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Individual Differences in the Development of Children's Arithmetic Fluency from Grades 2 to 3

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Running Head

Development of Children's Arithmetic Fluency

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Abstract

In the present research, we provide empirical evidence for the process of symbolic integration of number associations, focusing on the development of simple addition (e.g., 5 + 3 = 8), subtraction (e.g., 5 – 3 = 2), and multiplication (e.g., 5 x 3 = 15). Canadian children were assessed twice, in grade 2 and grade 3 (N = 244; 55% girls). All families were English-speaking, and parent education levels ranged from high school to postgraduate, with a median of community college. In grade 2, children completed general cognitive tasks (i.e., receptive vocabulary, working memory, nonverbal reasoning, and inhibitory control). In both grades, children completed single-digit addition and complementary subtraction problems. In grade 3, they completed single-digit multiplication problems and measures of applied mathematics, specifically, word-problem solving, algebra, and measurement. We found that addition and subtraction were reciprocally related (controlling for cognitive skills). Subtraction fluency predicted multiplication in grade 3, whereas addition fluency did not. In grade 3, both subtraction and multiplication fluency were predictors of applied mathematics, with multiplication partially mediating the relation between subtraction and applied mathematics performance. These findings support the view that learning arithmetic associations is a hierarchical process. As students practice each new skill, individual differences reflect the integration of the novel component into the developing associative network.

Word count: 211

Key Words: symbolic integration, addition, subtraction, multiplication, arithmetic, mathematics

Mathematical cognition involves the acquisition and use of a hierarchy of interrelations among abstract numerical symbols (Hiebert, 1988; Núñez, 2017). We propose that a process of *symbolic integration*, the creation of associations among numerals and other mathematical symbols, during learning is central to children's construction of mathematical understanding. As children acquire various associations among numerals, initially including ordinal, cardinal, and arithmetical connections, these associations form an increasingly interconnected mental network (Hiebert, 1988; Siegler & Lortie-Forgues, 2014; Xu et al., 2019; Xu & LeFevre, 2020). Failure to integrate number associations early may impede mathematical success in later grades (Jordan & Dyson, 2016; Rousselle & Noël, 2007). Moreover, children's integrated associations support the selection of increasingly efficient and flexible strategies for solving mathematical problems. For example, an understanding of the relations between addition and multiplication allows the equation 2 + 2 + 2 to be solved as 3 x 2. The goal of the present research was to explore the process of symbolic integration, focusing on the development of addition, subtraction, and multiplication associations for children in grades 2 and 3.

In the *hierarchical symbol integration* model (Xu et al., 2019), later-learned numerical associations build upon existing associations to create complex interconnected networks (Hiebert, 1988; Siegler & Lortie-Forgues, 2014). Initially, children acquire cardinal associations (e.g., 2 is bigger than 1 and smaller than 3) as they practice counting and comparing number sets. They also learn ordinal associations (e.g., 3 follows 2 and comes after 1) as they practice verbal counting and relative position. By grade 2, children have integrated cardinal and ordinal associations into a cohesive network that also supports their addition associations (e.g., 1 + 2 = 3; Lyons et al., 2014; Sasanguie & Vos, 2018; Xu & LeFevre, 2020). However, little is known about how children integrate their arithmetic associations in grades 2 and 3.

Integration and Differentiation of the Associative Mental Network

Development emerges as a result of a continuous process of integration and differentiation (Werner & Kaplan, 1956). In the context of mathematical knowledge, integration is necessary because the same numerals have a complex set of interrelations. Accordingly, differentiation also occurs in the associative network because only some number associations need to be activated in any given context (Campbell, 1994; Siegler & Chen, 2008; Xu et al., 2019). For example, children initially learn that 4 is larger than 3 and that 3 comes before 4. They eventually learn to associate 4 and 3 with 7 through addition, and with 1 through subtraction. Somewhat later, they learn that 4 and 3 are also associated with 12 through multiplication. Learning to associate 4 and 3 with 12 requires that children add to their existing associations that link 4 and 3 with other numbers (Siegler & Robinson, 1982). All of these associations must exist together in their mental network but need to be accessed selectively (Campbell, 1994; Campbell & Agnew, 2009; Campbell & Alberts, 2009).

Learning new associations, such as multiplication when it is first introduced, may require that children temporarily suppress the activation of other associations. Consistent with this view, when children in grade 3 (i.e., ages 8 through 10) started to learn multiplication, their addition skills were temporarily disrupted (Miller & Paredes, 1990). Similarly, children in grades 3 to 5 (i.e., ages 7 through 10) showed interference effects in a verification task -- that is, they were slower and more likely to make errors when multiplicative answers were given to addition problems (e.g., 2 + 3 = 6) or when additive answers were given to multiplication problems (e.g., 2 x 3 = 5; Lemaire et al., 1994). For children in grades 3 and 4, the interference effect was only evident at the end of the school year, after children had time to learn and practice multiplication. Children with dyscalculia (i.e., a specific learning disability related to acquiring mathematics, Butterworth, 2005) have difficulty suppressing associations among similar elements, such as related addition and multiplication facts, compared to their typically developing peers, suggesting that they face difficulties in storing new related arithmetic associations in memory (De Visscher & Noël, 2014a, 2014b; De Visscher & Noël, 2016). Interference effects persist among adults, who find it difficult to reject stimuli that highlight incorrect associations such as 2 x 3 = 5 (Campbell, 1995; Winkelman & Schmidt, 1974). In summary, differentiating and integrating various number associations is a crucial aspect of arithmetic development, but may be a source of challenge when children are learning and consolidating numerical associations.

Many influential models of arithmetic associations are based on the assumption that number associations are represented with varying degrees of accessibility in a cohesive mental network (Collins & Loftus, 1975). Adults show considerable evidence of having integrated networks of arithmetic facts (Ashcraft, 1992; Campbell, 1997; De Brauwer & Fias, 2011). Representational theories about how arithmetic associations are organized include the *Associative Network* model (Ashcraft, 1982), the *Distribution of Associations* model (Siegler, 1988), the *Network Interference* model (Campbell, 1994, 1995), the *Interacting Neighbours* model (Verguts & Fias, 2005), and the *Identical Elements* model (Rickard, 2005). Although the models make different detailed assumptions, they all include some kind of mechanism in which the strength of associations among problem elements is related to the speed and accuracy with which people respond to arithmetic stimuli (Ashcraft, 1992; Campbell, 1995; De Visscher & Noël, 2014b; Siegler, 1988). That is, the stronger the association, the more accessible the information and thus the faster and more accurately people respond. Moreover, number associations that are learned earlier, or practiced more, are also more strongly connected and thus more accessible than associations that are learned later or practiced less (see simulations by

Verguts & Fias, 2005). These models capture patterns of data from numerous studies in which arithmetic associations show competition and interference both between and within operations.

If the strength of the association to an arithmetic expression is strong, solvers directly access the association to that problem in memory to solve the problem (e.g., 6 x 6 = 36; Ashcraft, 1982; Campbell, 1994; LeFevre et al., 1996; Rickard, 2005). However, when associations between answers and arithmetic expressions are weak or not yet developed, children may use a partial retrieval strategy based on their accessible number associations, such as relying on counting to solve addition and subtraction problems (Groen & Parkman, 1972; Woods et al., 1975), or on repeated addition to solve multiplication problems (Imbo & Vandierendonck, 2008; Lemaire & Siegler, 1995; Siegler, 1988). They may also use strategies in which they combine operations, for example, solving 8 x 9 as 8 x 8 + 8 (LeFevre et al., 1996; Polspoel et al., 2017). By using direct or partial retrieval strategies over time, the associations between the operands and the correct answer are built and eventually become fully accessible in the mental network (Siegler, 1987). Furthermore, the integration process will be strengthened as solvers switch between strategies and operations to solve arithmetic problems.

Development of Addition, Subtraction, and Multiplication

Arithmetic associations develop continuously throughout elementary school, in a progression defined by the curriculum (i.e., addition precedes subtraction which precedes multiplication). Before they master arithmetic, however, children have considerable information about the cardinal and ordinal relations among numbers. Research has shown that fluency of access to ordinal associations (i.e., knowing that 2 precedes 3 and comes after 1) supersede fluency of access to cardinal associations (i.e., knowing that 3 is larger than 2) as the key predictors of addition performance from grades 1 to 2 (Sasanguie & Vos, 2018; Xu & LeFevre, 2020). The dominance of ordinal associations in predicting individual differences in addition in grade 2 (and beyond) reflects the integration of previously acquired number associations into a unified network (Lyons et al., 2014; Sasanguie et al., 2017; Sasanguie & Vos, 2018; Xu et al., 2019). However, little is known about how children's networks change as they integrate the various arithmetical operations, that is, addition, subtraction, multiplication, and division.

Addition and subtraction require understanding of additive composition and part-whole relations (Butterworth, 2005; Clark & Kamii, 1996; Nunes et al., 2016). These operations are complementary both procedurally and conceptually such that any subtraction problem can be transformed into an addition problem, and vice versa (Robinson, 2017; Robinson & Dubé, 2009, 2012). The complementary relations between addition and subtraction (e.g., 3 + 4 = 7, 7 – 4 = 3, 7 – 3 = 4) form the basis for computational shortcuts for subtraction problems based on addition associations (Bryant et al., 1999; Verschaffel et al., 2010). Accordingly, children's existing knowledge of addition should facilitate their acquisition of subtraction. More specifically, stronger complementary addition associations should increase the likelihood of correct responses to subtraction complements (see reviews in Siegler, 1987; Siegler & Shrager, 1984). Moreover, intervention studies show that training on subtraction-as-addition strategy (e.g., use a known addition association such as 8 + 4 = 12 to determine that the unknown subtraction association 12 – 4 = 8) promote children's subtraction skills (Baroody et al., 2014; Paliwal & Baroody, 2020). Evidence with adults is consistent with this claim. For example, practicing 3 + 2 facilitates access to 5 – 2 (Campbell & Agnew, 2009); similarly, practicing division equations facilitates their multiplication complements (Campbell & Alberts, 2009; De Brauwer & Fias, 2011). Thus, the complementary conceptual relation between addition and subtraction provides the basis for integrating these operations into the mental network.

By the time they start formal instruction in multiplication, children have two or more years of experience with addition and subtraction. Notably, multiplication is different from addition and subtraction because it involves representing quantities as composite units, rather than collections of units of one (Clark & Kamii, 1996; Harel & Confrey, 1994; Nunes et al., 2016; Steffe & Olive, 2010). For example, given a set of 16 objects, children whose thinking relies primarily on additive conceptual knowledge may mentally represent the objects as 16 individual items, or as a combination of 10 items and 6 items, whereas children with multiplicative conceptual knowledge are also able to represent the 16 objects as 4 composite units, with each consisting of 4 individual items. Making 4 units into one composite unit is a higher-level abstraction than thinking only of units of one. The transition from purely additive to more complex multiplicative representations is critical to the development of arithmetic skills (Harel & Confrey, 1994; Nunes et al., 2016).

The availability of accessible sequence and addition number associations in the mental network is crucial for facilitating the development of multiplication (Sherin & Fuson, 2005). For example, to use a "count-by" strategy to solve 5 x 3, children have to know the number sequence of 3s (i.e., 3, 6, 9, 12, 15) to get to 15, accessing second-order counting sequences to enhance multiplication. Alternatively, children can also solve 5 x 3 by first retrieving an accessible multiplication association, for example, 5 x 2 = 10, and then adding another 5 to get 15. In North America, teachers encourage children to use multiple strategies to solve multiplication problems when they are first introduced (NCTM, 2000). Thus, the accessibility of various previously acquired number associations can be harnessed to support the development of multiplication.

The Current Study

The main goal of the present study was to examine the developmental trajectories of addition, subtraction, and multiplication fluency in children from grades 2 to 3. Children were recruited from suburban areas in or near a large Canadian city. According to the Manitoba Curriculum (2013), by the end of grade 2, children are expected to understand the complementary relations between addition and subtraction. In grade 3, children are expected to continuously strengthen the complementary relations between addition and subtraction, and they also learn multiplication up to the five times table. Therefore, by the end of grade 3, children are expected to demonstrate some degree of integration of addition and subtraction and be able to further integrate the new competing associations they learn in multiplication.

The hierarchical symbol integration model provides a framework for understanding how various associations integrate over time in relation to the development of arithmetic (Xu et al., 2019; Xu & LeFevre, 2020). However, this model has not been previously tested with children who are integrating addition, subtraction, and multiplication associations during learning. Thus, we adopted the hierarchical symbol integration model for the present study to capture the hierarchical relations among basic arithmetical associations (i.e., single-digit addition, complementary subtraction associations, and single-digit multiplication). To the extent that operations are integrated, variability in the newest and thus less practiced operation (i.e., multiplication) would predict performance on more advanced mathematical tasks. In contrast, to the extent that these operations are differentiated because integration has not yet occurred, variability in subtraction or addition would also predict performance on more advanced mathematical tasks. These hypotheses are captured in Figure 1. First, we expected that children start to integrate addition and subtraction associations from grades 2 to 3. According to the hierarchical symbol integration model, integration is reflected in the prediction that variability in access to more advanced associations would supersede variability in fluent access to more basic associations to predict performance on more advanced mathematical tasks. Because addition is the basis for understanding subtraction, we hypothesized that addition in grade 2 would predict the growth of subtraction from grades 2 to 3 (H1a), whereas subtraction in grade 2 would not predict the growth of addition from grades 2 to 3 (H1b). A cross-lagged approach was used to investigate causal relations in the absence of experimental manipulation (Anderson & Kida, 1982). Second, in grade 3, we assumed that children have started to integrate addition and subtraction into a unified network. If subtraction captures the highest level of associative integration for children in grade 3, variability in access to subtraction should supersede variability in addition. Thus, we predicted that subtraction would predict multiplication in grade 3 (H2a), whereas addition would not predict multiplication in grade 3 (H2b).

The second goal of the present study was to explore how arithmetic associations are related to applied mathematics performance in grade 3. Three subtests (word-problem solving, algebra, and measurement) from *KeyMath* were administered (Connolly, 2007). The three tasks all had the same presentation format: Children were shown an image that corresponded to an orally presented mathematical problem. Using confirmatory factor analysis, an applied mathematics latent variable was created from these three subtests. To tap into the direct relation between arithmetic measures and applied mathematics, we controlled for cognitive processes that have known relations with applied mathematics performance. More specifically, we controlled for receptive vocabulary (Fuchs et al., 2015; Swanson et al., 2015), reasoning skills (Steel & Funnell, 2001; Primi et al., 2010), inhibitory control (LeFevre & Kulak, 1994; Miller & Paredes, 1990; Robinson & Dubé, 2013), and working memory (DeStefano & LeFevre, 2004; Raghubar et al., 2010).

As shown in Figure 1, because children start to learn multiplication in grade 3, and are thus just starting to integrate multiplication into their associative networks, we hypothesized that subtraction and multiplication would be more differentiated than integrated. Thus, after controlling for children's general cognitive skills (i.e., receptive vocabulary, reasoning skill, inhibitory control, and working memory), individual differences in subtraction (H3a) and multiplication (H3b) were expected to independently predict applied mathematics.

Method

Participants

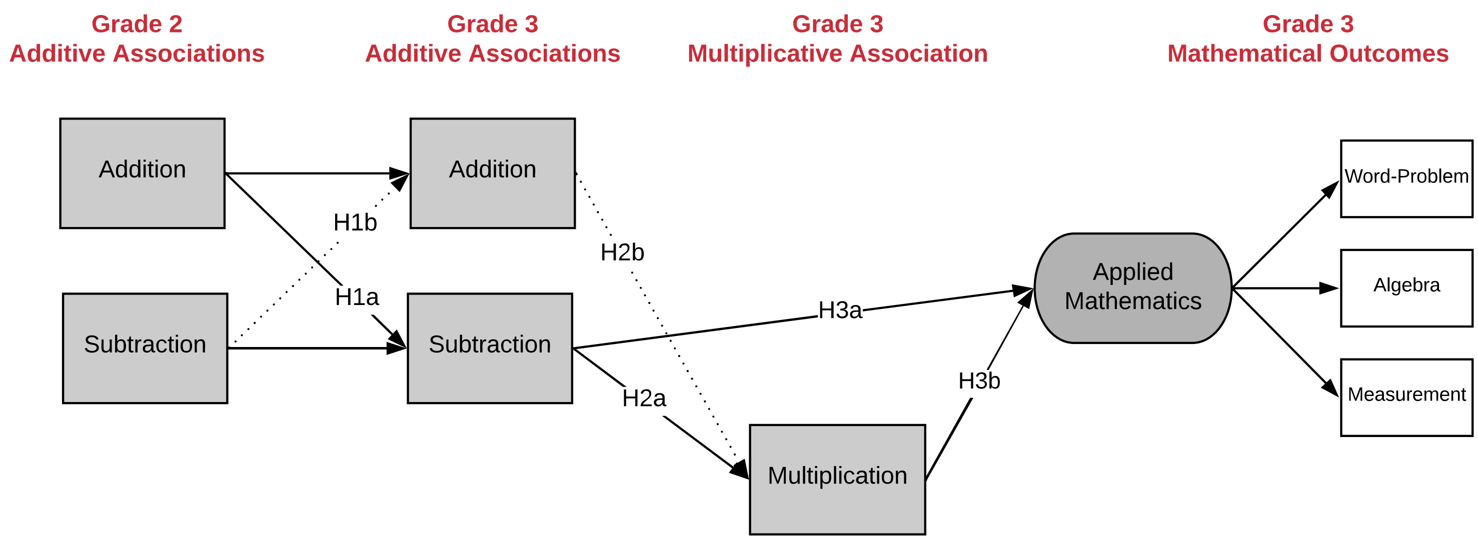
Following ethics approval from University of Winnipeg (Study title: Learning mathematics in a French immersion setting; Protocol #: HE08771), school principals were contacted. On approval of the principals, letters were sent home to parents, inviting children to participate. The data were collected as part of a larger project on children's language learning and mathematics achievement and included groups from four different research sites. We did not collect data on all of the key measures at the other sites for (e.g., algebra, measurement, and word-problem solving). Additionally, there were other differences across sites including language of the stimuli and educational background of the students. Thus, the data from only one site was included in the present analyses because other sites did not have equivalent data for at least one year on all of the measures of interest. Details about the larger project are available on the Open Science Framework (osf.io/428hp).

A group of 182 children were recruited near the end of grade 2 at Time 1 (i.e., April and May 2018) from seven public schools (82 boys; M_{age} = 7.8 years; SD = 0.29). Two of the seven schools were located in a small town outside the urban centre. Children were tested again near the end of grade 3 (from April to June 2019). A total of 35 children did not participate in grade 3 for personal reasons (e.g., changed schools or programs), but additional students from the same schools joined the study in grade 3. Thus, the final sample consisted of 244 children: In both grades 2 and 3, 147 participated; 35 participated only in grade 2; and 62 participated only in grade 3. In grade 3, there were 109 boys; $M_{\text{age}} = 8.9$ years; $SD = .13$.

Of the final sample, 90% of the children spoke English as their first language. Among the 10% of the students who reported English as their second language, first languages varied: German (2.5%), Russian (1.3%), Arabic (0.8%), Serbian (0.8%), Korean (0.8%), Bosnian (0.8%), Chinese (0.4%), French (0.4%), Yoruba (0.4%), Gujarati (0.4%), Punjabi (0.4%), and Polish (0.4%). Notably, 80% of these students reported speaking English at least half of the time at home. Parents' highest education level was collected for 229 mothers and 220 fathers. Among these parents, 14% of the mothers (11% of the fathers) had received a postgraduate degree, 31% of the mothers (26% of the fathers) had received an undergraduate degree, 29% of the mothers (33% of the fathers) had received a community college degree, 24% of the mothers (29% of the fathers) had received a high school diploma, and 2% of the mothers (3% of the fathers) had received less than a high school diploma (*Median* = community college degree for both mothers and fathers). Moreover, 152 children were enrolled in French immersion programs in which the instructional language is French, whereas 92 children were in English-instruction programs. There were no significant differences between immersion students and non-immersion students on any of the measures in the present study (ps > .05), except that parental education was higher for the immersion group than the non-immersion group for mothers (3.44 vs. 3.07), χ^2 (4, N = 229) = 12.48, p = .014, and for fathers, (3.26 vs. 2.90), χ^2 (4, N = 210) = 9.36, p = .053.

Figure 1

Summarized Hypotheses



Note. Dotted lines indicate expected non-significant paths; Working memory, reasoning, inhibitory control, and receptive vocabulary knowledge (not shown) are also expected to be predictive of the latent applied mathematical outcome.

Procedure

Written consent from parents was obtained at both time points. Most of the testing was done by five research assistants who had either completed or were working toward a Bachelor's degree in psychology, education, or developmental studies. Some testing was also completed by a high school internship student, an itinerant teacher, and a university professor (one of the authors of the study). All of the experimenters were provided with a detailed testing manual, including specific testing and scoring procedures in both written and video formats. The experimenters completed multiple training sessions (2 hours per session) during which they practiced the testing and scoring procedures and general principles of working with children.

Each child was tested individually in a quiet area of the school by an examiner. In both grades 2 and 3, children completed the measures in two 30-minute sessions. Tasks presented to children in the present paper were administered in English. The children in French Immersion completed a third session with measures in French (see Authors, DATE). The order of the test administration was fixed. After the testing sessions, children were given stickers in appreciation of their participation. Data were entered independently by three different research assistants and cross-checked for accuracy.

Measures

All of the cognitive measures were assessed in grade 2. Addition and subtraction were assessed in both grades 2 and 3. Multiplication and the three applied mathematical measures were assessed in grade 3. Reliabilities for each task (Cronbach's alpha) are reported in Table 1.

Cognitive Skills

Nonverbal Reasoning. The Matrix Reasoning task from the Weschler Intelligence Scale for Children-Fifth Edition (WISC-5; Wechsler, 2014) was used to assess children's nonverbal reasoning skills. Children were presented with an incomplete grid and they were asked to identify the missing part that properly completes the matrix. There were 34 trials, and testing was discontinued after three consecutive errors. The total score was the number of correct trials.

Receptive Vocabulary. The *Peabody Picture Vocabulary Test–Revised*, Form B (Dunn & Dunn, 2012) was adapted to reduce testing time. Children were presented with a card with four pictures. Upon hearing a word, children were asked to point to the picture that best corresponded to the word they had just heard. A total of 60 items from five subsets that were appropriate for children in grade 2 were used. The testing was discontinued when children made eight or more errors in a subset. The total score was the total number of correct items.

Working Memory. Three tasks (i.e., digit forward, digit backward, and spatial span) were used to assess children’s working memory. The two digit span tasks assessed verbal short-term and working memory, respectively, whereas the spatial span task assessed visual-spatial memory (Alloway et al., 2008).

Digit Span Forward. In the *Digit Span Forward* (WISC-5; Wechsler, 2014), children heard a series of numbers and were asked to repeat the numbers back in the same order they had just heard. There were two trials for each span, starting from a span length of two digits. If children correctly repeated all the numbers in the correct order for at least one of the two sequences per span, then the span length was increased by one digit. The test was discontinued when children were incorrect on both spans of a given length. The total score was the number of sequences that each child repeated correctly.

Digit Span Backward. The procedure for the *Digit Span Backward* (WISC-5; Wechsler, 2014) was identical to the *Digit Span Forward*, except that children were asked to repeat the numbers they heard in the reverse order. The total score was the number of sequences that each child repeated correctly.

Spatial Span. In this task, nine green dots were presented in a random pattern on an iPad screen (https://hume.ca/ix/pathspan/). Dots were arranged unsystematically, but the arrangement was the same on each trial. On each trial, a series of dots lit up one by one, after which children were asked to touch the dots in the order that they were illuminated. There were two trials for each span, starting from a span length of two dots. If at least one of the two trials was correctly reproduced, then the sequence length was increased by one dot. The task was discontinued when children made errors on both sequences for each sequence length. The total score was the number of sequences completed correctly.

Black and White Stroop. Children's inhibitory control skills were measured by a black and white Stroop task (Vendetti et al., 2015) that was implemented as an iPad application. In this task, a fixation cross appeared in the center of the screen for 500 ms, then a blank screen for 500 ms, followed by a visual stimulus (black square or white square) and an auditory color name (black or white). There were four blocks of trials, with eight trials in each block. The first and third blocks were always congruent trials where children heard a color name, and then they were asked to touch the square on the screen with the same color as fast as possible. The second and fourth blocks were always incongruent trials where children heard a color name, and then they were asked to touch the square on the screen with the different color as fast as possible. On each trial, children who failed to respond after 3 seconds were presented with the next trial. The score was an interference cost score calculated as the difference between mean adjusted response time (RT) for incongruent trials and mean adjusted RT for congruent trials. The adjusted RT for each block was calculated based on a linear integrated speed-accuracy score: $RT_{adjusted} = RT_{correct} + (PE \times [SD_{RT}/SD_{PE}])$, where $RT_{correct}$ is the response time on the correct trials, PE is the percent error, and SD is the standard deviation (Vandierendonck, 2017).

Arithmetic Skills

Addition. Children's knowledge of addition associations was measured by a paper-and-pencil measure (Chan & Wong, 2019). The addition task was comprised of 60 single-digit problems arranged in three columns (sums less than or equal to 17), and each problem was presented horizontally (e.g., 6 + 7 = __). Children were given one minute to write down as many answers as possible without skipping any questions. The score was the total number of questions answered correctly in one minute.

Subtraction. The procedure for the subtraction subset was identical to the addition subset. The 60 problems were the inverse of the aforementioned addition questions and they presented in a different order. The score was the total number of questions answered correctly.

Multiplication. The procedure for the multiplication subset was identical to the addition and subtraction subsets. Because children were not formally introduced to multiplication in grade 2, they completed the multiplication task only in grade 3. The 60 problems consisted of multiplication questions up to the 5 times table, with multiplicands ranging from 2 to 5 and multipliers ranging from 1 to 10. The score was the total number of questions answered correctly.

Applied Mathematics

Word-problem Solving. The Applied Problem Solving subtest of the *KeyMath* - third edition was used (Connolly, 2007). Children solved word problems that increased in difficulty. More specifically, they were assessed on identifying irrelevant information in word problems, inventing a word problem when given an arithmetic equation, and solving arithmetic word problems. We selected a subset of 12 questions by examining the standardized means and the standard deviations for the age group. The task was discontinued after three consecutive errors or if the child reached item 12. The score was the total number of questions answered correctly.

Algebra. The Algebra subtest of the *KeyMath* 3rd Edition was used (Connolly, 2007). Children solved pre-algebra and algebra problems that increased in difficulty. Children were assessed on (a) reasoning abilities in which children were asked to determine the missing number in equations or to identify number patterns involving addition and subtraction (e.g., Each box stands for the same number. Four squares equal to 8. What number belongs in the box?, (b) data analysis abilities in which children were asked to answer questions about data presented in charts, tables, and graphs, and (c) applied problem solving skills where children were asked to relate arithmetic to real life situations using their own strategies. We selected a subset of 15 questions by examining the standardized means and the standard deviations for the age group. Children started at item 7, and testing was discontinued after three consecutive errors or if they reached item 21. The score was the total number of questions answered correctly.

Measurement. A subset of questions was selected from the Measurement subtest of the *KeyMath* 3rd edition (Connolly, 2007). We designed additional problems that were similar to those found in the *KeyMath* inventory to balance the number of items in each domain (i.e., time, money, and temperature). Children solved a total of 24 problems. Items 1 to 6 assessed children's knowledge of time (e.g., What time does this clock show?). Items 7 to 19 assessed children's knowledge of money (e.g., Name the coins from left to right; How much money is this altogether?). The last six items assessed children's knowledge of temperature (e.g., What is the temperature shown in the picture?). Children were instructed to answer all of the questions. The score was the total number of questions answered correctly.

Data Analysis

In the present study, we conducted structural equation modeling using M*plus* (Muthén & Muthén, 1998) to investigate the process of symbolic integration, focusing on the development of addition, subtraction, and multiplication for children in grades 2 and 3. First, we specified two latent factors in the model: Working memory in grade 2 based on digit forward span, digit backward span, and spatial span, and applied mathematics in grade 3 based on word-problem solving, algebra, and measurement. Next, we constructed a path model between children's performance on addition, subtraction, multiplication, and the latent applied mathematics variable, controlling for children's receptive vocabulary, matrix reasoning, inhibitory control, and the working memory latent factor. Model fit was examined using a combination of the chi-square goodness of fit test ($p > .05$), comparative fit index (CFI > .95), root mean square error of approximation (RMSEA < .06), and standardized root mean square residual (SRMR <.08; Hu & Bentler, 1998). The M*plus* syntax for the final model is included in the supplementary materials.

To determine if there were differences between participants with complete data (i.e., participated in both grades, $n = 147$) and those with incomplete data (i.e., participated in one grade only, $n = 97$), *t*- tests and χ^2 tests were conducted to determine if the demographic and performance variables (i.e., French immersion status [yes/no], child's age, gender, parental education, forward digit span, backward digit span, spatial span, matrix reasoning, receptive vocabulary, Stroop, addition, subtraction, multiplication, word problem solving, pre-algebra, measurement) varied across groups. There were no significant differences between students with complete versus incomplete data. Thus, we are confident that our data meet the criteria for missing at random and thus the models were estimated by a full information maximum likelihood (FIML) method where all available information is used in all observations to find the optimal combination of estimates for the missing parameters (Enders, 2010). As an additional check on the results, we conducted sensitivity analyses based on the complete data (n = 147) using regression analyses (i.e., pairwise deletion was applied to the missing data). The results were similar to the ones estimated using the full information maximum likelihood method. Thus, we present the analyses in which we used FIML to account for missing data.

Results

Descriptive Statistics

Descriptive statistics are shown in Table 1. All measures were normally distributed except for receptive vocabulary and arithmetic. The vocabulary task was too easy for the children in grade 2: Ten children completed all of the items correctly. For the arithmetic measures, in contrast, performance was positively skewed, with a few children performing well above the mean of the sample. The distribution and the skew pattern across operations were similar at both time points, which may reflect the nature of this type of speeded task. None of the children reached ceiling performance (i.e., 60) on the arithmetic measures in either grades 2 or 3. Furthermore, in grade 2, although a small percentage of students had scores of 0 (2% for addition and 8% for subtraction), the majority of students answered between 4 and 12 items correctly for both addition and subtraction. This is consistent with our theory that students were still developing their addition and subtraction skills in grade 2, and thus could only answer a few questions correctly in a short time.

Outliers were defined as values with z-scores greater than +/-3.29 from the mean for the sample. One outlier was found for matrix reasoning, and two outliers were found for each of the following tasks: receptive vocabulary, Stroop, addition in grade 2, subtraction in grade 2, subtraction in grade 3, and multiplication in grade 3. Sensitivity analyses with and without these outliers showed similar patterns of results, and thus all of the children's data were included in the final analyses.

With two exceptions, there were no significant gender differences across the measures (*p*s > .05). Girls had higher scores on matrix reasoning than boys (16.7 vs. 15.40), $t(180) = -2.25$, $p = .025$, Cohen's $d = .34$, and boys had higher scores on subtraction than girls in grade 2 (5.67 vs. 4.33), $t(136.50) = 2.31$, $p = .022$, Cohen's $d = .29$. Gender was initially included as a control variable in the main analyses, but because it was not a predictor of any of the outcomes and did not influence the pattern of the results, it was not included in the final model.

As shown in Table 1, children improved from grades 2 to 3 on both addition, $F(1, 145) = 120.05$, $p < .001$, $\eta_p^2 = .45$, and subtraction, $F(1, 145) = 107.92$, $p < .001$, $\eta_p^2 = .43$. Thus, a cross-lagged framework was used to examine the bidirectional relations between the development of addition and subtraction skills of children from grades 2 to 3.

Structural Equation Modeling

The goal of structural equation modelling (SEM) is to understand the patterns of correlation among the variables in the model (Kline, 1998). Correlations among the measures for children are shown in Table 2. In both grades 2 and 3, significant inter-relations were present among all paths specified in the model. The final model fit was excellent, $\chi^2(56) = 55.33$, $p = .50$, SRMR = .05, CFI = 1.00, RMSEA = .00 (90% CI = [.00, .04]). The results are shown in Figure 2. For readability, control variables (receptive vocabulary, matrix reasoning, inhibitory control, and working memory) are not shown in the figure but are described in text. The R^2 values shown in Figure 2 for each outcome include variance predicted by the control measures. The factor loadings of the latent working memory variable based on digit forward span, digit backward span, and spatial span were all significant, .42, .46, and .32, respectively, *p*s < .001.

Table 1

Descriptive Statistics for all Variables

	N	*Min*	*Max*	*Mean*	*SD*	*Skew*	*Reliability*
Grade 2							
Reasoning [a]	182	3.00	24.00	16.12	3.94	-1.66	.84
Vocabulary [b]	182	9.00	36.00	29.73	4.90	-5.33	.77
Digit Forward Span [d]	182	4.00	15.00	8.17	2.17	2.04	.83
Digit Backward Span [d]	182	3.00	9.00	5.48	1.12	1.31	.80
Spatial Span [b]	174	0.00	13.00	5.55	2.55	-0.04	.82
Stroop [c]	176	-.59	1.29	0.42	.25	1.63	.76
Addition [a]	182	0.00	43.00	10.56	5.67	3.27	.82
Subtraction [a]	182	0.00	24.00	4.93	3.80	5.38	.76
Grade 3							
Addition [a]	208	1.00	37.00	15.04	6.74	4.12	.94
Subtraction [a]	206	0.00	34.00	8.22	5.56	4.94	.91
Multiplication [a]	202	0.00	37.00	7.29	5.72	5.85	.90
Word Problems [a]	209	0.00	12.00	5.57	2.42	1.45	.94
Algebra [a]	208	0.00	15.00	6.75	3.57	-0.50	.96
Measurement [a]	209	0.00	22.00	12.09	5.06	-0.47	.81

[a] Total correct; reliability was calculated based on individual item scores.

[b] Total correct; reliability was calculated using subset scores based on the first and second trial for each span length.

[c] Adjusted response time difference between congruent and incongruent trials; reliability was calculated based on the difference between mean response time on correct trials for incongruent trials and congruent trials.

[d] Total correct; test-retest reliability was obtained from the standardized WISC-V technical report (Wechsler, 2014).

Table 2

Correlations Among Measures in Grade 2 (G2) and Grade 3 (G3)

	Grade 2									Grade 3				
	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. Mother's Education	-													
2. Reasoning[a] G2	.08	-												
3. Vocabulary[a] G2	.13	.30***	-											
4. Digit Forward[a] G2	-.08	.13	.22**	-										
5. Digit Backward[a] G2	-.02	.19*	.16*	.17*	-									
6. Spatial Span[a] G2	-.01	.19*	.15	.10	.13	-								
7. Stroop[b] G2	-.01	-.00	.11	-.00	-.12	-.08	-							
8. Addition[a] G2	.24**	.18*	.10	.19**	.19**	.18*	-.11	-						
9. Subtraction[a] G2	.17*	.15*	.16*	.16*	.28***	.20**	-.07	.71***	-					
10. Addition[a] G3	.12	.22**	.06	.04	.20*	.10	-.20*	.54***	.46***	-				
11. Subtraction[a] G3	.18*	.18*	.15	.12	.25**	.09	-.12	.62***	.67***	.75***	-			
12. Multiplication[a] G3	.18*	.28***	.15	.16	.24**	.07	-.21*	.42***	.43***	.55***	.67***	-		
13. Word-Problem[a] G3	.16*	.31***	.53***	.37***	.27**	.15	-.12	.28**	.40***	.44***	.47***	.48***	-	
14. Algebra[a] G3	.22**	.37***	.45***	.36***	.24**	.20*	-.21*	.32***	.39***	.43***	.47***	.50***	.69***	-
15. Measurement[a] G3	.25***	.26**	.46***	.39***	.34***	.11	-.09	.34***	.45***	.40***	.45***	.45***	.68***	.65***

Note. $p < .05$*; $p < .01$**; $p < .001$***

[a] Total correct

[b] Adjusted response time difference between congruent and incongruent trials

Hypothesis 1

Hypothesis 1 was that addition in grade 2 would predict the growth of subtraction from grades 2 to 3 (H1a), whereas subtraction in grade 2 would not predict the growth of addition from grades 2 to 3 (H1b). This hypothesis was based on the assumption that addition provides the foundation upon which subtraction skills are later built. As shown in the cross-lagged portion of the model (see Figure 2), addition in grade 2 significantly predicted subtraction in grade 3, controlling for subtraction in grade 2, indicating that addition predicted the growth of subtraction. In contrast, subtraction in grade 2 also predicted addition in grade 3, controlling for addition in grade 2. We conducted the Satorra-Bentler scaled chi-square difference test to compare the model where the two cross-lagged paths were constrained to be equal in the model where the two paths were freely estimated (Satorra & Bentler, 2010). Results showed no statistical difference on the strength of the cross-lagged path coefficients between the constrained and unconstrained models, $\Delta\chi^2(1) = .03$, $p = .584$, suggesting that the path coefficients are statistically the same from addition to subtraction, and vice versa. Thus, despite finding an apparently weaker path from subtraction in grade 2 to addition in grade 3 than the reverse, the results show that development of the two operations are closely related. Thus, our first hypothesis was partially supported.

Hypothesis 2

Hypothesis 2 was that in grade 3, subtraction would predict multiplication (H2a), controlling for addition, whereas addition would not predict multiplication (H2b), controlling for subtraction. This hypothesis was supported. Subtraction in grade 3 was a significant predictor of multiplication in grade 3 (see Figure 2), whereas addition in grade 3 did not predict significant variance in multiplication. Children's reasoning skills were also related to multiplication (β

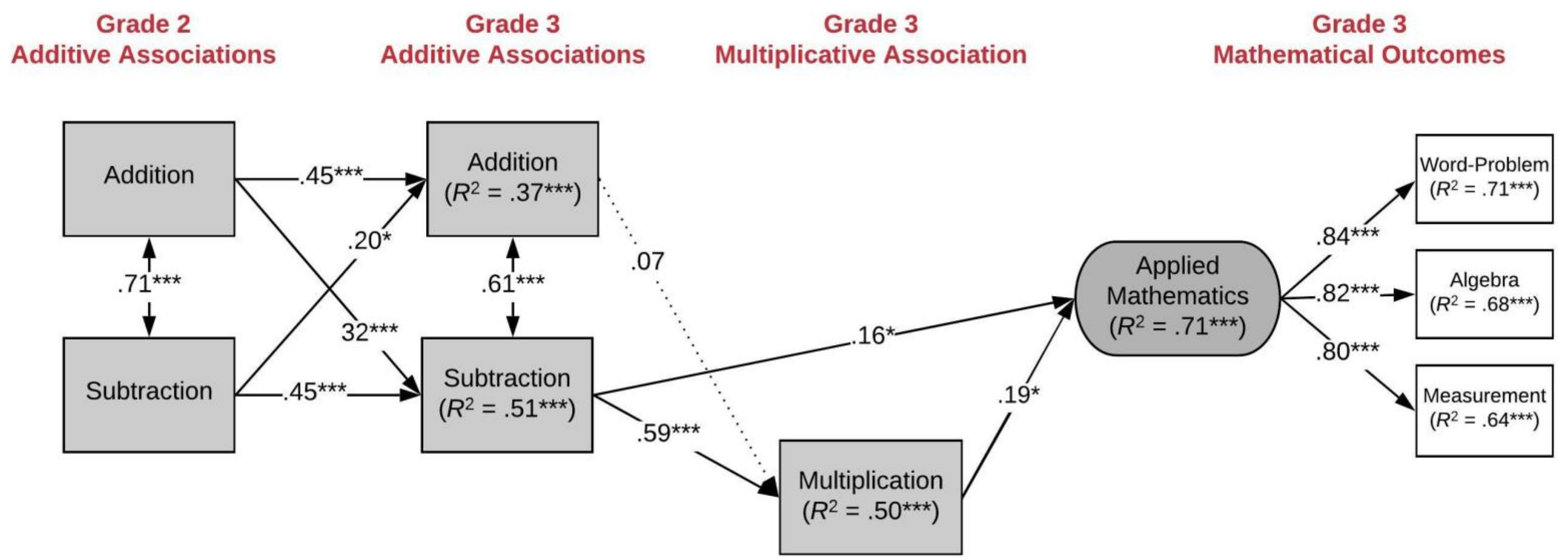
= .21, p = .001). These results are consistent with the integration model, in which addition associations become integrated with subtraction associations.

Hypothesis 3

Hypothesis 3 was that both subtraction and multiplication would predict the applied mathematics latent variable (H3a and H3b). This finding was expected because multiplication is the newest and the least practiced aspect of children's associative arithmetic network in grade 3, and thus subtraction and multiplication would not be fully integrated. As shown in Figure 2, applied mathematics was predicted by both multiplication and subtraction. Moreover, the indirect path from subtraction to applied mathematics through multiplication was significant (β = .11, p = .014), indicating that multiplication mediated the relations between subtraction and applied mathematics in grade 3. Furthermore, the applied mathematical latent variable was also predicted by receptive vocabulary (β = .29, p = .004) and working memory (β = .55, p < .001), but not by matrix reasoning (β = .04, p = .703) or inhibitory control (β = -.01, p = .886). Thus, our third hypothesis was supported.

Figure 2

Structural Equation Modeling for Children from Grades 2 to 3.



Note. The numbers on the arrows are the standardized coefficients. $*p < .05$, $***p < .001$.

Discussion

Arithmetic is a central mathematical skill and thus is a focus of instruction in the early grades of elementary school. In the present research, we explored the process of symbolic integration, focusing on addition, subtraction, and multiplication for students in grades 2 and 3. We hypothesized that the ongoing integration of arithmetic skills would predict a specific pattern of associations among addition, subtraction, and multiplication in this time period, when arithmetic skills are targeted in the curriculum. The results were consistent with the overall view that learning arithmetic associations is a hierarchical process. As each new skill is added and practiced, individual differences reflect the integration of the novel component. For students at this stage, addition and subtraction knowledge is more integrated than subtraction and multiplication knowledge. Furthermore, the finding that both subtraction and multiplication predicted applied mathematics performance suggested that integration of multiplication was partial because skill acquisition was in progress. In summary, our findings support the view that individual differences in fundamental arithmetic associations reflect ongoing integration of information into a network of symbolic digit associations (Xu et al., 2019; Xu & LeFevre, 2020).

Interpretations and Contributions of the Findings

We examined the longitudinal and bidirectional relations between addition and subtraction skills of students from grades 2 to 3. We found reciprocal relations between addition and subtraction, such that grade 2 skill in one operation predicted growth from grades 2 to 3 of the other operation. These findings are consistent with the view that addition and subtraction are complementary operations and that the development of addition and subtraction emerge through reciprocal interactions between these two operations (Robinson, 2017; Robinson & Dubé, 2009, 2012). Consistent with this conclusion, in several other studies, practice with one operation facilitated mastery of the other (Baroody et al., 2014; Campbell & Agnew, 2009; Paliwal & Baroody, 2020; Siegler, 1987). Similarly, teaching and practicing basic addition and subtraction expressions together yielded more improvement on both operations than teaching them separately (Buckingham, 1927). As an example, in China, children are first introduced to the concepts of addition and subtraction together, followed by practice which is designed to support mastery of basic addition and subtraction facts (Sun & Zhang, 2001). Moreover, Chinese first graders are expected to develop fluent access to the addition and subtraction associations before moving on to the more advanced operations of multiplication and division (Zhou & Peverly, 2005). In summary, our results are aligned with previous research demonstrating that experience with either operation can strengthen the associative pathways underlying both operations.

We found that multiplication and subtraction, but not addition, were significant predictors of more advanced mathematics (i.e., as indexed by a latent variable). However, multiplication partially mediated the relation between subtraction and advanced mathematics. Multiplication was a relatively new skill for the children in the present study, and they had learned only a subset of multiplication facts (i.e., up to the five times table). According to the hierarchical symbol integration model, the pattern of relations reflects the order of acquisition and hence, the degree of integration of associations in the hierarchical associative network (Xu et al., 2019). Thus, to test this model, it is important to study children who are in the process of learning arithmetic. In the present study, children were just beginning to learn multiplication and thus had not fully integrated multiplication into the network (Miller & Paredes, 1990). Accordingly, for these novice learners, both multiplication and subtraction were independent predictors of mathematics measures.

Multiplicative representations develop slowly (Clark & Kamii, 1996). Consistent with this view, the results of the present research indicate that the integration of multiplication into the network is still in progress for children in grade 3. Multiplication is constructed based on the additive representation, but it requires a more advanced level of abstraction (Harel & Confrey, 1994; Nunes et al., 2016; Sherin & Fuson, 2005). Although children show signs of multiplicative thinking as early as preschool (Nunes et al., 2016), even among middle-school children and adults, conceptual understanding of multiplication is incomplete (Dubé & Robinson, 2018; Robinson & LeFevre, 2012). Further research in which children's learning of arithmetic is followed over time is needed to test how all four arithmetic skills are related.

Limitations

One limitation of the present research is that children were only assessed twice, one year apart. With a larger number of timepoints and the participation of children in the later grades, we would presumably observe more complete integration of subtraction with addition and of multiplication with subtraction. Additionally, with more time points, latent curve modeling could be used to examine the developmental trajectories of growth across addition, subtraction, multiplication, and division. Although we had intended to collect data in 2020, when the children were in grade 4, the global pandemic interfered with this plan. In future studies, assessing arithmetic fluency across operations and at various key developmental points would be helpful in refining the hierarchical symbol integration model. Despite a reasonably large corpus of research on arithmetic fluency, only one study by Rinne et al. (2020) explored concurrent development of these skills longitudinally. The authors explored growth in each operation (i.e., addition, subtraction, and multiplication) from grade 3 to 5, but the relations among the operations were not examined. Instead, the focus was on whether reading fluency predicted growth in arithmetic skills, a pattern found only for multiplication. Thus, more research is needed to understand how children integrate different types of arithmetic associations longitudinally.

Implications for Instruction

The present study provides empirical evidence for the view that learning arithmetic associations is a hierarchical process. We show that by grade 3, subtraction superseded addition as the direct predictor of multiplication. Our findings suggest that fluent access to addition and subtraction is important when children are introduced to multiplication (Nunes et al., 2016), which is in turn a key skill in understanding rational numbers (i.e., proportional reasoning with decimals, percentage, and fractions; Hino & Kato, 2019; Thompson & Saldanha, 2003). In general, children’s prior knowledge in mathematics is an important influence on their responses to instruction (Kalyuga, 2007; Rittle-Johnson et al., 2009).

Arithmetic facts that share common digits (e.g., 4 + 2, 4 – 2, and 4 x 2) may provoke interference when children are developing their associative networks (Barrouillet et al., 1997; Miller & Paredes, 1990). For children with mathematics learning disabilities, it is especially difficult to efficiently manage appropriate associations and inhibit inappropriate ones (De Visscher & Noël, 2014b). Thus, for successful acquisition of arithmetic, children must capitalize on similarities and complementary associations between their existing knowledge and new associations. Consistent with this view, intervention studies have shown that addition and subtraction associations can be facilitated by teaching children to use strategies such as counting and derived facts based on a partial retrieval (reviewed by Dowker, 2003). However, for multiplication, direct retrieval is important for improvement; strategies such as repeated addition and skip counting are associated with a lack of improvement in adolescents across two time points (Suárez-Pellicioni et al., 2018) and with less-skilled performance among adults (LeFevre et al., 1996; Smith-Chant & LeFevre, 2003).

In many models of associative learning, earlier acquired associations are assumed to be the building blocks for higher-level associations (Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Xu et al., 2019). According to the hierarchical symbol integration model, the relations among different aspects of symbolic processing (e.g., classifying the order of digits such as 1, 2, 3, and arithmetic calculation) reflect the integration of numerical associations into a unified network (Xu et al., 2019; Xu & LeFevre, 2020). In the present study, we further extended this model to the integration of arithmetic associations during learning.

What are the implications of these findings for mathematics instruction? We can only speculate, because our research is necessarily tied to the educational experiences of the students in our sample. In many North American jurisdictions, a spiral curriculum is used in which topics are revisited multiple times across grades. Moreover, when a large number of topics are introduced each year, there is relatively little opportunity for either mastery or integration (Snider, 2004). The original proposal for spiral curricula in STEM topics (Bruner, 1960, as described in Ireland & Mouthaan, 2020) included the assumption that a prior topic would be mastered before a new topic is introduced. Thus, spiraling back to the topic would entail building on what is already mastered, not simply revisiting the same material (Ireland & Mouthaan, 2020). Theoretically, therefore, the implementation of a spiral curriculum would not be effective unless mastery is achieved at each level (Orale & Uy, 2018; Snider, 2004). The importance of mastering core topics is consistent with curricula in many Asian countries (see also Ma, 2010; Schmidt et al., 2002, 2018). The integration model is also consistent with the notion that learning is more effective when mastery of closely related concepts is facilitated.

Previous research has shown that Chinese children outperform their North American peers in international mathematics assessment such as PISA (Programme for International

Student Assessment; Fleischman et al., 2010). Young adults educated in China also showed more extensive integration of symbolic digit associations than their peers educated in Canada (Xu et al., 2019). Among the many differences in educational inputs between Asian and North American countries, such as the amount of time spent on mathematics instruction (Stevenson & Stigler, 1992) and the amount of mathematics knowledge of the teachers (Ma, 2010), the difference in instructional approaches may also partially explain the advantages of Chinese children (Snider, 2004; Schmidt et al., 2002, 2018). In the future, longitudinal research into students' acquisition and mastery of arithmetic associations is needed to expand the theory we have proposed and to provide a more nuanced understanding of effective arithmetic instruction.

Conclusion

Children need to understand the relations among addition, subtraction, and multiplication. This knowledge may be facilitated if instruction supports integration of these operations. For integration to be successful, however, children must have acquired strong addition and subtraction skills prior to being introduced to more advanced operations. According to the proposed hierarchical integration model, novel arithmetic associations should be learned in the context of existing knowledge because each operation is constructed and consolidated in relation to associations that are already present in the mental network.

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