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Divide and Conquer: Relations among Arithmetic Operations and Emerging Knowledge of Fraction Notation for Chinese Students in Grade 4

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Abstract

How do whole number arithmetic skills support students' understanding of fraction magnitude during the emerging stages of fraction learning? Chinese students in Grade 4 (N = 1038; M_{age} = 9.9 years; 55.6% boys) completed assessments of whole number arithmetic skills (i.e., addition, subtraction, multiplication, division), fraction mapping (i.e., connecting visual fraction representations to fraction notations), and fraction comparison (i.e., comparing magnitudes of fraction symbols). We found that division skills uniquely differentiated students who had a basic understanding of fraction notation (mappers) from students with no understanding of fraction notation (non-mappers). Furthermore, we found that division mediated the relations between all three other arithmetic operations (i.e., addition, subtraction, and multiplication) and fraction mapping performance for the mappers. For fraction comparison, there was evidence of the whole number bias for the majority of students. The present results highlight the importance of the mastery of division skills and its dominance in predicting individual differences in fraction mapping for Chinese students in grade 4.

Word count: 157

Key Words: symbolic integration, fractions, division, whole number arithmetic, mathematics

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Mathematics depends on an abstract symbol system, created to capture a wide array of numerical and functional associations (Thompson & Saldanha, 2003). Learning mathematics is a hierarchical process, in which the acquisition of basic concepts are the building blocks for more advanced concepts, and new concepts logically follow from prior ones (Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Xu et al., 2019). Students initially learn to use numerals to represent cardinal, ordinal, and arithmetic associations (Lyons et al., 2016; Merkley & Ansari, 2016; Sasanguie & Vos, 2018; Xu & LeFevre, 2020). As they add to their hierarchy of mathematical symbol knowledge, students integrate more advanced and abstract associations, such as rational numbers (Booth & Newton, 2012; Douglas et al., 2020). The abstractness and complexity of cognitive operations increases when number knowledge extends from integers to rational numbers, such as fractions (English & Halford, 1995; Thompson & Saldanha, 2003). Fraction knowledge is described as the “gatekeeper” for learning more advanced mathematics and science (Bailey et al., 2012; Booth et al., 2014; Booth & Newton, 2012; Siegler et al., 2011). Unfortunately, many students experience difficulties in understanding the relation between magnitude and fraction notation, especially at the initial stage of fraction learning (Hecht & Vagi, 2010; Siegler et al., 2011; Torbeyns et al., 2015). Difficulties in understanding fractions are not limited to North Americans (Siegler & Lortie-Forgues, 2017) but rather appear to be a global phenomenon, with difficulties also present in Europe (Gabriel et al., 2013; Meert et al., 2010b) and Asia (Chan et al., 2007).

In the present study, we focused on the early stages of fraction understanding. Participants were students in grade 4 who were born and educated in China. Consistently, Chinese students are at the top in overall performance on international mathematics assessments such as the Programme for International Student Assessment (OECD, 2018) and the Trends in International Mathematics and Science Study (Mullis et al., 2020). Despite their success, as in many other countries, Chinese students find fractions to be challenging. In particular, during the emerging stages of fraction learning, Chinese students have difficulty mastering the measurement concept of fractions, that is, the understanding that fractions can be seen as a number that can be infinitely partitioned, rather than a combination of two whole numbers (Jiang et al., 2020; Xin & Liu, 2014). An important question arises: How do whole number arithmetic skills support students' understanding of fraction magnitude during the emerging stages of fraction learning?

Why are Fractions Difficult to Learn?

Researchers and mathematics educators agree that one of the main sources of difficulty in learning fractions is interference from students' prior knowledge of the whole number system (see reviews by Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017). For example, when asked to compare magnitudes of two fractions (e.g., which fraction is larger, $\frac{5}{9}$ or $\frac{5}{6}$), both adults and children sometimes use a componential strategy that involves direct comparison of the numerators or denominators to identify the larger fraction (Bonato et al., 2007; DeWolf & Vosniadou, 2015; Meert et al., 2010a, 2010b). Use of a componential strategy reflects the *whole number bias,* that is, a tendency to process the whole number components of fractions discretely rather than processing fractions as a single magnitude holistically (Ni & Zhou, 2005). On this view, some of the difficulties that young learners have in understanding fractions may be explained by a conflict between the new information about fraction symbols and their prior knowledge of whole number symbols (Chi et al., 1994; Ni & Zhou, 2005; Siegler & Lortie-Forgues, 2017).

When learning about fractions, students exhibit misconceptions that can be linked to the whole number bias (Smith et al., 2005; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2004). For example, in the early stages of fraction learning, many students have difficulty explaining the mathematical role of the numerator and the denominator in representing fractions (Smith et al., 2005). Smith et al. anticipated that American students in grades 3 and 4 (ages 8-10 years) who understood fraction notation would explain the two numerals in a fraction using a division model, in which the denominator represents the number of pieces the whole is divided into, and the numerator represents the number of pieces of that size. However, when asked to explain why there are two numbers in a fraction (e.g., $\frac{1}{3}$), more than 70% of students gave inadequate explanations (e.g., “two numbers equal a fraction” or “top is the numerator, bottom is the denominator”), showing immature understanding of the fraction notation. In middle school (i.e., approximately ages 11-14 years), only slightly more than half of American students understood the connections between fractions and division, evidenced by providing adequate explanations (Levin, 2001). Thus, the conceptual understanding of the connection between fraction notation and division develops slowly, and even in middle school, some students continue to struggle with this connection.

Students’ misconceptions about fractions can also be observed when they attempt fraction comparison tasks (Braithwaite & Siegler, 2018; Meert et al., 2010b; Stafylidou & Vosniadou, 2004). In particular, students tend to consider each fraction as two unrelated whole numbers and then identify unit fractions with the smaller denominator as having the smaller magnitude (Ni & Zhou, 2005). For example, they may erroneously judge $\frac{1}{7}$ as being smaller than

$\frac{1}{9}$, because 7 is smaller than 9. Stafylidou and Vosniadou (2004) found that about one-third of Greek students in grades 5 and 6 (i.e., ages 10-11) believed that fraction magnitude increases (or decreases) as the magnitude of either the numerator or denominator increases (or decreases), showing that they treated the numerator and denominator as two independent whole numbers. Similarly, Belgian students in grades 5 and 7 (i.e., ages 10-12 years) responded more slowly and made more errors on fraction comparison problems with common numerators (e.g., $\frac{2}{7}$ versus $\frac{2}{5}$) than on fraction comparison problems with common denominators (e.g., $\frac{2}{7}$ versus $\frac{3}{7}$; Meert et al., 2010b). These results indicate that students experienced interference between the magnitudes of the whole number components and the magnitudes of the fractions. Given these misconceptions, it is imperative that young students learn to differentiate their prior whole number knowledge from their emerging knowledge of fractions during the learning process.

Integration and Differentiation of Fraction Knowledge

When students learn a new numerical concept, they initially go through a process of differentiation and then integration as that new concept connects with their prior numerical knowledge (Siegler & Lortie-Forgues, 2014). Integration of knowledge occurs when the acquisition of new associations does not simply replace previously learned associations, but rather, all of the acquired associations work interactively within a unified mental network (Siegler & Lortie-Forgues, 2014). In the first few years of elementary school, students learn to represent the cardinal, ordinal, and arithmetic associations among numerals (Sasanguie & Vos, 2018; Xu & LeFevre, 2020; Xu et al., 2021). Learning a new association may require that students temporarily suppress the activation of other previously learned associations, especially when these associations share numerical symbols (Miller & Paredes, 1990; De Visscher & Noël, 2016; Xu & LeFevre, 2020). For example, when students learn multiplication, they experience a temporary decline in their addition skills (e.g., providing the answer for 2×4 to the question $2 + 4$; Miller & Paredes, 1990). More generally, differentiating various kinds of numerical associations in different representational systems presents a challenge for young learners. This challenge is particularly evident when students are trying to integrate novel fraction magnitude knowledge into their existing numerical network because they need to overcome their whole number bias and change their conceptual understanding of number associations to a more abstract level (Ni & Zhou, 2005).

How do students integrate prior whole number knowledge with fraction knowledge? A variety of research suggests that multiplicative thinking plays a crucial role in the development of reasoning about fractions (e.g., Mack, 1993; Nunes et al., 2016; Steffe & Olive, 2010; Thompson & Saldanha, 2003). Multiplicative thinking requires students to understand number as units of units (Clark & Kamii, 1996; Harel & Confrey, 1994). For example, students with multiplicative conceptual knowledge can represent 9 objects as 3 composite units, each consisting of 3 individual items. Similarly, fractions can be expressed in multiple ways. Specifically, fractions represent a ratio between two quantities, measured in units of each other, that is, “A is $\frac{m}{n}$ as large as B” is essentially the same as “A is m times as large as $\frac{1}{n}$ of B” (Thompson & Saldanha, 2003). For example, if an apple is $\frac{1}{7}$ as large as a watermelon that also means that a watermelon is 7 times as large as an apple. From this view, both multiplicative thinking and fractions require students to understand that quantities can be measured flexibly in units of each other (Mack, 1993).

Division is the only whole number operation where the product of the operation can result in a fraction (Steffe & Olive, 2010; Thompson & Saldanha, 2003). Mathematically, fractions are a form of division (Harel & Confrey, 1994). More specifically, the connections between fractions and division include that a) fractions and division have different mathematics vocabulary for the same idea (e.g., For $\frac{m}{n}$, m is the dividend/numerator, and n is the divisor/denominator), b) the fraction bar is a division symbol (i.e., the bar between the numbers means to divide), and c) division can be written using fraction notations, and vice versa (i.e., $x = \frac{m}{n}$ or $x = m \div n$; Levin, 2001). Thus, knowledge of whole number division logically extends to fractions.

In China, the mathematics curriculum and textbooks emphasize the strong connections between division and fractions (Sun, 2019). In Chinese mathematics tradition, a fraction is defined as the result of a division, the remainder being taken as the numerator and the divisor as the denominator (Guo, 2010). Chinese mathematics textbooks also explicitly bridge whole number division and fraction problems together through problem invariants (Sun, 2019). Consider the following example: If you want to equally distribute three cakes to four people, how much cake does each person get? To answer this question, students are guided to compute the amount of cake per person by using division: $3 \div 4 = \frac{3}{4}$. The quotient represents the amount of cake in each share when 3 cakes are partitioned equally into 4 parts. Thus, any fraction can be seen as the quotient resulting from a division calculation. Students are taught to understand that the numerator of a fraction shares the same function as the dividend, and the denominator shares the same function as the divisor. Teachers go through a variety of different problems to help students conceptually understand the connections between division and fractions, and showcase that division can be written using fraction notation, and vice versa: $a \div b = \frac{(a)}{(b)}$ (Sun, 2019). More generally, making connections amongst various types of knowledge is a key feature in Chinese mathematics education (Li & Huang, 2013).

In previous studies with students in grades 4 through 8, division skills were related to both conceptual and procedural fraction knowledge (Hansen et al., 2015; Namkung et al., 2018; Siegler & Pyke, 2013; Stelzer et al., 2019, 2021). However, among these studies, only Namkung et al. included all four types of arithmetic operations with whole numbers, relying on a subset of items from a standardized math computation scale (i.e., with 18 items that covered the four operations). Thus, although division skills are assumed to be central to the integration of fraction knowledge into the mental network, the relative importance of division versus the other whole number arithmetic operations has not been thoroughly explored. In the present study, we used a separate test for each of the whole number arithmetic operations to tap into the integration of arithmetic skills and fraction knowledge within the mental network.

We adopted the hierarchical symbol integration model to capture the relations among whole number arithmetic skills and fraction knowledge. The central claim of this model is that individual differences in arithmetic associations reflect ongoing integration of knowledge into an associative network (Xu et al., 2019; Xu & LeFevre, 2020). Based on the assumptions of this integrated framework, we expected that all four whole number arithmetic operations would be related to fraction performance. However, when considering the four operations simultaneously, we hypothesized that division would predict unique variance in fraction tasks because division provides a distinct conceptual and procedural link between whole number knowledge and rational number knowledge (Levin, 2001; Siegler et al., 2012; Sun, 2019).

The Current Study

The goal of the present study was to examine the relations between whole number division skills and the understanding of fraction magnitude for Chinese students in grade 4. In China, mathematics education emphasizes the acquisition of foundational mathematics concepts and skills, and flexible application of these skills to a variety of problems (Ministry of Education, 2011). Notably, the educational system in China is highly centralized, and as such, teachers in all classrooms closely follow the mandatory national curriculum (Ni et al., 2014). According to the curriculum, students should be guided to develop an interconnected understanding of mathematical knowledge through repeated practice (Dahlin & Watkins, 2000; Ni et al., 2014). In China, fraction instruction begins in grade 3, focusing on the introduction to fraction notations (e.g., identifying, mapping, and comparing fractions) through the support with external representations such as pictures and words (Ministry of Education, 2011). Thus, most of the Chinese students in grade 4 were expected to understand that fraction represents a relationship between part(s) of a partitioned unit and whole (Xin & Liu, 2014).

Intensive emphasis on fraction instruction in China begins in the latter half of grade 4, when Chinese students start to learn to compare, order, and do arithmetic with fractions without the support of pictorial representations (Ministry of Education, 2011). Thus, in the first half of grade 4, students are expected to take a visual, partitioned external representation and use that to write a fraction in proper notation (e.g., a circle divided into two equal parts with one part shaded should be represented as $\frac{1}{2}$), a skill which we refer to as *fraction mapping*. However, they have not been taught to compare fraction magnitudes without the support of external representations (e.g., which is bigger, $\frac{1}{8}$ or $\frac{7}{8}$, $\frac{2}{3}$ or $\frac{5}{7}$).

In previous research, students' fraction mapping skills were related to more advanced understanding of fractions (Douglas, 2020; Hecht et al., 2003; Hecht & Vagi, 2010; Lewis, 2016; Mazzocco et al., 2013). For example, Douglas (2020) found that for students in grade 4 (i.e., ages 9-10), fraction mapping skills predicted more advanced fraction knowledge including performance on magnitude comparison, word problem, and estimation tasks. Moreover, Hecht and Vagi (2010) found that fraction mapping skills were related to fraction computation, word problems and estimation tasks of students with mathematical difficulties in grades 4 to 5. Furthermore, students in grades 4 through 8 (i.e., ages 9-13) with dyscalculia, a specific learning disability related to acquiring mathematics, performed poorly on simple fraction-based magnitude judgment tasks, such as those involving two fractions with common denominators or fractions equivalent to one-half (Mazzocco et al., 2013). More generally, the ability to map between symbol and referent is assumed to be the first step in the hierarchy of developing mathematical symbol competence (Hiebert, 1988).

In the present study, we included two fraction tasks to assess students' fraction knowledge: a fraction mapping task in which they were asked to map visual fraction representations to written fraction notation (see Appendix A), and a fraction comparison task in which they compared magnitudes of written fraction notation (see Appendix B). We asked two research questions regarding fraction understanding for Chinese students in grade 4.

Research Question 1: Do Chinese Students in Grade 4 Understand Fraction Notation?

Given the small amount of formal instruction that students have received about fractions in grade 4, they were expected to demonstrate only limited knowledge of fraction notation. Nonetheless, because of the design of the Chinese curriculum, students would have received instruction about fraction mapping and are expected to have mastered fraction mapping by the end of the first term in grade 4 (Xin & Liu, 2014). Consequently, Hypothesis 1 was that most of the students would be "mappers" who could connect visual representations to fraction notations and only a small portion of students would be "non-mappers" who showed no understanding of fraction notation.

Hypothesis 2 was that most students would not interpret written fractions as holistic magnitudes because they had not yet received relevant instruction and thus would perform poorly on fraction comparisons in which the relative magnitudes of the components were inconsistent with the relative magnitude of the whole fractions (e.g., $\frac{1}{8} < \frac{1}{7}$; Meert et al., 2010b). Although fraction mapping requires an understanding of the visual representations of fractions, this task does not require students to understand the magnitude of one fraction in comparison to another. That is, students do not need to understand that a smaller numerator and/or denominator alone does not provide information about the magnitude of a fraction. At the beginning of grade 4, students have not been taught a procedure to determine which is bigger, $\frac{2}{3}$ or $\frac{4}{7}$. As a result, we anticipated that fraction comparison scores would be low.

Research Question 2: What Role Do Division Skills Play in Students' Understanding of Fraction Notation?

We expected that performance on all whole number arithmetic tasks would be related to students' understanding of fraction notation (Siegler et al., 2011, 2013; Torbeyns et al., 2015). However, when considering the four operations simultaneously, we hypothesized that division skills would uniquely differentiate students with a basic understanding of fraction notation (mappers) from students with no understanding of fraction notation (non-mappers), whereas addition, subtraction and multiplication would not (Hypothesis 3). This hypothesis was guided by both the existing literature which indicates that division is more closely related, conceptually and procedurally, to fraction knowledge than are the other three operations (Hansen et al., 2015; Harel & Confrey, 1994; Stelzer et al., 2019; Thompson & Saldanha, 2003) and the Chinese curriculum which uses division to introduce fraction notation (Sun, 2019).

Moreover, in China, students in grade 4 have at least two years of experience with all four whole number arithmetic operations. Thus, we assume that students will have started to integrate the arithmetic operations into a unified network, and thus there would be considerable shared variance among the four operations. Based on the hierarchical symbol integration model (Xu et al., 2019), if division captures the highest level of integration, variability in the most advanced whole number arithmetic operation (i.e., division) would supersede variability in the more basic whole number arithmetic operations (i.e., addition, subtraction, or multiplication). For the group of students who have acquired some basic understanding of fraction notation, we hypothesized that addition, subtraction, and multiplication skills would predict unique variance in division skills, and furthermore, division skills would mediate the relations between fraction mapping (Hypothesis 4) and addition, subtraction, and multiplication.

Method

Participants

Following ethics approval from Shandong Normal University, school principals were contacted. Upon approval by the school principals, letters were sent home, inviting students to participate. A group of 1038 students in grade 4 (577 boys; M_{age} = 9.9 years; *SD* = .59) from two public elementary schools (24 classes) were recruited near the end of the first semester (December 2020). The schools were located in a town (with a population of over 6 million) in the northern part of China with an economic level approximately at the national average (National Bureau of Statistics, 2019). Information about socioeconomic status was not collected. However, parental education levels obtained from other cohorts from the two participating schools ranged from elementary school to postgraduate, with a median education level of a high school degree for both fathers and mothers, representative of low- to middle-SES level in China.

Procedure

Students were tested in one 45-minute group session during school hours in their classroom, with 37-46 students per class. Because the current study was conducted within the context of a larger project, a battery of questionnaires (e.g., mathematical anxiety, peer relationship, and perceived teacher support) and other measures (e.g., visuospatial reasoning tasks) were also administered during the testing session. Only measures that are relevant to the present research questions are reported and described below. Testing was administered by two experimenters who had either completed or were working toward a bachelor's degree in Education. Experimenters were provided with a detailed testing manual, including specific testing and scoring procedures. In each classroom, one experimenter focused on administration procedures (e.g., reading directions, keeping time), while the other experimenter circulated through the classroom to ensure students were following the testing instructions. Data were entered independently by 22 research assistants and cross-checked for accuracy.

Measures

Heidelberg Rechen Test

Students first completed five paper-and pencil tests from an adapted version of the standardized German *Heidelberg Rechen Test* (HRT; Haffner et al., 2005; adapted from Wu & Li, 2006). The adapted Chinese HRT is a measure of mathematical skills focusing on arithmetic and visuospatial reasoning skills for students in grades 1 through 6. In the present study, we focused on the arithmetic measures that are directly relevant to our research questions. Based on a large national assessment across seven regions in China (Wu & Li, 2005), the arithmetic subscale of the HRT has been reported to have excellent reliability for Chinese students in grade 4 (Cronbach's alpha was .88).

Writing Speed. The writing speed task from the Chinese adapted version of the HRT was used as a control variable. A total of 60 single Arabic digits ranging from 0 to 9 were presented in three columns, with 20 numbers in each column. Students were given 30 seconds to copy each number as quickly as possible. The score was the total number of digits students copied in 30 seconds.

Arithmetic skills. Students completed four tasks from the Chinese adapted version of the *arithmetical ability* subscale from the HRT*: addition, subtraction, multiplication*, and *division*. For each subset, questions were presented in two columns (20 questions per column). Students were given one minute to write down as many answers as possible, in order. Questions were ordered by increasing difficulty, although not systematically across operations. For addition, the first column contained of a mixture of single- and double-digit addends (e.g., 1 + 6, 13 + 7) with no sums greater than 20. The second column contained a mixture of single-, double-, and triple-digit addends (e.g., 6 + 16, 29 + 42, 256 + 464). For subtraction, the first column contained a mixture of single- and double-digit minuends and subtrahends (e.g., 4 – 1, 18 – 7, 15 – 13) with no minuends greater than 20. The second column contained a mixture of double- and triple-digit minuends, and single-, double- and triple-digit subtrahends (e.g., 15 – 8, 33 – 11, 120 – 22, 765 – 432). For multiplication, the first column contained single-digit multiplicands and multipliers (e.g., 3 × 1, 7 × 9). The second column contained a mixture of single- and double-digit multiplicands and multipliers, all less than 20 (e.g., 16 × 12, 19 × 9). For division, the first column contained single-and double-digit dividends and single-digit divisors (e.g., 6 ÷ 2, 30 ÷ 6). These problems were complementary to the problems found on a 10 x 10 multiplication table. The second column contained double- and triple-digit dividends and single-digit divisors (e.g., 45 ÷ 9, 450 ÷ 15). For each subset, the score was the total number of questions answered correctly in one minute.

Fraction Skills

To assess emerging fraction skills, students completed two paper-and-pencil tasks that were created for this study: fraction mapping and fraction comparison.

Fraction Mapping. The fraction mapping task was used to assess students' ability to connect fraction symbols with quantities. A total of 20 questions were presented in two columns (see the task in Appendix A). For each trial, students were presented with a picture and asked to write down the fraction that described the shaded portion of the picture. Students were given one minute to write down as many answers as possible, in order. The score was the total number of questions answered correctly in one minute. The internal reliability based on accuracy on individual trials was excellent, Cronbach's alpha = .98.

Fraction Comparison. The fraction comparison task was used to assess students' ability to connect fraction symbols with quantities without visual representations. A total of 20 questions were presented in two columns (see the task in Appendix B). For each trial, students were presented with two fraction symbols and asked to circle the fraction that was larger in magnitude. Student were given one minute to answer as many questions as possible in order. Notably, 11 questions were congruent such that the relative magnitude of the components (numerator and/or denominator) was consistent with the relative magnitude of the whole fractions (e.g., $\frac{1}{8} < \frac{7}{8}$; $\frac{4}{5} > \frac{2}{3}$). In contrast, the other 9 questions were incongruent such that the relative magnitude of the components was inconsistent with the relative magnitude of the whole fractions (e.g., $\frac{5}{9} < \frac{5}{6}$; $\frac{3}{4} > \frac{5}{7}$). The congruent and incongruent trials were presented unsystematically. The score was the total number of questions answered correctly in one minute.

The internal reliabilities were calculated for the two types of questions separately: Cronbach's alpha = .92 for the congruent trials, and Cronbach's alpha = .96 for the incongruent trials.

Analysis Plan

The goal of the present study was to investigate the relations between arithmetic skills (i.e., addition, subtraction, multiplication, division) and fraction understanding (i.e., fraction mapping and fraction comparison) for students in grade 4, controlling for gender and writing speed. Given that the four arithmetic operations were highly correlated (see Table 1), we first examined multicollinearity among these variables. A variance inflation factor (VIF) of 10 or more and/or a tolerance of 0.2 or less indicates an issue with multicollinearity (Field, 2017). In initial analyses, multicollinearity was not detected among the four operations (VIFs < 2.5; tolerance > 0.4) and thus was not a concern in the subsequent analyses. Given that students were from 24 classrooms, we conducted multilevel modeling to account for the hierarchical nature of the dataset using M*plus* (Muthén & Muthén, 1998).

Because most students were expected to be able to map visual representations to fraction notations whereas a small portion of students would show no understanding of fraction notation, we examined the distribution of the fraction mapping performance. As expected, we observed a bimodal distribution (see Figure 2). Thus, we conducted a logistic mixed-effect model analysis containing class as a random effect to examine which factors differentiated students with some knowledge of fraction notations (mappers) from students with no knowledge of fraction notations (non-mappers). Next, we examined performance on the number comparison task. However, performance was poor (see details in Results) and thus this task was not included in further analyses. Lastly, for the mappers, we conducted a multilevel path analysis containing class as a random effect to examine the hierarchical relations among the more basic whole number arithmetic (i.e., addition, subtraction, and multiplication), whole number division, and fraction mapping performance. Model fit was examined using a combination of the chi-square goodness of fit test ($p > .05$), comparative fit index (CFI > .90), root mean square error of approximation (RMSEA < .06), and standardized root mean square residual (SRMR < .08; Hu & Bentler, 1999).

A small percentage of data (< 0.5%) were missing for each of the variables (see Table 1). Given the extremely small number of missing cases compared to the sample size, these missing cases are unlikely to be influential in the interpretation of the results even if those cases were not missing at random (Enders, 2010). The final path model was estimated by a full information maximum likelihood method where all available information is used in all observations to find the optimal combination of estimates for the missing parameters (Enders, 2010).

In the present study, we did not collect data on students' general intelligence. However, a subgroup of students ($n = 568$) completed a paper-and-pencil version of the Raven's progressive matrices test (Raven, 1938) as an index of their nonverbal intelligence when they were in grade 3. Additional analyses based on a subgroup of the sample ($n = 568$) show that the results were very similar after controlling for students' nonverbal intelligence (see Supplementary Material). Thus, the present analyses included all of the students ($N = 1038$) without controlling for nonverbal intelligence.

Results

Descriptive Statistics

Descriptive statistics and correlations among variables are shown in Table 1. All measures were significantly correlated with each other. The correlations between fraction mapping and whole number arithmetic skills (i.e., the four operations) indicate medium to large associations between these variables (i.e., .25 to .37; Funder & Ozer, 2019). The smaller correlation of .25 between fraction mapping and multiplication compared to the other three operations may be related to low variability in multiplication performance for Chinese students (see Figure 1). In contrast, the correlations between fraction comparison and whole number arithmetic skills had small effect sizes (i.e., .09 to .17; Funder & Ozer, 2019). Violin plots for the tasks of interest are shown in Figures 1 and 2. All measures were normally distributed, however, the patterns of distribution of scores varied across the measures. Outliers were defined as values with z-scores greater than ±3.29 from the mean for the sample. Outliers were found for the following tasks: writing speed (4), addition (4), subtraction (5), multiplication (13), division (5), and fraction comparison (13). Sensitivity analyses with and without these outliers showed the same patterns of results, and thus all of the data were included in the final analyses.

Gender Differences

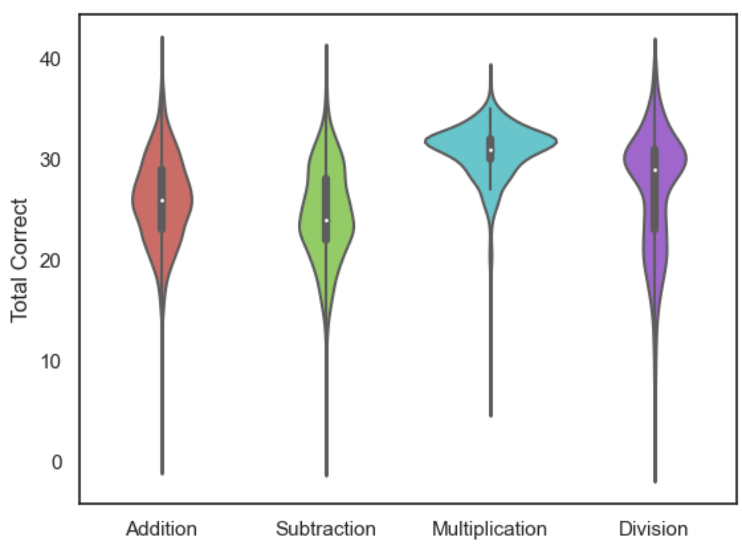
Boys had higher scores than girls on addition (26.3 versus 25.6, $t(1{,}035) = 2.42$, $p = .016$, Cohen's $d = .16$), subtraction (25.0 versus 24.1, $t(1{,}035) = 2.71$, $p = .004$, Cohen's $d = .17$), and multiplication (31.0 versus 30.4, $t(1{,}035) = 3.04$, $p = .001$, Cohen's $d = .19$). Girls had higher scores than boys on fraction mapping (10.9 versus 9.7, $t(1{,}033) = -3.25$, $p = .001$, Cohen's $d = .21$). There were no significant gender differences for division, writing speed, or fraction comparison (ps > .05). We note that these differences are small; statistical significance reflects the large sample size. Overall, boys and girls achieved similar scores, with differences of two points or less across all measures. Nonetheless, in the subsequent analyses, we controlled for gender.

Arithmetic Skills

Students performed well on all four arithmetic operations (see Table 1 and Figure 1) in that most completed at least 20 of the 40 problems in the one-minute time limit. Number correct for each operation was analyzed in a one-way repeated measures ANOVA. Performance varied with operation, $F(3, 3111) = 797.07$, $p < .001$, $\eta^2_{partial} = .44$. Post hoc pairwise comparisons using the Bonferroni adjustment revealed that students performed the best on multiplication, followed by division, addition, and subtraction (all pairwise comparisons were significant, *p*s < .001). The patterns of distribution for addition, subtraction and division were highly similar (see Figure 1). In contrast, for multiplication, students performed substantially better (i.e., 4-6 points higher). Moreover, approximately 75% of the students correctly answered between 30 and 35 multiplication questions, however, 1% of the students had scores of less than 20. This finding reflects that the majority of the Chinese students have memorized the multiplication table by grade 4, as a result of extensive practice.

Figure 1

Violin Plots for the Addition, Subtraction, Multiplication and Division Tasks



Note. The white dot is the median, and the black bar in the center of the plot shows the interquartile range.

Table 1

Descriptive Statistics and Correlations Among Measures. Correlations Below the Diagonal are For All Students (n = 1038); Correlations Above the Diagonal are For Mappers (n = 846).

	N	*M*	*SD*	Skew	Min	Max	1	2	3	4	5	6	7
1. Writing Speed	1037	43.50	10.89	-0.16	2	60	-	.23	.19	.18	.14	.22	.09
2. Addition	1036	25.98	4.43	-0.40	1	40	.25	-	.70	.52	.64	.34	.19
3. Subtraction	1037	24.61	4.83	-0.33	1	39	.22	.71	-	.52	.69	.34	.15
4. Multiplication	1036	30.75	2.95	-2.26	6	38	.21	.59	.56	-	.53	.29	.11
5. Division	1035	26.88	6.03	-0.87	1	39	.17	.66	.70	.56	-	.37	.13
6. Fraction Mapping	1036	10.20	5.67	-0.63	0	20	.17	.32	.31	.25	.37	-	.11
7. Fraction Comparison	1035	10.62	2.94	-1.18	0	20	.09	.17	.16	.09	.11	.14	-

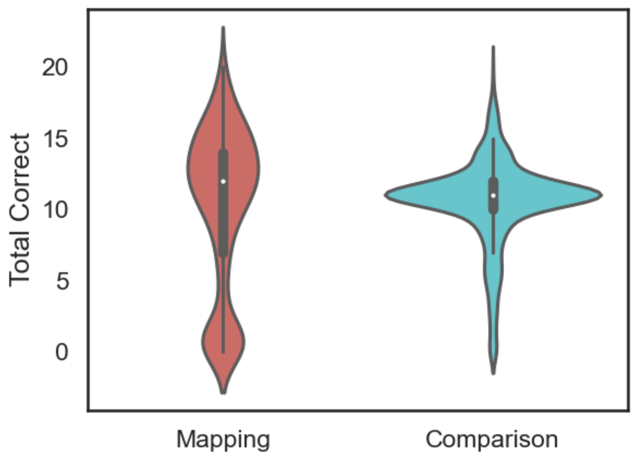
Note. All correlations were statistically significant. Correlations > .10 are significant at $p < .001$; correlations > .07 are significant at $p < .01$. Maximum possible scores were 60 for writing speed, 40 for the arithmetic operations, and 20 for the fraction measures.

Research Question 1: Do Chinese Students in Grade 4 Understand Fraction Notation?

For fraction mapping, one group of students showed a normal distribution, but another group, who scored “0” or “1” on the task, were clustered at the bottom end of the range (see Figure 2). In this bottom cluster, 82 students had a score of 0 (8%) and 110 students had a score of 1 (11%). Further examination of individual trial performance for students who scored “1” revealed that 91 students had reversed the numerator and denominator for most of the questions attempted (e.g., writing $\frac{2}{5}$ as $\frac{5}{2}$). Because the fourth trial had the same numerator and denominator (i.e., $\frac{4}{4}$), these students likely accidentally produced a correct response. Moreover, 13 students who scored “1” only correctly responded to $\frac{1}{2}$, one of the most familiar fractions. Thus, although these students did not obtain a score of “0” their single correct response suggests they did not understand the connection between fraction symbols and the quantities they represent. In the following analysis, the group who achieved scores greater than 1 were classified as “mappers”, whereas students who scored “0” and “1” were classified as “non-mappers”. The most common errors for the non-mappers were inverting the numerator and denominator (i.e., writing $\frac{2}{5}$ as $\frac{5}{2}$; $n = 125$) or providing a whole number response based on counting the shaded, unshaded, or total parts (e.g., writing $\frac{2}{5}$ as 2, 3 or 5; $n = 32$).

Figure 2

Violin Plots for the Fraction Mapping and Fraction Comparison Tasks



Note. The white dot is the median, and the black bar in the center of the plot represents the interquartile range.

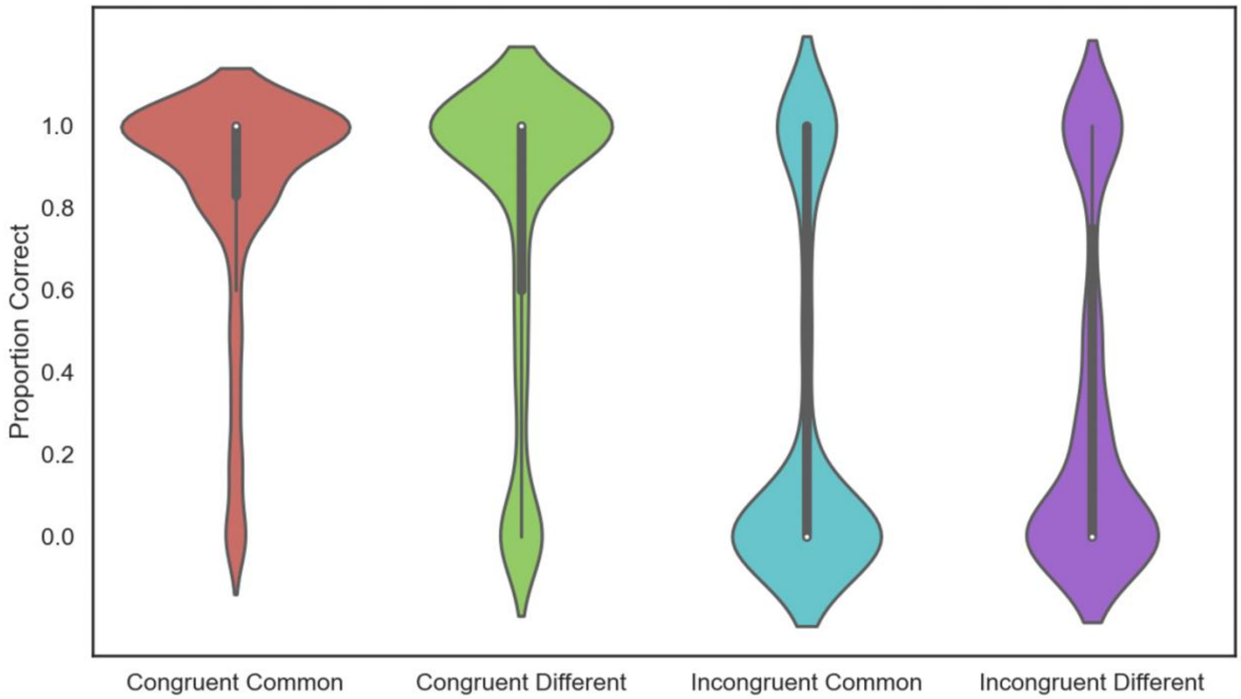
On the fraction comparison task, many students (75%) attempted all 20 trials. However, a large proportion of students (65%) only answered about half of the trials correctly (i.e., scores ranged from 10 to 12; see Figure 2). Notably, the fraction comparison task can be divided into two types of trials: congruent and incongruent. On congruent trials (n = 11), the relative magnitude of the components was consistent with the relative magnitude of the whole fraction (e.g., $\frac{1}{6} < \frac{5}{6}$; $\frac{5}{7} > \frac{2}{3}$; see Appendix B). On five of the congruent trials, the fraction pairs had common denominators (e.g., $\frac{1}{6} < \frac{5}{6}$; see trials A1, A2, A5, A9, A10), and on the other six trials, the fraction pairs had different denominators (e.g., $\frac{5}{7} > \frac{2}{3}$; see trials A12, A13, A14, A15, A16, A17). On incongruent trials (n = 9), the relative magnitude of the components was inconsistent with the relative magnitude of the whole fraction (e.g., $\frac{5}{8} < \frac{5}{6}$; $\frac{2}{3} > \frac{4}{7}$). Five of these incongruent trials had common numerators (e.g., $\frac{5}{8} < \frac{5}{6}$; see trials A3, A4, A6, A7, A8), whereas the other four trials had different numerators and denominators (e.g., $\frac{2}{3} > \frac{4}{7}$; see trials A11, A18, A19, A20).

As shown in Figure 3, the distribution on each of the four types of trials was extremely skewed. The median proportion correct for both types of congruent trials was 1, indicating that many of the students had perfect scores. In contrast, the median proportion correct for both types of incongruent trials was 0, indicating that the majority of the students consistently selected the incorrect fraction as the larger magnitude. Thus, most students consistently selected the fraction with larger components, regardless of trial type. Overall, these Chinese grade 4 students failed to consider the relation between the numerator and denominator, suggesting that they did not understand how fraction symbols denote magnitude. Thus, fraction comparison was excluded from subsequent analyses.

In summary, in support of our first and second hypotheses, we found that most students could map visual representations to fraction notations and thus had some basic understanding of fraction notation, however, they have difficulty assessing fraction magnitudes without visual representations. Instead, their performance on the comparison task is based on the magnitude of the components of fractions, not on the magnitude of the whole fraction (Braithwaite & Siegler, 2018; Meert et al., 2010b; Stafylidou & Vosniadou, 2004).

Figure 3

Violin Plots for the Four Types of Fraction Comparison Trials



Note. The white dot is the median, and the black bar in the center of the plot represents the interquartile range.

Research Question 2: What Role Do Division Skills Play in Students' Understanding of Fraction Notation?

We also hypothesized that division performance would differentiate mappers (i.e., scores greater than 1 on the fraction mapping task; n = 844), from non-mappers (i.e., scores of 0 or 1 on the fraction mapping task; n = 192). A logistic mixed-effect model containing classroom (n = 24) as a random effect was fit to the data to predict whether students were mappers (coded as 1) or non-mappers (coded as 0). First, an intercept-only model was fit which contained a classroom variable (Level 2). We found that the intra-class correlation coefficient was .04, indicating low variability among the classrooms. Then, a second model was fit with the addition of Level 1 variables (i.e., gender, writing speed, addition, subtraction, multiplication, and division). To compare the intercept-only and final model, a likelihood ratio test was conducted to see if there was significant improvement (Baayen et al., 2008). The log likelihood difference test indicated that the final model was significantly better than the intercept-only model, $p < .001$.

As shown in Table 2, girls were more likely than boys to understand fraction notation. Importantly, when considering all four arithmetic operations in the model, students' division skills uniquely differentiated students who had shown some understanding of fraction notations and students who did not. This effect was small (Chen et al., 2010) but significant: For a one unit increase in division, the odds of a student being classified as a mapper were 1.62 times higher. Thus, in support of our third hypothesis, division predicted unique variance in students' understanding of fraction notations.

Table 2

Mixed Effects Model for Fraction Mapping (n = 1036)

Parameter	Estimate	*SE*	*t*	*p-value*	Odds Ratio
Intercept	1.38***	0.11	12.18	< .001	--
Gender	0.63***	0.18	3.43	.001	1.88
Writing Speed	0.03	0.09	0.30	.786	1.03
Addition	0.25	0.14	1.73	.084	1.28
Subtraction	-0.06	0.13	-0.46	.656	0.95
Multiplication	-0.04	0.10	-0.40	.691	0.96
Division	0.48***	0.15	3.30	.001	1.62

Notes. $p \leq .001$***; Gender was coded as 0 (Boys) and 1 (Girls). Continuous variables were standardized. Higher odd ratio values indicate that the predictor is associated with a higher probability of being as mappers.

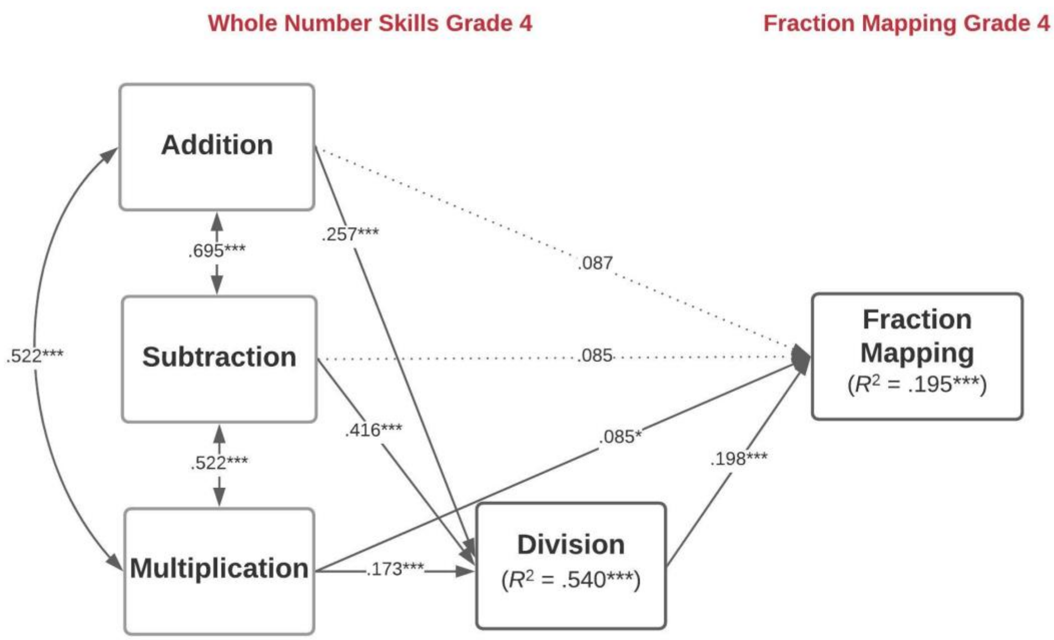
Next, we examined the hierarchical relations among whole number arithmetic (i.e., addition, subtraction, multiplication, and division), and fraction mapping performance after excluding the students who showed no understanding on the fraction mapping task (i.e., the mappers; $n = 844$). We hypothesized that students' division skills would mediate the relations between addition, subtraction and multiplication, and fraction mapping performance, controlling for gender and writing speed. To test this hypothesis, we fit a multilevel path model with classroom as a random effect. First, an intercept-only model was fit which contained a classroom variable (Level 2). The intra-class correlation coefficient was .09, indicating modest variability on the fraction mapping performance among the classrooms. Then, a second model was fit with the addition of Level 1 variables (i.e., gender, writing speed, addition, subtraction, multiplication, and division). The log likelihood difference test indicated that the final model was significantly better than the intercept-only model, $p < .001$. The final model fit was excellent, $\chi^2(3) = 4.67$, p

= .198, SRMR = .011, CFI = .999, RMSEA = .026. The model is shown in Figure 4. For readability, control variables (i.e., gender and writing speed) are not shown in the figure but are described in text. The R^2 values shown in Figure 3 for each outcome include variance predicted by the control measures.

As hypothesized, addition, subtraction, and multiplication skills each predicted unique variance in division skills. Furthermore, both multiplication and division skills predicted unique variance in fraction mapping, controlling for gender ($\beta = .090$, $p = .003$) and writing speed ($\beta = .147$, $p < .001$). The indirect effects from addition ($\beta = .051$), subtraction ($\beta = .082$) and multiplication ($\beta = .034$) to fraction mapping through division were all significant, ps < .01. These results suggest that for Chinese students in grade 4, division skills fully mediated the relations between fraction mapping skills and both addition and subtraction. In contrast, division skills partially mediated the relation between multiplication and fraction mapping skills.

Figure 4

Path Analysis for Arithmetic and Fraction Mapping for the Mappers (n = 844)



Note. Numbers on the arrows are standardized coefficients. Dashed lines represent non-significant paths.

**p* < .05, ****p* < .001

As shown in Figure 4, despite the similar magnitudes of the standardized path coefficients from addition, subtraction, and multiplication to fraction mapping, only the direct path from multiplication to fraction mapping was significant. Notably, the overall variability of multiplication (*SE* = .039) was smaller compared to that of addition (*SE* = .046) and subtraction (*SE* = .048), which explains why the path from multiplication to fraction mapping was significant (*p* = .028) whereas those from addition and subtraction to fraction mapping were not (*p* = .062 and *p* = .077, respectively). Altogether, whole number arithmetic (i.e., the four operations) explained approximately 20% of the variance in fraction mapping. Despite the importance of whole number arithmetic skills more generally, division predicted unique variance in fraction mapping, and more importantly, the strength of the path coefficient for division was twice that of the other three operations. Taken together, these results generally support the final hypothesis such that addition, subtraction, and multiplication skills predicted division skills, which in turn predicted fraction mapping performance for the mappers.

Discussion

How do whole number arithmetic skills support students' understanding of fraction magnitude during the emerging stages of fraction learning? In the present study, Chinese students in grade 4 completed four types of whole number arithmetic tasks (i.e., addition, subtraction, multiplication, and division) and two tasks that were designed to assess students' knowledge of fraction notation and fraction magnitude. Consistent with research with students from around the world (Chan et al., 2007; Gabriel et al., 2013; Namkung et al., 2018; Siegler & Lortie-Forgues, 2017), we found that the Chinese students in this study had difficulty understanding fractions: About one-fifth of students did not understand fraction notations, and most students showed a whole number bias when they were comparing fraction symbols.

Learning mathematics is a hierarchical process, in which earlier acquired associations are the building blocks for higher-level associations (Hiebert, 1988; Núñez, 2017; Siegler & Lortie-Forgues, 2014; Xu et al., 2019). According to the hierarchical symbol integration model, students need to integrate various associations into a unified structure, selectively retrieving the specific associations needed to solve specific mathematical problems (Xu & LeFevre, 2020). In the present study, we extended this model to the integration of whole number arithmetic and fraction knowledge. We found that the most advanced whole number arithmetic operation (i.e., division) mediated the relations between the more basic whole number arithmetic associations (i.e., addition, subtraction, or multiplication) and predicted whether students had acquired some basic understanding of fraction notation. Further, the strength of the path coefficient from division to fraction mapping was twice that of the other three operations, highlighting the importance of division in the understanding of fraction mapping.

Bridging Whole Number Knowledge and Fractions via Division

We found that about one-fifth of the students were not able to produce correct fraction notation for visual representations (i.e., non-mappers), suggesting that they have limited knowledge about symbolic fractions. About 82% of the non-mappers either inverted the numerator and denominator of the fractions or provided a whole number response based on counting the shaded, unshaded, or total parts, indicating that they did not understand the conceptual part-whole relation reflect in fraction notation. This immature understanding of fraction notation was also observed in American students in grades 3 and 4 (Smith et al., 2005). More interestingly, when considering all four types of whole number arithmetic in one model, we found that division uniquely differentiated the mappers and non-mappers (Table 2), although the effect size was small. More generally, students with stronger division skills were more likely to be mappers, consistent with other work showing that division is important for fraction understanding (Hansen et al., 2015; Namkung et al., 2018; Siegler & Pyke, 2013; Stelzer et al., 2019; Sun, 2019; Thompson & Saldanha, 2003).

In large longitudinal studies in the United States and United Kingdom, Siegler et al. (2012) found that students' knowledge of fractions and mastery of whole number division in elementary school (ages 10-12 years) were both significant predictors of mathematics achievement five to six years later, controlling for other whole number arithmetic skills (addition, subtraction, and multiplication), general intelligence, working memory, family education and family income. Siegler et al. speculated that the acquisition of whole number division is a more advanced skill than other arithmetic operations and thus it is more challenging to master. Based on this speculation, for older students we might expect to see a wider range of individual differences for division performance in comparison to addition, subtraction, or multiplication. Thus, consistent with the present research, division skills play a central role in students' acquisition of more advanced mathematics.

We found a significant path from multiplication to fraction mapping in the overall analysis that was not significant when we controlled for nonverbal intelligence (see Supplementary Materials). Multiplication and division are complementary, both procedurally and conceptually (e.g., $3 \times 6 = 18$; $18 \div 3 = 6$; Robinson, 2017). In studies of North American adults, division solutions often reference multiplication knowledge (Mauro et al., 2003). In contrast, there is limited variability in students' multiplication performance in this sample (see Figure 2) and it is very likely that students were solving all operations by retrieving answers from memory. In China, students memorize the multiplication table through extensive practice and achieve a very high level of fluency. In other countries where there is less emphasis on students' achieving very high fluency in multiplication, we would expect greater variability in multiplication fluency among students at this age and that both multiplication and division would predict variance in fraction mapping.

The limited fraction knowledge of grade 4 students was also observed in the fraction comparison task. We found that most of the students performed poorly on the fraction comparison task when the relative magnitudes of the components and whole fractions were inconsistent (i.e., incongruent trials), suggesting that they were probably using the magnitude of the denominators to make their decisions. Thus, consistent with prior studies, emergent fraction learners processed the whole number components of fractions discretely rather than processing the magnitude of the fraction holistically. This misconception is known as the whole number bias (Ni & Zhou, 2005). Students' ability to overcome the interference of the whole number bias is crucial to their integration of novel fraction magnitude knowledge into their existing numerical network (Vosniadous et al., 2008).

Without a solid understanding of fraction magnitude, students may experience difficulties in learning fraction arithmetic (Lortie-Forgues et al., 2015). For example, American students in grade 4 systematically misapplied rules of whole number arithmetic to fraction addition and subtraction problems, particularly on problems with different denominators (e.g., $\frac{1}{3}+\frac{3}{4}=\frac{4}{7}$; Schumacher & Malone, 2017). Difficulty with fraction arithmetic was also observed for American students from grades 4 to 8 when they were estimating the magnitudes of fraction sums on a 0-1 number line, suggesting they had poor conceptual understanding of fraction arithmetic (Braithwaite et al., 2018). Thus, understanding how to help students overcome the whole number bias during the emerging stages of fraction learning is an important goal of future research.

Implications for Mathematics Instruction

Our research is closely tied to the educational experiences of students in China and therefore may not generalize to students in other educational systems. In China, mathematics education has been viewed as essential to the Chinese culture for over 2000 years. Chinese students have consistently demonstrated excellent performance in all aspects of mathematics in international assessments (e.g., Mullis et al., 2020). Although there are many elements that contribute to the successful mathematics education in China, the coherent, precisely defined national curriculum may have a major impact (Zhang & Padilla, 2016). In most Western countries, a spiral curriculum is used in which many topics are introduced each year and revisited numerous times across grades (Ireland & Mouthaan, 2020; Snider, 2004; Schmidt et al., 2002, 2018). In contrast, except for a few difficult topics that are arranged in a spiral way, China uses a linear curriculum in which a few topics are taught in great depth each year, with minimal repetition in later years (Li et al., 2009; Li & Huang, 2013). More specifically, the Chinese mathematics curriculum is sequentially organized such that students are taught different topics thoroughly, breaking down difficult topics into small steps, along with systematic practices that vary in difficulty, allowing students to master the core skills and concepts (Li & Huang, 2013). Importantly, a central theme in the Chinese mathematics curriculum is making connections between previous knowledge and newly acquired knowledge, rather than treating mathematics as a collection of isolated concepts and procedures (Sun, 2019).

Despite the overall success of the Chinese educational curriculum, fractions are conceptually challenging for young learners. Prior to formal schooling, children may have some experiences with mathematical vocabularies involving specific fractions (e.g., a half), for example, when they share food or toys. However, they are unlikely to understand fractions until later in elementary school, given the abstractness and complexity of the proportional reasoning skills that underlie fraction knowledge (English & Halford, 1995; Mack, 1993; Ni & Zhou, 2005; Thompson & Saldanha, 2003). In support of this notion, we found that grade 4 students in China – a country with top performance on international assessments – exhibited difficulty in understanding symbolic fractions, evidenced by their erroneous performance on fraction mapping and comparison tasks.

Prior to grade 4, the Chinese mathematics curriculum primarily focuses on the integration of whole number arithmetic skills, which become the foundation of the more advanced fraction concepts (Ministry of Education, 2011). Aligned with the Chinese curriculum, our results show that prior knowledge of whole number division supported students' understanding of fraction magnitude during the emerging stages of fraction learning. These results generally support the view that the mastery of closely related concepts is crucial for mathematics learning (Ma, 2010). With respect to fraction learning, we posit that formal education of fractions should be introduced after students have integrated whole number arithmetic skills into their numerical associative network. Explicit instruction on the connections between fractions and division draws students' attention to the fact that the numerator and denominator of a fraction have the same functions as the dividend and divisor in division, knowledge which prepares students to conceptually understand fractions (Sun, 2019).

Limitations and Future Research

The present study was focused on the relations between whole number arithmetic and emerging fraction knowledge. One limitation therefore was that we did not consider other cognitive skills that have well-established relations with mathematics achievement. For example, Jordan et al. (2013) found that language skills (i.e., receptive vocabulary and word reading), nonverbal reasoning, attention, and whole number line estimation skills in grade 3 were all related to fraction knowledge in grade 4 (i.e., fraction mapping, fraction comparison and fraction number line). Thus, considering the relations between both domain-general and domain-specific cognitive skills and fraction knowledge may further understanding of knowledge that supports fraction learning.

The present study is important because it captures fraction knowledge during the emerging stages of fraction learning. However, in future research, following students from grades 4 to 6 would provide insights into how fraction knowledge expands as students master new fraction concepts, such as fraction arithmetic and fraction comparison, and help identify the prerequisite skills required to master more advanced fraction concepts. Concrete pictorial representations of fractions can be helpful during the early stages of fraction learning; however, they are not a replacement for conceptual instruction on the connections between fractions and division. Concrete representations become difficult for students to use when fractions become more complex, for example, interpreting improper fractions (e.g., how is it possible to take 7 out of 3 things; Sun, 2019) or converting fractions into proportions (e.g., if there are 4 boys and 3 girls in a class, the proportion of boys to girls is $\frac{4}{3}$; Thompson & Saldanha, 2003). Thus, instruction focused on mastering basic concepts and integrating previous knowledge with newly acquired knowledge is key for fraction success.

Conclusion

Students' existing whole number knowledge is the foundation for acquiring fraction understanding (Ni & Zhou, 2005; Siegler et al., 2011). Despite a large corpus of research on fraction learning, our study is the first to provide empirical evidence for the important role of division in connecting whole number knowledge with fraction knowledge for Chinese novice fraction learners. Within the framework of the hierarchical symbol integration model (Xu et al., 2019; Xu & LeFevre, 2020), we show the relative importance of division versus other whole number arithmetic operations for children's understanding of symbolic fraction notation. We also show that the whole number bias is evident in fraction comparison performance even for Chinese students who have demonstrated some understanding of the mapping between fraction magnitude and fraction notation. These results suggest that, beyond conceptual understanding of fraction notation, students also need to overcome interference between the relative magnitude of the fraction components and the relative magnitude of the whole fractions. In summary, the present results support the view that learning mathematics is a hierarchical process: Specifically, we found support for the view that division is a central organizing concept for rational number processing.

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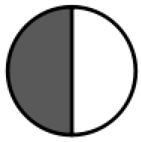
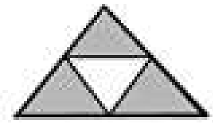
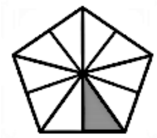
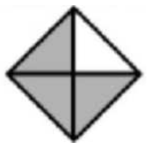
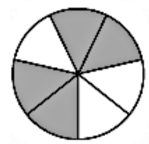
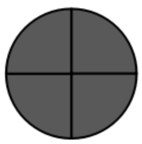
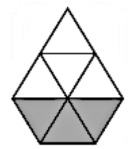
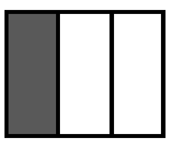
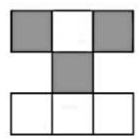
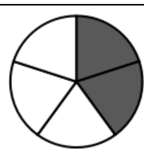
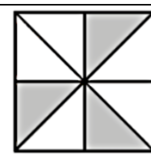
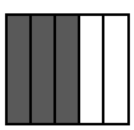
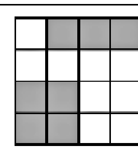
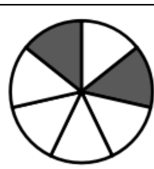
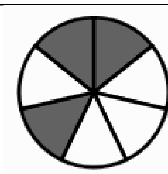
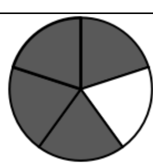
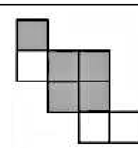
Appendix A: Fraction Mapping Stimuli

Translated Instructions: Write down the fraction of the shaded area in order

A1			A11		
A2			A12		
A3			A13		
A4			A14		
A5			A15		
A6			A16		
A7			A17		
A8			A18		
A9			A19		
A10			A20		

Appendix B: Fraction Comparison Stimuli

Translated Instructions: Circle the larger fraction in each pair in order

A1	$\frac{1}{8}$	$\frac{7}{8}$	**A11**	$\frac{2}{3}$	$\frac{1}{4}$
A2	$\frac{3}{4}$	$\frac{1}{4}$	**A12**	$\frac{4}{5}$	$\frac{2}{3}$
A3	$\frac{3}{5}$	$\frac{3}{7}$	**A13**	$\frac{1}{3}$	$\frac{3}{4}$
A4	$\frac{5}{9}$	$\frac{5}{6}$	**A14**	$\frac{7}{8}$	$\frac{3}{5}$
A5	$\frac{1}{6}$	$\frac{5}{6}$	**A15**	$\frac{3}{4}$	$\frac{5}{6}$
A6	$\frac{1}{9}$	$\frac{1}{8}$	**A16**	$\frac{1}{4}$	$\frac{2}{7}$
A7	$\frac{3}{4}$	$\frac{3}{5}$	**A17**	$\frac{3}{8}$	$\frac{1}{3}$
A8	$\frac{2}{5}$	$\frac{2}{3}$	**A18**	$\frac{2}{3}$	$\frac{4}{7}$
A9	$\frac{5}{7}$	$\frac{4}{7}$	**A19**	$\frac{5}{8}$	$\frac{2}{3}$
A10	$\frac{7}{9}$	$\frac{8}{9}$	**A20**	$\frac{3}{4}$	$\frac{5}{7}$

Note: Congruent trials are A1, A2, A5, A9, A10, A12, A13, A14, A15, A16, A17.

Incongruent trials are A3, A4, A6, A7, A8, A11, A18, A19, A20.