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Pathways to Learning Mathematics for Students in French-Immersion and English-Instruction Programs

Chang Xu[1]

Sabrina Di Lonardo Burr[2]

Sheri-Lynn Skwarchuk[3]

Heather Douglas[2]

Anne Lafay[4]

Helena P. Osana[4]

Victoria Simms[6]

Judith Wylie[5]

Erin A. Maloney[7]

Jo-Anne LeFevre[1,2]

[1] Department of Psychology, Carleton University

[2] Department of Cognitive Science, Carleton University

[3] Faculty of Education, University of Winnipeg

[4] Department of Education, Concordia University

[5] School of Psychology, Queen's University Belfast

[6] School of Psychology, Ulster University

[7] School of Psychology, University of Ottawa

Corresponding Author: Chang Xu

Email Address: ChangXu@cmail.carleton.ca

Department of Psychology

Carleton University

1125 Colonel By Drive, Ottawa, K1S 5B6

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Pathways to Learning Mathematics for Students in French-Immersion and English-Instruction Programs

Abstract

Canadian students enrolled in either French-immersion or English-instruction programs were followed from grades 2 to 3 (M_{age} = 7.8 years to 8.9 years; N = 244; 55% girls). In each grade, students completed two mathematical tasks that required oral language processing (i.e., word-problem solving and number transcoding from dictation) and two that did not (i.e., arithmetic fluency and number line estimation). Students in both English-instruction (n = 92) and French-immersion programs (n = 152) completed tasks in English. Students in French-immersion programs also completed word-problem solving and transcoding tasks in French. The models were framed within the Pathways to Mathematics model, with a focus on the linguistic pathways for students in English-instruction and French-immersion programs. For tasks with oral language processing, performance in grade 3 was predicted by students' English receptive vocabulary for both English-instruction and French-immersion students, even when French-immersion students were tested in French, controlling for performance in grade 2. In contrast, for tasks without oral language processing, receptive vocabulary in either English or French did not predict performance in grade 3, controlling for performance in grade 2. These results have implications for teaching mathematics within the context of immersion education.

Word count: 193

Key Words: immersion, first language, additional language, mathematics, Pathways to Mathematics model

Educational Impact and Implications Statement [30-70 words]

English-speaking Canadian students (grades 2 and 3) who attended either English-instruction or French-immersion programs had similar mathematics performance, suggesting that learning mathematics in a second language does not impede students' mathematical learning. Regardless of the language of instruction, students' proficiency in English, not French, was the dominant predictor of their progress in mathematical tasks that involved oral language.

Across the world, students are taught mathematics in many different languages. Moreover, sometimes the instructional language for mathematics is different from students' home language, such as when children immigrate to a new country or when students participate in language immersion programs (de Araujo et al., 2018). Much of the literature on the role of language for students who learn mathematics in an additional language has been focused on students in minority or immigrant language contexts, where they are learning the majority language at school but speaking another language at home (de Araujo et al., 2018). Immersion programs, in contrast, are chosen by parents to enhance students' linguistic and cultural knowledge and to provide the opportunity for them to become bilingual (Baig, 2001). In these cases, the language of instruction is not typically the majority language of the community. In the current study, we investigated how language proficiency is related to mathematics learning from grade 2 to grade 3 for students participating in either English-instruction or French-immersion programs in Canada. These grades are of particular interest because, although mathematics lessons are conducted in French in immersion programs, students are still developing their French language skills. Children's mathematics knowledge is related to future employment prospects and socio-economic status (Ansari, 2015; Bynner & Wasik, 2009), and thus there are practical, social, and economic reasons to study how mathematics skills are acquired in classrooms where students learn mathematics in an additional language.

Decades of research has shown that the mathematics achievement of students enrolled in Canadian French-immersion programs is equivalent to that of peers instructed in English in Canada (Barik & Swain, 1976; Lambert, 1974; Lindholm-Leary & Genesee, 2014; Turnbull et al., 2001). Similar results have been found in other countries such as the United States for students in Spanish immersion (Steele et al., 2017; Watzinger-Tharp et al., 2018), in China for students in English immersion (Cheng, 2012), and in Germany for students in English immersion (Fleckenstein et al., 2019). In contrast, Irish students who were learning mathematics in Irish as an additional language outperformed their English-learning peers (Murray, 2010). Many factors may influence whether the language of instruction is related to students' mathematics learning. From a cognitive perspective, becoming multilingual may enhance students' cognitive skills, particularly inhibitory processes, and thus potentially enhance or support their learning in general (Bialystok, 2011; Genesee, 2015). From a sociocultural perspective, discrepancies between the languages used at home versus in school may have negative consequences, including, for example, reducing students' participation in classroom activities (de Araujo et al., 2018). In the present research, we focused on cognitive factors that may influence mathematics learning and contrasted students in French-immersion with those in English-instruction programs.

Pathways to Mathematics: Cognitive Correlates of Learning

The present research was framed within the Pathways to Mathematics model (LeFevre et al., 2010). According to the Pathways model, three categories of cognitive skills contribute to early mathematics development: quantitative, linguistic, and attentional/working memory skills (LeFevre et al., 2010; Sowinski et al., 2015). Quantitative skills refer to children's ability to efficiently determine exact quantities and are measured using subitizing (i.e., the ability to rapidly identify a small set of objects), counting, or number comparison (e.g., which is more, 4 or 7?). Students' quantitative knowledge, especially when accessed via written number symbols, is consistently related to mathematics achievement (De Smedt et al., 2009, 2013; Hawes et al., 2019; LeFevre et al., 2010; Revkin et al., 2008; Schneider et al., 2017; Vanbinst et al., 2015; Yun et al., 2011).

Students' attentional and working memory skills are required for controlling, regulating, and actively maintaining relevant information for mathematical tasks (see reviews in Allen et al., 2019; Friso-van den Bos et al., 2013; LeFevre et al., 2005; Raghubar et al., 2010). In accord with Baddeley's model (Baddeley, 2001), working memory includes the phonological loop, the visual-spatial sketchpad, and the central executive. The processes attributed to the central executive include inhibition, shifting, and updating (Raghubar et al., 2010). More generally, working memory is a correlate of mathematics performance and educational achievement across the elementary school years (Alloway et al., 2009; Gathercole et al., 2006; Swanson & Beebe-Frankenberger, 2004).

Of most interest in the present context is the linguistic pathway. LeFevre and colleagues (2010) found that 5- and 6-year-old students' receptive vocabulary and phonological awareness in kindergarten predicted individual differences in performance for a variety of mathematical outcomes two years later (e.g., arithmetic, word-problem solving, geometry, and number line estimation; see also Sowinski et al., 2015). Furthermore, other researchers have found that language skills predict mathematics skills and achievement (e.g., Purpura & Ganley, 2014; Singer & Strasser, 2017; Vukovic & Lesaux, 2013). In a meta-analysis, there was a significant relation between language skills and mathematics performance (r = .42; Peng et al., 2020). The consistent and persistent relations between language and mathematical achievement suggest that knowledge of the instructional language might be a central factor in students' mathematical development.

The Relations Between Language Demands and Mathematics Performance

Beyond the Pathways to Mathematics model, further evidence supporting the relation between language and mathematics comes from additional-language learners whose language of instruction at school is different from the language they use at home (de Araujo et al., 2018). The specific language demands of mathematics problems may interfere with or complicate mathematics learning for additional-language learners (Abedi & Lord, 2001; Authors, in press; de Araujo et al., 2018; Wolf & Leon, 2009). For example, additional-language learners attending English-instruction programs in grades 2 and 3 in the United States experienced challenges when solving word problems that used complex linguistic phrasing such as passive language or included a conditional clause (Banks et al., 2016). Moreover, additional-language learners in grades 4 to 8 in the United States also experienced challenges completing large-scale state mathematics assessments when teachers used more general academic vocabulary (e.g., *based on*, *consequently,* and *substantially*) in classrooms (Wolf & Leon, 2009). In sum, knowledge of the instructional language used for mathematics may influence students' learning and performance.

Even students learning mathematics in their first language rely on general language skills for mathematics tasks that require oral or written language processing. For example, students' receptive vocabulary is related to their performance on mathematics word problems (Fuchs et al., 2015; Harvey & Miller, 2017; LeFevre et al., 2010; Méndez et al., 2019; Swanson et al., 2015). When students are presented with a word problem, they need to encode the words so that they comprehend the meaning of the problem and produce an appropriate strategy to solve it (Daroczy et al., 2015; Fuchs et al., 2015; Mayer, 2004). Thus, multilingual students solving mathematical word problems presented in their most proficient language understand the problem better, use more advanced strategies, and provide more in-depth responses for open-ended questions (Ambrose & Molina, 2014; Bernardo, 2002; Telli et al., 2018). This advantage of solving mathematics problems in their most proficient language has been shown for Yoruba-speaking students in grades 1 to 4 (Adetula, 1989), for Vietnamese-speaking students in grade 4 (Clarkson, 2006), and for Spanish-speaking students in grades 4 and 5 (Domínguez, 2011).

Beyond word problems, other mathematical tasks require students to use oral language knowledge. For example, transcoding Arabic numbers from dictation requires that students listen to a spoken number word and write down the Arabic digits corresponding to that number. Performance on transcoding tasks has been linked to mathematics achievement in grades 1 to 6 (Geary et al., 2000; Moeller et al., 2011; Moura et al., 2013). Successful transcoding depends on students' number-word vocabulary and their ability to generate number words within a syntactic structure (Anglin et al., 1993; McCloskey et al., 1986; Skwarchuk & Anglin, 2002). Students need to learn a small set of basic number words (i.e., the number words for units, teens, decades, and so on) and combine these elements to construct additional number words (Skwarchuk & Anglin, 2002).

Transcoding performance varies with the specific number-word structures for different languages (Imbo et al., 2014; Klein et al., 2013; Seron & Fayol, 1994). For example, English has a base-10 structure (e.g., 80 is eighty in English, which implies eight tens), whereas French has a mixture of base-10 (e.g., 31 is *trente et un* in French, that is three tens and one) and base-20 structures (e.g., 71 is *soixante et onze* in French, that is, six tens plus eleven). Through analyzing the types of transcoding errors students make, researchers have found that a base-20 number structure used in French for numbers between 60 and 100 is structurally more complex for students in grade 2 (i.e., 7-years-old) than a base-10 number system (Authors, under review; Barrouillet et al., 2004; Imbo et al., 2014; Seron & Fayol, 1994). Because of the different structures across languages, students who speak English as their first language may experience challenges when they are learning to transcode in French and rely on translating to English to perform transcoding tasks. Therefore, number-word vocabulary and transcoding abilities in the students' first language (English) may support the development of transcoding in their second language (French).

In comparison to mathematical tasks with linguistic demands, language skills may be less involved in mathematical tasks that are presented as written numerals. Consistent with the *encoding-complex* model (Campbell, 1994), solvers are faster and more accurate when solving arithmetic problems presented in familiar numeric format (e.g., 3 + 4) than in other formats (e.g., verbal or written formats such as three + four; Bernardo, 2001; Campbell et al., 1999; Campbell & Epp, 2004; Salillas & Wicha, 2012). Accordingly, when adults solve simple addition and multiplication problems presented in numeric format, they are likely to rely on direct retrieval of arithmetic facts (Geary et al., 1993; LeFevre et al., 1996). These findings suggest that language may be minimally involved when people are solving simple arithmetic problems presented in numeric format. Furthermore, students' quantitative skills are highly correlated with their arithmetic performance (Schneider et al., 2017). Taken together, students' quantitative skills, rather than their language proficiency, should support the development of simple arithmetic skills.

Similarly, number line estimation is another numeric task that may not include an oral component. In the number-to-position version of the task, students estimate the position of a number on a visual line where only the endpoints are labelled. Performance on the number line task improves as students gain more mathematical knowledge (e.g., Ashcraft & Moore, 2012; Barth & Paladino, 2011; Laski & Siegler, 2007; LeFevre et al., 2013; Muldoon et al., 2013) and is strongly correlated with other mathematical skills for school-aged students above six years of age (Schneider et al., 2018). In contrast, language proficiency is not related to performance on the number line task for school-aged students (Namkung & Fuchs, 2016; Praet & Desoete, 2014). Therefore, students' quantitative skills, rather than their language proficiency, should be involved in the development of number line estimation.

The Current Research

In the present study, we examined how language proficiency in English and French was related to mathematics development for grade 2 and 3 students enrolled in French-immersion programs in Manitoba, Canada. For comparison, we also assessed the language and mathematical skills of students in the same communities who were instructed in English. In most cases, students in both groups spoke English at home. All lived in communities that were predominately English speaking.

In Canada, English and French are official languages of the federal government. In all provinces and territories except Quebec, English is the most spoken language. According to the 2016 Canadian census, 74% versus 23% of Canadians speak English versus French at home (Statistics Canada, 2016). The goal of language immersion programs is to expose students to a language of instruction in school that they may not speak at home. Because educational authority occurs provincially/territorially in Canada, participation rates in French immersion vary by province/territory, ranging from 7-32%, with the national average falling at 11.4%. In Manitoba, the province where the current study was carried out, 14% of students are enrolled in French Immersion programs (Canadian Parents for French, 2019).

In early immersion programs in Manitoba, students receive all of their kindergarten instruction in French and most (i.e., 75-80%) of their elementary instruction (i.e., grades 1 to 6) in French. Importantly, the instructional language of mathematics is French in all grades. Before formal schooling, children have either fully or partially built their fundamental number knowledge at home (e.g., rote counting, identifying numerals; Skwarchuk et al., 2014). After entering formal schooling, students in immersion programs, who likely acquired their fundamental number knowledge in their first language, enter a language community in which they acquire additional mathematical knowledge, some of which is specific to French. One consequence of learning mathematics in school in an additional language is that when students are presented with numerals in a mathematical problem, they may think in terms of their first language (e.g., English) rather than the language of instruction (e.g., French). Notably, although the students in immersion programs in the current study had two or three years of formal education in French, their use of French outside of school was minimal because English is the primary language used in oral communication, reading, writing, and media. In fact, in Manitoba in general, 71% of residents report English as their mother tongue (Statistics Canada, 2016). Thus, students in French-immersion programs are less proficient in French than in their first language (i.e., English).

To compare how mathematical development differs for students enrolled in French-immersion and English-instruction programs, a group of students enrolled in English-instruction programs were recruited from the same communities as a comparison group. Research questions were framed within the Pathways to Mathematics model (LeFevre et al., 2010; Purpura & Ganley, 2014; Sowinski et al., 2015; Träff et al., 2017; Vukovic & Lesaux, 2013). Specifically, although language proficiency was the central focus of this study, quantitative skills and working memory were also evaluated as potential predictors of mathematics performance and learning. We wanted to ensure that any differences in mathematics performance between students in English-instruction and French-immersion programs were not the result of differences in intelligence. Thus, the Matrix Reasoning task from the Weschler Intelligence Scale for Children was administered as a proxy measure for nonverbal intelligence (Wechsler, 2014).

Research Questions

In the present study we investigated how linguistic skills are related to mathematical development for students in English-instruction and French-immersion programs. To examine the relations, we consider two types of mathematical tasks: tasks that require substantial oral language processing and tasks that require minimal oral language processing.

We first examined the linguistic pathways for word-problem solving and number transcoding tasks, both of which were orally presented to students in grades 2 and 3, and thus require oral language processing during encoding. In the present study, French-immersion students completed these tasks twice (in both English and French, at different times). English-instruction students completed these tasks only in English. Across groups, we expected that receptive language skills would be related to mathematics performance for tasks that require oral language processing (Hypothesis 1). For students in both English-instruction and French-immersion, we hypothesized that English receptive vocabulary would predict performance on these mathematical tasks. French vocabulary was not expected to predict additional unique variance in mathematics performance in either English or French because students' access to the meanings of vocabulary words in their second language has been found to be slower and more effortful than access to the meanings in their first language (de Araujo et al., 2018).

We also examined the linguistic pathways for mathematical tasks that were presented in numeric format and required written responses. These tasks had minimal language demands during encoding and response. Thus, we expected that receptive language skills would not be related to performance on arithmetic and number line tasks (Hypothesis 2).

Method

The data analyzed in this paper are part of a larger project, spanning four sites (Ontario, Quebec, Manitoba, and Northern Ireland), on language learning and mathematics achievement.

Only students from Manitoba were included in the present analysis because the home and instructional language experiences were different across sites. Details on the larger project are available on the Open Science Framework (OSF): https://osf.io/428hp/?view_only=ee8dbeb6a7fa4c43ae0b2c06a462326c.

Participants

After obtaining ethics approval from the university and the school division, school principals were contacted. Following principal and teacher approval, letters were sent home inviting students to participate. Students (N = 182; M_{age} = 7.8 years; SD = 0.29; 82 boys) from seven public schools in Manitoba were recruited near the end of the 2018 school year. Three of the schools were immersion milieu schools in which all school activities, including announcements and instruction, are in French. The milieu model is one of many French immersion models that exist across Canada. Four schools were non-immersion schools where, except for French class, all subjects are taught in English.

Students were tested at the end of grade 2 (April and May of 2018) and again at the end of grade 3 (i.e., April, May, and June of 2019). Between grades 2 and 3, 35 students dropped out of the study for personal reasons. In grade 3, 62 new students from the same schools were recruited for the study. The final sample consisted of 244 students (152 French-immersion), with 147 students participating in both grades 2 and 3.

Parents (N = 241) reported the primary language spoken and the amount of English-language exposure at home. Ninety percent of parents reported that English was their child's first language. Parents of 23 students reported first languages that were not English (i.e., German, Russian, Arabic, Serbian, Korean, Bosnian, Chinese, French, Yoruba, Gujarati, Punjabi, and Polish). The proportion of students whose parents reported English as their first language did not differ between the French-immersion and English-instruction groups, $\chi^2(1, N = 241) = 1.47$, $p = .23$. Additionally, parents reported on the frequency with which they spoke English at home (1 = not at all, 5 = always); 93% of parents reported that students always (5) or often (4) spoke English at home, and 7% reported that students sometimes (3) spoke English at home. The distribution of speaking their first language at home did not differ for the French-immersion and English-instruction groups, $\chi^2(4, N = 241) = 4.92$, $p = .43$. No significant differences were found across the language demographic measures for French-immersion versus English-program students (*p*s > .05). Furthermore, analyses comparing the modeling results with and without the students whose home language was not English did not differ. Because the inclusion of students whose first language is not English is more representative of the real classroom environment, all students were retained in the subsequent analyses.

Measures

Cognitive Skills

In grade 2, students in both English-instruction and French-immersion programs completed the nonverbal reasoning measure and measures of the quantitative, linguistic, and attentional/working memory pathways, all in English. French-immersion students also completed a receptive vocabulary measure in French in grade 2.

Quantitative Pathway. The quantitative pathway was assessed with two tasks: number comparison and subitizing (i.e., rapid naming of quantities 1 to 3). Students' knowledge of the magnitude of Arabic numbers was assessed using a number comparison task (Bigger Number App). The task consists of 26 experimental trials, on which two single-digit numbers (1-9) are presented on an iPad. The distance between the two numbers was manipulated such that 13 trials have a small distance ranging from 1 to 3 (e.g., 2-4) and 13 trials have a large distance ranging from 4 to 7 (e.g., 3-8; Bugden & Ansari, 2011). Students were asked to choose the numerically larger number as quickly and accurately as possible. Students were given three seconds to respond, after which the next trial automatically appeared. Scoring was calculated by mean accuracy in percent/mean RT in seconds. The internal reliability (Cronbach's α) based on RT for correct trials was .94 for students in each of the English-instruction and French-immersion programs.

Subitizing efficiency was assessed by asking students to orally name quantities of one, two, or three dots as quickly as possible. For practice, students named a single row of stimuli (six sets of one, two, or three dots) and were provided feedback if they produced an incorrect response. They were encouraged to respond quickly and accurately. After the practice, students completed two pages of stimuli with sets of dots arranged in four rows with six sets of dots in each row. Research assistants noted the time (in seconds) and errors that students produced on each page. If students did not provide a response for more than two seconds on a stimulus, they were prompted to move to the next item. Scoring was calculated as the number of correct items per page (maximum 24) divided by response time in seconds per page. Internal reliability (Cronbach's α) based on performance on each page was .87 and .80 for students in English-instruction and French-immersion programs, respectively. More details about this measure are available in Authors (date). The stimuli and instructions can be downloaded from OSF link (see above).

Linguistic Pathway. The linguistic pathway was measured with receptive vocabulary. English receptive vocabulary was measured using an adapted form of the *Peabody Picture Vocabulary Test–Revised*, Form B (Dunn & Dunn, 2012). French receptive vocabulary was measured using an adapted form of the *Échelle de vocabulaire en images Peabody*, Form A

(Dunn et al., 1993). For both the English and French version of this task, students were presented with four pictures on a single page. Upon hearing a word, students were required to point to the picture that best corresponded to the target word. Adapted versions of both receptive vocabulary tasks were created to reduce the amount of time needed for testing. We pre-selected subsets of questions by examining the standardized means and the standard deviations for the age group being assessed. For each test, the number of items was selected based on the students' grade level. In the present study, three subsets of 12 items were selected for a total of 36 items per test. Testing was discontinued when a child made eight or more errors in a single set. Scoring was the total number of correct responses. The internal reliability (Cronbach's α) for the English task among the three subsets for students in the English-instruction and French-immersion programs was .70 and .81, respectively. The internal reliability for the French task among the three subsets for French-immersion students was .83.

Attentional/Working Memory Pathway. The attentional/working memory pathway was assessed with three tasks: digit span forward, digit span backward, and spatial span. The digit span forward and backward tasks assess verbal short-term and working memory, whereas the spatial span task assesses visual-spatial memory (Alloway et al., 2008).

In the *Digit Span Forward* (WISC-V; Wechsler, 2014), students heard a series of numbers and were asked to repeat the numbers back in the order in which they were delivered. Starting with a span length of two digits, each span is presented for two trials. If the numbers are repeated in the correct order on at least one of the two sequences in the span, the span length is increased by one digit. Testing is discontinued when students repeat both spans of a given length incorrectly. The score was the total number of sequences correctly recalled. Test-retest reliability from the standardized WISC-V technical report was .83 (Wechsler, 2014).

The *Digit Span Backward* (Wechsler, 2014) has the same procedure as the *Digit Span Forward*, except students must repeat the numbers back in the reverse order in which they were delivered. Test-retest reliability from the standardized WISC-V technical report was .80 (Wechsler, 2014).

In the *Spatial Span* task (https://hume.ca/ix/pathspan/), nine green dots were presented on an iPad. Dots were arranged randomly with the same arrangement on each trial. A series of dots lit up one by one in varying patterns, and children attempted to reproduce the pattern by tapping on the dots in the same order. Two sequences were presented, starting at a sequence length of two. If the sequence was correctly repeated for at least one of the two sequences in the span, the span length was increased by one. Testing was discontinued when students produced incorrect responses on both spans of a given length. The score was the total number of sequences correctly repeated (maximum possible score of 18). Internal reliability (Cronbach's α) based on the subscores of first and second trials at each length for students in the English-instruction and French-immersion programs was .84 and .80, respectively.

Matrix Reasoning. Students completed a Matrix Reasoning task from the Weschler Intelligence Scale for Children-Fifth Edition (Wechsler, 2014). On each trial, they were presented with an incomplete grid and asked to select (from five options) the item that best completes the grid. There were 34 trials, two of which were practice items, and testing was discontinued after three consecutive errors. The total score was the number of correct trials. The internal reliability (Cronbach's α) was .84 and .81 for students in the English-instruction and French-immersion programs, respectively.

Mathematics Performance

Students completed four measures of mathematics performance in both grades 2 and 3. Word-problem solving, transcoding, arithmetic fluency, and number line estimation were administered and completed in English for both groups of students. Students in French-immersion programs also completed French versions of the word-problem solving and transcoding tasks.

Word-Problem Solving. The word-problem solving task was an adapted form of the Applied Problem Solving subtest of the KeyMath 3rd Edition (Connolly, 2007). To avoid practice effects for students in French-immersion programs, two forms of this subtest (Form A and Form B) were administered. Each form had similar types of questions, but different numerical values. For the English testing sessions, students in the English-instruction and French-immersion programs completed Form B in grade 2 and Form A in grade 3. For the French testing sessions, students in the French-immersion programs completed Form A of the French translated version of the subtest in grade 2 and Form B in grade 3.

In both the English and French versions of the task, students were asked to solve 12 mathematics problems of increasing difficulty. On each trial, the researcher read a mathematical word problem to the student. If a student made three consecutive incorrect responses, the task was discontinued. The scoring was the number of correct responses, with possible scores ranging from 0 to 12. The internal reliability (Cronbach's α) for the English version of the task was .78 and .74 in grade 2 and .94 and .94 in grade 3 for students in the English-instruction and French-immersion programs, respectively. The internal reliability for the French version of the task was .75 in grade 2 and .96 in grade 3 for the French-immersion students.

Transcoding. Students' ability to transcode number words to numerals was assessed through a transcoding task. In this task, students heard a number word and they were asked to write it down in numeral form (see stimuli on the OSF). In grade 2, students were presented with 20 trials: one one-digit number, four two-digit numbers, 12 three-digit numbers, and three four-digit numbers. Notably, there were two sets of stimuli for this task (Version A and Version B). Two sets of stimuli were chosen to complement a digit naming task administered in grade 2 that was part of the larger study but not reported here. Version A and Version B were highly similar, and no differences were found between the two versions in either English, $t(180) = -1.15$, $p = .25$, or French, $t(106) = .13$, $p = .90$.

In grade 3, students were presented with up to 30 trials, 6 trials for each set of three-, four-, five-, six-, and seven-digit numbers. Trials were discontinued when a child incorrectly responded to all trials in a set. Reliability was calculated based on the accuracy of individual trials. The internal reliability (Cronbach's α) for the English version of the task was .94 in grade 2 and .81 in grade 3 for students in the English-instruction programs and .95 in grade 2 and .87 in grade 3 for students in French-immersion programs. The internal reliability for the French version of the task was .94 in grade 2 and .84 in grade 3. The lower reliabilities in grade 3 may have been due to the inclusion of a larger range of more difficult numbers for this age group. Transcoding is a common measure of students' knowledge of the number system (Barrouillet et al., 2004; Moura et al., 2015). The validity of the current implementation was supported by the finding that, as expected, the students in French Immersion made more errors on the stimuli with complex decade components (e.g., 392, 3072) than those without complex decades (e.g., 834, 1545; Authors, under review).

Arithmetic Fluency. Students' arithmetic fluency was assessed using a paper-and-pencil calculation fluency test (Chan & Wong, 2020). The assessment consists of two pages: 60 single-digit addition problems with sums less than or equal to 17 (page 1) and 60 single-digit subtraction problems with minuends less than or equal to 17 (page 2). Both pages are arranged in a matrix of 20 rows by 3 columns. Students are given one minute per page to solve as many problems as they can from top to bottom without skipping any problems. Scores were the total number of correctly solved problems on both pages. Children's performance on addition and subtraction was highly correlated for both English-instruction and French-immersion students in grade 2 and grade 3, *p*s < .001.

Number Line Estimation. Students' number line estimation skills were assessed using the Estimation Line app on an iPad (https://hume.ca/ix/estimationline/). In this task, students were presented with a horizontal line on a screen with two labeled endpoints: 0 on the left endpoint and 1,000 on the right endpoint. Students were asked to estimate the position of a target number ranging from 0 to 1,000 by tapping the line where they think the target number belongs. After the child tapped the screen, a red vertical mark was displayed to show their estimate. Prior to the experimental trials, students completed three practice trials. Students were presented with 24 experimental trials in random order. Because there is no standard set of items for number line estimation, we chose target numbers that were equally spread out across the number range (see stimuli on the OSF). Performance on the number line task was significantly related to students' performance on the other mathematical measures and has been used frequently in studies of mathematical development (see also Sasanguie et al. 2011; Schneider et al., 2018). In sum, it has substantial validity as an index of magnitude processing (Siegler, 2016).

For each trial, percent absolute error (PAE) was calculated using the formula (|Child's estimate – Target Number| / 1,000) × 100. Scoring was the mean PAE across all trials. Internal reliability based on the PAE of the individual trials was .87 and .86 in grade 2 and .86 and .89 in grade 3 for the students in the English-instruction and French-immersion programs, respectively.

Procedure

Research assistants who had either completed or were working toward completion of a bachelor's degree in psychology or education administered the assessments. All of the research assistants were provided a detailed testing manual, which included specific testing and scoring procedures in both written and video formats. The research assistants completed two or three training sessions (2-3 hours per session) during which they practiced the testing and scoring procedures and were instructed on the general principals of working with students. Students were individually tested by a research assistant in a quiet area of their school. Students in English-instruction classes participated in two 30-minute sessions whereas students in French-immersion classes participated in three 30-minute sessions, one of which was conducted in French. The order of the test administration was fixed. To prevent them from switching between languages from task to task, French immersion students completed all of the French language tasks in one session with a French-speaking research assistant. As a token of appreciation for participating, students received stickers after each session.

Results

Analysis Plan

In the present study, we conducted path analyses using M*plus* (Muthén & Muthén, 1998) to investigate the developmental pathways for students who learn mathematics in an additional language from grades 2 to 3. Three types of cognitive skills were specified as predictors of the mathematical outcomes: quantitative skills, language, and working memory. Principal component analyses (PCA) were conducted to create two component scores subsequently labelled *quantitative skills* and *working memory* based on the full sample. The *quantitative* PCA included two measures: number comparison and subitizing (factor loadings = .83), accounting for 68.6% of the variance. The *working memory* PCA included three measures: digit span forward, digit span backward, and spatial span (factor loadings of .67, .70, and .58, respectively), accounting for 42.4% of the variance. Best and Miller (2010) included working memory (storage and processing) as one of three components of executive functions. Component scores were used in subsequent analyses.

Next, we conducted path analyses, examining developmental trajectories for each group and each mathematical outcome. Given the robust and highly replicated findings from developmental studies showing direct relations between mother's educational attainment and children' academic outcomes and cognitive development (Magnuson, 2007), we control for mother's education in all subsequent analyses to capture the relations between the variables of interest as purely as possible. Model fit was examined using a combination of the chi-square goodness of fit test ($p > .05$), comparative fit index (CFI > .95), root mean square error of approximation (RMSEA < .06), and standardized root mean square residual (SRMR <.08; Hu & Bentler, 1998).

Effect of School

To examine a potential effect of school, we tested an intercept-only multilevel model containing school ($n = 7$) as a random effect. We found that the intra-class correlation coefficients were low for the outcome variables in grade 3:English problem solving (.044), French problem solving (.002), English transcoding (.035), French transcoding (.007), arithmetic fluency (.002), and number line estimation (.007). The low intra-class correlation coefficients indicate low variability among the schools for these measures. Thus, a cluster effect of school is not a concern in the present study and classroom was not included in further analyses.

Missing Data

In the present study, some students only participated in one wave of data collection (i.e., 35 students participated in grade 2 only and thus did not provide data for outcome measures in grade 3; sixty-two students participated in grade 3 only and thus did not provide data for the cognitive and mathematical measures in grade 2). A series of *t*-tests and χ^2-tests were conducted on performance variables (i.e., cognitive skills and mathematics performance measures) and demographic variables (i.e., language of instruction, age of child, gender of child, and parental education level) to determine if there were differences between students who participated in both grades ($n = 147$) versus those who only participated in either grade 2 or grade 3 ($n = 97$). No significant differences were found between these groups on these measures. Thus, because we are confident that our data meet the criteria for "missing at random" (Enders, 2010), we used a full information maximum likelihood method that uses all the information of the observed data to estimate the missing parameters of the models. More specifically, this estimation method relies on an iterative process, repeating the log-likelihood computations many times to find the optimal combination of estimates for the missing parameters that maximize the log-likelihood and thus produces the best fit to the data (Enders, 2010). As an additional check on the results, we conducted sensitivity analyses based on the complete data (i.e., $n = 147$) using regression analyses. The sensitivity analysis yielded similar results to the ones estimated using the full information maximum likelihood method. Thus, we present the results using all the data with full information maximum likelihood missing data estimation.

Group Characteristics

Comparison Analyses

To examine the equality of the two instruction groups beyond the pathways of interest, we tested for differences in age, nonverbal intelligence, and mother's education. Students enrolled in the French-immersion and English-instruction programs did not differ by: age in grade 2 (M_{age} 7:10 vs. 7:10), $t(180) = .76$, $p = .45$; age in grade 3 (M_{age} 8:10 vs. 8:10), $t(240) = -.60$, $p = .55$; proportion of girls and boys, $\chi^2(1, N = 244) = .31$, $p = .58$; or nonverbal intelligence ($M = 16.04$ vs. 16.24), $t(180) = .35$, $p = .73$.

Mothers ($n = 229$) and fathers ($n = 220$) were asked to provide information about their highest education level, ranging from 1 (less than a high school diploma) to 5 (postgraduate degree). Mother's education was significantly higher for students enrolled in French-immersion programs than for students in English-instruction programs, $\chi^2(4, N = 229) = 12.48$, $p = .014$, however, father's education did not significantly differ across the two groups, χ^2 $(4, N = 210) = 9.36$, $p = .053$. This pattern is consistent with the general finding that parents of the students enrolled in the Canadian French-immersion programs are generally from middle-class families (see a review in Slavin & Cheung, 2005). Furthermore, according to Statistics Canada (2016), 32% of adults in Winnipeg aged 25 to 64 have a university degree, whereas 21% have a community-college degree. The data revealed that the education levels of parents of students in the French-immersion program more closely aligned with the percentages reported in the census (i.e., the median for mothers was a university degree and for fathers was a community college degree), whereas the education levels of parents of students in the English-instruction programs were slightly lower than the Winnipeg average (i.e., the median for both mothers and fathers was a community college degree).

Descriptive Statistics

Except for English receptive vocabulary (grade 2), English transcoding (grade 2), arithmetic fluency (grades 2 and 3), and number line estimation tasks (grade 3), the measures were normally distributed (i.e., z skewness < |3.29|). No outliers (z > |3.29|) were present in the

French-immersion group, whereas in the English-instruction group, data for two students were outliers for English receptive vocabulary in grade 2 and data for two students were outliers for arithmetic fluency in grade 2. Sensitivity analyses with and without these outliers showed similar patterns of results, and thus all the students' data were included in the final analyses.

There were only a few significant gender differences across the measures. Boys made more accurate estimates in the number line task than girls in both grade 2 (M_{PAE} =18.0% vs. 21.1%), $t(175) = -2.54$, $p = .01$, Cohen's $d = .38$, and grade 3 (M_{PAE} =12.5% vs. 16.6%), $t(205) = -4.19$, $p < .001$, Cohen's $d = .60$. Additionally, boys had higher scores on the French transcoding task than girls in grade 2 (5.41 vs. 3.59), $t(106) = 2.81$, $p = .006$, Cohen's $d = .53$. Gender was included as a control variable in the subsequent models that involved the number line task and the French transcoding task.

As shown in Table 1, and consistent with the literature (Barik & Swain, 1976; Lambert, 1974; Lindholm-Leary & Genesee, 2014; Turnbull et al., 2001), students in English-instruction and French-immersion programs performed equally well on all of the mathematical measures, *p*s > .05, regardless of the degree to which tasks required oral language processing. Moreover, students in the two groups did not differ in quantitative, first language (i.e., English receptive vocabulary), or working memory skills, *p*s > .05.

<Insert Table 1 Here>

We analyzed students' mathematics improvement from grades 2 to 3 for word-problem solving, arithmetic fluency, and number line estimation in 2 (grade: 2, 3) by 2 (program: English-instruction, French-immersion) mixed ANOVAs. Students' performance improved across grade for word-problem solving in English (3.4 vs. 5.7), $F(1, 145) = 188.80$, $p < .001$, $\eta_p^2 = .57$, arithmetic fluency (15.3 vs. 24.5), $F(1, 144) = 155.56$, $p < .001$, $\eta_p^2 = .52$, and showed decreased error on number line estimation (19.9% vs. 14.8%), $F(1, 140) = 71.08$, $p < .001$, $\eta_p^2 = .34$. The interactions between grade and program were not significant for any analysis, suggesting that the improvement in these tasks from grades 2 to 3 was similar for students in French-immersion and English-instruction programs. For word-problem solving in French, students in the French-immersion group improved from grades 2 to 3 (3.4 vs. 5.9), $F(1, 68) = 130.56$, $p < .001$, $\eta_p^2 = .66$. The transcoding task included different types of stimuli in grades 2 and 3, and thus transcoding performance in grade 3 for English-instruction and French-immersion students was analyzed with an ANCOVA, controlling for transcoding performance in grade 2. We found no main effect of program, $F(1, 144) = 2.15$, $p = .52$, $\eta_p^2 = .02$, suggesting similar performance in each group. In summary, students in the two programs showed similar overall performance and similar patterns of improvement in their mathematics skills across grades.

Correlations

Correlations among the measures for students in both groups are shown in Table 2. In both groups, quantitative skills were significantly correlated with arithmetic and number line performance in both grades. In contrast, quantitative skills were more modestly related to word-problem solving, with no significant correlation observed with word-problem solving for French-immersion students in grade 3. Working memory was correlated with all mathematical measures for both groups, except for arithmetic in grade 3. Finally, for both groups, English vocabulary scores were strongly related to word-problem solving and transcoding, moderately related to number line performance, but not correlated with arithmetic scores for either group in either grade. These patterns are consistent with the view that word-problem solving and transcoding implicate oral language processes whereas number line and arithmetic are less demanding of language skill.

For students in French immersion, quantitative skills and working memory were also related to the mathematical tasks with French language demands (i.e., word-problem solving and transcoding). The correlations were similar for the English and French versions for those in French-immersion programs. English vocabulary skills were also consistently related to the English and French mathematical tasks, although the correlations were somewhat lower for the French mathematics measures. French vocabulary was correlated with all mathematics performance measures, except for the French transcoding task in grade 3. Thus, for students in French immersion, cognitive skills were related to French mathematics performance measures in similar ways as they were to English mathematics performance. Further analyses are necessary to explore both the similarities and differences across groups.

<Insert Table 2 Here>

Path Analyses

In the present study, we constructed our models based on the Pathways to Mathematics model (LeFevre et al., 2010; Sowinski et al., 2015). For ease of interpretation, a summary of both direct and indirect pathways for all models can be found in Table 3.

Hypothesis 1. We predicted that for students in English-instruction programs, English receptive vocabulary in grade 2 would be related to performance on mathematical tasks that require oral language processing in grades 2 and 3; for students in French-immersion programs, English receptive vocabulary would be related to performance on both English and French mathematical tasks that require oral language processing in grades 2 and 3. Path analyses for performance on mathematical tasks requiring oral language processing are shown in Figures 1 and 2; analyses for tasks not requiring oral language processing are shown in Figures 3 and 4.

<Insert Table 3 Here>

Word-Problem Solving. The model for students in English-instruction programs is shown in Figure 1a. Model fit was excellent, $\chi^2(1) = 1.00$, $p <.001$, SRMR = 0, CFI = 1.0, RMSEA = 0 (90% CI = [0, 0]). English word-problem solving in grade 2 was predicted by all three cognitive skills, including English vocabulary. Moreover, performance on English word-problem solving in grade 3 was predicted by English receptive vocabulary but not by quantitative skills or working memory, controlling for performance in grade 2.

The model for students in French-immersion is shown in Figure 1b. Model fit was excellent, $\chi^2(2) = 2.41$, $p = .30$, SRMR = .01, CFI = 1.0, RMSEA = .04 (90% CI = [0, .17]). English word-problem solving in grade 2 was predicted by all three cognitive skills, including English vocabulary, and French word-problem solving in grade 2 was predicted by the same cognitive skills. French vocabulary did not predict unique variance in either English or French word-problem solving. Note that the relation between English and French word-problem solving is also significant in this model. Moreover, controlling for performance in grade 2, performance on both English and French word-problem solving in grade 3 was predicted by English vocabulary but not French vocabulary, working memory, or quantitative skills. Thus, as hypothesized, students' first-language proficiency is related to their ability to solve word problems presented in either their first or second language.

<Insert Figure 1 Here>

Transcoding. The model for students in English-instruction programs is shown in Figure 2a. Model fit was excellent, $\chi^2(1) = .19$, $p = .66$, SRMR = .01, CFI = 1.0, RMSEA = 0 (90% CI = [0, .21]). English transcoding in grade 2 was predicted by quantitative skills and working memory, but not by English receptive vocabulary. Moreover, controlling for performance in grade 2, performance on English transcoding in grade 3 was predicted by English receptive vocabulary and quantitative skills, but not working memory.

The model for French-immersion programs is shown in Figure 2b. Model fit was excellent, $\chi^2(4) = 2.25$, $p = .69$, SRMR = .01, CFI = 1.0, RMSEA = 0 (90% CI = [0, .09]). English transcoding in grade 2 was predicted by all three cognitive skills, including English vocabulary, and French transcoding in grade 2 was predicted by quantitative skills and working memory, but not English vocabulary. French vocabulary did not predict unique variance in either English or French transcoding. Note that the relation between English and French transcoding is also significant in this model. Moreover, controlling for performance in grade 2, performance on both English and French transcoding in grade 3 was predicted by English vocabulary but not French vocabulary, working memory, or quantitative skills. Thus, as hypothesized, students' first-language proficiency is related to their ability to transcode numbers presented in either their first or second language. Overall, for students learning mathematics in an additional language, the linguistic pathway for both French and English outcomes was driven by their English (i.e., home) language skills.

<Insert Figure 2 Here>

Hypothesis 2. We predicted that language skills would not be related to students' performance on tasks with minimal oral language processing. In this study, arithmetic fluency and number line estimation did not require oral language processing, although students could have transcoded from written to oral language mentally.

Arithmetic Fluency. The model for students in English-instruction programs is shown in Figure 3a. Model fit was excellent, $\chi^2(1) = 1.52$, $p = .224$, SRMR = .02, CFI = .99, RMSEA = .08 (90% CI = [0, .30]). Arithmetic fluency in grade 2 was predicted by working memory and quantitative skills, but not receptive vocabulary. In grade 3, only arithmetic in grade 2 was a significant direct predictor of arithmetic. As predicted, language skills are not strongly related to simple arithmetic performance when input and response are both numeric.

The model for students in French-immersion programs is shown in Figure 3b. Model fit was excellent, $\chi^2(1) = .09$, $p = .76$, SRMR = 0, CFI = 1.0, RMSEA = 0 (90% CI = [0, .15]). Similar to the students in the English-instruction programs, arithmetic fluency in grade 2 was predicted by working memory and quantitative skills, but not receptive vocabulary in either English or French. In grade 3, only grade 2 arithmetic was a significant predictor of grade 3 performance. These results are the same as those for students in the English-instruction programs.

<Insert Figure 3 Here>

Number Line Estimation. The model for students in English-instruction programs is shown in Figure 4a. Model fit was excellent, $\chi^2(1) = .56$, $p = .46$, SRMR = .01, CFI = 1.0, RMSEA = 0 (90% CI = [0, .25]). Number line performance in grade 2 was predicted by working memory and quantitative skills, but not receptive vocabulary. In grade 3, quantitative skills were the only significant predictor of number line performance in grade 3, controlling for performance in grade 2. These results are consistent with the assumption that language skills are not central to number line performance when no oral language processing is required.

The model for students in French-immersion programs is shown in Figure 4b. Model fit was excellent, $\chi^2(1) = .81$, $p = .37$, SRMR = .01, CFI = 1.0, RMSEA = 0 (90% CI = [0, .21]). Number line estimation in grade 2 was predicted by working memory, but not quantitative skills, English receptive vocabulary, or French receptive vocabulary. In grade 3, in addition to grade 2 number line performance, quantitative skills and English receptive vocabulary predicted number line estimation.

<Insert Figure 4 Here>

The results for arithmetic fluency were consistent with our hypothesis that receptive vocabulary (in either English or French) would not be a significant predictor of tasks with minimal language processing requirements. However, the results for number line estimation were more complex than we expected. For students in French Immersion, English vocabulary and quantitative skills both predicted grade 3 number line performance. Post-hoc analyses were conducted to provide more detailed information about these patterns (see Table A1 in Appendix A). For children to successfully estimate on a number line, they must connect numerals to proportions of the number line (LeFevre et al., 2013). Thus, they need to understand how a target numeral is related to the endpoints of the line (e.g., that 500 is halfway between 0 and 1,000). In the present study, the poor performance on the 0-1,000 number line task in grade 2 (mean PAE of 20%) may be reflective of the range of the number line; students may have had difficulty with the 0-1000 number line because it exceeded the range of numbers students are formally taught, according to the Manitoba curriculum. In contrast, in grade 3, the mean PAE had decreased to approximately 15%, indicating that more students understood the proportional relations between numerals and positions on the 0-1,000 number line. This developmental pattern matches the curriculum expectations about number range.

Consistent with the view that this improvement was related to a greater understanding of the numerals from 0 to 1,000, correlations between transcoding and number line estimation were high for students in both English-instruction (r = -.59) and French-immersion programs (r = -.56). Exploratory regression analyses revealed that above and beyond the three cognitive skills, English transcoding in grade 3 was a significant predictor of number line estimation in grade 3, controlling for number line estimation in grade 2. This pattern was found for students in both the

English-instruction and French-immersion groups. Moreover, for students in French immersion, English receptive vocabulary was not a significant predictor of number line performance when transcoding was included in the model. This finding suggests that knowledge of the verbal number system in English, rather than English receptive vocabulary more generally, supported children's number line performance in grade 3. Thus, even though number line performance ostensibly did not require oral language processing, it was closely linked to students' understanding of the verbal and written number language.

Discussion

The goal of the present study was to investigate the developmental linguistic pathways for students who learn mathematics in an additional language from grades 2 to 3 based on the framework of the Pathways to Mathematics model (LeFevre et al., 2010). We found that cognitive skills and mathematics performance were similar for students in French-immersion and English-instruction programs. More importantly, for mathematical tasks involving oral language processing (i.e., word-problem solving and transcoding), first language proficiency (i.e., English receptive vocabulary) predicted performance in grade 3 for students in French-immersion programs, controlling for performance in grade 2. French vocabulary skills were correlated with performance and with English vocabulary but did not predict unique variance in mathematical performance. This pattern held even for mathematical tasks where the presentation and response language was French. In contrast, for mathematical tasks with minimal oral language demands (e.g., arithmetic and number line estimation), students' quantitative skills and their knowledge of the number system, rather than their general language skills, were most strongly related to performance in grades 2 and 3.

For all measures in grades 2 and 3, there were no significant differences in performance for students enrolled in French-immersion programs compared to those in English-instruction programs. Furthermore, students in both French-immersion and English-instruction programs had similar improvements in mathematics performance from grades 2 to 3, suggesting that students learning mathematics in an additional language learned at the same rate as those learning in their first language. Similarly, previous studies with students in Canadian French-immersion programs have shown that learning mathematics in an additional language does not put students at a disadvantage (Barik & Swain, 1976; Lambert, 1974; Lindholm-Leary & Genesee, 2014; Turnbull et al., 2001).

The Linguistic Pathway

Mathematical Tasks that Require Oral Language Processing

For mathematical tasks that required oral language processing (i.e., word-problem solving and transcoding), we found that grade 2 receptive vocabulary in English, students' home language, was related to concurrent mathematics performance and predicted mathematics performance in grade 3, controlling for performance in grade 2. For students in French-immersion programs, both English and French receptive vocabulary were moderately correlated with tasks that required oral language processing (see Table 2). This finding suggests that language abilities, more generally, are important for mathematics (Peng et al., 2020). Moreover, English and French receptive vocabulary were highly correlated (i.e., $r = .66$). These correlations are consistent with the Developmental Interdependence Hypothesis (Cummins, 1979), which suggests that underlying cognitive or academic proficiency is shared across languages and that transfer of both linguistic and conceptual elements occurs for languages with similar roots, such as English and French[1] (Cummins, 2005).

According to the Developmental Interdependence Hypothesis (Cummins, 1979), when students are exposed to both languages, in either a bilingual or immersion context, experience with either language can facilitate the development of proficiency in both languages. Support for this hypothesis comes from research on the development of reading skills for French-immersion students (e.g., Archambault et al., 2019; Côté et al., 2021; Jared et al., 2010; see a review in Genesee & Jared, 2008). Across these studies, language skills in English, such as phonological awareness, letter-sound knowledge, grammatical ability, and rapid automatized naming, were significant predictors of reading ability in French. In an intervention study of at-risk French-immersion students in grade 1, Côté et al. (2021) found that an English reading intervention led to improvements on both English and French measures of word reading (i.e., regular, exception, and pseudowords). Similarly, in their intervention study, Archambault et al. (2019) found that, for French-immersion students in grade 3, a French reading fluency intervention led to improvements in both French and English reading fluency. Together, these intervention studies provide support for causal models of cross-linguistic transfer in bilingual learners.

Few researchers have investigated mathematics learning for French-immersion students. At the primary school level, when children in French-immersion programs were taught mathematics in French, they had equivalent or better performance on standardized mathematics tests administered in English than their peers in English-instruction programs with one exception: French-immersion students tended to have worse performance on arithmetic word

[1] French has roots in Latin and Greek and English shares many of these antecedents, in part because it was influenced by French. Much English vocabulary has French roots whereas common words are often Germanic.

problems than their English-instruction counterparts (Genesee, 1987; Sawin & Lapkin, 1982). However, after students in French-immersion programs learned to read in English, this difference disappeared. Furthermore, Bournot-Trites and Reeder (1999) followed two cohorts of French-immersion students from grades 4 through 7. The first cohort received 80% of their instruction, including mathematics instruction, in French. The second cohort received 50% of their instruction in French and 50% of their instruction, including mathematics instruction, in English. On mathematics tests in English, the 80% French-instruction cohort outperformed the 50% French-instruction cohort. Bournot-Trites and Reeder concluded that higher intensity of second-language instruction led to an increase in French proficiency which in turn assisted students in acquiring a strong understanding of mathematical concepts. Thus, students who had acquired their mathematics knowledge through French instruction performed successfully on English mathematics tests. More generally, these results support the Developmental Interdependence Hypothesis, highlighting that learning experiences in either language can facilitate development in both languages.

In the present study, students in French-immersion programs had less extensive receptive vocabulary in French than in English because of their life-long exposure to English at home. Thus, when the two vocabulary measures were considered simultaneously, only the unique contributions of English vocabulary predicted the development of mathematics for tasks that required oral language processing. The Threshold Hypothesis (Cummins & Swain, 1986) indicates that "linguistic, cognitive, and academic advantages are associated with high levels of proficiency in *both* first and additional languages" (p. xvi). The similar patterns for English-instruction and French-immersion students suggest that even when presented tasks in French, students' proficiency in English was the dominant predictor of performance, presumably because students are not yet fluent in French. More studies with older students are needed to explore the role of the language of instruction and other languages (e.g., languages spoken in the home) in mathematics learning. In summary, for immersion students in the early years of elementary school, individual differences in mathematical tasks with substantial oral language demands may be more strongly related to oral language skills in their first language than in the language of instruction. Furthermore, because English is the language of the home, it is likely that many or most students receive support in English for their mathematics learning at home.

Word-problem solving requires knowledge of vocabulary, because students need to encode the words to access the mathematics (Daroczy et al., 2015; Fuchs et al., 2015; Mayer, 2004). Similarly, transcoding also requires knowledge of number vocabulary because students use a small set of number words as a basis to construct additional number words within a syntactic structure (Anglin et al., 1993; McCloskey et al., 1986; Skwarchuk & Anglin, 2002). Thus, when completing tasks, students in French-immersion programs may either need to, or prefer to, process the information in their more proficient language (English) than in their less proficient language (French). This issue of which language people use on mathematical tasks as their competency changes needs further research. In summary, links between number language and mathematics performance may depend on a variety of experiential factors, including age of learning the additional language as well as instructional experiences (Frenck-Mestre & Vaid, 1993; Salillas & Wicha, 2012; Van Rinsveld et al., 2017).

Our findings that English vocabulary (i.e., home language) had a stronger relation to mathematical tasks that required oral language processing than did French vocabulary (i.e., instructional language) appears to conflict with findings from the literature on reading and writing acquisition where students' instructional language predicts literacy learning (Murphy, 2018). For example, Mancilla-Martinez and Lesaux (2010) and Kieffer (2012) found that for Spanish-speaking students attending English-instruction schools in the United States, students' instructional language in kindergarten, rather than their home language, predicted the growth of reading comprehension. Notably, Kieffer suggested that much of the variation in English reading comprehension that was explained by Spanish oral language skills was the same variation explained by English oral language skills, supporting Cummins' (1979) view that there is an underlying language proficiency that is not specific to a particular language.

Our findings can be interpreted similarly to those of Kieffer (2012). Both English and French vocabulary skills were correlated with linguistic mathematical outcomes and were highly correlated with each other – thus, much of the explained variance in performance was shared between English and French vocabulary. Our findings that unique variance is attributed to English rather than French vocabulary presumably reflects the differences between the circumstances and characteristics of students in immersion programs compared to those learning in a minority language, including factors such as language dominance in the society, socio-economic status of families, and the perceived value of learning the first versus the second language (de Araujo et al., 2018; Murphy, 2018). Further work with older students is needed to determine whether the dominance of English language skills as a predictor of learning mathematics in French persists once students develop a higher level of proficiency in French.

Mathematical Tasks with Minimal Oral Language Processing

For mathematical tasks that did not require oral language processes (i.e., written arithmetic and number line tasks), patterns of performance were similar, but not identical, for students in English-instruction and French-immersion programs. As expected, on the single-digit arithmetic task presented in numeric format, neither English nor French vocabulary predicted performance for students in French immersion. This finding is consistent with previous work showing that phonological awareness, rather than vocabulary, is closely related to the development of single-digit arithmetic skills, presumably because simple arithmetic requires the activation of verbal codes for the corresponding number words and associated arithmetic facts stored in memory (De Smedt et al., 2010; Hecht et al., 2001; Krajewski & Schneider, 2009; Simmons & Singleton, 2008; Singer & Strasser, 2017).

Unexpectedly, we found that English vocabulary predicted number line performance in grade 3 for students enrolled in French-immersion programs. We had hypothesized that general language abilities would not be directly related to number line performance because no oral language processing was required. Additional analyses revealed that English transcoding mediated the relation between vocabulary and number line performance in grade 3, controlling for performance in grade 2. Transcoding may be asemantic in that students may use rules to translate between spoken and written number forms (Barrouillet et al., 2004), but it may also index students' knowledge of relations between multi-digit numbers and magnitudes. This knowledge is also required for accurate estimation on a number line that includes two- and three-digit numbers. For example, children have to differentiate the tens and hundreds units of the target number (e.g., Helmreich et al., 2011) and they must understand what the tens and hundreds positions represent (i.e., that the "9" in 90 is ten times larger than the single-digit number 9). They must then apply this knowledge of number magnitude to estimating the proportion of the line (Dietrich et al., 2016). Thus, both transcoding (i.e., math-specific vocabulary) and number line performance are related to students' underlying representations of multi-digit numbers (Helmreich et al., 2011; Moeller et al., 2009).

The Non-Linguistic Pathways: Quantitative Skills and Working Memory

The focus of the current study was on the linguistic pathway because we were interested in how language skills played a role in learning mathematics in a second language. Nonetheless, quantitative skills and working memory are also important for mathematics performance. In the present study, quantitative skills and working memory were related to *concurrent* performance for both groups of students (see reviews in De Smedt et al., 2013; Schneider et al., 2018). However, both pathways only uniquely predicted the changes in math performance from grades 2 to 3 for some mathematics outcomes (see Table 3 for details).

Mathematical development is hierarchical, with more basic numerical associations leading to the development of more complex mathematical concepts (Merkley & Ansari, 2016; Núñez, 2017; Xu & LeFevre, 2020). Within this hierarchy, quantitative skills are foundational for numerical tasks. Thus, in the present study, students with better quantitative skills may have relied on fluent access to the relative magnitude of digits during calculation or when they were transcoding verbal number words to digits. Furthermore, controlling for performance in grade 2, quantitative skills predicted number line performance in grade 3 for both groups, which is consistent with the view that children's knowledge of the relative magnitude of digits is a necessary precursor for placing numbers on a number line (see a meta-analysis in Schneider et al., 2018).

A substantial amount of research shows that working memory is related to mathematics performance (DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Raghubar et al., 2010). For example, working memory is important for extracting the meaning of text in word problems, for maintaining and updating the results of intermediate calculations during arithmetic tasks, and for inhibiting information that is not relevant to ongoing processing (Allen et al., 2019; Friso-van den Bos et al., 2013; Peng et al., 2016; Raghubar et al., 2010). The results of the present study stand in contrast to other studies, where working memory predicted mathematical skills longitudinally (LeFevre et al., 2013; Van der ven et al., 2012; Viterborai et al., 2015). Notably, our measure of working memory did not assess all the types of executive function processes that may be involved in children's mathematical development (Friso-van den Bos et al., 2013; Peng et al., 2016). Thus, in future longitudinal studies, working memory should be assessed comprehensively and both working memory and mathematics should be assessed at multiple time points. Overall, consistent with the hypotheses of the Pathways to Mathematics model, the patterns of relations varied across cognitive skills and depended on the mathematical outcome being measured. Important to the present study, the strength and significance of pathways differed based on the oral language processing demands of the task, but the non-linguistic pathways were similar for students in English-instruction and French-immersion programs.

Limitations and Implications for Future Research

The results highlight the importance of first-language development for students in French-immersion programs. Because these students have much less extensive experience in French than in English, learning mathematics in French may require mediation from their existing mathematical knowledge in English. Notably, the design of the present study does not allow us to provide a definitive answer about how students in the French-immersion programs learn and process mathematical knowledge. Collecting more comprehensive information about how language processing is involved in mathematics learning will be important in future work. For example, Powell et at. (2017) found that math-specific vocabulary is more important than general vocabulary for mathematics learning in grade 5. On this view, math-specific vocabulary in French may be a stronger predictor of students' developing mathematical skills for middle-school students and beyond.

A limitation of the present study is that we only included receptive vocabulary measures as a proxy of students' language skills. Broader aspects of language skill also influence mathematical learning, such as phonological awareness, syntactical structure, academic vocabulary, and oral comprehension (Abedi & Lord, 2001; Banks et al., 2016; de Araujo et al., 2018; Fuchs et al., 2015; Hecht et al., 2001; Powell et al., 2020; Wolf & Leon, 2009). Thus, future research should include multiple measures to investigate the complex role of language skills in the process of learning mathematics within the context of immersion programs.

Educational Implications

The present study is the first to focus on the cognitive factors related to learning mathematics within the context of French-immersion education in Canada. Our findings suggest that some immersion students may rely on their first language to learn and process mathematical content presented in an additional language, possibly translating or switching between the two languages (Adetula, 1989; Clarkson, 2016; Cuevas, 1984; Domínguez, 2011). Others may rely on their dominant language or the language in which they learned a specific mathematical skill (Van Rinsveld et al, 2015, 2017; Bernardo, 2001; Campbell & Epp, 2004; Salillas & Wicha, 2012).

Because exposure to first language begins in infancy, and first language development often continues outside of school, bilingual students may process some numerical tasks in their first language, as opposed to the language of instruction (Domínguez, 2011). Conversely, language selection for any given task for bilingual students may depend on second language proficiency and relative dominance of the two languages (see a review in Kroll et al., 2006). That is, until additional language learners become highly proficient in their additional language, they will likely need to actively inhibit their first language when selecting a language with which to respond (Kroll et al., 2006). The results of the present study show that in grades 2 and 3, students in the immersion programs had similar cognitive, mathematical, and first language skills as their peers in English-instruction programs and their English-language vocabulary skills were similarly important to their mathematics performance. As they progress through school, however, students' expertise in French improves and thus their reliance on their first language skills may decrease. For example, based on the Threshold Hypothesis (Cummins & Swain, 1986), as students' second language proficiency improves, so should their understanding of new mathematical concepts taught in a second language (Bournot-Trites & Reeder, 2001). In future, multi-year longitudinal studies should be carried out to examine the developmental trajectories of learning mathematics in an additional language across different immersion contexts.

For educators of students in early immersion programs, we offer a few suggestions. First, students may need well-developed proficiency in mathematics in their first language (i.e., knowledge of number words and other mathematics vocabulary) to support learning mathematical content presented in their additional language. This proficiency is especially important for tasks that involve oral language processing if students shift from their additional to their first language to solve problems. Second, given that children's knowledge of their first language is the best predictor of individual differences in mathematical tasks that involve oral language processing, teachers should be aware of the additional processing demands for students to understand content in another language and thus, should be sensitive to students with weak verbal abilities in their proficient language. More specifically, teachers need to understand that students' difficulties may not stem from any difficulties with the mathematics concepts or with additional language skills but may instead reflect difficulties with first language skills. Finally, teachers in immersion contexts may need to encourage the use of both languages (English and French) when students discuss the meanings of mathematical concepts and help them to connect mathematical concepts with their everyday experiences in both languages (Domínguez, 2011). Language proficiency in students' first language is an important support for learning academic skills in an additional language (see also Slavin & Cheung, 2005).

Conclusion

In conclusion, we found that the mathematics performance and the development of mathematical knowledge were very similar for students in French-immersion and English-instruction programs. Proficiency in English (i.e., their primary language) was important for all students in the present research, especially for mathematical tasks that involved oral language processing. Further research is needed to understand how French-language proficiency may influence mathematical learning for immersion students in later grades.

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Tables and Figures

Table 1

Performance on English (E) and French Measures (F) for Students in English-instruction and French-immersion Programs

	English-instruction					French-immersion					Comparison		
	Min	*Max*	*Mean*	*SD*	*z-Skew*	*Min*	*Max*	*Mean*	*SD*	*z-Skew*	*t*	*df*	*d*
Grade 2													
Number Comparison	.08	1.23	.82	.16	-3.28	.50	1.40	.83	.15	2.53	-.62	175	.04
Subitizing	.82	2.28	1.54	.30	.67	.84	2.27	1.51	.25	.34	.76	178	.11
Vocabulary (E)	12	36	29.45	4.77	-4.23	9	36	29.92	4.99	-6.09	-.64	180	.10
Vocabulary (F)	-	-	-	-	-	2	27	15.36	6.61	-2.51	-	-	-
Digit Forward	4	15	8.39	2.39	1.16	4	13	8.02	2.00	1.34	1.14	180	.17
Digit Backward	3	9	5.65	1.14	1.41	3	8	5.37	1.09	.42	1.66	180	.25
Spatial Span	0	12	5.58	2.68	.24	0	13	5.53	2.47	-.35	.13	172	.02
Word-Problem (E)	0	9	3.22	2.13	2.92	0	9	3.61	2.29	2.58	-1.78	180	.18
Word-Problem (F)	-	-	-	-	-	0	10	3.39	2.10	2.76	-	-	-
Transcoding (E)	1	20	15.82	5.45	-5.04	3	20	15.40	5.17	-4.88	.87	180	.08
Transcoding (F)	-	-	-	-	-	2	20	12.14	6.13	-1.41	-	-	-
Arithmetic	2	61	15.96	9.75	9.51	1	40	15.18	8.11	3.68	.59	180	.09
Number Line	5.73	43.12	21.05	8.12	1.39	4.12	44.94	18.87	8.27	2.54	1.74	175	.27
Grade 3													
Word-Problem (E)	0	12	5.16	2.25	1.88	0	11	5.82	2.48	.41	-1.93	207	.28
Word-Problem (F)	-	-	-	-	-	0	11	5.85	2.58	.05	-	-	-
Transcoding (E)	0	28	12.71	5.43	.30	0	30	13.67	6.31	.58	-1.11	207	.16
Transcoding (F)	-	-	-	-	-	0	30	10.42	6.67	3.01	-	-	-
Arithmetic	4	62	24.17	12.72	4.20	2	53	22.61	10.86	2.73	.94	206	.14
Number Line	4.54	34.59	15.76	6.79	2.53	3.09	34.76	14.27	7.38	3.40	1.44	205	.21

Table 2

Correlations Among Measures for French-immersion Students (Above Diagonal) and English-instruction Students (Below Diagonal)

	Grade 2											Grade 3					
		WM	Quant.	Vocabulary		Word problems		Transcoding		Arith.	NL	Word Problems		Transcoding		Arith.	NL
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Mother's Education	-	-.11	.04	.07	.17	.22*	.17	.04	.13	.17	-.04	.11	.30**	.10	.15	.13	.02
2. Working Memory (G2)	.09	-	.17	.28**	.35***	.40***	.38***	.46***	.29**	.34***	-.37***	.40***	.36**	.34**	.25*	.19	-.34**
3. Quantitative Skills (G2)	.16	.13	-	-.05	-.06	.23*	.24*	.30**	.23*	.46***	-.19	.13	.19	.24*	.23*	.37***	-.27*
4. Vocabulary (G2/E)	.23	.27*	.11	-	.66***	.41***	.37***	.40***	.26**	.18	-.25**	.55***	.56***	.40***	.34***	.18	-.41***
5. Vocabulary (G2/F)	-	-	-	-	-	.42***	.37***	.32***	.27**	.24*	-.26**	.40***	.47***	.29**	.18	.23*	-.30**
6. Word-Problem (G2/E)	.24*	.41***	.27*	.51***	-	-	.65***	.42***	.44***	.57***	-.43***	.62***	.74***	.58***	.53***	.50***	-.45***
7. Word-Problem (G2/F)	-	-	-	-	-	-	-	.47***	.53***	.53***	-.47***	.64***	.71***	.63***	.54***	.55***	-.47***
8. Transcoding (G2/E)	.25*	.46***	.30*	.21	-	.42***	-	-	.54***	.43***	-.37***	.60***	.59***	.59***	.53***	.36***	-.50***
9. Transcoding (G2/F)	-	-	-	-	-	-	-	-	-	.50***	-.33***	.40***	.48***	.42***	.43***	.42**	-.49***
10. Arithmetic (G2)	.32**	.31**	.49***	.08	-	.26*	-	.33**	-	-	-.53***	.40***	.50***	.40***	.51***	.70***	-.48***
11. Number Line (G2)	-.12	-.39***	-.34**	-.21	-	-.26*	-	-.58***	-	-.33**	-	-.44***	-.42***	-.37***	-.36***	-.36***	.57***
12. Word-Problem (G3/E)	.20	.46***	.31*	.49***	-	.63***	-	.47***	-	.30*	-.36**	-	.70***	.69***	.61***	.52***	-.55***
13. Word-Problem (G3/F)	-	-	-	-	-	-	-	-	-	-	-	-	-	.72***	.66***	.58***	-.61***
14. Transcoding (G3/E)	.26*	.42***	.43***	.49***	-	.57***	-	.53***	-	.35**	-.48***	.57***	-	-	.76***	.53***	-.56***
15. Transcoding (G3/F)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	.56***	-.44***
16. Arithmetic (G3)	.28*	.23	.49***	.02	-	.41**	-	.33*	-	.64***	-.29*	.50***	-	.40***	-	-	-.46***
17. Number Line (G3)	-.16	-.38**	-.44***	-.31*	-	-.43***	-	-.38**	-	-.37**	.62***	-.57***	-	-.59***	-	-.39***	-

Note. $p < .05$*; $p < .01$**; $p < .001$***; Shaded cells are French measures.

Table 3

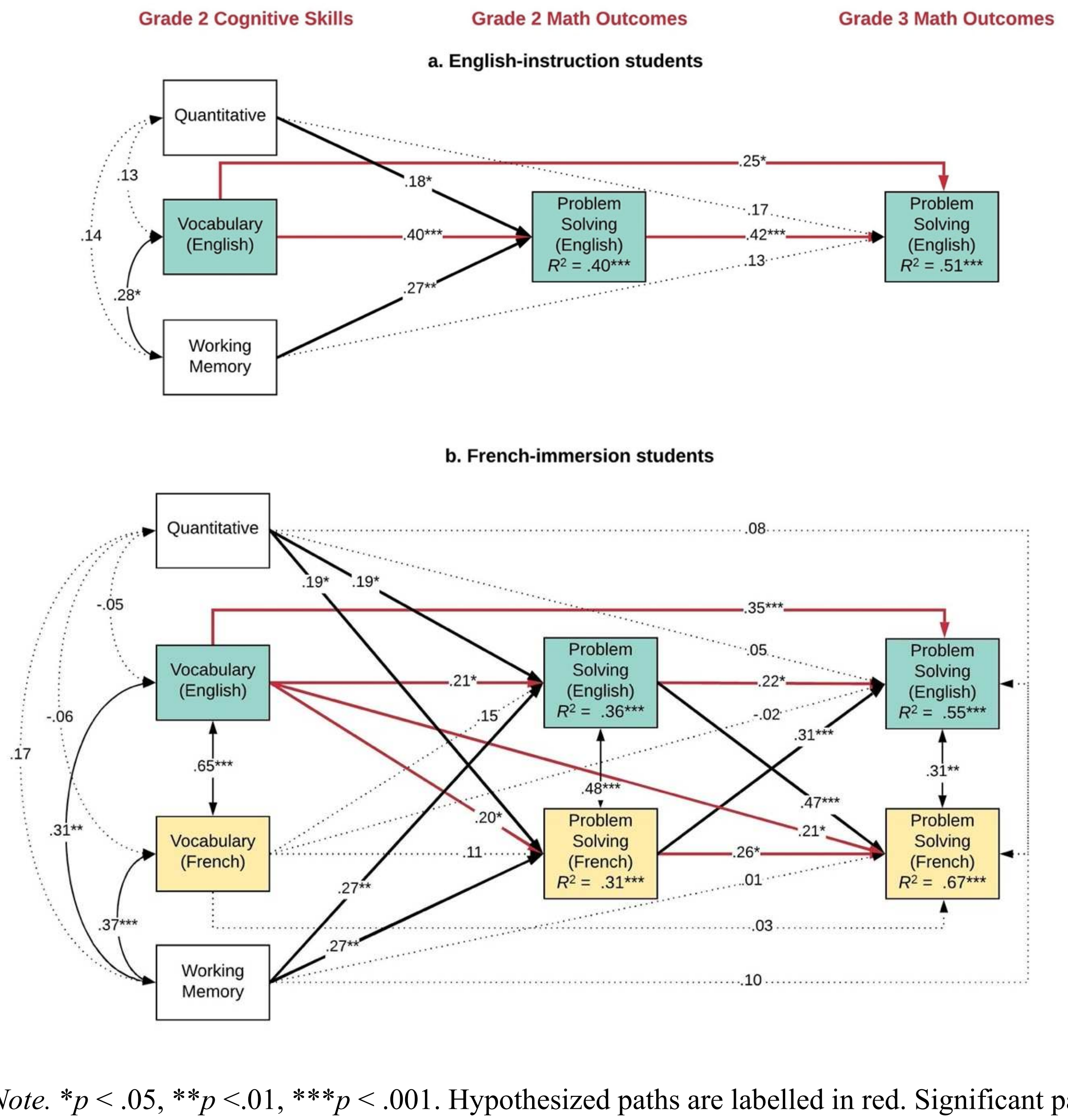
Summary of Patterns of Significance for Path Models of English and French Mathematical Outcomes for Students in the English-Instruction (E) and French-Immersion (FI) Programs

	Mathematical Outcomes by Input Language for Each Group									
	Word problem solving			Transcoding			Arithmetic		Number Line	
	English		French	English		French	English		English	
Predictors	E	FI	FI	E	FI	FI	E	FI	E	FI
Grade 2										
Quantitative Skills	✓	✓	✓	✓	✓	✓	✓	✓	✓	×
English Vocabulary	✓	✓	✓	×	✓	×	×	×	×	×
French Vocabulary	-	×	×	-	×	×	-	×	-	×
Working Memory	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Grade 3 (Growth)										
Quantitative Skills	×	×	×	✓	†	×	†	†	✓	✓
English Vocabulary	✓	✓	✓	✓	✓	✓	×	×	×	✓
French Vocabulary	-	×	×	-	×	×	-	×	-	×
Working Memory	†	×	†	†	†	×	†	†	†	†

Note. The "✓" indicates a significant path in that model. The "-" indicates that the variable was not applicable or tested in the model (see text for details). The "×" indicates non-significant path, $p > .05$. The † indicates a significant indirect path only.

Figure 1

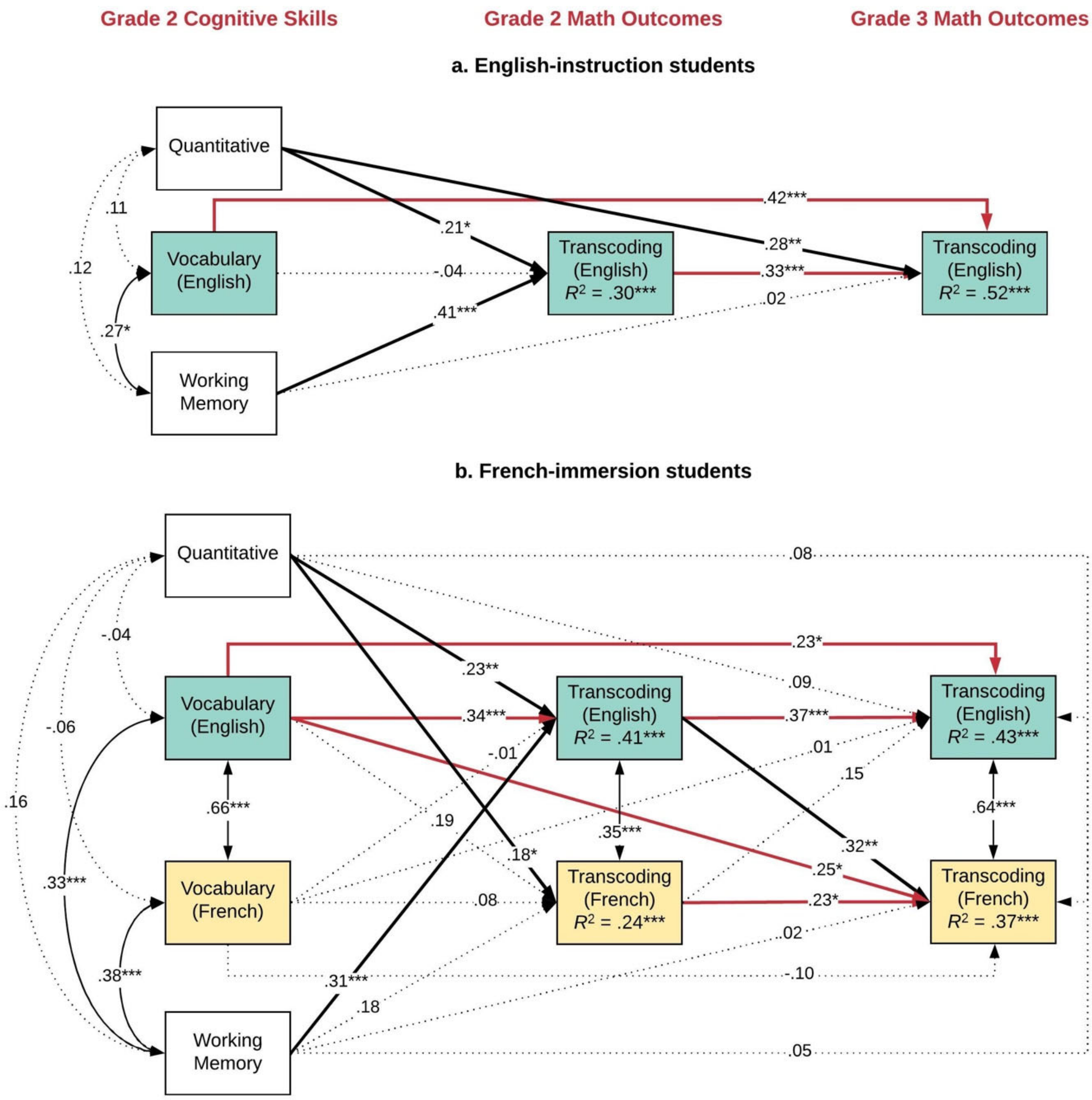
Word-Problem Solving from Grades 2 to 3 for Students in English-Instruction (a) and French-Immersion Programs (b)



Note. $*p < .05$, $**p <.01$, $***p < .001$. Hypothesized paths are labelled in red. Significant paths are represented by solid arrows. Non-significant tested paths are represented by dashed arrows. For English-instruction students, mother's education was controlled for the concurrent English problem solving (β_{edu} =.10, p =.27) and for concurrent English problem solving (β_{edu} = .22, p = .01) and French problem solving (β_{edu} = .20, p = .02) for French-immersion students.

Figure 2

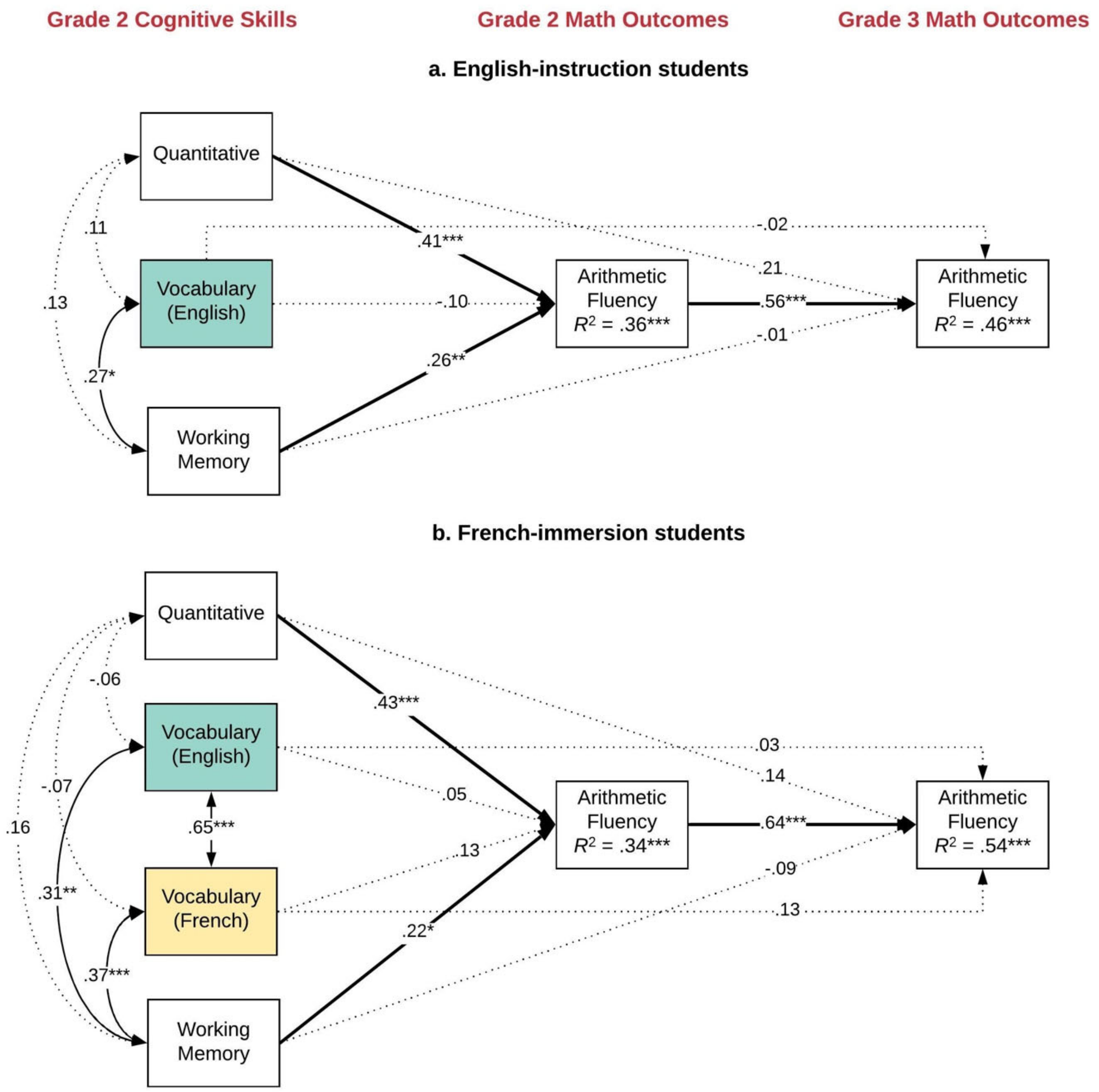
Transcoding from Grades 2 to 3 for Students in English-Instruction (a) and French-Immersion Programs (b)



Note. *p < .05, **p <.01, ***p < .001. Hypothesized paths are labelled in red. Significant paths are represented by solid arrows. Non-significant tested paths are represented by dashed arrows. For English-instruction students, mother's education was controlled for the concurrent English transcoding (β_{edu} =.17, p =.08). For French-immersion students, mother's education and gender were controlled for concurrent English transcoding (β_{edu} = .04, p =.63; β_{gender} = -.23, p = .002, respectively) and French transcoding (β_{edu} = .14, p = .15; β_{gender} = -.23, p = .01, respectively).

Figure 3

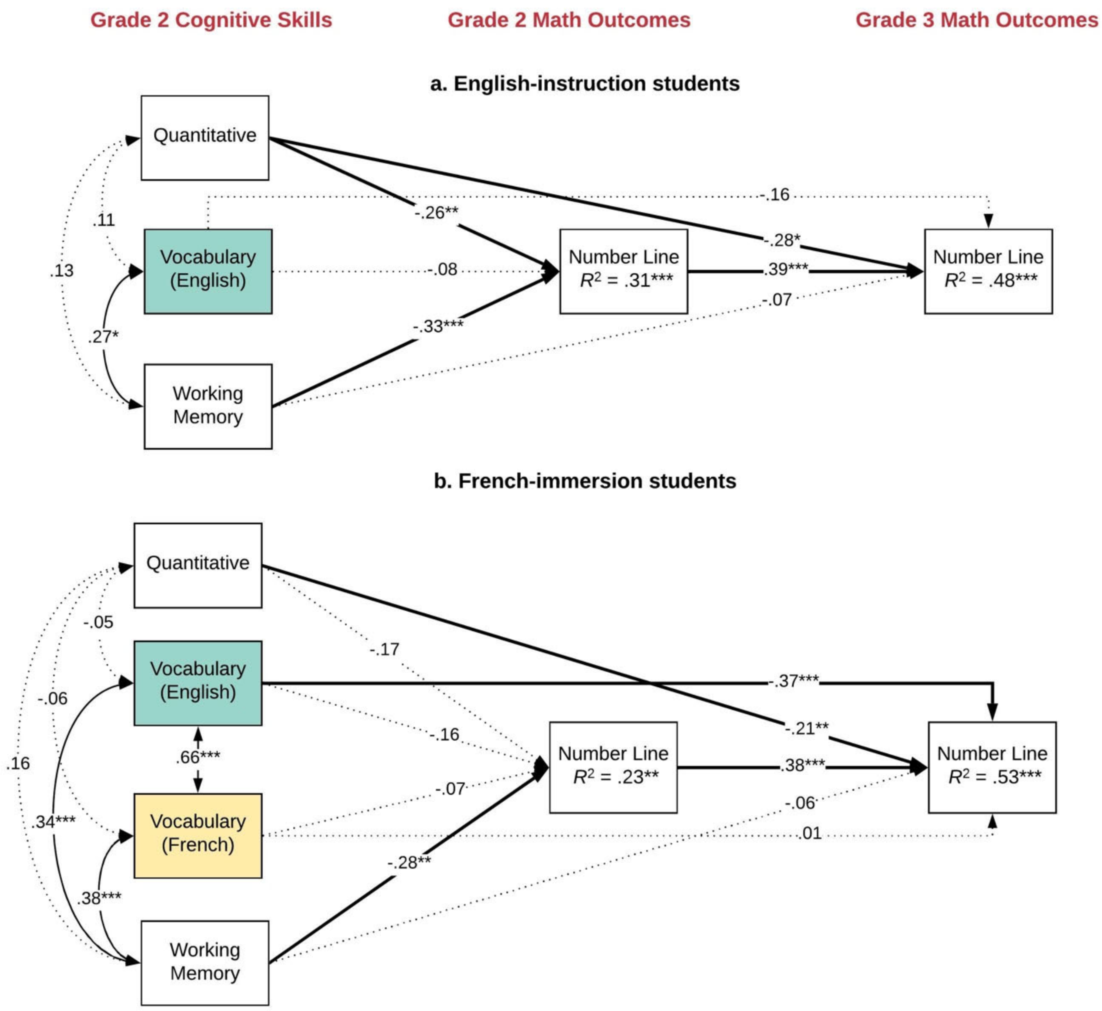
Arithmetic Fluency from Grades 2 to 3 for Students in English-Instruction (a) and French-Immersion Programs (b)



Note. *p < .05, **p <.01, ***p < .001. Significant paths are represented by solid arrows. Non-significant tested paths are represented by dashed arrows. Mother's education was controlled for the concurrent arithmetic fluency for English-instruction (β_{edu}=.23, p =.001) and for French-immersion students (β_{edu}= .16, p = .05).

Figure 4

Number Line Estimation from Grades 2 to 3 for Students in English-Instruction (a) and French-Immersion Programs (b)



Note. **p* < .05, ***p* <.01, ****p* < .001. Significant paths are represented by solid arrows. Non-significant tested paths are represented by dashed arrows.

Mother's education was controlled for the concurrent number line estimation for English-instruction (β_{edu} =-.03, *p* =.79) and for French-immersion students (β_{edu} = -.01, *p* =.96). Gender was also controlled for the concurrent and growth of number line estimation for English-instruction (β_{gender} = .26, *p* =.006; β_{gender} = .19, *p* = .06) and French-immersion students (β_{gender} = .12, *p* = .17; β_{gender} = .24, *p* = .001).

Appendix A

Table A1

Hierarchical Regression of Number Line Estimation in Grade 3 for Students in English-instruction and French-immersion Programs

	English-instruction [a]				French-immersion [b]			
	β	*t*	*p*	*Unique r²*	*β*	*t*	*p*	*Unique r²*
Model 1								
Quantitative Skills in Grade 2	**-.24***	**-2.05**	**.046**	**.05***	**-.24***	**-2.39**	**.019**	**.05***
English Vocabulary in Grade 2	-.15	-1.29	.204	.02	**-.25***	**-2.49**	**.015**	**.06***
Working Memory in Grade 2	-.07	-.57	.574	.00	-.07	-.63	.530	.00
Number Line in Grade 2	**.45*****	**3.65**	**.001**	**.14*****	**.40*****	**3.70**	**<.001**	**.13*****
Model 2								
Quantitative Skills in Grade 2	-.14	-1.16	.253	.01	-.18	-1.76	.082	.03
English Vocabulary in Grade 2	-.05	-.39	.701	.00	-.14	-1.36	.177	.02
Working Memory in Grade 2	-.03	-.21	.835	.00	-.03	-.29	.773	.00
Number Line in Grade 2	**.39****	**3.20**	**.002**	**.10****	**.34****	**3.27**	**.002**	**.09****
English Transcoding in Grade 3	**-.31***	**-2.23**	**.030**	**.05***	**-.36***	**-2.43**	**.017**	**.05***
French Transcoding in Grade 3	-	-	-	-	.07	.48	.634	.00

Note. Squared semi-partial correlations indicate unique r^2 within that specific model tested. $p < .05$*, $p < .01$**, $p < .001$***

[a] In Model 1, $R^2 = 46.1\%$, $F(4, 50) = 10.69$, $p < .001$; in Model 2, $R^2 = 51.1\%$, $F(5, 49) = 10.23$, $p < .001$; $\Delta R^2 = 5.0\%$, $p = .030$.

[b] In Model 1, $R^2 = 36.0\%$, $F(4, 70) = 9.86$, $p < .001$; in Model 2, $R^2 = 43.2\ \%$, $F(6, 68) = 8.61$, $p < .001$; $\Delta R^2 = 7.1\%$, $p = .018$.