance and the home environment, a preponderance of small heterogeneous samples, few studies conducted outside of Westernized countries, limited or variable (across studies) assessment of home environments, untested interactions between parent characteristics and home activities, and an emphasis on concurrent studies, usually of children in preschool, kindergarten, or Grade 1. As described below, the present research addresses some of these concerns and advances the field, in particular, by using a longitudinal design and a large sample from a non-Western country.

Numeracy Activities. Parents provide a range of home numeracy experiences at home. These experiences have been categorized by researchers as formal (or direct), and informal (or indirect, e.g., Elliott & Bachman, 2017; Skwarchuk et al., 2014). In general, the term *formal* refers to experiences that have a didactic focus, where parents intentionally choose activities in which they teach their children specific math-related skills, such as mental addition or recognizing numbers (LeFevre et al., 2009). Some authors have further differentiated formal experiences into basic or mapping and advanced or operational activities (Dunst et al., 2017; Skwarchuk et al., 2014; Susperreguy et al., 2018). *Mapping activities* are those in which parents focus on teaching simple numerical activities related to the different ways of representing numbers (e.g., identifying numbers). *Operational activities* include more advanced activities in which children manipulate digits or quantities (e.g., practicing mental math, comparison of quantities). *Informal* activities, on the other hand, refer to activities in which children's learning of numerical or mathematical skills occurs incidentally (Skwarchuk et al., 2014; Sonnenschein et al., 2012). Examples of informal activities include engaging in board games with math-relevant content, playing cards, or cooking. These informal activities may provide opportunities for children to learn numerical or mathematical skills, but learning about math is presumably not the central goal of the activity.

Children's early mathematical skills correlated with both formal and informal home numeracy activities. In terms of formal activities, Thompson, Napoli, and Purpura (2017) found that home activities that are explicitly centered on math, such as counting objects, printing numbers, or using number activity books predicted the early numeracy skills of preschoolers. Similarly, others have found that the frequency of formal numeracy activities predicted children's enumeration skills (Mutaf Yıldız, Sasanguie, De Smedt, & Reynvoet, 2018) and arithmetic skills (Vasilyeva, Laski, Veraksa, Weber, & Bukhalenkova, 2018). Operational numeracy activities predicted symbolic number knowledge for prekindergarten (Susperreguy et al., 2018) and kindergarten children (Skwarchuk et al., 2014). In contrast, parents' informal numeracy practices predicted nonverbal arithmetic (Skwarchuk et al., 2014) and children's arithmetic with objects (Susperreguy et al., 2018). However, none of the studies reviewed here addressed the question of whether these home numeracy activities relate to changes in children's early numeracy learning over time.

Longitudinal predictions of home numeracy. Most of the studies on the home numeracy environment (HNE) evaluated the HNE and children's skills concurrently, or they assessed the HNE at one time point and predicted children's numeracy skills at a later time point. Only a few researchers have studied the relation between the HNE and growth in children's numeracy skills. Anders et al. (2012) found that the HNE predicted numeracy skills of German children concurrently in the first year of preschool, but did not predict growth in numeracy skills from the beginning to the end of preschool. Niklas and Schneider (2014) studied older German children and reported that the HNE predicted early number skills and growth in mathematical abilities from kindergarten to the end of Grade 1. Similarly, with Dutch children, Kleemans, Segers, and Verhoeven (2018) found that kindergarten home numeracy experiences predicted arithmetic with large problem sizes in Grade 2, after taking into account prior arithmetic skills in Grade 1. Thus, there is some evidence for a relation between the HNE in kindergarten and growth in arithmetic skills, though it is specific to samples of German and Dutch children, and does not distinguish among types of home numeracy activities.

None of the longitudinal studies separately tested the influences of formal and informal numeracy activities. Anders et al. (2012) and Kleemans et al. (2018) used composite measures of the HNE that included a range of activities, whereas Niklas and Schneider (2014) only captured children's exposure to informal activities (i.e., reports of playing games with numerical content). The distinction between types of home numeracy activities is relevant in longitudinal studies because it allows for the evaluation of whether and how specific home numeracy activities predict growth of numeracy skills at different ages, potentially clarifying some of the inconsistent results of concurrent studies. Some activities may be relevant at some ages but not at others (Thompson et al., 2017). Furthermore, children's development of numeracy skills may be influenced by the cumulative effects of both types of home numeracy experiences (Bradley, Caldwell, & Rock, 1988).

Formal and informal numeracy activities could also differentially contribute to the development of more complex skills over time. This possibility is suggested by the research on the home literacy environment (HLE), where informal and formal home literacy activities (i.e., shared book reading at home, and teaching about reading and writing, respectively) are differentially related to children's outcomes in the early years of schooling, but they both predict more complex skills in later years. Specifically, shared book reading is correlated with word comprehension (Flack, Field, & Horst, 2018), whereas parental literacy teaching is related to letter knowledge (Manolitsis et al., 2013), letter identification (Hood, Conlon, & Andrews, 2008), and early literacy skills (Sénéchal & LeFevre, 2002; Sénéchal & Young, 2008). These initial distinct influences of formal and informal activities on literacy outcomes converged over time to predict reading comprehension in older children (Manolitsis et al., 2013; Sénéchal & LeFevre, 2002). Thus, longitudinal research on the HLE suggests that both formal and informal home literacy activities may differentially contribute to children's learning of complex literacy skills such as reading comprehension. Based on these findings from home literacy research, similar longitudinal patterns may emerge for early numeracy.

Current Research

The main goal of the present study was to evaluate whether different aspects of the HNE would predict growth in children's early math outcomes in the transition from the beginning of pre-kindergarten to the end of kindergarten. This study extends prior work and addresses many of the concerns about variability in the field outlined by Elliott and Bachman (2017) by examining longitudinal relations between numeracy activities at home and growth of kindergarteners' mathematical skills with a large group of children from a non-Westernized country (Chile) as they transition from pre-kindergarten to kindergarten. Moreover, in the present research we explored the relations between different aspects of the home numeracy environment (i.e., formal and informal), and several different early math skills (Mutaf Yıldız et al., 2018), thereby addressing the limitations of using monolithic assessments of both children's experiences and their developing skills.

To explore the longitudinal associations between home numeracy activities and children's mathematical skills, we assessed specific aspects of children's early mathematical skills. In contrast, most of the studies of home numeracy and early mathematical skills only included global or composite measures of mathematics, often standardized assessments that do not provide information on specific math skills or on growth in these skills. By incorporating specific assessments, we were able to evaluate math constructs and add constructs at appropriate times throughout the study (i.e., adding arithmetic at Time 3 because this is when it would be developmentally appropriate to do so). Specifically, we included (a) outcomes that involve written digits, where children need to use symbolic number knowledge (i.e., symbolic outcomes: number comparison, number line estimation, and arithmetic skills), and (b) outcomes that do not involve written digits, where children could use quantity knowledge, though it is not necessary to solve the task (i.e., non-symbolic number comparison, where children may recognize that one set of dots has more dots than another, or children may count the dots to identify which set of dots has more than the other). To further explore these links, we also included (c) an outcome for which children may rely on either (or both) written digits or quantity knowledge (i.e., applied problem solving). Given the developmental course of number knowledge, we expected that some of the growth in these skills would be predicted by the numeracy experiences children encounter at home.

The current analyses are based on data from a study that included three testing points: beginning of pre-kindergarten (i.e., the first of a two-year preschool program prior to entering first grade; Time 1), end of pre-kindergarten (Time 2), and one year later when children were finishing kindergarten (Time 3). Because the purpose of this study was to evaluate whether different types of home numeracy activities predict growth in children's numeracy skills from pre-kindergarten to kindergarten, we focused on the children's outcomes at the end of kindergarten (i.e., Time 3). Detailed analyses of the HNE and the children's early math skills in pre-kindergarten were reported in Susperreguy et al. (2018). Examination of the children's results at the end of kindergarten, almost two years after parents reported on their numeracy activities at home, allowed us to model how the home environment relates to growth in mathematical skills from the end of pre-kindergarten to the end of kindergarten. Thus, one of the main contributions of the current study is that it analyzes the longitudinal influence of the early home numeracy environment on children's developing skills (i.e., the *growth* in numeracy skills). Another important contribution is that we distinguish between skills that emphasize written digits and quantity knowledge at the end of kindergarten, and we assessed the influence of both formal and informal home numeracy activities.

We hypothesized that home numeracy activities, both formal and informal, would predict the growth in mathematical outcomes from the end of pre-kindergarten to the end of kindergarten. Based on research on the home *literacy* environment (HLE) (Manolitsis et al., 2013; Sénéchal & LeFevre, 2002), we expected that informal home numeracy activities (i.e., shared number-game play) would predict growth in all math outcomes. Home literacy research tells us that shared storybook reading (i.e., informal HLE) influences word learning (Flack et al., 2018) and growth in vocabulary knowledge (Sénéchal & LeFevre, 2014), and it has an ongoing influence on reading enjoyment and reading comprehension (Sénéchal, 2006). Following this research, we predicted that shared number-game experience in pre-kindergarten (as indexed by game exposure) would be related to growth in mathematical skills that require written digits, as well as in those skills where symbolic number knowledge is not necessarily involved. That is, we hypothesized that the wide range of specific activities involved in number game play, such as counting, naming digits, doing simple arithmetic, comparing quantities, spatial activities, and so on, will be persistently related to a variety of children's developing mathematical skills (LeFevre et al., 2009). Presumably, a preschool home environment rich in exposure to informal numerical and mathematical activities will continue to be a kindergarten home environment rich in these activities, providing continued support for children's mathematical development.

We also predicted that parents' reports of formal activities (i.e., operational activities) in pre-kindergarten would predict growth in mathematical outcomes from the beginning to the end of kindergarten. We predicted an ongoing influence of pre-kindergarten operational activities on kindergarten skills that employ written digit knowledge, assuming that early operational home activities (e.g., practicing mental arithmetic) will reflect parents' orientation towards providing specific academic activities (i.e., the home numeracy curriculum; Sénéchal & LeFevre, 2014). We also predicted an influence of operational activities on the growth of children's performance at the end of kindergarten on a task that does not involve written digits (i.e., comparing dot quantities less than nine). This prediction is based on the assumption that children will use their symbolic knowledge and strategies (i.e., counting, addition, subitizing), skills that are fostered by operational activities, as they learn to focus on exact rather than approximate quantities in numerical tasks (Lyons, Bugden, Zheng, De Jesus, & Ansari, 2018; Merkley & Ansari, 2016).

Finally, we assumed that early number skills (measured at the start of pre-kindergarten) will also predict growth in all of the mathematical outcomes, mainly because early number skills (i.e., verbal counting, number naming, and cardinality) are key precursors for further math learning (Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Purpura, Baroody, & Lonigan, 2013). Accounting for these early number skills, measured concurrently with the home activities, provides a stringent test of the changes in children's specific mathematical outcomes during the pre-kindergarten and kindergarten year.

Method

Participants

The present study is part of a large research project investigating factors that predict children's early math skills prior to entering Grade 1. Children (all Spanish monolingual) and their parents (85% mothers) were recruited from seven schools in the greater urban area of Santiago, Chile. Thus, in Year 1 of the study, all children were enrolled in kindergarten preparation (i.e., half-day pre-kindergarten program) at the elementary school they attended in Year 2. The children were recruited from schools that served a wide range of socioeconomic groups, including low, middle, and high SES families. Hence, parental education varied greatly, ranging from having a high school diploma or less (26%) to having attended or completed university (43%). The median level of educational attainment was having graduated from the equivalent to a community college or professional institute.

Procedure

This study received ethics approval from the Ethics Committee of the Faculty of

Education at the Pontificia Universidad Católica de Chile. First, schools were recruited for the study. Once principals agreed to participate, informed consent was obtained from parents for both themselves and their children. Children were asked if they wanted to participate and were only tested if they agreed. Parents completed questionnaires at schools during group-parent-teacher conferences at Time 1. Children completed assessments at three time points in two individual sessions in a quiet place at their schools. Each testing session lasted approximately 25 minutes. All testing was conducted by trained research assistants, who were supervised by the first author. Testing at Time 1 (i.e., early in the 2016 fall term) lasted for two months. Children had an average of two months of school experience at Time 1, which took place early in the 2016 school year (Time 1; N = 419, 48% male; M_{age} = 4:7 years:months, range 3:4 – 5:7). Approximately 7-8 months later, at the end of the 2016 school year, children were assessed for a second time (Time 2; N = 406). Finally, children were assessed for a third time approximately 9-10 months later, at the end of the 2017 school year (Time 3; N = 368, 47% male, M_{age} = 5 years: 10 months, range 4:9 to 6:6 months). Of the 419 children included at Time 1, 13 did not participate in Time 2, and 51 did not participate in Time 3 either because they had left the school or had prolonged absences due to health issues.

Materials

For complete descriptions of measures, please refer to Susperreguy et al. (2018).

Parental factors. Parents completed a Spanish-translated version of the Skwarchuk et al. (2014) parent questionnaire. As a part of the larger project, parents reported on the frequency of home numeracy and literacy activities they participated in with their children, their attitudes towards math and literacy, and the numeracy and literacy skills expectations for their children to acquire by Grade 1. Although the focus of this study is on home numeracy, we included the questions about home literacy from the Skwarchuk et al. (2014) parent questionnaire to reduce the focus on numeracy and allow parents to indicate their participation in a wider-range of potentially familiar activities. Demographic information was also collected in the parental questionnaire, which was administered via paper-and-pencil. Based on a principal component analysis (PCA), factors were created for attitudes and expectations, for both numeracy and literacy. See Susperreguy et al. (2018) for full numeracy and literacy descriptives and details of the scales.

Home activities.

Numeracy - Mapping and operational activities. Parents reported the frequency with which they engaged in formal home numeracy activities with their children, using a five-points scale ranging from rarely or never (0) to most days per week (4). Based on the results of a principal component analysis (PCA), two uncorrelated components (factor scores) were created: mapping activities and operational activities. Mapping activities consisted of five items (Cronbach's alpha = .81) that related to identification and representations of numbers (i.e. "I help my child to recite numbers in order," "We sing counting songs," "I teach my child to recognize printed numbers," "I ask about quantities," and "I encourage the use of fingers to indicate how many"). Operational activities involved five questions (Cronbach's alpha = .81) about manipulation of digits or quantities (i.e., "I help my child to learn simple sums," "I encourage my child to do math in his or her head," "We talk about time with clocks and calendars," "I help my child weigh, measure, and compare quantities," and "We play games that involve counting, adding or subtracting").

Numeracy - Shared number-game play. We used a shared board-game checklist (Skwarchuk et al., 2014) to measure parents' knowledge of number games as a proxy measure of informal home activities related to number and mathematics. Following Skwarchuk et al.'s procedure, we asked parents to select the games they recognized from a list that included real games, some with numerical content and some without numerical content, and invented game titles to serve as foils. The real game titles were selected from commercial games that were available in Chile. The number-game score was calculated by subtracting the number of invented number games that parents recognized from the total of real number games that parents recognized, and standardizing the corrected total score.

Literacy activities - Code-related and meaning-related. Following the work by Skwarchuk et al. (2014), parents reported the frequency with which they engaged in ten home literacy activities. Two activity components were identified using PCA: code-related and meaning-related activities. Code-related activities (Cronbach's alpha = .84) consisted on five questions focused on print (i.e., "I help my child read words," "I ask my child to point to letters or words when we read," "I help my child print words," "I help my child sing or recite the alphabet", and "I teach my child to recognize printed letters"). Meaning-related activities (Cronbach's alpha = .79) involved five items related to oral language (i.e., "I teach my child the sounds of the letters," "We identify words on signs," "I introduce new words and their definitions to my child," "We make up rhymes in songs," and "I ask questions when we read together").

Children's numeracy measures. Because the present study is part of a larger project, several other tasks were administered (Montoya et al., 2019). Table 1 includes descriptive statistics for the children's tasks.

Early number skills. Three tasks – verbal counting, number identification, and object counting – were chosen to assess some of the key facets of early number knowledge (Purpura & Lonigan, 2015). These tasks were administered at Time 1. In the *verbal counting* task, children were asked to count as high as they could until the child made a mistake. Scoring was the highest number counted. In the *number identification* task (adapted from Purpura & Ganley, 2014), children were shown 17 digits ranging from 1 to 28 (i.e., 1, 2, 9, 7, 4, 6, 3, 5, 8, 10, 17, 15, 12, 11, 22, 28, and 23) on flashcards and asked, “What number is this?” The total number of correct responses was the score for this task. The *object counting task* was adapted from Purpura and Ganley (2014). Children were given 20 tokens and were asked to put a given number of tokens on a sheet of paper. There were seven trials (i.e., 3, 4, 6, 8, 10, 11, and 16) and scoring was the number of correct responses. Raw scores on these three tasks were entered into a PCA, whose solution resulted in a factor called *early number skills*.

Non-symbolic mathematical outcomes. At Times 2 and 3, children completed a mathematical task requiring knowledge of quantities, but not written digits.

Number comparison – non-symbolic. The numeracy screener task (Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013), which includes symbolic (digits) and non-symbolic (dots) components, was used to assess number comparison. In the non-symbolic part of the task, children are presented with a sheet of paper that has various pairs of dots. Using a pencil, they must cross off the larger quantity of dots (e.g., 6 dots vs. 3 dots). They have a maximum of 120 s to complete the task. The score was the number of correct responses divided by time taken in seconds.

Symbolic mathematical outcomes. Children completed three mathematics measures involving written digit knowledge. The first two were administered at Times 2 and 3. Given the developmental course of arithmetic skills in young children, the task assessing arithmetic fluency was only included at Time 3.

Number comparison – symbolic. In the symbolic part of the numeracy screener task

(Nosworthy et al., 2013), children must correctly cross off the larger of two digits (e.g., What is larger, 7 or 2?). Maximum time allowed was 120 s. The score was the number of correct responses divided by the time taken to in seconds.

Number Line estimation. We administered an iPad version of the number line task (https://hume.ca/ix/estimationline.html), ranging from 0 (on the left) to 10 (on the right). Children had to quickly and accurately mark where the number goes on the line. The score was the mean absolute error (i.e., error between the target and the selection) across trials.

Arithmetic Fluency. An iPad application (MadMinute) was used to assess the fluency of single-digit addition with sums less than or equal to nine. Children completed two practice problems prior to completing the 1-minute timed task. Children indicated the correct response to the problems by touching the answer on the screen. Scoring was the number of correct sums.

Mathematical outcome including both symbolic and non-symbolic knowledge.

Applied problem solving. Children completed the Problemas Aplicados (Applied Problems) subtest of the Batería III Woodcock-Muñoz (Muñoz-Sandoval, Woodcock, McGrew, & Mather, 2005). For this task, children answer mathematical problems that become progressively difficult as the task advances. Testing is discontinued after six consecutive errors. The score is the total number of correct responses.

Children's literacy measures. As a part of the larger project, children completed two standardized literacy measures, both at Times 2 and 3. These tasks were included in the current analyses to evaluate whether the home numeracy activities predict growth of skills in the math domain, or whether they predict performance across domains (i.e., predict literacy outcomes).

Receptive vocabulary*.* Children completed the Spanish adaptation of the PPVT-R test of receptive vocabulary (Dunn, Padilla, Lugo, & Dunn, 1986). In this task, children must select a picture that represents the target word, from a selection of four pictures. Testing is discontinued after six errors in a set of eight items, and scoring is the total number of correct responses.

Letter-word identification. Decoding skills were measured with the Identificación de Letras y Palabras (Letter–Word Identification) subtest of the Batería III Woodcock-Muñoz (Muñoz-Sandoval et al., 2005). For this task, children identify letters, syllables, and words. Testing is discontinued after six consecutive errors and scoring is the total number of correct responses.

[Table 1 goes here]

Results

Descriptive Analyses

Correlations among parental factors, home numeracy activities, and children's outcomes are in Table 2. Detailed analyses of the relations among attitudes, expectations, and activities for literacy and numeracy are provided in Susperreguy et al. (2018). For the purposes of this study, we will briefly summarize these correlations. Parents were more positive towards literacy than towards numeracy, and they also had higher expectations for children's literacy than numeracy achievements. More-educated parents recognized more number games than less-educated parents. Moderate correlations were found between the frequency of literacy activities and numeracy activities. Importantly, for home numeracy, operational and informal activities but not mapping activities were reliably correlated with children's performance. Thus, the models tested in the present research evaluated only the relations between children's outcomes and operational and informal activities.

Early number skills were moderately correlated with all numeracy outcomes, both at Time 2 and Time 3. Parental education was positively correlated with early number skills at

Time 1 and with applied problem solving and symbolic number comparison at both Time 2 and Time 3, and with arithmetic fluency at Time 3. As expected, given the developmental course of numeracy skills, the same outcome measured at Time 2 and Time 3 was positively correlated over time, ranging from $r = .49$ for non-symbolic number comparison to $r = .65$ for applied problem-solving.

Number line performance, however, was an exception because it was not correlated with parental education, was not correlated with other home numeracy factors, and the correlation between the number line task at Time 2 and Time 3 was low. This low correlation could reflect children's poor performance on this task. Thus, we did not analyze number line estimation, but address the topic in the Discussion section.

[Table 2 goes here]

Model Development

The relations among home numeracy factors, including parental factors (i.e., parental education and parental attitudes and expectations toward math), early number skills at Time 1, and math outcomes at Time 2 were discussed in detail in Susperreguy et al. (2018). In the present analysis, we focused on growth in children's outcomes from the end of pre-kindergarten (i.e., Time 2) to the end of kindergarten (i.e., Time 3). Through examination of missing data pattern frequencies, missing data from Time 1 to Time 3 was determined to be missing at random. As a result, the models were estimated by a full information maximum likelihood (FIML) method; instead of deleting observations with missing values, all available information is used in all observations to estimate the model. In the current paper, we focused on examining the predictive longitudinal relations between operational and informal home numeracy activities and mathematical outcomes at Time 3. We tested four different models, one for each outcome, using structural equation modeling in Mplus Version 7 (Muthén & Muthén, 1998-2012). Confidence intervals for coefficients were calculated with bias-corrected bootstrapping (1000 samples). Each model is illustrated in a figure, showing the path coefficients. Significant indirect effects are described in the figure captions. A summary of the overall pattern of results is shown in Table 3.

Non-symbolic comparison. As hypothesized, after accounting for the effect of parental factors, children's early number skills at Time 1, and their non-symbolic number comparison skills at Time 2, shared number-game play uniquely predicted children's performance on the non-symbolic number comparisons at Time 3 ($\beta = .095$, $p = .049$, $CI = [.005, .189]$). Similarly, after controlling for the parental factors, their early number skills at Time 1, and their non-symbolic number comparison skills at Time 2, operational activities uniquely predicted children's performance on the non-symbolic number comparisons at Time 3, ($\beta = .136$, $p = .003$, CI = [.047, .227]). Indirect paths from both shared-number game play and operational activities to non-symbolic number comparison skills at Time 3 were also significant. Thus, both shared number-game play and operational activities predicted the growth of non-symbolic number comparison skills.

[Figure 1 goes here]

Symbolic number comparison. As shown in Figure 2, after accounting for parental factors, early number skills at Time 1, and symbolic number comparison skills at Time 2, shared number-game play did not uniquely predict symbolic number comparisons at Time 3 ($\beta = .074$, $p = .101$, CI = [-.019, .162]), but the indirect path was significant. In contrast, after accounting for parental factors, early number skills at Time 1, and symbolic number comparison skills at Time 2, operational activities directly predicted symbolic number comparison skills at Time 3 ($\beta = .114$, $p = .013$, CI = [.025, .206]). The indirect path from operational activities to the symbolic number comparisons at Time 3 was also significant. Together, our findings show that operational activities predicted the growth of symbolic number comparison skills from Time 2 to Time 3, whereas shared number-game play was only indirectly related to the growth of symbolic number comparisons.

[Figure 2 goes here]

Arithmetic fluency. We tested whether shared number-game play and operational activities predicted arithmetic fluency at Time 3, controlling for children's symbolic number comparison skills at Time 2 (see Figure 3). This analysis does not evaluate growth because fluency was not measured at Time 2, however, symbolic number knowledge is a known correlate of arithmetic in children (Göbel, Watson, Lervåg, & Hulme, 2014; Lyons et al., 2014) and requires related skills such as number recognition and knowledge of magnitudes (Jordan et al., 2009) . As shown in Figure 3, both shared number-game play (β = .093, p = .028, CI = [.031, .210]) and operational activities (β = .122, p = .006, CI = [.015, .174]) directly predicted arithmetic fluency at Time 3, after accounting for parental factors, children's early number skills at Time 1, and symbolic number comparison skills at Time 2. Further, the indirect paths from operational activities and shared number-game play to arithmetic fluency at Time 3 were also significant. These findings support the hypothesis that operational activities and shared number-game play are related to children's arithmetic fluency almost two years later.

[Figure 3 goes here]

Applied problem solving. After controlling for parental factors, children's early number skills at Time 1, and their applied problem-solving skills at Time 2, neither shared number-game play (β = .002, p = .969, CI = [-.080, .083]) nor operational activities (β = .068, p = .070, CI = [-.004, .142]) directly predicted applied problem-solving skills at Time 3 (see Figure 4). However, the indirect paths from shared number-game play and operational activities to applied problem-solving at Time 3 were significant.

[Figure 4 goes here]

Domain-specificity of home activities

Do formal home activities influence performance across domains? The evidence is mixed. Some researchers have reported that home numeracy activities predict both mathematics and reading outcomes (Huntsinger, Jose, & Luo, 2016), and that the home literacy environment predicts numeracy skills in addition to literacy skills (Anders et al., 2012). More recently, Napoli and Purpura (2018) reported that the HNE may influence not only numeracy outcomes, but vocabulary development. Others, however, have not found such broad cross-domain influences. For example, LeFevre, Polyzoi, Skwarchuk, Fast, and Sowinski (2010b) found that for Canadian children, only the frequency of home numeracy activities, not home literacy factors, predicted numeracy outcomes. Similarly, we (Susperreguy et al., 2018) did not observe a cross-domain influence for the relations between the home literacy environment and mathematical outcomes at Time 2: Home literacy activities did not predict mathematical outcomes, and home numeracy activities did not predict literacy outcomes at the end of pre-kindergarten. Accordingly, we re-examined this question using the current longitudinal dataset for the Time 3 outcomes. As shown in Table S1 in Supplementary materials, the frequency with which parents reported doing meaning-based literacy activities with their preschoolers was not correlated with the Time 3 mathematical outcomes. However, the reported frequency of code-based literacy activities was correlated with the symbol-based and combined math outcomes (i.e., symbolic number comparison, arithmetic fluency, and applied problem-solving). Similarly, reported frequency of operational math activities was similarly correlated with vocabulary knowledge and letter-word identification.

Regression analyses were used to determine whether there was specificity of links between home activities and children's growth in numeracy and literacy performance: a) Are literacy growth outcomes predicted by operational numeracy activities? and b) Are numeracy growth outcomes predicted by code-based literacy activities? (See Supplementary materials, Tables S2 and S3.) The results supported strong within-domain specificity of home numeracy activities: Operational numeracy activities did not predict growth in literacy outcomes and code-based literacy practices did not predict growth in mathematical outcomes.

Summary

The current findings extend previous work (Mutaf Yıldız et al., 2018; Skwarchuk et al., 2014; Vasilyeva et al., 2018) on concurrent relations between home numeracy activities and children's mathematical outcomes by showing that there are persistent effects of home numeracy activities on the growth of children's mathematical performance. Home numeracy activities predicted growth in skills directly and indirectly through earlier performance. Both operational activities and shared number-game play predicted children's growth on the non-symbolic comparison task. Operational activities predicted growth on the symbolic number comparison task, and both operational activities and shared number-game play predicted arithmetic fluency at Time 3. Finally, although applied problem-solving growth was mainly explained by children's prior achievement on that measure, home numeracy activities (i.e., operational activities and shared number-game play) were indirectly related to children's performance at Time 3. The results showed strong within-domain influences of home activities on numeracy and literacy outcomes, despite cross-domain correlations among the home factors.

Discussion

Our results demonstrate that informal and operational numeracy activities at home are longitudinally related to children's acquisition of fundamental mathematical skills and knowledge at school. The present research is one of the few studies of the relation between home numeracy environment and growth of the early math skills. It is also one of a few studies of the home numeracy environment of Spanish-speaking children in a Latin-American country. To predict growth, we controlled for parental factors, early math skills (measured at the beginning of pre-kindergarten), and prior achievement on the specific outcomes assessed at the end of pre-kindergarten. We used a large sample of children, who were homogeneous in terms of country and educational experience, and a variety of measures to assess early mathematical skills longitudinally. Thus, our findings add to the growing body of research demonstrating that the HNE is an important factor in the development of early mathematical skills (Anders et al., 2012; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010; Mutaf Yıldız et al., 2018; Skwarchuk et al., 2014).

The persistent effects of home numeracy environments on the growth of children's mathematical outcomes were related to both shared number-game play (i.e., informal) and parents' reports of operational (i.e., formal) activities. Specifically, both informal and operational activities were indirectly related to all four of the numeracy outcome measures at the end of kindergarten (Time 3), indicating that the effects of home numeracy activities on the Time 3 outcome were mediated by children's performance at the beginning of pre-kindergarten (Time 1) or the end of pre-kindergarten (Time 2), or both. Moreover, at Time 3, operational activities had direct effects on kindergarteners' symbolic and non-symbolic comparison, and on arithmetic fluency. Informal activities (i.e., parents' knowledge of number-related games) also had direct effects on non-symbolic comparison and arithmetic fluency at the end of kindergarten. Prior studies of home numeracy activities and children's performance, found that informal and formal activities were differentially related to children's specific numeracy outcomes (Mutaf Yıldız et al., 2018; Skwarchuk et al., 2014). For example, Skwarchuk et al. (2014) found that informal activities at age 4 were related to children's non-symbolic arithmetic skills at age 5, whereas formal activities at age 4 were related to children's knowledge of written digits at age 5. In contrast, our findings show that, as numeracy tasks become more complex and require a combination of written digit and quantity knowledge, children's home experiences are related to tasks where both kinds of knowledge can be used. For example, for the non-symbolic comparison task used in this study, children may subitize the quantities less than 4, and then use either counting or addition strategies to add the additional dots (i.e., for 5 dots, 3-4-5 or 3 + 2; Lyons et al., 2018; Merkley & Ansari, 2016). Thus, both informal and formal home numeracy activities are relevant for developing skill in numerical tasks and both may contribute independently to children's growth in early numeracy outcomes.

The current results mirror the findings from the home literacy literature, where initially-differentiated links between home activities and specific skills (Hood et al., 2008) converge to predict more complex skills (i.e., reading comprehension) later in school (Manolitsis et al., 2013; Sénéchal & LeFevre, 2002). Our results suggest that both operational and informal home numeracy activities measured at the beginning of the pre-kindergarten year were related to the development of complex numeracy skills such as arithmetic fluency at the end of kindergarten. For example, operational activities (e.g., "I encourage my child to do math in his or her head") involve engaging in the manipulation of symbolic or non-symbolic quantities, which promotes performance on arithmetic tasks. Similarly, number games involve a wide range of skills, both symbolic (e.g., recognizing and comparing digits, counting) and non-symbolic (e.g., comparing quantities, subitizing). Furthermore, the strategies children use will mature as children become more familiar with numbers (Schiffman & Laski, 2019). On number board games, for example, children begin to count spaces by twos and then apply this strategy when adding numbers. Thus, both informal and formal home numeracy activities when children are starting pre-kindergarten may help them develop a range of numeracy skills over the course of the two preparatory years prior to entering Grade 1. Of note, nevertheless, is that both kinds of activities had independent effects in the present longitudinal analyses, showing that a broad assessment of the home numeracy environment is important to fully capture the role of home experiences in children's learning over time.

The longitudinal effects of home numeracy activities may indicate that parents who engage in numeracy-related activities when their children are in preschool continue to engage in math-related activities as their children develop. Because the reported activities and the knowledge of number games were tailored to the age of the children at the beginning of the study, parents probably adjust the level of activities as their children get older (Thompson et al., 2017). In other words, the activities in which parents engage with their children presumably are consistent with children's educational needs over time, as well as with their developmental levels of performance. This consistency could explain why HNE measured early in preschool predicts growth in numeracy outcomes at the end of kindergarten (see Sénéchal & LeFevre, 2014, for evidence of parent responsiveness in literacy development). Although our evidence is correlational, it is consistent with the view that parents' home activities are sensitive to children's developing skills, and that these activities foster gains in early number skills. Future research could include assessments of the home numeracy environment over time to elucidate how and if parents adapt their activities as children get older.

Our results support a domain-specific longitudinal link between the home environment (i.e., home literacy versus home numeracy activities) and children's outcomes (LeFevre et al., 2010a). Operational numeracy activities did not predict growth in literacy outcomes (i.e., word reading and vocabulary), and code-based literacy activities did not predict growth in numeracy outcomes. Instead, although there were cross-domain correlations among the home activities in our study, only domain-specific home activities were uniquely predictive of domain-specific growth in the outcomes. In contrast, Napoli and Purpura (2018) found that the frequency with which parents read number story books to their children was related to numeracy performance. Number storybook reading may specifically support learning of math- and spatial-words such as *before*, *lower*, or *most* (Purpura, Napoli, Wehrspann, & Gold, 2017).

In general, relations between children's language-related experiences at home and early mathematical outcomes may reflect the important role of language processes in children's mathematical learning (LeFevre et al., 2010a; Purpura, Hume, Sims, & Lonigan, 2011; Purpura et al., 2017). Because language exposure was not assessed in the current research, we cannot directly test these relations. Furthermore, the home literacy measure used by Napoli and Purpura (2018) asked parents to report on number book reading, and thus may partially confound home literacy and home numeracy activities (LeFevre et al., 2009). In future research, number and non-number storybook exposure should be assessed independently to evaluate the role of these activities on children's outcomes over time. Also, children's exposure to activities that integrate both literacy and numeracy activities should also be assessed (e.g., games tapping into math language, numbers, or both; baking or working through a recipe).

Although children in this study were asked to complete a number line task, we did not present analyses of those data because the home numeracy activities were not related to children's growth on this task. Moreover, children in this study performed very poorly overall on the number line task, which is consistent with research showing that many kindergarten children find the task very difficult unless they have explicit relevant training or relatively advanced numeracy skills (Siegler & Ramani, 2009; Xu & LeFevre, 2016, 2018). Similar to our findings, a recent meta-analysis by Schneider et al. (2018) showed that the correlation between number line performance and other skills increased with age, and that the lowest correlation was for children under six years.

We did not find direct effects of the home numeracy activities on the growth of applied problem-solving skills; the only effects observed were indirect, through early skills and prior achievement. There was little room for improvement on the applied problem-solving task over the one-year interval because it is a standardized test suitable for a wide age range. Future studies should use more sensitive measures of mathematical problem solving to understand the role of the HNE in the development of these skills among kindergarten children.

In line with other research on home numeracy and early math skills, mapping activities did not relate to growth in children's early mathematical skills (Skwarchuk et al., 2014; Susperreguy et al., 2018). Mapping activities, which are focused on representation and identification of numbers, might be most relevant early in development (Dunst et al., 2017). Some of the variability across different studies in the relations between home numeracy activities and children's performance might relate to inconsistent selection of relevant activities in relation to children's ages (Thompson et al., 2017). More generally, parental numeracy activities that foster relatively more advanced aspects of children's knowledge such as math operations, or that engage children in a wide range of activities (e.g., as in math-related games) may be most important in fostering success over time.

There is considerable variability in the effect sizes for the relations between home numeracy activities and children's early mathematical outcomes (Dunst et al., 2017; Elliott & Bachman, 2017). The present study may help researchers to identify some of the sources of this variability. We used multiple measures of home numeracy activities that had been developed extensively in a completely different population, and implemented our work in a longitudinal design with a large sample of children from the same country and within the same age range. Our findings of moderate but persistent relations between home numeracy activities and growth in kindergarten children's early mathematical skills supports the view that home activities are relevant for understanding early mathematical learning in this age range under the conditions that existed in this research. The educational experiences of these children were structured and consistent across schools, which may also be important in the stability of the findings. Overall, we suggest that consideration of the many factors that may increase variability across samples is critical in interpreting the relations between home numeracy experiences and children's early mathematical performance.

What are the implications of the current findings for parents? Our findings demonstrate that informal and operational activities were related to the growth of children's basic numeracy skills, specifically number comparison and arithmetic. Number comparison is a fundamental marker of mathematical skill in the early grades (Lyons et al., 2014; Vanbinst, Ghesquière, & De Smedt, 2012), and arithmetic skills are key for later math success (Duncan et al., 2007). Thus, our results have implications for educators and other stakeholders in that they provide a basis for recommending that parents become involved in specific math activities with their children at home to promote their children's math skills over time. Everyday numeracy interactions like number-game play (Ramani & Siegler, 2008) provide exposure to math-relevant language and activities that contribute to early math development (Levine et al., 2010; Purpura et al., 2017). These findings also align with those coming from research on home literacy, where exposure to shared reading has a long-lasting effect (Hood et al., 2008; Mol & Bus, 2011; Sénéchal & LeFevre, 2002). Furthermore, operational activities in which children manipulate numbers and engage in mental operations also create a context that promotes math skills (Berkowitz et al., 2015). Identification of similar patterns of results as in other studies of children from different cultures underscores the importance of parental input in children's early learning.

Future Research

The current work is correlational in nature, and we cannot rule out the possibility that parents whose children have more advanced numeracy skills may report more advanced activities, or that parents who engage in more relevant activities do so because their children are interested in math (Lukie, Skwarchuk, LeFevre, & Sowinski, 2014). Studies that include ongoing observations of parent-child interactions could illuminate important features of the evolving home environment (Vandermaas-Peeler, Way, & Umpleby, 2002). In our study, we followed children for a period of almost two years, from the beginning of pre-kindergarten to the end of kindergarten. It could be very informative to evaluate whether these effects continue through the transition to first grade.

Conclusion

In conclusion, home numeracy activities predicted growth in children's numeracy skills during the transition from pre-kindergarten to kindergarten. Our study demonstrates sustained predictive relations between growth in children's numeracy skills and the frequency of operational and informal home numeracy activities at home. Thus, given that children's early individual differences in numeracy are stable and predictive of later performance, the current findings support the view that parents' promotion of numeracy skills at home will support children's numeracy development.

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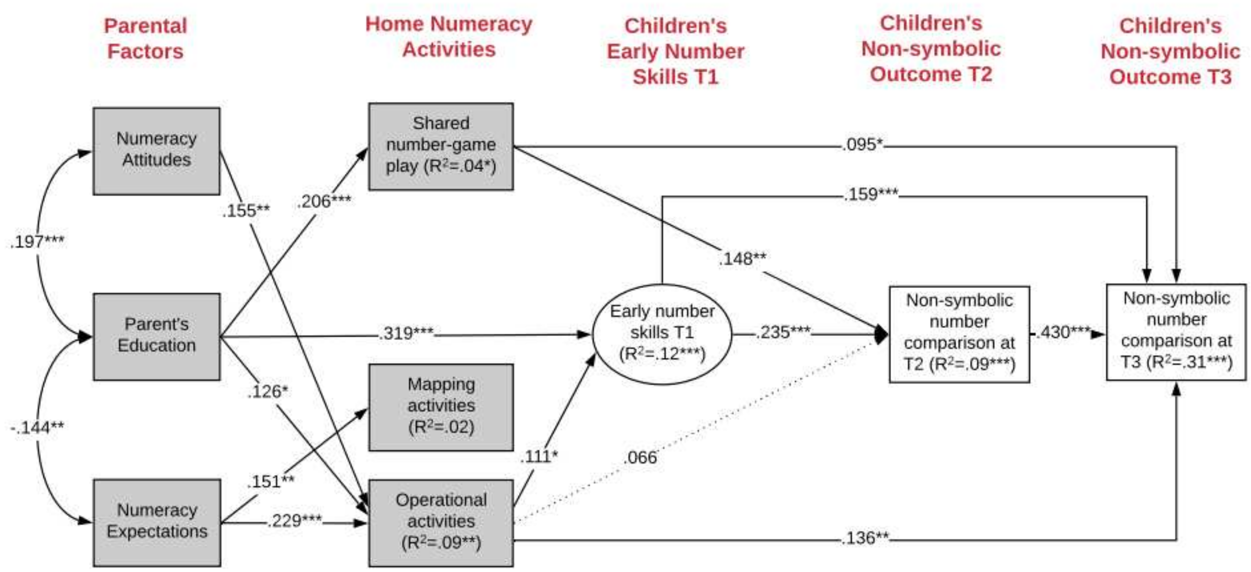
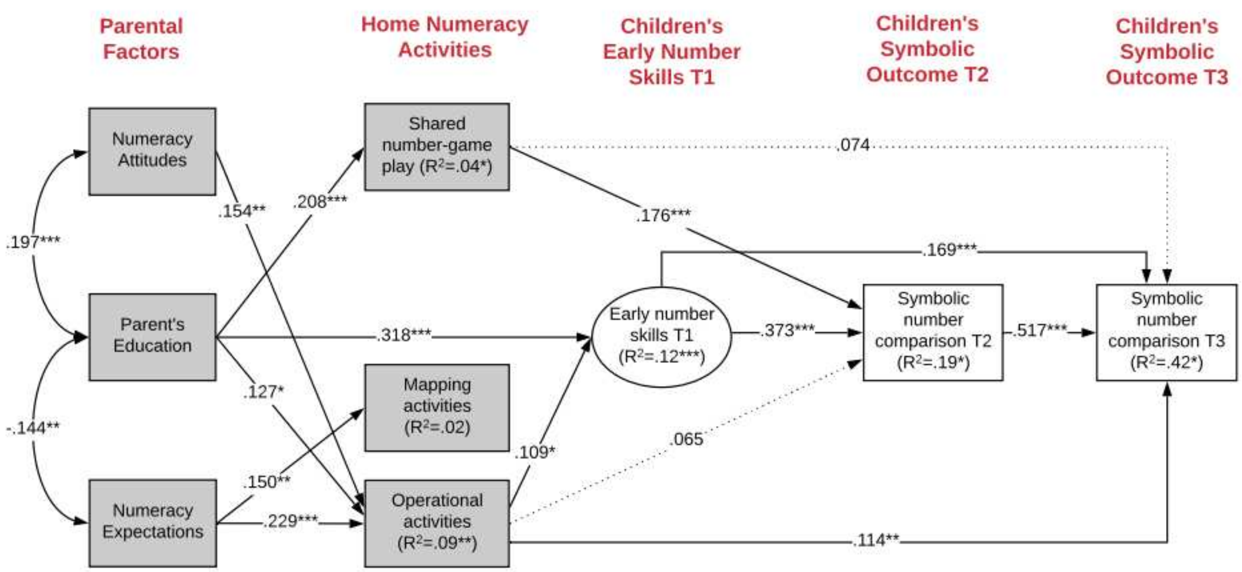
Figure Legends and Captions

Figure 1. Home Numeracy Model predicting non-symbolic number comparison. Model fit statistics: χ^2 (18) = 20.387, p =. 3115, SRMR = .028, CFT = .991, RMSEA = .018, CI = [0, .049]. Numbers on direct paths are standardized coefficients. There were significant indirect paths from shared number game play through non-symbolic comparison at Time 2, β = .064**, CI = [.021, .109] and from operational activities through early number skills at Time 1, β = .018, CI = [.002, .038], and through early number skills at Time 1 and non-symbolic comparison at Time 2, β = .011, CI = [.002, .024].



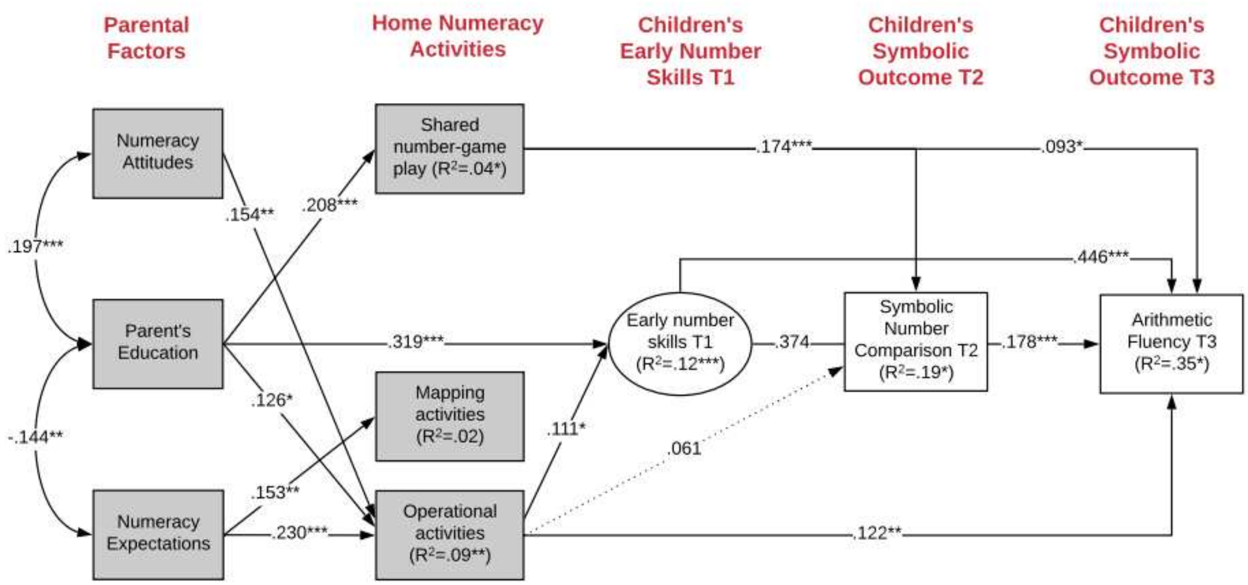
*p < .05, **p < .01, *** p < .001

Figure 2. Home Numeracy Model predicting symbolic number comparison. Model fit statistics: χ^2 (18) = 23.508, *p* =. 1718, SRMR = .030, CFT = .974, RMSEA = .027, CI = [0, .054]. Numbers on paths are standardized coefficients. There was a significant indirect path from shared number game play through symbolic comparison at Time 2, β = .091**, CI = [.044, .143] and from operational activities through early number skills at Time 1, β = .018*, CI = [.003, .037], and through early number skills at Time 1 and symbolic comparison at Time 2, β = .021, CI = [.003, .045].



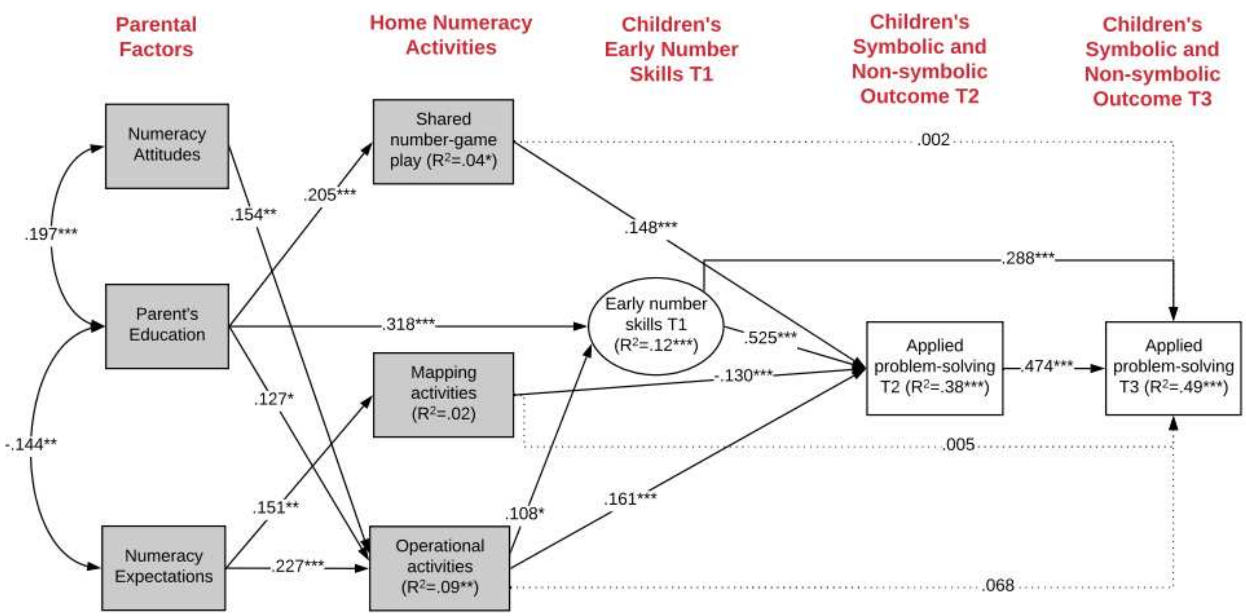
*$p < .05$, **$p < .01$, *** $p < .001$

Figure 3. Home Numeracy Model predicting arithmetic fluency. Model fit statistics: χ^2 (18) = 21.679, *p* =. 2465, SRMR = .030, CFT = .989, RMSEA = .022, CI = [0, .051]. Numbers on paths are standardized coefficients. There was a significant indirect path from shared number game play through symbolic comparison at Time 2, β = .031**, CI = [.031, .149] and from operational activities through early number skills at Time 1, β = .050*, CI = [.019, .251], and through early number skills at Time 1 and symbolic comparison at Time 2, β = .007, CI = [.003, .046].



p* < .05, ***p* < .01, * *p* < .001

Figure 4. Home Numeracy Model predicting applied problem solving. Model fit statistics: χ^2 (17) = 20.378, *p* =. 2553, SRMR = .031, CFT = .994, RMSEA = .022, CI = [0, .052]. Numbers on paths are standardized coefficients. There were significant indirect paths from *shared number game play* through applied problem solving at Time 2, β = .070**, CI = [.146, .503] and from *operational activities* through early number skills at Time 1, β = .031*, CI = [.015, .279], through applied problem solving at Time 2, β = .076*, CI = [.163, .534], and through early number skills at Time 1 and applied problem solving at Time 2, β = .027, CI = [.015, .240].



*p < .05, **p < .01, *** p < .001

Tables

Table 1

Descriptive Statistics for Children's Measures

	Time 1				Time 2				Time 3			
	M	*SD*	Range	*Reliability*	*M*	*SD*	Range	*Reliability*	*M*	*SD*	Range	*Reliability*
Verbal counting[1]	13.2	8.5	0-69	r = .53	--	--	--	--	--	--	--	--
Number identification[2]	6.2	4.2	0-17	α = .89	--	--	--	--	14.8	5.19	0-26	α = .75
Give N Task (Object counting) [2]	3.1	2.2	0-7	α = .82	--	--	--	--	--	--	--	--
Number comparison												
Symbolic[4]	--	--	--	--	0.22	0.10	.00-.48	n/a	.32	.10	.03-.64	n/a
Non-symbolic[4]	--	--	--	--	0.27	0.07	.01-.47	n/a	.34	.07	.10-.60	n/a
Number line[3]	--	--	--	--	2.4	1.3	.3-6.3	α = .72	1.73	.96	.3-5.6	α = .80
Applied problem-solving	11.4	3.5	0-27	α = .65	15.1	3.8	5-26	α = .70	18.5	4.5	7-31	α = .68
Letter-word identification[2]	3.5	2.1	0-13	α = .64	8.2	8.9	1-73	α = .71	16.5	13.5	2-73	α = .85
Vocabulary (TVIP)[2]	40.5	13.0	0-70	α = .62	49.1	12.0	4-79	α = .65	62.4	11.6	12-95	α = .56
Arithmetic fluency[2]	--	--	--	--	--	--	--	--	3.86	2.53	0-12	n/a

Note. For Time 1, N = 388 to 415, for Time 2, N = 403 to 406, for Time 3, N = 368.

[1]Highest count. [2]Total correct trials. [3]Mean absolute error. [4]Number correct/second.

Table 2

Correlations among Control Measures, Parental Factors, Home Numeracy Activities, and Children's Numeracy Performance

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1. Age	-0.01	**0.19**	0.04	-0.01	-0.02	0.10	**0.11**	**0.37**	**0.20**	**0.16**	**0.24**	-0.07	**0.20**	**0.22**	**0.22**	-.04	**0.24**
2. Gender		**-0.12**	-0.09	0.07	0.03	0.01	-0.02	*-0.11*	0.06	**0.17**	-0.02	0.00	-0.01	0.03	-0.04	0.02	**-0.16**
3. Parent education			**0.20**	**-0.14**	-0.09	*0.12*	**0.20**	**0.33**	**0.17**	*0.12*	**0.23**	-0.05	**0.17**	*0.12*	**0.26**	-0.05	**0.23**
4. Attitudes[a]				0.00	-0.01	**0.18**	*0.11*	**0.17**	0.02	-0.01	*0.11*	-0.04	0.09	0.09	**0.17**	0.04	**0.12**
5. Expectations[a]					**0.15**	**0.21**	-0.03	-0.06	-0.03	-0.01	0.00	*-0.13*	0.05	0.05	0.06	-0.09	0.05
6. Mapping activities[a]						0	0.07	0.01	-0.06	-0.04	*-0.12*	-0.01	0.01	0.00	-0.03	0.02	-0.05
7. Operational activities[a]							0.03	**0.15**	*0.12*	*0.11*	**0.25**	**-0.17**	**0.20**	**0.20**	**0.22**	-0.08	**0.21**
8. Shared number-game play								**0.15**	**0.23**	**0.18**	**0.23**	-0.06	**0.23**	**0.20**	**0.16**	0.02	**0.21**
9. Early number skills[a] (T1)									**0.41**	**0.27**	**0.57**	**-0.29**	**0.39**	**0.30**	**0.56**	**-0.22**	**0.54**
10. Numb.com: sym (T2)										**0.63**	**0.54**	**-0.30**	**0.61**	**0.48**	**0.42**	**-0.16**	**0.38**
11. Numb.com: non-sym (T2)											**0.41**	**-0.20**	**0.50**	**0.49**	**0.32**	**-0.14**	**0.22**
12. App. problem-solving (T2)												**-0.27**	**0.50**	**0.38**	**0.65**	**-0.17**	**0.48**
13. Number line (T2)													**-0.25**	**-0.14**	**-0.27**	**0.23**	**-0.20**
14. Numb.com: sym (T3)														**0.66**	**0.53**	**-0.30**	**0.51**
15. Numb.com: non-sym (T3)															**0.29**	**-0.13**	**0.37**
16. App. problem-solving (T3)																**-0.29**	**0.56**
17. Number line (T3)																	**-0.22**
18. Arithmetic fluency (T3)																	

Notes. Italicized numbers indicate $p < .05$, **bolded** numbers indicate $p < .01$ [a] factor/latent scoring described in the methods section. T1= Time 1 measure; T2= Time 2 measure. T3= Time 3 measure.

Table 3

Summary of evidence for direct and indirect relations between home numeracy activities (informal, operational) and growth in early mathematical abilities

	Home Numeracy Factors			
	Informal (parents' knowledge of number games)		Operational Activities	
Outcomes	Direct	Indirect	Direct	Indirect
Non-symbolic Comparison	✓	✓[2]	✓	✓[1,2,3]
Symbolic Comparison		✓[2]	✓	✓[1,3]
Arithmetic fluency	✓	✓[2]	✓	✓[1,3]
Applied Problems		✓[2]		✓[1,2,3]

Note. Significant paths between predictors (informal and operational activities) and Time 3 performance are shown as check marks. Significance in this table is based on confidence intervals; p values are also reported in the text and figures.

[1] through Time 1 early number skills; [2] through Time 2 skill; [3] through both Time 1 early number skills and Time 2 skills.