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Walking Another Pathway: The Inclusion of Patterning in the Pathways to Mathematics Model

Sabrina M. Di Lonardo Burr[1], Chang Xu[2], Heather Douglas[1], Jo-Anne LeFevre[1,2], & María Inés Susperreguy[3,4]

[1]Department of Cognitive Science, Carleton University

[2]Department of Psychology, Carleton University

[3]Faculty of Education, Pontificia Universidad Católica de Chile

[4]Millennium Nucleus for the Study of the Development of Early Math Skills (MEMAT)

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Abstract

According to the Pathways to Mathematics model (LeFevre et al., 2010), children's cognitive skills in three domains – linguistic, attentional, and quantitative – predict concurrent and future mathematics achievement. We extended this model to include an additional cognitive skill, patterning, as measured by a non-numeric repeating patterning task. Chilean children who attended schools of low or high socioeconomic status (*N* = 98; 54% girls) completed cognitive measures in Kindergarten (M_{age} = 71 months) and numeracy and mathematics outcomes one year later in Grade 1. Patterning and the original three pathways were correlated with the outcomes. Using Bayesian regressions, after including the original pathways and mother's education, we found that patterning skills predicted additional variability in applied problem solving and arithmetic fluency, but not number ordering, in Grade 1. Similarly, patterning skills were included in the best model for applied problem solving and arithmetic fluency, but not for number ordering, in Grade 1. In accord with the hypotheses of the original Pathways to Mathematics model, patterning varied in its unique and relative contributions to later mathematical performance, depending on the demands of the tasks. We conclude that patterning is a useful addition to the Pathways to Mathematics model, providing further insights into the range of cognitive precursors that are related to children's mathematical development.

Word count: 211 words

Keywords: patterning, cognitive precursors, numeration, mathematics, Chile

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In daily life, children are exposed to a variety of patterns, that is, sequences that progress according to a rule. Patterns can be non-numeric, involving colours, shapes, sizes, and pictures (e.g., colour sequences in floor tiles) or alphanumeric, involving letters or numbers (e.g., house numbers on a street that increase by two). Within a sequence, elements can either repeat (e.g., a string of alternating red and green lights) or grow (e.g., the counting sequence, 1 2 3 4…). Children's performance on repeating patterning tasks is related to their concurrent and later mathematics achievement (Fyfe et al., 2019; MacKay & De Smedt, 2019; Papic et al., 2011; Rittle-Johnson et al., 2015, 2019; Wijns, Torbeyns, Bakker et al., 2019; Wijns et al., 2021; Zippert et al., 2019, 2020). Moreover, the relation between patterning and mathematics achievement remains after controlling for other cognitive precursors, such as working memory, verbal, and spatial skills (Rittle-Johnson et al., 2019; Wijns, Torbeyns, De Smedt et al., 2019; Zippert et al., 2020). Thus, non-numeric repeating patterning may be a core cognitive precursor of mathematical learning.

Mathematics has been defined as the "science of patterns" (Steen, 1988, p. 616). Clements and Sarama (2014) stated that "Patterning is the search for mathematical regularities and structures. Identifying and applying patterns helps bring order, cohesion, and predictability to seemingly unorganized situations and allows … generalizations beyond the information [provided]" (p. 215). Similarly, Charles (2005) proposed that patterning should be one of the "big ideas" in mathematical curricula because the logic of mathematical ideas often mirrors the predictable and repetitive nature of the patterns. Accordingly, Zippert et al. (2020) theorized that repeating patterning knowledge may help children's mathematics, broadly, as well as specific numeracy skills, including (a) mastery of the numeration system, by helping them detect the patterns in numbers and learn the underlying rules, and (b) the successor principle (i.e., when one object is added to a set of N, the set now contains N + 1 objects). Based on these definitions and Zippert et al.'s theorical model, there is reason to believe that patterning is an important cognitive precursor of early mathematical development.

The Pathways to Mathematics Model

LeFevre et al. (2010) noted a gap in the mathematics literature. Specifically, there was a need to "[identify] a consistent and coherent set of cognitive skills that are predictive of early numeracy development (LeFevre et al., 2010; p. 1754). Accordingly, they proposed the Pathways to Mathematics model, a theoretical framework outlining the relations among three cognitive precursor skills – attentional, linguistic and quantitative – and early mathematical development (LeFevre et al., 2010; Sowinski et al., 2015). The model was originally conceptualized to contrast the roles of domain-general (e.g., spatial attention, linguistic) and domain-specific (e.g., quantitative) skills in mathematical development from preschool to the early years or primary school. The precursor skills were defined, in part, on neuropsychological work (Dehaene et al., 2003, 2004; Spelke & Dehaene, 1999) which showed that numerical processing involves three separate neural circuits in the parietal lobe (i.e., circuits related to language processing, attention, and quantity/magnitude). The model was evaluated using longitudinal data from 4- and 5-year-old Canadian children, who were followed for two years (LeFevre et al., 2010).

Considering the evidence that repeating patterning is related to children's early mathematical learning and Zippert et al.'s (2020) theoretical model, in which repeating patterning knowledge predicts future math knowledge beyond working memory, the goal of the present research was to determine whether the Pathways to Mathematics model should be extended to include non-numeric patterning as a cognitive precursor. More specifically, in the present study we assessed whether repeating patterning explained additional variance in children's mathematics and numeracy outcomes above and beyond the original three pathways. The pathways were operationalized as described below.

The Attentional Pathway

In LeFevre et al. (2010), the attentional pathway was assessed with spatial span (i.e., visual-spatial working memory). Sowinski et al. (2015) argued that attentional skills more generally (i.e., including working memory and executive functions) are central to understanding mathematical development (LeFevre et al., 2005; Peng, Namkung, Barnes, et al., 2016; Raghubar et al., 2010). Thus, they adopted Baddeley and Hitch's (1974) working memory model as a theoretical framework for the attentional pathway for 7- to 9-year-old children (see also Bull et al., 2008). In the present research, we followed the original model and used a single measure of spatial span to capture the attentional pathway because spatial working memory is particularly relevant for young children and mathematics (Bull et al., 2008; Caviola et al., 2020; Van de Wijer-Bergsam et al., 2015).

The Linguistic Pathway

The linguistic pathway also has an important role in the Pathways to Mathematics model. There are many different aspects of children's language ability that are related to mathematics, including vocabulary and phonological awareness (Peng et al., 2020). For example, receptive vocabulary in preschool is predictive of later mathematics achievement (Harvey & Miller, 2017; Hornung et al., 2014; Passolunghi & Lanfranchi, 2012). Receptive vocabulary is especially important for mathematical problem-solving tasks where children need to comprehend the meaning of the problem to construct a solution (Fuchs et al., 2021; Harvey & Miller, 2017; LeFevre et al., 2010; Swanson et al., 2015). In the present study, we used receptive vocabulary to index the linguistic pathway.

The Quantitative Pathway

The quantitative pathway captures children's ability to access the links between representations of quantity and numerals. Early quantitative skills, such as subitizing, support children's acquisition of the links between quantities and symbols (Feigenson et al., 2004; LeFevre et al., in press). Accordingly, LeFevre et al. (2010) used subitizing to index the quantitative pathway for 4- and 5-year-old children. For 7- and 8-year-old children, Sowinski et al. (2015) included subitizing and two other measures in the quantitative pathway, the speed of counting of sets of 4 to 6 objects and symbolic number comparison (e.g., Which number is bigger, 4 or 7?). The decision to expand the quantitative pathway to include symbolic number comparison arose from both theory and research. Theoretically, symbolic number comparison requires children to access quantity information through symbols and thus is related to, but more advanced than, subitizing and counting (Lyons & Ansari, 2015). Empirically, symbolic number comparison skills in Kindergarten predict children's math skills in Grade 1 and beyond (Hawes et al., 2019; Locuniak & Jordan, 2008; Nosworthy et al., 2013). Thus, Sowinski et al. argued that symbolic number comparison is an appropriate measure to include in the quantitative pathway. Based on this argument, in the present study, symbolic number comparison in Kindergarten was used to represent the quantitative pathway.

Support for the Pathways to Mathematics Model

In the original study of the Pathways to Mathematics model, LeFevre et al. (2010) proposed two hypotheses: 1) The three pathways would contribute independently to early numeracy performance; 2) The three pathways would vary in their unique and relative contributions to later mathematical performance, dependent on the demands of the tasks. In support of their hypotheses, all three pathways were significantly correlated with early numeracy measures for 4- and 5-year-olds, but different pathways predicted unique variance for different numeracy outcomes two years later. More specifically, all three pathways predicted significant unique variability in later numeration, calculation, and number line measures, whereas only the linguistic and quantitative pathways predicted significant unique variance in magnitude comparison, and only the linguistic and attentional pathways predicted significant unique variance in geometry and measurement. In multiple regressions, the variability in numeracy and mathematics outcomes explained by the three pathways ranged from 26% to 56%.

Since the original study, additional support has been found for the Pathways to Mathematics model across a range of samples and measures. Cirino (2011) investigated the relations among the three pathways and single-digit addition with Kindergarten children in the United States. With confirmatory factor analysis, Cirino found a five-factor structure for quantitative precursors (i.e., non-symbolic comparison, symbolic comparison, symbolic labelling, rote counting, and counting knowledge) and a two-factor structure for linguistic precursors (i.e., phonological awareness and rapid automatized naming). Together, quantitative, linguistic, and attentional skills predicted 55% of the variance in single-digit addition, with symbolic quantitative skills mediating the effects of the domain-general pathways.

Beyond Kindergarten, Sowinski et al. (2015) tested the Pathways to Mathematics model with 7- and 8-year-old Canadian children, using composite measures to index the three pathways. They found that all three pathways accounted for unique variance in backward counting and arithmetic fluency, but only the quantitative and linguistic pathways accounted for unique variance in children's knowledge of calculation and the number system. The variability in math outcomes explained by these three predictors ranged from 27% to 47%.

For 7- to 9-year-old Canadian children, Xu et al. (2021) found that all three pathways accounted for unique variance in word problem solving, explaining 37% and 56% of the variance for first- and second-language learners, respectively. For arithmetic fluency, the three pathways accounted for 42% and 38% of the variance for first-and second-language learners, respectively, with the quantitative pathway and attentional/working memory pathway (for first-language learners) accounting for unique variance. In contrast, for word reading, the three pathways accounted for 22% and 23% of the variance for first-and second-language learners, respectively, but only the linguistic pathway and attentional/working memory pathway (for first-language learners) accounted for unique variance.

Träff et al. (2018) used path analysis to test the Pathways to Mathematics model with Swedish 9- and 10-year-olds. Notably, in addition to the three original pathways Träff et al. also included an approximate quantitative pathway and a verbal working-memory pathway that was separate from their spatial processing pathway. Their mathematical outcomes included a hierarchical structure of symbolic number and arithmetic skills, such that symbolic number processing was assumed to predict single-digit arithmetic, which was assumed to predict multi-digit calculation, which was assumed to predict arithmetic word problem-solving. With respect to the cognitive pathways, the linguistic pathways directly predicted single-digit arithmetic and arithmetic word-problem solving; the approximate quantitative pathway directly predicted single-digit arithmetic and multi-digit calculation; and the spatial and verbal working-memory pathways directly predicted arithmetic word problem-solving. In total the model explained 40% to 58% of the variance in the mathematical outcomes.

In summary, the results of these studies support the Pathways to Mathematics model as a useful framework for understanding individual differences in mathematical skills and reinforce the notion that contributions of the individual pathways vary depending on the outcome measure. Quantitative, linguistic, and attentional skills are consistently linked to mathematical outcomes, regardless of language or culture, with systematic patterns depending on the specific type of mathematical task. Thus, the Pathways to Mathematics model captures a consistent and coherent set of cognitive skills that are predictive of early numeracy and mathematics development. However, the Pathways to Mathematics model, as originally formulated, did not include measures of reasoning or other non-verbal cognitive skills, such as non-numeric repeating patterning. Subsequently, repeating patterning has been found to consistently correlate with mathematical achievement among young children (Rittle-Johnson et al., 2013, 2015, 2019; Zippert et al., 2019, 2020, 2021) and predict later mathematical achievement (Fyfe et al., 2015; Rittle-Johnson et al., 2015). Thus, expanding the Pathways to Mathematics model to include repeating patterning among the key cognitive precursors might enhance the model and help strengthen the theoretical framework for linking a consistent set of cognitive precursors to later mathematics achievement.

Patterning as a Cognitive Precursor to Mathematics

The theoretical argument in favour of patterning as a core precursor skill for mathematics is that numerical knowledge is essentially a complex set of relations, connected by rules. Rittle-Johnson et al. (2019) suggested that "identifying, extending, and describing predictable sequences (patterns) in objects and numbers are core to mathematical thinking" (p. 168). Consistent with this view, non-numeric repeating patterning has been linked both concurrently and longitudinally to a range of numeracy and mathematics outcomes, such as counting, calculation, arithmetic fluency, and applied problem solving (Burgoyne et al., 2017; Fyfe et al., 2015; Peng, Namkung, Fuchs, et al., 2016; Rittle-Johnson et al., 2015; Spencer et al., 2020; Zippert et al., 2019, 2020). Importantly, in support of including an additional pathway in the Pathways to Mathematics model, the relation between patterning and mathematics achievement remains after controlling for other related domain-general abilities, such as working memory, relational knowledge, verbal ability, and spatial skills (Miller et al., 2016; Rittle-Johnson et al., 2019; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2019, 2020). Thus, non-numeric repeating patterning may be a core cognitive precursor of mathematical learning that is independent of other cognitive skills.

The Present Study

The goal of the present study was to test an expanded Pathways to Mathematics model that includes patterning as a core cognitive predictor of later mathematics achievement. Specifically, our focus was to determine whether non-numeric repeating patterning predicts mathematics and numeracy outcomes (i.e., applied problem solving, arithmetic fluency, number ordering) above and beyond the original pathways. We assessed patterning with a non-numeric repeating patterning task (Rittle-Johnson et al., 2019). Non-numeric repeating patterning tasks are an appropriate measure of patterning skills for children transitioning between Kindergarten and Grade 1 (e.g., Burgoyne et al., 2017; Fyfe et al., 2015; Rittle-Johnson et al., 2015; Zippert et al., 2019; 2020). We hypothesized that the three original pathways and patterning, measured in Kindergarten, would be correlated with numeracy and mathematics outcomes in Kindergarten and Grade 1. We further hypothesized that, with the addition of patterning, the expanded Pathways to Mathematics model would explain additional variance in numeracy and mathematics outcomes.

We used Bayesian regressions to address our research questions (Faulkenberry et al., 2020). First, we determined whether patterning explained additional unique variance in the numeracy and mathematics outcomes beyond the three original pathways. Second, we determined which of the four pathways predicted unique variance for each of the outcomes. Because the four cognitive precursors are presumably correlated and thus explain shared variance, we did not make specific hypotheses about the strength of each of the cognitive precursors in predicting the outcomes.

We tested the expanded model with two mathematics outcomes. *Applied problem solving* is a broad measure of mathematics designed to assess numeracy knowledge, reasoning, and problem solving with quantities. *Arithmetic fluency* is a symbolic task commonly used as an outcome measure in studies with primary school children. Zippert et al. (2020) found that patterning was a significant predictor of both broad mathematics achievement and specific numeracy skills (i.e., counting to 100). Thus, in addition to the two mathematics outcomes, we also tested the expanded model with a symbolic numeracy outcome, number ordering.

Number ordering is a measure of children's ability to order a series of visually presented digits (Lyons et al., 2014). To our knowledge, number ordering has not been used as an outcome measure in other patterning studies. However, patterning skills are related to numeracy outcomes that involve order knowledge, such as counting to 100 (Zippert et al., 2020), suggesting that patterning may also be related to identifying number order. Number ordering is related to number comparison (Xu & LeFevre, 2021), but requires additional knowledge beyond the mappings between symbols and magnitudes (Lyons & Ansari, 2015). Empirically, many children in Kindergarten have trouble making ordinal judgments beyond the count list (Hutchison et al., 2022). Between Kindergarten and Grade 1, children's understanding of what it means for a sequence to be “in order” improves dramatically as they extend the concept of ordinality beyond the count list to nonadjacent sequences. However, even in Grade 1, children are still in the process of developing conceptual and procedural skills that are related to ordinality (Finke et al., 2021; Hutchison et al., 2022; Xu & LeFevre, 2021). Thus, performance on ordinal tasks, such as number ordering and order judgment tasks (Lyons & Ansari, 2015), taps into a range of individual differences in symbolic number knowledge, decision making, and strategic processes (Hutchison et al., 2022; Muñez et al., 2022; Vogel et al., 2021; Vos et al., 2021; Xu & LeFevre, 2021). In the present study we aimed to identify the cognitive precursors that predict ordinal understanding as measured by number ordering.

Method

The participants in the present study were Chilean children in Kindergarten and Grade 1. Chilean children’s mathematical development has not previously been explored within the Pathways to Mathematics framework. We assume that the relations between cognitive precursors and numeracy and mathematics outcomes are universal. That is, regardless of where children are educated or what language they speak, attentional, linguistic, quantitative, and patterning skills are expected to predict mathematics achievement. Thus, we did not have any culture-specific hypotheses.

In the Chilean preschool curriculum, patterning is included under the Mathematical Thinking nucleus (MINEDUC, 2019). This nucleus includes learning related to several aspects of math, including numbers and non-numeric patterns. Thus, we assume that children in the present research had some experience with patterning in school. Although we do not measure patterning in Grade 1, we acknowledge that patterning continues to be included in the Chilean curriculum in Grade 1. Under the learning goal, “Patterning and Algebra”, children are expected to be able to recognize, describe, create, and continue repetitive patterns and numerical patterns up to 20 by the end of Grade 1.

Participants and Procedure

The data analyzed in this paper are part of a short-term longitudinal study of the development of mathematics skills that involved 367 children in Kindergarten through Grade 3 (Susperreguy et al., 2022). In the present analyses, we included only children who were in Kindergarten ($N = 98$; $M_{age} = 71$ months; $SD = 4.4$; 54% girls) during the first wave of data collection because the patterning measure was only administered in Kindergarten. Children were tested over two sessions in the second semester of Kindergarten and again one year later in the second semester of Grade 1. In Grade 1, 94 children were retained.

Ethics approval for the study was received from the Scientific Ethics Committee of Social Sciences, Arts and Humanities at the Pontificia Universidad Católica de Chile. Families were recruited from five schools (two high-SES and three low-SES schools) in the urban metropolitan area of Santiago, Chile. Upon agreement from the principals of the schools, parents and their children were invited to participate.

The low-SES schools were selected according to an index of school vulnerability (Índice de Vulnerabilidad Escolar, IVE). This index is calculated annually by the Chilean Ministry of Education and corresponds to the percentage of students at each school classified as vulnerable (i.e., students who are either living in poverty conditions or at risk of school failure/dropout; Agencia de Calidad de la Educación, 2015). Higher percentages represent a higher degree of vulnerability. The three low-SES schools in the present study had IVE ratings ranging from 74% to 85% in 2018. The high-SES schools did not receive government subsidies because they were private schools. In the present study, approximately half of the children attended low-SES schools (49.5%) and half of the children attended high-SES schools (50.5%). Mother's education was assessed on a 12-point scale, ranging from "completed primary school" (1) to "doctorate degree" (12). In the present sample, the median level of education was "some university" (9). School SES was highly correlated with mother's education, *r*(96) = .72, BF_{10} = 1.21e+14. In the subsequent analyses we control for mother's education because it was more strongly correlated with child outcomes than was school SES (see Results for details).

Materials

All cognitive skills were measured in Kindergarten. Except for number ordering, which was assessed in Grade 1 only, all mathematics outcome measures, both those analyzed in the present study (i.e., applied problem solving and arithmetic fluency), and those that were part of the larger longitudinal project, were administered in Kindergarten and Grade 1. Due to floor effects in Kindergarten and a bimodal distribution in Grade 1, Calculations, an additional task from the WJ III Tests of Achievement (Muñoz-Sandoval et al., 2005), and number naming were administered but not included in the subsequent analyses. Similarly, a backward counting task was administered in Grade 1, but a high proportion of children successfully counted backward from 20 (67%), thus backward counting was not included in the subsequent analyses.

Cognitive Skills

Spatial Span. In the *PathSpan* iPad task (https://hume.ca/ix/pathspan/), children are presented with nine green dots on the tablet screen. The dots light up one-by-one. Children must attempt to reproduce the pattern of lights by tapping on the dots in the same order. Children begin the task with a practice trial of a 2-dot sequence. After the practice trial, testing begins with a 2-dot sequence. There are three trials for each dot-sequence length (ranging from 2 to 9 dots). Testing is discontinued when children are incorrect on all three trials for a given dot-sequence length. Scoring was the total number of correctly reproduced trials. Internal reliability based on the subscores of first, second, and third trials at each length was .88.

Receptive Vocabulary. The Hispanic-American adaptation of the Peabody Picture Vocabulary Test–Revised (Dunn et al., 1986) was administered to children in Kindergarten. In this task, children are presented with four pictures and the experimenter orally presents a word. Children must point to the picture that best represents the spoken word. The starting point is determined by the child's age, with testing discontinued when a child makes six errors in a set of eight stimuli. Scoring was the total number of correct words. The split half reliability among the eight sets that the majority of the children attempted (94%) was .67.

Number Comparison. In this iPad task ("Bigger Number" app), children are presented with two single-digit numbers (e.g., 3 and 7) on the screen and asked to touch the larger number as quickly and accurately as possible. Children have a maximum of three seconds to make a response before the task automatically advances to the next trial. There are 26 items in total, with half of the trials having a small distance between the two numbers (i.e., a difference of 1, 2, or 3) and half of the trials having a large distance between the two numbers (i.e., 4, 5, 6, or 7). Scoring was calculated using a linear integrated speed-accuracy score (LISAS; $\mathrm{RT}_{\mathrm{adj}} = \mathrm{RT}_{\mathrm{correct}} + \mathrm{PE} \times [SD_{\mathrm{RT}}/SD_{\mathrm{PE}}]$), where $\mathrm{RT}_{\mathrm{correct}}$ is the mean response time on the correct trials and percentage error (PE) is weighted by the ratio of the standard deviations of the correct response time and the percentage of error (Vandierendonck, 2017). The internal reliability based on RT for correct trials was .88.

Patterning. Patterning was tested using an adapted version of the Teacher-based Repeating Patterning Assessment (Rittle-Johnson et al., 2019) which was developed using patterning worksheets from websites with resources for early-childhood educators. The original measure was modified to make it more difficult because the children in our sample were older. See Appendix A for items and adaptations. In this task, children are presented with patterns of objects as pictures and they are asked to complete, extend, or match the patterns shown in the pictures. The task consists of 10 items of increasing difficulty. Children attempted all 10 items. Scoring was the total number of correct patterns with possible scores ranging from 0 to 10. The internal reliability, based on the accuracy of individual trials, was .80.

Mathematics and Numeracy Outcomes

Applied Problem Solving. Applied problem solving was measured using the Applied Problems subtest of WJ III Tests of Achievement (Muñoz-Sandoval et al., 2005). Math problems are read aloud to the children, each accompanied by a picture. Children provide a verbal response to the problems. This task measures numeracy knowledge, problem solving, and quantitative reasoning. For example, children may be asked, “Show me four fingers”, “How many bananas are there?” or “If there are 10 bananas in a bag and 4 bananas are eaten, how many bananas would be left in the bag?” The starting point depends on the grade of child, with items increasing in difficulty as the task progresses. In the present study, all children started at the same point. Testing is discontinued after six consecutive errors or failures to respond. Scoring was the total number of correct responses. Cronbach’s α based on the items where 75% of children attempted to respond were .66 and .69 in Kindergarten and Grade 1, respectively.

Arithmetic Fluency. In this pencil-and-paper subtest of the WJ-III Tests of Achievement (Muñoz-Sandoval et al., 2005) children are given three minutes to complete addition, subtraction, and multiplication problems with operands ranging from 0 to 10. There are two pages in total, with 80 items per page. Items are presented in rows of 10, with the first six rows consisting of a mixture of addition and subtraction problems. Multiplication problems are not introduced until the seventh row and thus no children in the present study attempted any multiplication problems. Scoring is the total number of correct calculations solved within three minutes, summing across the two pages, with possible scores ranging from 0 to 160. Children in Kindergarten found this task very difficult, with 70% of children scoring 3 or less. Thus, arithmetic fluency in Kindergarten was excluded from analyses. Cronbach's α, based on the items where 75% of children responded, was .77 in Grade 1.

Number Ordering. In this iPad task ("Number Ordering" app) children are presented with three numbers on the screen. They must tap the numbers in ascending order as quickly and accurately as possible. Children have a maximum of seven seconds to make a response before the task automatically advances to the next trial. Children complete three practice trials and feedback is provided as needed to ensure they understand the task. There are 24 experimental trials in total with digits ranging from 1 to 9. Of the 24 experimental trials, 12 trials are ordered sequences (e.g., 1 3 7) and 12 trials are unordered sequences (e.g., 3 1 7). Scoring was calculated based on the adjusted RT using the LISAS method described above. The internal reliability based on RT for correct trials was .78 in Grade 1.

Results

Analysis Plan

We used a Bayesian approach to analyze the data. There are several advantages of Bayesian approaches: Researchers can quantify evidence on a continuous scale, make claims about the likelihood of both the null and alternative hypothesis given the evidence, and integrate testing with estimation (Faulkenberry et al., 2020; Jarosz & Wiley, 2014). The Bayes factor, BF_{01}, is "a ratio that contrasts the likelihood of the data fitting under the null hypothesis with the likelihood of fitting under the alternative hypothesis" (Jarosz & Wiley, 2014, p. 3). For example, a Bayes factor of 4.0 indicates that the data are four times more likely to occur under the null hypothesis than the alternative hypothesis. Taking the inverse, BF_{10}, puts the Bayes factor in terms of the alternative hypothesis (e.g., $BF_{01} = 4.0$, $BF_{10} = 1/4.0 = 0.25$). The interpretation of the strength of the evidence for the null or alternative hypothesis is in accordance with Jeffreys' (1961) guidelines: Bayes factors between 1-3 suggest *anecdotal* evidence, 3-10 suggest *substantial* evidence, 10-30 suggest *strong* evidence, 30-100 suggest *very strong* evidence, and greater than 100 suggest *decisive* evidence for the hypothesis (see Table 4 of Jarosz & Wiley, 2014). All Bayesian analyses were conducted in JASP (JASP Team, 2021).

Descriptive Statistics

Testing occurred across two sessions and thus occasionally a child missed a session or a task could not be completed. Two scores were removed for spatial span due to experimenter error. Moreover, one extreme outlier (i.e., z-scores > $|3.29|$) was observed for number ordering (z-score = 4.59). The descriptive statistics after removing the outlier and the number of children who completed each task are shown in Table 1. The mean, standard deviation, and skew values did not show any evidence of ceiling or floor effects (i.e., all skew values were < $|1.00|$). Thus, there was sufficient variability to proceed with further analyses.

Correlations

Correlations among the measures are shown in Table 2. The correlations of children's age and gender with the cognitive precursors and mathematics and numeracy outcomes were more likely under the null than the alternative hypothesis. In contrast, except for number ordering and spatial span, there was substantial to decisive evidence in favour of true non-zero correlations between mother's education and the cognitive precursors and mathematics and numeracy outcomes. Thus, only mother's education was controlled in the subsequent analyses.

Table 1

Descriptive Statistics for all Measures

Measures	*N*	*Min*	*Max*	*Mean*	*SD*	*Skew*
Kindergarten						
Spatial Span[a]	95	0.00	13.00	5.63	3.03	0.16
Receptive Vocabulary[a]	98	44.00	85.00	62.23	10.28	0.13
Number Comparison[b]	96	1.11	3.51	1.99	0.53	0.82
Patterning[a]	98	0.00	10.00	6.13	2.83	-0.37
Applied Problems[a]	98	8.00	27.00	17.64	4.13	0.04
Grade 1						
Applied Problems[a]	93	12.00	32.00	23.57	3.56	-0.98
Arithmetic Fluency[a]	93	1.00	39.00	14.73	8.08	0.39
Number Ordering[b]	92	2.13	6.31	4.11	0.92	0.44

Note. [a]Total correct; [b]Adjusted RT (in seconds)

There was strong to decisive evidence in favour of true non-zero correlations among the mathematics and numeracy outcomes. With respect to the pathways, there was strong to decisive evidence that linguistic and patterning skills were moderately correlated with all numeracy and mathematics outcomes. In contrast, the strength of the correlations varied for the quantitative and spatial pathways, depending on the task (see Table 2). Overall, these patterns support the fundamental assumptions of the Pathways to Mathematics model that linguistic, attentional, and quantitative skills measured in Kindergarten are related to mathematical skills in primary school (LeFevre et al., 2010) and the research showing that patterning measured in Kindergarten is correlated with mathematical skills and other cognitive predictors (Zippert et al., 2020).

Table 2

Correlations Among Measures in Kindergarten (K) and Grade 1 (G1)

Measure	1	2	3	4	5	6	7	8	9	10
1. Age	-									
2. Gender[a]	-.12	-								
	0.26									
3. Mother's Education	-.02	-.08	-							
	0.13	0.17								
4. Spatial Span[b] K	-.03	-.07	.15	-						
	0.13	0.16	0.37							
5. Vocabulary[b] K	.05	-.10	.37	.32	-					
	0.14	0.20	**135.87**	**15.77**						
6. Number Comparison[c] K	.09	-.14	-.40	-.18	-.24	-				
	0.19	0.31	**429.48**	0.53	**1.86**					
7. Patterning[b] K	.19	-.08	.29	.32	.45	-.23	-			
	0.74	0.17	**8.03**	**16.50**	**6315.20**	**1.59**				
8. Applied Problems[b] K	.20	.05	.31	.21	.40	-.21	.38	-		
	0.81	0.14	**13.63**	0.94	**479.36**	0.92	**159.80**			
9. Applied Problems[b] G1	.03	.12	.38	.30	.37	-.27	.45	.60	-	
	0.14	0.24	**161.19**	**6.61**	**95.90**	**3.87**	**2491.14**	**4.49e+7**		
10. Arithmetic Fluency[b] G1	.17	.19	.27	.37	.46	-.36	.53	.54	.61	
	0.47	0.61	**3.78**	**61.69**	**5192.25**	**46.18**	**325240.6**	**608015.1**	**1.50e+8**	
11. Number Ordering[c] G1	.02	.03	-.19	-.13	-.34	.39	-.32	-.32	-.26	-.45
	0.13	0.14	0.68	0.28	**32.55**	**178.37**	**12.86**	**14.16**	**2.71**	**2465.79**

Note. [a]Binary scores: for gender "1" = boys and "0" = girls; [b]Total correct; [c]Adjusted RT. Estimated Bayes Factors (BF_{10}) are presented underneath the correlations. $BF_{10} > 1$ are bolded, indicating stronger support for the alternative than the null hypothesis.

Regression Analyses

We conducted Bayesian regressions that included as covariates: the linguistic pathway (receptive vocabulary), the attentional pathway (spatial span), the quantitative pathway (number comparison), and the patterning pathway (non-numeric repeating patterning). In JASP, linear regression procedures do not allow for missing data, thus, listwise deletion was used in all subsequent analyses. There were two goals with these analyses. First, we wanted to determine if patterning explained additional variability above and beyond the original pathways. To accomplish this goal, we specified a null model that included the three original pathways (i.e., linguistic, attentional, and quantitative) and mother's education as covariates. Then, we compared this null model, which we refer to as our *baseline* model, to a model that contained the original pathways, mother's education, and patterning as covariates. We report the R^2 values for both models as well as the Bayes factor for the model that includes patterning, compared to the baseline model.

Second, we wanted to determine which of the covariates would best predict children's mathematics and numeracy outcomes, also considering mother's education as a covariate. Because we had five covariates, we tested and compared 32 models (2^5), formed by considering all possible submodels of our five covariates. For each outcome, the best (i.e., most probable) model is determined and the other 31 models are compared to the best model using the Bayes factor, BF_{01}. Unlike conventional frequentist testing, Bayesian analyses provide us with the advantage of stating positive evidence for a model without a covariate (i.e., the null case). To assess the predictive adequacy for each of the 32 models, we used a multivariate Cauchy distribution (see Rouder & Morey, 2012) as our default prior. We further specified that all prior model probabilities should be uniform. Thus, the prior probability for each model, P(M), is .031

(1/32).

For each regression analysis we provide two summary tables. In the first table, we list the five most probable models in decreasing order of posterior model probability (i.e., the best or most probable model is listed first). As stated above, for all models, the prior probability, P(M), is .031. For each of these models, we report: i) the posterior model probabilities, that is, $P(M|data)$; ii) the factor by which the prior odds for a given model have been updated to produce the posterior odds for that model, that is, BF_M; iii) the Bayes factor for a given model compared to the best (i.e., most probable) model, that is, BF_{01}; and iv) the R^2 value. The information in this table allows us to determine which covariates predict each mathematics outcome.

In a second table, we report model-averaged posterior summaries for each of the regression coefficients: i) the estimates of mean and standard deviation for each covariate; ii) the posterior inclusion probabilities, that is, $P(incl|data)$; iii) the inclusion Bayes factor, that is, BF_{Inc} (i.e., the factor by which the prior odds for including a covariate are increased after we have observed the data); and iv) the 95% credible interval. Notably, the prior probability of including each covariate (i.e., mother's education, receptive vocabulary, spatial span, patterning, and number comparison) in the 32 models is .50 (each covariate appears in exactly 16 of the 32 models). The information in this table allows us to determine the size of the effect of each covariate.

Applied Problem Solving

We first compared our baseline model (i.e., the original three pathways and mother's education) to a model that also included patterning. The baseline model explained 27% of the variance in applied problem solving in Grade 1. When patterning was added to the model, there was very strong evidence that this model was better than the baseline model ($BF_{10} = 40.05$). The model that included patterning explained 36% of the variance in applied problem solving. Thus, the inclusion of a patterning pathway improved the Pathways to Mathematics model.

Next, we determined the best (i.e., most probable) model, this time only including the intercept in the null model. As shown in Table 3, the most probable model includes mother's education and patterning skills as predictors of applied problem solving in Grade 1. Notably, however, there was only anecdotal evidence to suggest that this model, M_1, was better than M_2 and M_3 ($1 < BF_{01} < 3$). There was substantial evidence ($BF_{01} > 3$) to suggest that M_1 was better than M_4 and M_5. Overall, the data have increased model odds for M_1 by a factor of 15.23. Collectively, mother's education and patterning skills explain 33% of the variance in applied problem solving. Moreover, all five models listed in Table 3 contain patterning as a predictor of applied problem solving and we have increased our belief ($BF_M > 1$) in all five of these models.

Table 3

The Five Most Probable Models for Applied Problem Solving in Grade 1 (n = 88)

Models		P(M|data)	BF_M	BF_{01}	R^2
M_1:	Mother's Education + Patterning Skills	.329	15.23	1.00	.33
M_2:	Mother's Education + Patterning Skills + Linguistic Skills	.179	6.76	1.84	.35
M_3:	Mother's Education + Patterning Skills + Attentional Skills	.133	4.74	2.49	.34
M_4:	Mother's Education + Patterning Skills + Linguistic Skills + Attentional Skills	.060	1.98	5.49	.36
M_5:	Mother's Education + Patterning Skills + Quantitative Skills	.060	1.97	5.52	.30

In Table 4, the odds for including mother's education and patterning skills are increased after observing the data (P(incl|data) > 0.50; $BF_{Inc} > 1$) whereas the odds for including linguistic, attentional, and quantitative skills are decreased (P(incl|data) < 0.50; $BF_{Inc} < 1$). The odds for including patterning skills as a predictor of applied problem solving have increased by a factor of 154.09. In sum, based on the information presented in Tables 3 and 4, there is decisive evidence to suggest that patterning skills predict applied problem solving.

Table 4

Inclusion Probabilities for Covariates in the Applied Problem Solving Regression

Coefficient	*M* (SD)	P(incl|data)	BF_{Inc}	95% CI
Intercept	23.48 (0.32)	1.00	1.00	[22.82, 24.02]
Patterning Skills	0.50 (0.14)	.99	154.09	[0.20, 0.74]
Mother's Education	0.26 (0.16)	.84	5.21	[0.00, 0.47]
Linguistic Skills	0.22 (0.04)	.39	0.64	[-0.0001, 0.10]
Attentional Skills	0.04 (0.08)	.28	0.39	[-0.01, 0.24]
Quantitative Skills	0.00 (0.28)	.16	0.19	[-0.58, 0.53]

Applied problem solving was measured in both Kindergarten and Grade 1. Thus, to evaluate the change in applied problem solving from Kindergarten to Grade 1, we conducted additional regressions in which we also included applied problem solving in Kindergarten as a covariate in the baseline model. The baseline model explained 46% of the variance in applied problem solving in Grade 1. When we compared the baseline model to the model that also included patterning, there was substantial evidence that the baseline model was better (BF_{01} = 4.16). With the inclusion of patterning skills, the model explained 46% of the variance in applied problem solving. Thus, the inclusion of a patterning pathway in the Pathways to Mathematics model did not explain additional variance in the change in applied problem solving from Kindergarten to Grade 1.

Next, we determined the best (i.e., most probable) model, this time only including the intercept and applied problem solving in the null model. As shown in Table 5, the most probable model of applied problem solving in Grade 1 includes mother's education and patterning skills. There was anecdotal evidence to suggest that the most probable model, M_1, was better than M_2, M_3, and M_4 but substantial evidence to suggest that M_1 was better than M_5. Overall, the data have increased model odds for M_1 by a factor of 9.86. Collectively, mother's education, patterning skills, and applied problem solving in Kindergarten explain 45% of the variance in applied problem solving in Grade 1. Moreover, all five models listed in Table 5 contain patterning as a predictor of applied problem solving and we have increased our belief in all five of these models.

Table 5

The Five Most Probable Models for Applied Problem Solving in Grade 1, Controlling for Applied Problem Solving in Kindergarten (n = 88)

Models		P(M|data)	BF_M	BF_{01}	R^2
M_1:	Mother's Education + Patterning Skills	.241	9.86	1.00	.45
M_2:	Patterning Skills	.215	8.49	1.12	.42
M_3:	Mother's Education + Attentional Skills + Patterning Skills	.090	3.06	2.68	.46
M_4:	Attentional Skills + Patterning Skills	.082	2.77	2.95	.43
M_5:	Linguistic Skills + Patterning Skills	.059	1.95	4.08	.43

In Table 6, the odds for including mother's education and patterning skills are increased after observing the data whereas the odds for including linguistic, attentional, and quantitative skills are decreased. The odds for including patterning skills as a predictor have increased by a factor of 12.79. Thus, although patterning skills did not account for additional unique variance after the original pathways were considered, patterning alone accounted for a similar proportion of variance to all three of the original pathways combined. In sum, based on the information presented in Tables 5 and 6, there is decisive evidence to suggest that patterning skills predict the change in applied problem solving from Kindergarten to Grade 1.

Table 6

Inclusion Probabilities for Covariates in the Applied Problem Solving Regression, Controlling for Applied Problem Solving in Kindergarten

Coefficient	*M* (SD)	P(incl|data)	BF_{Inc}	95% CI
Intercept	23.48 (0.29)	1.00	1.00	[22.84, 23.99]
Applied PS Kindergarten	0.36 (0.09)	1.00	1.00	[0.20, 0.53]
Patterning Skills	0.34 (0.15)	.93	12.78	[0.00, 0.58]
Mother's Education	0.11 (0.13)	.53	1.12	[0.00, 0.38]
Attentional Skills	0.04 (0.08)	.29	0.41	[-0.02, 0.25]
Linguistic Skills	0.01 (0.02)	.21	0.26	[-0.01, 0.07]
Quantitative Skills	-0.02 (0.26)	.16	0.19	[-0.99, 0.35]

Arithmetic Fluency

We first compared our specified baseline model to a model that also included patterning. The baseline model explained 33% of the variance in arithmetic fluency. When patterning was added to the model, there was decisive evidence that this model was better than the baseline model (BF_{10} = 112.38). With the inclusion of patterning, the model explained 43% of the variance in arithmetic fluency. Thus, the inclusion of a patterning pathway improved the Pathways to Mathematics model.

Next, we determined the best (i.e., most probable) model, this time only including the intercept in the null model. In Table 7, the most probable model includes linguistic, attentional, and patterning skills as predictors of arithmetic fluency in Grade 1. There was anecdotal evidence to suggest that M_1 was better than M_2 and substantial evidence to suggest that M_1 was better than M_3, M_4, and M_5. Overall, the data have increased model odds for M_1 by a factor of 14.20. Collectively, linguistic, attentional, and patterning skills explain 42% of the variance in arithmetic fluency in Grade 1. Moreover, all five models listed in Table 7 contain patterning skills as a predictor of arithmetic fluency and we have increased our belief in all five of these models.

Table 7

The Five Most Probable Models for Arithmetic Fluency in Grade 1 (n = 88)

Models		P(M|data)	BF_M	BF_{01}	R^2
M_1:	Linguistic Skills + Attentional Skills + Patterning Skills	.314	14.20	1.00	.42
M_2:	Linguistic Skills + Patterning Skills	.227	9.11	1.38	.39
M_3:	Linguistic Skills + Attentional Skills + Patterning Skills+ Quantitative Skills	.100	3.46	3.13	.43
M_4:	Linguistic Skills + Patterning Skills + Quantitative Skills	.075	2.51	4.20	.40
M_5:	Mother's Education + Linguistic Skills + Attentional Skills + Patterning Skills	.068	2.25	4.64	.42

In Table 8, the odds for including patterning, linguistic, and attentional skills are increased after observing the data whereas the odds for including mother's education and quantitative skills are decreased. The odds for including patterning skills as a predictor of arithmetic fluency have increased by a factor of 382.50. In sum, based on the information presented in Tables 7 and 8, there is decisive evidence to suggest that patterning skills predict arithmetic fluency.

Table 8

Inclusion Probabilities for Covariates in Arithmetic Fluency Regression

Coefficient	*M* (SD)	P(incl|data)	BF_{Inc}	95% CI
Intercept	14.57 (0.64)	1.00	1.00	[13.48, 15.93]
Patterning Skills	1.11 (0.29)	1.00	382.50	[0.66, 1.73]
Linguistic Skills	0.17 (0.09)	.87	6.49	[0.00, 0.31]
Attentional Skills	0.29 (0.29)	.61	1.57	[-0.00, 0.82]
Quantitative Skills	-0.37 (0.92)	.25	0.34	[-3.23, 0.07]
Mother's Education	0.03 (0.12)	.18	0.22	[-0.09, 0.43]

Number Ordering

We first compared our specified baseline model to a model that also included patterning. The baseline model explained 20% of the variance in number ordering. When patterning was added to the model, there was only anecdotal evidence that the model that included patterning was better (BF_{10} = 1.66). With the inclusion of patterning, the model explained 22% of the variance in number ordering. Thus, the inclusion of a patterning pathway did not substantially improve the Pathways to Mathematics model.

Next, we determined the best (i.e., most probable) model, this time only including the intercept in the null model. In Table 9, the most probable model includes linguistic and quantitative skills as predictors of number ordering in Grade 1. However, there was only anecdotal evidence to suggest that M_1 was better than M_2 and M_3, both of which include patterning skills. There was substantial evidence to suggest that M_1 was better than M_4 and M_5. Overall, the data have increased model odds for M_1 by a factor of 12.00. Collectively, linguistic and quantitative skills explain 20% of the variance in number ordering in Grade 1. With respect to patterning skills, 3 of the 5 most probable models contained patterning skills as a predictor of number ordering and we have increased our belief ($BF_M > 1$) in all three of these models. However, Table 9 shows that when patterning skills are included as a predictor above and beyond linguistic and quantitative skills, the model only explains an additional 2% of the variance.

Table 9

The Five Most Probable Models for Number Ordering in Grade 1 (n = 87)

Models		P(M|data)	BF_M	BF_{01}	R^2
M_1:	Linguistic Skills + Quantitative Skills	.279	12.00	1.00	.20
M_2:	Linguistic Skills + Patterning Skills + Quantitative Skills	.127	4.52	2.19	.22
M_3:	Patterning Skills + Quantitative Skills	.114	3.99	2.45	.18
M_4:	Quantitative Skills	.093	3.16	3.01	.14
M_5:	Mother’s Education + Linguistic Skills + Patterning Skills + Quantitative Skills	.057	1.88	4.89	.20

In Table 10, the odds for including linguistic and quantitative skills are increased after observing the data whereas the odds for including mother's education, attentional skills, and patterning skills are decreased. The odds for including patterning skills as a predictor of number ordering have decreased by a factor of 1.39 (1/0.72). In sum, based on the information presented in Tables 9 and 10, there is anecdotal evidence to suggest that patterning skills should be excluded as a predictor of number ordering.

Table 10

Inclusion Probabilities for Covariates in Number Ordering Regression

Coefficient	*M* (SD)	P(incl|data)	BF_{Inc}	95% CI
Intercept	4.11 (0.09)	1.00	1.00	[3.92, 4.31]
Quantitative Skills	0.46 (0.23)	.90	9.10	[0.00, 0.82]
Linguistic Skills	-0.01 (0.01)	.68	2.11	[-0.04, 0.001]
Patterning Skills	-0.03 (0.04)	.42	0.72	[-0.12, 0.00]
Mother's Education	0.00 (0.02)	.18	0.22	[-0.02, 0.05]
Attentional Skills	-0.00 (0.01)	.18	0.22	[-0.04, 0.03]

Discussion

The goal of the present research was to test an expanded version of the Pathways to Mathematics model (LeFevre et al., 2010) in which patterning was included as an additional precursor. The original model included three cognitive precursors –attentional, linguistic, and quantitative – which predicted children's concurrent mathematical skills in preschool or Kindergarten and their more advanced skills two years later (see also Sowinski et al., 2015). In the present paper, we added a fourth cognitive pathway, patterning, that was assessed with a non-numeric repeating patterning task. In previous studies, patterning skills were related to both concurrent and later numeracy skills and mathematics achievement for 4- to 6-year-old children (Burgoyne et al., 2019; Fyfe et al., 2019; Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2019; Zippert et al., 2020). We propose that the extended Pathways to Mathematics model provides a more complete picture of the cognitive precursors that are related to early mathematical development.

The Original Pathways

Was there evidence for the three original pathways in the present research? There were medium to large correlations between the three original pathways and the mathematics outcomes (i.e., applied problem solving and arithmetic fluency) (Funder & Ozer, 2019). There were large correlations between the quantitative and linguistic pathways and the numeracy outcome, whereas there was a weak correlation between the attentional pathway and number ordering. The original pathways along with mother's education predicted 27%, 33%, and 20% of the variance in applied problem solving, arithmetic fluency, and number ordering in Grade 1, respectively. These are similar results to those in the original Pathways to Mathematics study, where the model accounted for between 26% and 56% of the variance across a range of numeracy and mathematical outcomes (LeFevre et al., 2010). Below we elaborate on each pathway.

The Attentional Pathway

We found evidence for the inclusion of spatial attention as a predictor of arithmetic fluency. Consistent with this idea, Sowinski et al. (2015) and Xu et al. (2021) found that the attentional pathway predicted unique variance in arithmetic. Beyond arithmetic, previous studies have found that spatial working memory is an important predictor of mathematics, in general, when children are in primary school but becomes less important as children get older (Caviola et al., 2020; Van de Wijer-Bergsam et al., 2015). In the present study, although the children were in primary school, there was no evidence for the inclusion of spatial attention as a predictor of number ordering or applied problem solving. Notably, spatial span does not capture verbal working memory or other executive functions that may be important for mathematical development (Peng, Namkung, Barnes et al., 2016). Thus, additional tasks, such as measures of verbal working memory or inhibition, may have strengthened the contribution of this pathway.

The Linguistic Pathway

The Pathways to Mathematics model posits that vocabulary skills are required for mathematical tasks that involve number system knowledge (e.g., number names, number structure) and/or articulatory processes (e.g., verbally presented word problems, speaking the answer to problems; see also Zhang & Lin, 2015; Xu et al., 2021). As in LeFevre et al. (2010), the linguistic pathway was a consistent predictor of mathematics in the present study. Linguistic skills were moderately correlated with all numeracy and mathematics outcomes and included as a predictor in the best model for all outcomes except for the change in applied problem solving from Kindergarten to Grade 1. These results support the view that linguistic skills, specifically vocabulary, are closely linked to mathematical skills (Lin & Peng, 2021; Peng et al., 2020). Moreover, these results are consistent with the findings from previous studies of the Pathways to Mathematics model in which linguistic skills predicted symbolic numeracy skills (LeFevre et al., 2010), arithmetic skills (Sowinski et al., 2015; Träff et al., 2018) and word problem-solving (Xu et al., 2021).

The Quantitative Pathway

With the exception of applied problem solving in Kindergarten, quantitative skills were correlated with all numeracy and mathematical outcomes. Quantitative skills were a predictor in the best model for number ordering and there was only anecdotal evidence for excluding these skills as a predictor of arithmetic fluency. However, there was substantial evidence to exclude quantitative skills as a predictor of applied problem solving in Grade 1. Thus, the quantitative pathway may be most strongly connected to tasks that involve efficient access to symbolic written numbers, as is required for number ordering and arithmetic fluency. Consistent with these findings, in most of the research showing correlations between number comparison and math performance for Kindergarten and Grade 1 children, written arithmetic was the outcome measure (Schneider et al., 2017). Moreover, previous studies of the Pathways to Mathematics model have found that quantitative skills predict arithmetic (LeFevre et al., 2010; Sowinski et al., 2015; Xu et al., 2021).

In summary, the findings were consistent with the hypotheses of the Pathways to Mathematics model. The three original pathways contributed independently to early numeracy performance but varied in their unique and relative contributions to later mathematical performance, dependent on the demands of the tasks.

The Patterning Pathway

Should patterning be included in an expanded Pathways to Mathematics model? Yes. Patterning was included in the best model for both applied problem solving and arithmetic, explaining an additional 9% and 10% beyond the baseline model (i.e., original pathways and mother’s education), respectively. Other studies have found that patterning is related to a range of mathematical skills (e.g., Burgoyne et al., 2019; Fyfe et al., 2017; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2020). Like the other pathways, the patterning pathway was correlated with all outcomes but was not uniformly related to all mathematics and numeracy outcomes. Below we discuss the findings for the patterning pathway for each outcome.

Applied Problem Solving

Patterning skills predicted applied problem solving in Grade 1 above and beyond the original Pathways to Mathematics model. With respect to the change in applied problem solving from Kindergarten to Grade 1, patterning did not improve the baseline model. However, it was included as a predictor in the best model. Moreover, the best model, which included mother's education, applied problem solving in Kindergarten, and patterning, explained the same amount of variance in applied problem solving in Grade 1 as the baseline model. These findings are consistent with the findings of Zippert et al. (2020) who found that patterning measured in preschool significantly predicted broad mathematics one year later, above and beyond working memory. Patterning presumably measures children's ability to detect and apply information about relations among quantities or symbols, skills that are fundamental for mathematical thinking (Charles, 2005; Steen, 1988). These relational reasoning skills are crucial for solving applied problems in which children must make conceptual connections between real-world situations and the appropriate mathematical operations (Clement, 1982).

Arithmetic Fluency

Patterning skills predicted applied arithmetic fluency in Grade 1 above and beyond the original Pathways to Mathematics model. The finding that patterning skills predict arithmetic fluency is consistent with the findings of other research on calculation (Fyfe et al., 2017; MacKay & De Smedt, 2019) and arithmetic skills (Burgoyne et al., 2017, 2019). Fyfe et al. (2017) suggested that calculation skills are supported by children's ability to identify and generalize predictable sequences, both in objects and numbers. Early patterning skills may be important for later calculation skills and arithmetic fluency because these mathematical tasks require reasoning skills to develop procedural and conceptual knowledge (e.g., Kindrat & Osana, 2018).

Number Ordering

Consistent with Zippert et al.'s (2020) theoretical model, patterning was moderately correlated with number ordering in Grade 1. However, patterning did not predict number ordering above and beyond the original pathways nor was it included in the best model for number ordering. Similarly, in examining the relations between patterning and numeracy skills, Zippert et al. (2020) found that patterning predicted children's knowledge of the count sequence to 100, but not knowledge of the successor principle. Moreover, Zippert et al. (2021) found that tutoring in repeating patterning knowledge, with or without training in a specific numeracy skill, did not lead to improvements in either numeracy or general mathematics knowledge.

Although number ordering has not previously been included as an outcome in the Pathways to Mathematics papers or patterning papers, we included this task because it is a foundational numeracy skill that is still developing in Grade 1 (Hutchison et al., 2022; Lyons et al., 2014; Xu & LeFevre, 2021). In our opinion, number ordering is an example of the type of knowledge that Zippert et al. (2020) argued would be supported by patterning skills. For example, Zippert et al. (2019) suggested that "the reason for the link to numeracy [from patterning] may be that patterning skills involve deducing underlying rules in the sequence of objects, and numeracy knowledge also requires deducing underlying rules from examples…" (p. 755).

Repeating patterning likely did not emerge as a probable cognitive precursor of number ordering because the other three pathways captured the same variance as patterning (i.e., as reflected in the simple correlations). Nevertheless, the best model for number ordering only explained 20% of the variance, suggesting that further research on the development of number ordering skills is needed. At this age, children may rely on both familiarity with the count list and paired number comparisons (i.e., their stored knowledge of the links between quantities and symbols) to support speed and accuracy in this task, rather than on their knowledge of numerical patterns (Hutchison et al., 2022; Lyons & Ansari, 2015; Lyons et al., 2014). Order skills continue to change both qualitatively and quantitatively beyond Grade 1: Hutchison et al. (2022) found that children's conceptual understanding of number order beyond the count list improved dramatically between Kindergarten and Grade 1 and Lyons et al. (2014) found that order judgments did not predict arithmetic until Grade 3. In summary, more theoretical and empirical evidence is needed to understand the development of ordering skills and how these skills relate to patterning and other cognitive precursors.

Summary

Like the other pathways, the patterning pathway was correlated with all outcomes but was not uniformly related to all mathematics and numeracy outcomes. In other studies, patterning has been related to a range of mathematical skills (e.g., Burgoyne et al., 2019; Fyfe et al., 2017; Wijns, Torbeyns, Bakker, et al., 2019; Zippert et al., 2020). Thus, the inclusion of patterning in the Pathways to Mathematics model has strong support both from the present analyses and from the literature.

What Role Does Patterning Play in Mathematical Development?

Patterning was often the strongest predictor of numeracy and mathematical outcomes. Presumably, patterning is a strong predictor because of the overlap between patterning and mathematics, with both involving "identifying, extending, and describing predictable sequences in objects and numbers" (Zippert et al., 2020, p. 2). However, despite the strong relation between patterning and mathematics, there is no consensus in the literature about the role of patterning in mathematics. Is patterning a domain-general or domain-specific skill? In support of the domain-general view, Alexander et al. (2016) discussed the importance of relational reasoning, which they defined as the capacity to perceive patterns, to a wide range of academic outcomes (including mathematics). In contrast, in support of the domain-specific view, Miller et al. (2016) found that patterning was part of a four-factor model of informal mathematical ability (i.e., patterning, number and operations, measurement, and geometry). More extensive studies which develop the notion of patterning beyond preschool may help to address the role of patterning in mathematical development.

The Pathways to Mathematics Model separates domain-general from domain-specific pathways. In the original model, the linguistic and attentional/working memory pathways are domain general, that is, the underlying individual differences are important for a range of outcomes, whereas the quantitative pathway is domain specific, that is, the individual differences are primarily important for mathematical tasks that involve written number symbols. To address whether patterning is a domain-general or domain-specific skill, research is needed that considers a broader range of mathematics and non-mathematics outcomes (e.g., reading). Nonetheless, in the present study there was substantial evidence to suggest that patterning is a cognitive correlate of numeracy and mathematics outcomes for children in Grade 1. Thus, regardless of whether patterning is its own independent domain-specific pathway or whether it is part of a broader domain-general reasoning pathway, patterning improved the predictive validity of the Pathways to Mathematics model.

Limitations and Future Research

The present study has some limitations. Cognitive skills, including patterning, were only assessed in Kindergarten with a single measure. Assessing cognitive skills with multiple measures and at multiple time points would provide information about the possible reciprocal relations between changes in cognitive skills and growth in numeracy and mathematics outcomes (Peng & Kievet, 2020). Moreover, by including additional measures of patterning, linguistic, attentional, and quantitative skills, a more robust model could be tested (Sowinski et al., 2015; Zhang & Lin, 2015).

Another limitation of this study is that some numeracy and mathematics outcomes were either only measured in Grade 1 (e.g., number ordering) because these skills are too advanced for Kindergarten children (Hutchison et al., 2022) or could only be analyzed in Grade 1 (i.e., arithmetic fluency) due to floor effects in Kindergarten. Thus, we could only examine the change in performance for applied problem solving. In the future, following children with more sensitive math measures and across more time points would allow for a better understanding of how early cognitive precursors relate to later mathematics achievement.

Implications and Conclusion

The goal of the present research was to determine whether patterning should be included as a core cognitive precursor in the Pathways to Mathematics model. Patterning may predict mathematics because both skills involve learning about rules and associations. For mathematics, these rules and associations are first identified among numbers and then among more complex concepts. Supporting this view, patterning was correlated with numeration skills (i.e., number ordering) and more complex mathematics (i.e., problem solving and arithmetic). Moreover, beyond the original pathways, patterning predicted applied problem solving and arithmetic fluency, although there was no strong evidence for the inclusion of patterning as a predictor of number ordering skills.

Overall, this study expanded research on the Pathways to Mathematics model and on patterning to children educated in South America. Consistent with research in North America and Europe, we found support for the Pathways to Mathematics model and for the relations between patterning and later mathematics outcomes. Thus, there is evidence to support the generalizability of the expanded version of the Pathways to Mathematics model: Modifying the model to include patterning as an additional cognitive precursor may provide greater insights into individual differences in performance across mathematical tasks.

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Appendix A

Items from the adapted version of the Teacher-Based Patterning Task (Rittle-Johnson et al., 2019) in order of administration.

Original Item Type	**Adapted Item Type**	**Adapted Item**	**Percentage Correct**
What's next AB	What's next ABB		56.1
What's next ABC	What's next ABBC		64.3
Missing AB	Missing ABBC		52.0
Missing ABC	Missing ABC		66.3
Missing ABB	Missing ABCB		44.9
Extend AB	Extend AAB		62.2
Extend AABB	Extend AABB		77.6
Extend ABC	Extend ABBC		38.8
Match AB	Match ABB		77.6
Match ABBB	Match AABC		73.5